Theory of Nematic Fractional Quantum Hall States

Yizhi You,[1] Gil Young Cho,[1] and Eduardo Fradkin[1,2]
[1]Department of Physics and Institute for Condensed Matter Theory, University of Illinois at Urbana-Champaign, 1110 West Green Street, Urbana, Illinois 61801-3080, USA
[2]Kavli Institute for Theoretical Physics, University of California Santa Barbara, California 93106-4030, USA
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We derive an effective field theory for the isotropic-nematic quantum phase transition of fractional quantum Hall states. We demonstrate that for a system with an isotropic background the low-energy effective theory of the nematic order parameter has $z = 2$ dynamical scaling exponent, due to a Berry phase term of the order parameter, which is related to the nondissipative Hall viscosity. Employing the composite fermion theory with a quadrupolar interaction between electrons, we show that a sufficiently attractive quadrupolar interaction triggers a phase transition from the isotropic fractional quantum Hall fluid into a nematic fractional quantum Hall phase. By investigating the spectrum of collective excitations, we demonstrate that the mass gap of the Girvin-MacDonald-Platzman mode collapses at the isotropic-nematic quantum phase transition. On the other hand, Laughlin quasiparticles and the Kohn collective mode remain gapped at this quantum phase transition, and Kohn's theorem is satisfied. The leading couplings between the nematic order parameter and the gauge fields include a term of the same form as the Wen-Zee term. A disclination of the nematic order parameter carries an unquantized electric charge. We also discuss the relation between nematic degrees of freedom and the geometrical response of the fractional quantum Hall fluid.

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I. MOTIVATION AND INTRODUCTION

Strongly correlated electronic systems have a strong tendency to have liquid-crystal-like ground states (e.g., crystals, smectics or stripes, and nematics) that break spontaneously translation and rotational invariance to varying degrees [1]. Typically, these states arise as the result of the competition between repulsive Coulomb interactions and effective attractive interactions that arise from the disruption of strongly correlated states in systems with microscopic repulsive interactions. In two-dimensional electron gases (2DEGs) in large magnetic fields, these effects are even stronger since the kinetic energy of the electron is completely quenched in a uniform perpendicular magnetic field, and hence, interaction effects are dominant. For these reasons, in addition to incompressible quantum Hall states (FQH), integer or fractional, electronic liquid crystal phases are generally expected to occur in these systems [2].

Theoretically, several Hartree-Fock studies [3–6] (and effective field theories [7]) have predicted stripe phases, as well as "bubble" and other crystalline states [8], in addition to the expected Wigner crystals [9–13]. These phases are expected to become exact ground states for very weak magnetic fields [14], and in effective field theories [7] Similarly, (compressible) nematic phases have been found in variational wave-function calculations [15–17] and also in phenomenological hydrodynamic theories [18]. Exact diagonalization studies of small systems have found evidence of short-range stripe order in a Landau level [19]. For a recent review on electronic nematic phases, see Ref. [20].

Experiments in the second Landau level, $N = 2$ (and in the first Landau level, $N = 1$, in tilted fields), have established the existence of compressible states of the 2DEG with an extremely large transport spatial anisotropy with a marked temperature dependence [21–23], a nematic Fermi fluid [2,24]. In these experiments, the anisotropy was probed by a small in-plane component of the magnetic field that breaks rotational invariance explicitly. However, the strong temperature dependence of the anisotropy implies that that the in-plane field reveals a strong tendency to break rotational invariance spontaneously. Thus, the measured anisotropy of the transport can be regarded as the response to the in-plane field exactly in the same way as the magnetization is the response to a Zeeman field in a magnet. In this sense, the anisotropy versus in-plane field curves can be regarded as the equation of state of the 2DEG (or, rather, the nematic susceptibility). On the other hand, given the absence of pinning effects observed in this regime, the linearity of their I-V curves at low voltages, and the scaling behavior exhibited by the data, one can readily conclude that these states are regarded as (compressible) electron nematic states [24] rather than stripes (or unidirectional charge-density waves), or "bubble" phases (i.e., multidirectional charge-density wave states), expected from Hartree-Fock calculations [3–6]. To date, the compressible nematic state in the $N = 2$ Landau level near filling fraction $\nu = 9/2$ is the best documented case of a nematic phase in any electronic system [20].

More recent magnetotransport experiments in the first, $N = 1$, Landau level have shown that incompressible the fractional quantum Hall state with filling fraction $\nu = 7/3$ have a pronounced temperature-dependent anisotropy in their longitudinal transport. As in all experiments of this type (see, e.g., the review of Ref. [20]), the anisotropy is seen in the presence of a weak symmetry-breaking field (here, the in-plane component of the magnetic field), which reveals a pronounced (but smooth) rise of the transport anisotropy as the temperature is lowered below some characteristic value. Since the symmetry is broken explicitly, these experiments provide evidence for a large temperature-dependent nematic susceptibility in these fluid states [25]. These experiments strongly suggest that, at least in the $N = 1$ Landau level, the FQH phases the 2DEG may be proximate to a phase transition to an incompressible nematic state inside the topological fluid phase, i.e., a nematic FQH state. The notion of a nematic FQH state was actually suggested early on by Balents [26]. However, this concept did not attract significant attention until the recent experiments of Xia and co-workers, which suggested the existence of strong nematic correlations, became available [25].

The experiments of Xia and co-workers motivated Mulligan and co-workers [27,28] to formulate a theory of a quantum phase transition inside the $\nu = 7/3$ FQH phase, from an isotropic fluid to a nematic FQH state interpreted as a quantum Lifshitz transition. The theory of Mulligan *et al.* uses as a starting point the effective field theory of a Laughlin isotropic FQH fluid with filling fraction $\nu = 1/m$ (with m an odd integer) whose effective Lagrangian is that of a (hydrodynamic) gauge field a_μ, with a Maxwell and a Chern-Simons term [29,30]. In this picture, the FQH quantum Lifshitz transition occurs when the coefficient of the electric field term of the Maxwell-like term of the effective action of the hydrodynamic gauge field vanishes, and can be regarded as a Chern-Simons version of the quantum Lifshitz model [31].

While this theory successfully predicts many aspects of the experiment (in particular, the anisotropy), it has several difficulties, the most serious of which is that in a Galilean invariant system the coefficient of the term for the hydrodynamic electric field is fixed by Kohn's theorem [32]. Although this restriction can be violated by a relatively small amount by Landau level mixing effects [28], it is unlikely to become large enough to trigger a Lifshitz transition to a nematic state. Another puzzling aspect is that the Chern-Simons Lifshitz theory of Mulligan *et al.* also applies to the integer Hall states. However, barring large enough Landau level mixing effects, it is hard to see how a system in the integer quantum Hall regime may break spontaneously rotational invariance. The experiment of Xia *et al.* has also prompted several studies of integer and fractional quantum Hall states in systems in which the anisotropy is built in explicitly in the geometry of the two-dimensional surface in which the electrons reside [33], including wave functions for states with fixed anisotropy [34].

Maciejko and co-workers [35] recently proposed an effective field theory of the spontaneous breaking of rotational invariance in a nematic state in the FQH regime with the form of a nonlinear sigma model on the noncompact target space $SO(2,1)^+$ manifold of the rotational degrees of freedom and the amplitude of the local nematic order parameter. They proposed that the nematic transition is triggered by a softening of the intra-Landau level Girvin-MacDonald-Platzman (GMP) collective mode of the FQH fluid [36]. A key result from this work is the observation that, due to the breaking of time-reversal invariance in the FQH fluid, the dynamics of the nematic fluctuations is governed by a Berry phase term, whose coefficient they conjectured to be essentially the same as the (nondissipative) Hall viscosity of the FQH fluid [37–39]. Maciejko and co-workers also further an interpretation of nematic fluctuations as a fluctuating geometry (making contact with ideas put forward by Haldane on the existence of geometric degrees of freedom in the FQH liquid [33,40,41]). Similar ideas were discussed by two of us in the context of a nematic transition in a spontaneous anomalous quantum Hall state [42], and earlier on by one of us in a theory of thermal melting of the pair-density-wave superconducting state [43]. The conjectured connection between the nematic fluctuations in the FQH fluid and the Hall viscosity strongly suggests a relation with theories of the geometric response of these topological fluids [44–48], which we further elaborate below.

In this paper, we address several open aspects of this problem that have remained unexplained. One of the issues is the origin of the nematic quantum phase transition which Maciejko *et al.* argued could be due to a softening of the GMP collective mode. Here, we will show that the GMP mode can become gapless at wave vector $\boldsymbol{q} = 0$ if the effective interactions among the electrons are sufficiently attractive in the quadrupolar channel. It is known that in a Fermi liquid, a sufficiently attractive effective interaction in the quadrupolar channel (i.e., a sufficiently negative charge-channel Landau parameter F_2) can trigger a nematic instability through a Pomeranchuk instability, which results in a spontaneous quadrupolar distortion of the Fermi surface [49]. Here, we postulate that at long wavelengths, in addition to the long-range Coulomb interaction, there is an attractive short-range quadrupolar interaction. Such an effective interaction can arise due to the softening of the short-distance Coulomb interaction in Landau levels $N \geq 1$. In fact, an early numerical study by Scarola and co-workers [50] of the effective interactions of composite fermions [51] showed that in Landau levels with $N \geq 1$ there is a strong tendency for the FQH liquid to become unstable (and was interpreted as an exciton instability.) From the point of view of symmetry breaking, a $\boldsymbol{q} = 0$ ("exciton") quadrupolar condensate is equivalent to an instability to nematic state since they break the same spatial symmetries. The other focus of this work is to clarify the relation between the nematic fluctuations (and possible order) in the FQH fluid to the response of this fluid to changes on the actual background geometry of the surface on which the 2DEG resides. This is an important question since quantities such as the Hall viscosity measure the response to shear deformations of the geometry, and this is not quite the same as the nematic response, although, as we show below, they are related.

In order to study the quantum nematic phase transition in a FQH fluid, we first generalize the fermion Chern-Simons theory of the FQH states [52] to include the effects of the attractive quadrupolar interaction and show that indeed there can be a quantum phase transition inside all Jain states of the FQH provided the quadrupolar interaction is sufficiently attractive. In our treatment we also include the coupling to the background geometry of the 2D surface on which the 2DEG resides. We then use our recent results presented in Ref. [47] to show that the quadrupolar interaction couples to both the so-called statistical gauge field (of the fermion Chern-Simons theory) and the spin connection of the geometry. Our first main result is the derivation of the effective action for the nematic degrees of freedom, which, as expected, has the form proposed by Maciejko *et al.* The fluctuations of the nematic order parameter are strongly coupled to the GMP mode of the FQH fluid (which has quadrupolar character), and the nematic quantum phase transition is triggered when the $\boldsymbol{q} = 0$ component of this mode becomes gapless. Furthermore, the dynamics of the nematic degrees of freedom is controlled by a Berry phase term and, hence, has a dynamical critical exponent $z = 2$. However, its coefficient is not the Hall viscosity of the FQH fluid (as conjectured in Ref. [35]) but is given, instead, by the Hall viscosity of the effective integer Hall effect of the composite fermions. Nevertheless, the Hall viscosity of the system (both in the isotropic and in the nematic phase), defined as the response to the shear deformation of the underlying geometry, is the same as the Hall viscosity of the FQH fluid obtained in Ref. [38] (and recently rederived by us [47]). These results are reminiscent of the previous study by two of us [42], where we studied the effective theory of the phase transition between an isotropic Chern insulator and a nematic Chern insulator. We also demonstrate that in this theory the nematic transition is reached while the Kohn mode remains unaffected in both phases and at the phase transition. In addition, we show that the components of the nematic order parameter can be used to define an effective spin connection (which is effectively the same as the "nematic gauge field" phenomenologically introduced in Ref. [35]) and that it couples to an external electromagnetic probe field through a term with the form of the Wen-Zee term [53], a result also anticipated by Maciejko *et al.* We also derive the effective action for the spin connection of the background geometry and show that it has the same form (with the same universal coefficients) in both phases. Finally, we use our effective field theory to investigate the properties of a disclination of the nematic order parameter in the nematic phase, and show that it carries a fractional (but nonuniversal) electric charge and that the Hall viscosity is modified by the disclination, which agrees with the a symmetry-based argument of Ref. [35].

This paper is organized as follows. The theory of spontaneous rotational symmetry breaking is developed in Sec. II. After summarizing the fermion Chern-Simons gauge theory in Sec. II A, in Sec. II B we introduce the quadrupolar interaction and its coupling with the statistical gauge field and with the spin connection of the background metric. In Sec. III, we derive the effective Landau-Ginzburg theory from the fermion Chern-Simons theory, and in Sec. IV, we show that there is a quantum phase transition to a nematic phase for sufficiently strong attractive quadrupolar coupling. In Sec. V, we discuss the behavior of the Goldstone mode of the broken orientational symmetry in the nematic phase and the nature of the disclinations. The coupling to the background geometry is developed in Sec. VI. Our conclusions are presented in Sec. VII. In the appendixes, we present details of the calculation of the effective field theory. In Appendix A, we present the calculation of the nematic correlators, and in Appendix B we present the calculation of mixed correlators of nematic and gauge fields. A proof of gauge invariance is given in Appendix C, and the nematic collective excitations are derived in Appendix D.

II. SPONTANEOUS BREAKING OF ROTATIONAL SYMMETRY IN FQH STATES

A. Composite Fermion theory of FQH states

Here, we begin with a short review of the composite fermion theory of a FQH state [51,52], specializing in the simpler case of the Laughlin state at filling $\nu = 1/3$, which can be easily generalized to the other states in Jain sequence $\nu = p/(2sp+1)$, where $s, p \in \mathbb{Z}$. Let us consider a theory of electron field Ψ in two space dimensions in a uniform magnetic field. The action for this system is

$$\mathcal{S} = \int d^2x dt \left[\Psi^\dagger(x) D_0 \Psi(x) - \frac{1}{2m_e} (\boldsymbol{D}\Psi(x))^\dagger \cdot (\boldsymbol{D}\Psi(x)) \right] - \frac{1}{2} \int d^2x' d^2x dt V(|\boldsymbol{x} - \boldsymbol{x}'|) \Psi^\dagger(x)\Psi(x)\Psi^\dagger(x')\Psi(x'), \tag{2.1}$$

in which $D_\mu = \partial_\mu + iA_\mu$ is the covariant derivative of the electron, m_e is the mass of the electron, and we have set the Planck constant $\hbar$, the speed of light c, and the electric charge e to unity. The four-fermion term encodes the two-body interaction between the electrons. The electromagnetic gauge field A_μ can be written as $A_\mu = \bar{A}_\mu + \delta A_\mu$, where $\bar{A}_\mu$ is for the uniform magnetic field $\bar{B} = \epsilon^{ij}\partial_i \bar{A}_j$ perpendicular to the plane and δA_μ is the probe field to measure the response of the FQH state.

The average electron density $\bar{\rho}$ and the uniform external magnetic field $\bar{B}$ are related to each other through the filling fraction ν,

$$\bar{\rho} = \frac{\nu}{2\pi}\bar{B} = \frac{1}{6\pi}\bar{B}, \tag{2.2}$$

where we have set $\nu = 1/3$ for the leading Laughlin state. For a general Jain state, the filling fraction is $\nu = p/(2sp+1)$, where s and p are two integers. The Laughlin FQH state with $\nu = 1/3$ can be pictorially understood as the liquid state of the electrons in which, on average, each electron is bound with the two flux quanta. For a general Jain state, each electron is bound to $2s$ flux quanta and becomes a composite fermion [51].

This is a problem of strongly coupled electrons and cannot be tackled directly using weak coupling methods. To make progress, we follow Ref. [52] and consider the equivalent system obtained by coupling the system of interacting electrons to the (dynamical) Chern-Simons term of the statistical gauge field a_μ, using minimal coupling. (For a detailed discussion, see Ref. [30].) The action of the equivalent problem is

$$\begin{aligned}\mathcal{S} = &\int d^2x dt \left[\Psi^\dagger(x) D_0 \Psi(x) - \frac{1}{2m_e}(\boldsymbol{D}\Psi(x))^\dagger \cdot (\boldsymbol{D}\Psi(x))\right] \\ &-\frac{1}{2}\int d^2x' d^2x dt [V(|\boldsymbol{x}-\boldsymbol{x}'|)\Psi^\dagger(x)\Psi(x)\Psi^\dagger(x')\Psi(x')] \\ &+\frac{1}{8\pi}\int d^2x dt \epsilon^{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda,\end{aligned} \tag{2.3}$$

where $D_\mu = \partial_\mu + iA_\mu + ia_\mu$ is a new covariant derivative that includes the minimal coupling to both the electromagnetic field A_μ and the statistical field a_μ. This is the exact mapping of the original problem defined by the action of Eq. (2.1). The Chern-Simons term binds the two flux quanta to the electron and turns the electron into the composite fermion [51,54].

We now consider uniform states that can be described using the average field approximation in which we smear out the two flux quanta bound to the electron over the two-dimensional plane. This translates as choosing the average part of $\bar{a}_\mu$ to partially cancel the external magnetic field $\bar{A}_\mu$. For the $\nu = 1/3$ Laughlin state, the effective field is

$$\bar{A}_\mu + \bar{a}_\mu = \frac{1}{3}\bar{A}_\mu. \tag{2.4}$$

Thus, the composite fermion Ψ is subject to the magnetic field $\bar{A}_\mu + \bar{a}_\mu$ which is $1/3$ of the magnetic field experienced by the electron. The composite fermion is in the integer quantum Hall effect at the filling $\nu = 1$, and in effect is weakly coupled. We can write out the Lagrangian of the composite fermion:

$$\begin{aligned}\mathcal{S} = &\int d^2x dt \left[\Psi^\dagger(x) D_0 \Psi(x) - \frac{1}{2m_e}(\boldsymbol{D}\Psi(x))^\dagger \cdot (\boldsymbol{D}\Psi(x))\right] \\ &-\frac{1}{2}\int d^2x' d^2x [V(|\boldsymbol{x}-\boldsymbol{x}'|)\Psi^\dagger(x)\Psi(x)\Psi^\dagger(x')\Psi(x')] \\ &+\frac{1}{8\pi}\int d^2x dt \epsilon^{\mu\nu\lambda} \delta a_\mu \partial_\nu \delta a_\lambda.\end{aligned} \tag{2.5}$$

Here and below, we denote by D_μ,

$$D_\mu = \partial_\mu + i\frac{1}{3}\bar{A}_\mu + i\delta a_\mu + i\delta A_\mu, \tag{2.6}$$

the covariant derivative of the composite fermion (again, for the $\nu = 1/3$ Laughlin state). The fields δa_μ and δA_μ are the fluctuation of the gauge fields about their average values. Furthermore, the density fluctuation of the electron $\delta\rho = \Psi^\dagger\Psi - \bar{\rho}$ is bound with the flux of δa_μ:

$$\delta\rho(x) = \frac{1}{4\pi}\delta b(x) = \frac{1}{4\pi}\epsilon^{ij}\partial_i \delta a_j. \tag{2.7}$$

This makes the density-density interaction between the electrons to be quadratic in the gauge field δa_μ. As the action is quadratic in the composite fermion field Ψ, we can integrate out the fermion and the fluctuating part δa_μ of the statistical gauge field to obtain the effective theory for the gauge field δA_μ. From the effective theory of δA_μ, one can calculate the electromagnetic Hall response and find the collective excitations of the FQH state [55], which, with some caveats, agree at long wavelengths with the experiments and numerical calculations.

B. Quadrupolar interaction

The composite fermion theory summarized above is rotationally invariant and cannot describe a nematic FQH state. Thus, we look for a new ingredient for the composite fermion theory to describe the nematic state and the transition toward the nematic state from the isotropic state. Since the density-density interaction of electrons and the Chern-Simons term (at the level of the bare action of the composite fermion theory) cannot induce the spontaneous breaking of the rotational symmetry, we look for an interaction that can favor the anisotropic state rather than the isotropic state. To this end, we follow the approach of the nematic Fermi fluid of Ref. [49] and add a quadrupolar interaction term $\mathcal{S}_q$ to the action of the form

$$\mathcal{S}_q = -\frac{1}{2}\int dt \int d^2x d^2x' F_2(|\boldsymbol{x}-\boldsymbol{x}'|)\mathrm{Tr}[Q(x)Q(x')], \tag{2.8}$$

where $F_2(|\boldsymbol{x}-\boldsymbol{x}'|)$ is the Landau interaction in the quadrupolar channel whose spatial Fourier transform is

$$F_2(\boldsymbol{q}) = \frac{F_2}{1+\kappa \boldsymbol{q}^2} \tag{2.9}$$

and $\kappa > 0$ parametrizes the interaction range. The coupling constant F_2 (i.e., the Landau parameter) has units of energy $\times$ (length)^6. Here, we introduce the 2×2 traceless symmetric tensor $Q(x)$:

$$Q(x) = \Psi^\dagger(x)\begin{pmatrix} D_x^2 - D_y^2 & D_xD_y + D_yD_x \\ D_xD_y + D_yD_x & D_y^2 - D_x^2 \end{pmatrix}\Psi(x). \tag{2.10}$$

Here, D_x and D_y are the spatial covariant derivatives defined in Eq. (2.6).

The full action (including the quadrupolar interaction $\mathcal{S}_q$) is manifestly rotationally invariant. In the case of a Fermi liquid, for large enough attractive quadrupolar interactions, $F_2 < 0$, there is a Pomeranchuk instability that results in the spontaneous breaking of rotational invariance and the development of a nematic phase [49]. Here, too, if $F_2 < 0$ and large enough in magnitude, the quadrupolar coupling can induce a transition to an anisotropic phase by developing the finite expectation value of $Q(x)$. When $\langle Q\rangle \neq 0$, the continuous rotational symmetry $O(2)$ of the two-dimensional space is broken down to C_2 generated by the discrete π rotation of the plane. However, in the case of a Fermi fluid at zero external magnetic field, the nematic phase leads to the spontaneous distortion of the Fermi surface and the development of an anisotropic effective mass for the quasiparticles in the anisotropic state. Furthermore, in the absence of a coupling to the underlying lattice, the resulting nematic phase is a non-Fermi liquid. In the case at hand, although there is no Fermi surface to begin with, at the level of mean field theory, nematicity is also manifest as an effective anisotropy of the effective mass of the composite fermions.

Now, we include the quadrupolar interaction in the the (fermionic) Chern-Simons theory of the FQH states [52] of Eq. (2.3):

$$\begin{aligned}\mathcal{S} = &\int d^2x dt\left[\Psi^\dagger(x)D_0\Psi(x) - \frac{1}{2m_e}(\boldsymbol{D}\Psi(x))^\dagger\cdot(\boldsymbol{D}\Psi(x))\right]\\ &-\frac{1}{32\pi^2}\int d^2x' d^2x dt V(|\boldsymbol{x}-\boldsymbol{x}'|)\delta b(x)\delta b(x')\\ &+\frac{1}{8\pi}\int d^2x dt \epsilon^{\mu\nu\lambda}\delta a_\mu\partial_\nu\delta a_\lambda\\ &-\frac{1}{2}\int dt\int d^2x d^2x' F_2(|\boldsymbol{x}-\boldsymbol{x}'|)\mathrm{Tr}[Q(x)Q(x')].\end{aligned} \tag{2.11}$$

Here, we use the Chern-Simons constraint (i.e., the "Gauss law") to represent the fluctuating density $\delta\rho$ of the composite fermion in terms of the fluctuating statistical field δb, which results in density-density interaction quadratic in the statistical gauge field.

However, the quadrupolar interaction cannot be written as a quadratic form in the statistical gauge field δa_μ. Instead, we perform a Hubbard-Stratonovich decoupling transformation to rewrite the quadrupolar interaction term $\mathcal{S}_q$ in terms of two fields M_1 and M_2 (which can be regarded as the two real components of a 2×2 real symmetric matrix field). After decoupling, the action $\mathcal{S}_q$ of Eq. (2.8) takes the form

$$\begin{aligned}\mathcal{S}_q = \int d^2x dt\Bigg[&\frac{1}{4F_2m_e^2}\boldsymbol{M}^2 - \frac{\kappa}{4F_2m_e^2}\sum_{i=1,2}|\nabla M_i|^2\\ &+\frac{M_1}{m_e}\Psi^\dagger(D_x^2-D_y^2)\Psi + \frac{M_2}{m_e}\Psi^\dagger(D_xD_y+D_yD_x)\Psi\Bigg].\end{aligned} \tag{2.12}$$

Here, we introduce suitable factors of the electron mass m_e to make the Hubbard-Stratonovich fields M_1 and M_2 dimensionless. F_2 is the coupling constant of the quadrupolar interaction of Eq. (2.10).

It is apparent that in Eq. (2.12) M_1 and M_2 play the role of the order parameters for the nematic phase. These fields couple to the stress tensor of the composite fermions and thus play a role analogous to a background metric. In this sense, we can regard the nematic fluctuation as providing a "dynamical metric" that modifies the local geometry of the composite fermions [42,43].

Thus, we end up with the following action for the composite fermions coupled to the Chern-Simons gauge field, with a density-density interaction and a quadrupolar interaction,

$$\begin{aligned}\mathcal{S} = &\int d^2x dt\left[\Psi^\dagger(x)D_0\Psi(x) - \frac{1}{2m_e}(\boldsymbol{D}\Psi(x))^\dagger\cdot(\boldsymbol{D}\Psi(x))\right]\\ &-\frac{1}{32\pi^2}\int d^2x' d^2x dt V(|\boldsymbol{x}-\boldsymbol{x}'|)\delta b(x)\delta b(x')\\ &+\frac{1}{8\pi}\int d^2x dt \epsilon^{\mu\nu\lambda}\delta a_\mu\partial_\nu\delta a_\lambda\\ &+\int d^2x dt\Bigg[\frac{1}{4F_2m_e^2}\boldsymbol{M}^2 + \frac{\kappa}{4F_2m_e^2}\sum_{i=1,2}|\nabla M_i|^2\\ &+\frac{M_1}{m_e}\Psi^\dagger(D_x^2-D_y^2)\Psi + \frac{M_2}{m_e}\Psi^\dagger(D_xD_y+D_yD_x)\Psi\Bigg],\end{aligned} \tag{2.13}$$

where, again, the covariant derivatives are given in Eq. (2.6). Thus far, we have not made any approximations. In the next section, we discuss the uniform states that result by treating this theory in the average field approximation and by considering the effects of fluctuations at the one loop (RPA) level.

It turns out that, from the theory we define, the resulting quantum phase transition is strongly first order and to a state with maximal nematicity (and without Landau quantization). To avoid this pathological limit, and to make the nematic phase stable (and accessible by a continuous quantum phase transition), we introduce an extra term in the kinetic energy part of the action of the form

$$S_6 = -\alpha \int d^2x dt \Psi^\dagger \left(\frac{-\boldsymbol{D}^2}{2m_e} - \frac{\bar{\rho}\pi}{m_e} \right)^3 \Psi, \tag{2.14}$$

where, once again, $\boldsymbol{D}$ stands for the space components of the (full) covariant derivative and $\boldsymbol{D}^2$ is the covariant Laplacian. A term of a similar type was introduced by Oganesyan *et al.* [49] in their theory of the nematic Fermi fluid formed by a Pomeranchuk instability. Here, too, this (technically irrelevant) term insures that the nematic state is stable, provided the coupling constant α is large enough (as we see below). For other ranges of α, the quantum phase transition becomes first order, as happens in theories of the electronic nematic transition in lattice systems [56]. Although the addition of this term complicates the calculation somewhat, it does not change the physics in any essential way. We note that this term commutes with the gauge-invariant kinetic energy and, consequently, it has the same eigenstates. Thus, this term changes only the eigenvalues, but it does not induce Landau level mixing.

C. Symmetries

The action of Eq. (2.13) has two important symmetries. One is local gauge invariance, under which the Fermi field $\Psi(x)$ and the gauge field $a_\mu(x)$ transform as

$$\Psi'(x) = e^{-i\Lambda(x)}\Psi(x), \qquad a'_\mu(x) = a_\mu(x) + \partial_\mu \Lambda(x), \tag{2.15}$$

where $\Lambda(x)$ is a (smooth) gauge transformation.

The second symmetry is invariance under the coordinate transformation of global rotations in real space,

$$x'_i = R_{ij}(\varphi) x_j, \tag{2.16}$$

where $R_{ij}(\varphi)$ is the 2×2 rotation matrix by an angle of φ. The Fermi field is invariant (a scalar) under rotations, $\Psi(Rx') = \Psi'(x)$. However, the invariance of the action of Eq. (2.13) under global rotations requires that the Hubbard-Stratonovich fields $\boldsymbol{M}$, which are conjugate to the nematic order parameter field Q_{ij} of Eq. (2.10), transform not as a vector under rotations but as a director, i.e., a vector in without a direction. This means that it transforms under a rotation by twice the rotation angle in real space,

$$M'_i = R_{ij}(2\varphi) M_j. \tag{2.17}$$

Under this transformation, the Hubbard-Stratonovich field is invariant under a rotation by π. Similarly, the nematic order parameter, i.e., the traceless symmetric 2×2 matrix field of Eq. (2.10), transforms as a tensor under rotations by an angle of 2φ.

III. EFFECTIVE FIELD THEORY OF NEMATIC ORDER PARAMETER

The full action of Eq. (2.13) is a quadratic form in the composite fermions. These fermionic fields can be integrated out, allowing us to obtain an effective field theory for the nematic order parameter M_1 and M_2 coupled to the gauge fields. This procedure is safe provided one is expending about a saddle point state with a finite energy gap. The resulting effective Lagrangian can be decomposed as the three parts

$$\mathcal{L} = \mathcal{L}_a + \mathcal{L}_M + \mathcal{L}_{a,M}, \tag{3.1}$$

where $\mathcal{L}_a$ and $\mathcal{L}_M$ include only the fluctuating gauge fields $\delta a + \delta A$ and only the nematic order parameter M_i, $i = 1, 2$, and $\mathcal{L}_{a,M}$ represents the coupling between the gauge fields and the nematic order parameter.

For clarity, we discuss the three parts, $\mathcal{L}_a$, $\mathcal{L}_M$, and $\mathcal{L}_{a,M}$, of the full effective theory separately. Here, we briefly show what we can learn from the three parts before describing the details of each term. $\mathcal{L}_a$ is the effective Lagrangian for the statistical gauge fields of the isotropic FQH states [52]. $\mathcal{L}_M$ is the effective Lagrangian for the nematic order parameters. It has the conventional Landau-Ginzburg form supplemented by a topological Berry phase term. We demonstrate that there is a continuous phase transition if the quadrupolar interaction F_2 is bigger than a critical value. Furthermore, we show that there is a Berry phase term for the nematic order parameter, which is similar to the Hall viscosity term, and the Berry phase term makes the quantum critical point have the dynamical exponent $z = 2$. From $\mathcal{L}_{a,M}$, we see that there is a topological term, similar to the Wen-Zee term [53], which describes the response to the curvature induced by disclination (not the deformation from the background geometry). In addition, $\mathcal{L}_{a,M}$ also contains an anisotropic Maxwell term that represents the coupling of the Kohn collective mode to the nematic order parameter fields [42].

A. Gauge field Lagrangian: $\mathcal{L}_a$

Here, we consider the term of the effective Lagrangian of Eq. (3.1) that includes only the gauge fields a_μ and δA_μ. This part of the effective action does not know the nematic order parameter, and so it should be the same effective action of the gauge fields as in the isotropic FQH states [52]:

$$\mathcal{L}_a = -\frac{1}{2}(\delta a_\mu + \delta A_\mu)\Pi^0_{\mu\nu}(\delta a_\nu + \delta A_\nu) + \frac{\varepsilon^{\mu\nu\lambda}}{8\pi}\delta a_\mu \partial_\nu \delta a_\lambda. \tag{3.2}$$

Here, $\Pi^0_{\mu\nu}(x-y)$, given by

$$\Pi^0_{\mu\nu}(x-y) = -i\frac{1}{Z_F}\frac{\delta^2 Z_F}{\delta a_\mu(x) a_\nu(y)} = \langle j_\mu(x) j_\nu(y)\rangle, \quad (3.3)$$

is the bare polarization tensor of the integer quantum Hall state of the composite fermion, and it is given in Ref. [52], whose results we use. The current-current time-ordered correlators shown in Eq. (3.3) are computed in the free-composite fermion theory and Z_F is the partition function of composite fermions with an integer number of filled effective Landau levels. In the low-energy and long-wavelength limit, $\Pi^0_{\mu\nu}(\boldsymbol{q},\omega)$ is given by

$$\begin{aligned}
\Pi^0_{00}(\boldsymbol{q},\omega) &= -\frac{1}{\pi}q^2\frac{m_e}{\bar{b}(1+\alpha\bar{\omega}_c^2)},\\
\Pi^0_{0j}(\boldsymbol{q},\omega) &= -\frac{1}{\pi}q_j\omega\frac{m_e}{\bar{b}(1+\alpha\bar{\omega}_c^2)} + \frac{i}{\pi}\epsilon^{jk}q_k,\\
\Pi^0_{j0}(\boldsymbol{q},\omega) &= -\frac{1}{\pi}q_j\omega\frac{m_e}{\bar{b}(1+\alpha\bar{\omega}_c^2)} - \frac{i}{\pi}\epsilon^{jk}q_k,\\
\Pi^0_{ij}(\boldsymbol{q},\omega) &= -\frac{1}{\pi}\delta_{ij}\omega^2\frac{m_e}{\bar{b}(1+\alpha\bar{\omega}_c^2)} - \frac{i}{\pi}\epsilon^{ij}\omega\\
&\quad -\frac{(q^2\delta_{ij}-q_iq_j)}{m_e(1+\alpha\bar{\omega}_c^2)},
\end{aligned} \quad (3.4)$$

where $\bar{b} = B/3$ for the $\nu = 1/3$ Laughlin state. Here, we include in the results of Ref. [52] the corrections due to the extra term in the kinetic energy of Eq. (2.14).

B. Order parameter Lagrangian: $\mathcal{L}_M$

We now obtain the Lagrangian for the nematic fields $\mathcal{L}_M$ of Eq. (3.1) to the quartic order in the nematic order parameter by calculating one-loop Feynman diagrams. We see that $\mathcal{L}_M$ exhibits the isotropic-anisotropic phase transition and the quantum phase transition. To calculate $\mathcal{L}_M$, we need to compute the two-point and four-point correlators of $N_i = -i(\delta Z/\delta M_i)$, and the calculation is done in Appendix A. Here, for simplicity, we discuss only the case of the FQH state at the filling $1/3$. However, as discussed in Appendix A, it is straightforward to generalize the calculations to the other states in Jain sequence $\nu = p/2sp+1$, $p,\, s \in \mathbb{Z}$.

Integrating out the composite fermion and expanding about the low-energy limit, i.e., taking the lowest terms in frequency ω and momentum $\boldsymbol{q}$, we obtain the effective theory of the nematic fluctuations,

$$\begin{aligned}
\mathcal{L}_M &= \frac{\epsilon^{ij}\bar{\rho}}{2(1+4\alpha\bar{\omega}_c^2)^2}M_i\partial_0 M_j - r\boldsymbol{M}^2\\
&\quad -\frac{\bar{\kappa}}{2}(\nabla M_i)^2 - \frac{u}{4}(\boldsymbol{M}^2)^2,
\end{aligned} \quad (3.5)$$

where $\boldsymbol{M}^2 = M_1^2 + M_2^2$.

In the effective Lagrangian of Eq. (3.5), we ignore two physically significant corrections terms. The Lagrangian of Eq. (3.5) is invariant under the $O(2)$ symmetry of arbitrary global rotations in the order parameter space, i.e.,

$$M_i \to R_{ij}(\phi)M_j, \quad (3.6)$$

where $R_{ij}(\phi)$ is the 2×2 rotation matrix by an arbitrary angle ϕ. However, as we saw in Sec. II C, the only symmetry (aside from gauge invariance) is a combination of a rotation in space by an angle φ and a rotation in the order parameter space by 2φ, which leave $\boldsymbol{M} \equiv -\boldsymbol{M}$ invariant. This means that the larger symmetry of the Lagrangian of Eq. (3.5) is only approximate and that the Lagrangian must contain terms that reduce the symmetry accordingly. In fact, the effective Lagrangian allows for an extra (formally irrelevant) operator of the form

$$\mathcal{L}_{\rm SO} = -\lambda((\boldsymbol{M}\cdot\nabla)\boldsymbol{M})^2, \quad (3.7)$$

which is invariant under joint rotations in real space and in the order parameter space (and is formally a "spin-orbit-type" coupling). Such terms are well known to arise in the free energy of classical liquid crystals [57,58].

The resulting effective Lagrangian of Eq. (3.5) has the same form as the effective theory of the nematic order parameter in a Chern insulator [42], and of the effective field theory of the nematic FQH state of Maciejko and collaborators [35]. Moreover, upon defining the complex field $\Phi = M_1 + iM_2$, it is easy to see that the Lagrangian of Eq. (3.5) is equivalent to the Lagrangian of a 2D dilute Bose gas (with r playing the role of the chemical potential and u the contact interaction). As in Refs. [35,42], the effective theory of the nematic order parameter field contains a Berry phase term associated with the non-dissipative response of the quantum Hall effect, which relates to the Hall viscosity. This term makes time and space scale differently, and the associated quantum critical point has the dynamical exponent $z = 2$.

In our discussion, we have neglected the role of the symmetries of the underlying lattice. While lattice effects are irrelevant (and unimportant) for the topological properties of the FQH fluids, they do matter for the nematic fluctuations and ordering. In the case of GaAs-AlAs heterostructures, the 2DEG resides on surfaces that have a tetragonal C_4 symmetry. The extra terms of the Lagrangian that break the symmetry from the full continuous rotations down to C_4 are proportional to $M_1^2 - M_2^2$ and $2M_1M_2$. In Sec. V A we show that these terms gap out the Goldstone modes of the nematic phase.

The parameters entering into the effective Lagrangian of Eq. (3.5) are obtained by a direct calculation of the correlators, and are found to be

$$r = -\frac{1}{4F_2 m_e^2} - \frac{\bar{\omega}_c}{2\pi \bar{l}_b{}^2 (1+4\alpha\bar{\omega}_c^2)},$$
$$\bar{\kappa} = -\frac{\kappa}{2F_2 m_e^2} - \frac{1}{\pi}\left[\frac{1}{(1+\alpha\bar{\omega}_c^2)} + \frac{1}{2(1+4\alpha\bar{\omega}_c^2)} + \frac{2}{(1+9\alpha\bar{\omega}_c^2)}\right],$$
$$u = \frac{\bar{b}\bar{\omega}_c}{4\pi}\frac{1}{(1+4\alpha\bar{\omega}_c^2)^2}\left[\frac{1}{4(1+4\alpha\bar{\omega}_c^2)} - \frac{3}{4(1+16\alpha\bar{\omega}_c^2)}\right]. \tag{3.8}$$

Here, $\bar{\omega}_c = \bar{b}/m_e$ and $\bar{l}_b = \sqrt{3}\ell_0$ (for the Laughlin state at $\nu = 1/3$) are the effective cyclotron frequency and the effective magnetic length of the composite fermion, where $\ell_0 = B^{-1/2}$ is the magnetic length.

From these results, we can also see that the nematic order parameter will condense only when the quadrupolar interaction is attractive and larger in magnitude than the critical value:

$$|F_2^c| = \frac{\pi \bar{l}_b{}^2}{2\bar{\omega}_c m_e^2}(1+4\alpha\bar{\omega}_c^2). \tag{3.9}$$

Furthermore, since $u > 0$, the quantum phase transition is continuous and the nematic state is stable.

From Eq. (2.10), it is clear that the nematic order parameters formally couple to the quadrupole density in the same way as the background metric couples to the energy-momentum tensor [although the extra term in the kinetic energy of Eq. (2.14) does not couple to the nematic fields]. We can regard the nematic order parameters as a "dynamical spatial metric" that modifies spatial components of the metric tensor. From this observation, one may naively expect that the prefactor Berry phase term in Eq. (3.5) may be the Hall viscosity of the FQHE $\eta_H = \bar{\rho}/2\nu$ when $\alpha = 0$.

However, for $\alpha = 0$, the prefactor of the Berry phase term of Eq. (3.5) is the Hall viscosity term of the integer quantum Hall state at $\nu = 1$, and not of the actual Hall viscosity of the fractional quantum Hall state. See the discussion in Sec. VI. This difference originates in the fact that the "dynamical metric" associated with the nematicity and the background metric are not equivalent. For FQH states, the nematic order parameters couple only with the stress energy tensor, while the background metric not only couples with the stress energy tensor, but also appears in the form of a spin connection, as discussed in detail in Ref. [47]. In the composite fermion or composite boson theories, when we attach flux to the electron to form a composite particle, each flux quantum attached to the particle induces the additional angular momentum 1/2. This makes the composite particle couple to the spin connection though the particle is a scalar and not a spinor. The orbital spin then couples to the local geometry to the spin connection much in the same way as relativistic fermions do. Thus, after we perform flux attachment to describe the FQH fluids, the composite fermion resulting from the flux attachment will minimally couple with the spin connection ω_μ, as shown explicitly in Ref. [47]. The coupling through the spin connection with the background geometry is the origin of the difference between the nematic order parameter and the (deformed) background metric. The derivation of the correct Hall viscosity from the background metric deformation through the composite fermion theory is reported elsewhere [47].

The results of this section can be easily generalized to all the states in the Jain sequence $\nu = p/2p+1$, with the effective Lagrangian density. However, when p goes to infinity, the theory approaches the half-filled Landau level and the gap vanishes. In this regime, the system becomes a non-Fermi liquid and the effective Lagrangian for the gauge field given by Eq. (3.2) now has a Landau damping term [59,60]. In this limit, at least formally, this theory is a generalization of the theory of the nematic quantum phase transition in Fermi fluids [49] to describe the compressible nematic quantum fluid at half-filled Landau levels (see Ref. [20] and references therein.)

C. Order parameter and gauge field Lagrangian: $\mathcal{L}_{a,M}$

Here, we derive the third term of the effective Lagrangian of Eq. (3.1), $\mathcal{L}_{a,M}$, that describes the coupling between the gauge fields and the nematic order parameters. This part of the effective Lagrangian will be important later for investigating the quantum numbers and statistics of the disclinations of the nematic phase.

In the presence of nematic order, the natural coupling between the nematic order parameter and the gauge field is as a local anisotropy of the Maxwell term. Since the order parameter acts as the spatial components of a metric tensor [42], the indices of the field strength tensor f_{ij} of the gauge fields contract with the (inverse of) metric spatial tensor g^{ij}. The resulting terms in the effective Lagrangian are

$$\mathcal{L}_{a,m} = \frac{m_e 2M_1}{4\pi\bar{b}(1+4\alpha\bar{\omega}_c^2)}(\partial_x\delta\tilde{A}_0 - \partial_0\delta\tilde{A}_x)^2 - \frac{m_e 2M_1}{4\pi\bar{b}(1+4\alpha\bar{\omega}_c^2)}(\partial_y\delta\tilde{A}_0 - \partial_0\delta\tilde{A}_y)^2 + \frac{m_e M_2}{\pi\bar{b}(1+4\alpha\bar{\omega}_c^2)}(\partial_x\delta\tilde{A}_0 - \partial_0\delta\tilde{A}_x)(\partial_y\delta\tilde{A}_0 - \partial_0\delta\tilde{A}_y), \tag{3.10}$$

where $\delta\tilde{A} = \delta A + \delta a$. These terms are second order in derivatives and are time-reversal and parity invariant.

However, there are contributions to $\mathcal{L}_{a,M}$ that are first order in derivatives and hence break time reversal and parity. These contributions have the form of a Wen-Zee term [53,61]. The Wen-Zee term can be understood as the response of the FQH states to a change of the geometric curvature: the curvature will trap the gauge charge. While this term can be ignored if the nematic order is uniform in space, it has interesting consequences for the charge and the statistics of the disclination in the nematic phase.

To obtain the Wen-Zee term for the nematic order parameter, we perform calculation of one-loop diagrams with one current and one and two nematic fields,

$$\mathcal{L}_{wz} = -\frac{1}{2} T_{i\mu} M_i (\delta a_\mu + \delta A_\mu + 2Z_\mu) + \frac{1}{3} R_{ij\mu} M_i M_j (\delta a_\mu + \delta A_\mu), \quad (3.11)$$

where $T_{i\mu}$ and $R_{ij\mu}$ denote the following three-point (time-ordered) correlators of the composite fermions,

$$T_{i\mu}(\boldsymbol{r}, t) = i2 \frac{1}{Z_F} \frac{\delta Z_F}{\delta M_i \delta a_\mu} = -i \langle N_i(\boldsymbol{r}, t) j_\mu(0, 0) \rangle,$$
$$R_{ij\mu}[\boldsymbol{r}_i, t_i] = -3i \frac{1}{Z_F} \frac{\delta Z_F}{\delta M_i \delta M_j \delta a_\mu} = -\langle N_i(\boldsymbol{r}_1, t_1) N_j(\boldsymbol{r}, t_2) j_\mu(\boldsymbol{r}_3, t_3) \rangle, \quad (3.12)$$

where the correlators are time-ordered functions of the free-composite fermion theory, and

$$Z_0 = 0,$$
$$Z_x = (\delta a_x + \delta A_x) M_1 + (\delta a_y + \delta A_y) M_2$$
$$Z_y = (\delta a_x + \delta A_x) M_2 - (\delta a_y + \delta A_y) M_1. \quad (3.13)$$

The correlators of Eq. (3.12) are computed explicitly in Appendix B.

After calculating the above correlators, we obtain the coupling between the geometric curvature induced by the nematic fields and the statistical gauge field, which explicitly has the form of a Wen-Zee term,

$$\mathcal{L}_{wz} = \frac{1}{4\pi} \epsilon^{\mu\nu\rho} \omega_\mu^Q \partial_\nu (\delta a_\rho + \delta A_\rho), \quad (3.14)$$

where ω_μ^Q (with $\mu = 0, x, y$) is the effective spin connection induced by the local nematic order parameters, i.e.,

$$\omega_0^Q = \frac{\epsilon^{ij}}{(1 + 4\alpha \bar{\omega}_c^2)^2} M_i \partial_0 M_j,$$
$$\omega_x^Q = \frac{\epsilon^{ij}}{(1 + 4\alpha \bar{\omega}_c^2)^2} M_i \partial_x M_j - t(\partial_x M_2 - \partial_y M_1),$$
$$\omega_y^Q = \frac{\epsilon^{ij}}{(1 + 4\alpha \bar{\omega}_c^2)^2} M_i \partial_y M_j + t(\partial_x M_1 + \partial_y M_2), \quad (3.15)$$

where

$$t = \frac{2}{1 + \alpha \bar{\omega}_c^2} - \frac{2}{2 + 8\alpha \bar{\omega}_c^2}. \quad (3.16)$$

The spin connection ω^Q of the nematic order parameter is different from the spin connection of the background geometry. The meaning of the spin connection can be clarified by looking at its curl,

$$\partial_x \omega_y^Q - \partial_y \omega_x^Q \propto \frac{1}{2} \sqrt{g} R, \quad (3.17)$$

where R is the geometric curvature of the dynamical metric induced by the nematic order parameters M_i. Here, g, the determinant of the metric, is given by $g = 1 - 4\boldsymbol{M}^2$.

Here, the coupling term between the "spin connection" and gauge fields in Eq. (3.14) has a similar form to the Wen-Zee term of Ref. [53]. However, the coefficient in Eq. (3.14) is not the orbital spin of the FQH state. Instead, this coefficient is equal to the orbital spin of the integer quantum Hall state at $\nu = 1$ (when $\alpha = 0$). This can be easily understood from the composite fermion theory because the composite fermions effectively are the integer quantum Hall state and any response at the mean-field approximation of the composite fermion will be the same as that of the integer quantum Hall phase. This fact still remains true even after integrating out the statistical gauge field. The difference again comes from the nonequivalence between the nematic order parameter and the background metric. When we attach Chern-Simons flux to the fermion in a background metric, the orbital spin induced by the flux attachment also gives rise to an additional geometry-gauge coupling term, which has the form of a Wen-Zee term [47]. For the nematic order parameter, the coupling between the gauge field and the nematic order parameters comes only from the composite fermion that forms an integer quantum Hall effect (IQHE). The derivation of the correct Wen-Zee terms through the composite fermion theory is reported in Ref. [47].

D. Full effective action

We are now ready to present the full effective Lagrangian of Eq. (3.1) in terms of the gauge fields and nematic order parameters. It is given by

$$\mathcal{L} = \frac{\bar{\rho} \epsilon^{ij}}{2(1 + 4\alpha \bar{\omega}_c^2)^2} M_i \partial_0 M_j - r\boldsymbol{M}^2 - \frac{\bar{\kappa}}{2} (\nabla M_i)^2 - \frac{u}{4} (\boldsymbol{M}^2)^2 + \frac{1}{4\pi} \epsilon^{\mu\nu\rho} \omega_\mu^Q \partial_\nu (\delta a_\rho + \delta A_\rho) - \frac{1}{2} \Pi_{\mu\nu}^0 (\delta a_\mu + \delta A_\mu)(\delta a_\nu + \delta A_\nu) + \frac{1}{8\pi} \varepsilon^{\mu\nu\lambda} \delta a_\mu \partial_\nu \delta a_\lambda + \frac{1}{24\pi} \varepsilon^{\mu\nu\lambda} \omega_\mu^Q \partial_\nu \omega_\rho^Q. \quad (3.18)$$

In the last line, we have added the gravitational Chern-Simons term of the induced spin connection of the nematic fields, where we use the results of Ref. [45].

IV. CONDENSATION OF THE GMP MODE AT THE NEMATIC PHASE TRANSITION

The FQH fluids have several types of collective excitations [36,55]. The Kohn mode is a cyclotron collective mode related to inter-Landau-level particle-hole excitations. If the system has Galilean invariance, the energy of the Kohn mode at zero momentum depends only on the bare mass of the electron and is insensitive to any other microscopic detail [32]. In a FQH fluid, the Kohn mode is not the lowest energy collective excitation and at finite wave vector $\boldsymbol{q}$ can (and does) decay to lower-energy modes. On the other hand, the lowest energy collective mode, the GMP mode, is stable. This mode is a quadrupolar intra-Landau-level fluctuation, and at long wavelengths, it can be regard as a fluctuating quadrupole with structure factor $\sim \boldsymbol{q}^4$ (instead of $\boldsymbol{q}^2$ as in the case of the Kohn mode) (see Refs. [55,62]). Therefore, for a FQH state with the quadrupolar interaction, we can expect that the interaction can substantially change the behavior of the GMP mode by mixing with the nematic fluctuations (which are also quadrupolar). In this section, we consider the behavior of the GMP collective excitation of the FQH state at and near the quantum phase transition between the isotropic state and the nematic state.

To get the spectrum of the collective excitations, we need the full polarization tensor of the electromagnetic response [52]. To this end, we first calculate the polarization tensor of the composite fermions,

$$\Pi^0_{\mu\nu} = -i\frac{1}{Z_F}\frac{\delta^2 Z_F}{\delta a_\mu \delta a_\nu}, \tag{4.1}$$

where Z_F is the partition function of the composite fermions. Because the composite fermion system is in an integer quantum Hall ground state, the poles of $\Pi_{\mu\nu}$ correspond to the Landau levels spaced by $\bar{\omega}_c$, the effective cyclotron frequency of the composite fermions [modified by the contributions of the extra terms of Eq. (2.14)].

Next, we compute the change in $\Pi_{\mu\nu}$ due to the effects of both the quadrupolar interaction and the density-density interaction and determine the full polarization tensor for the external electromagnetic field $K_{\mu\nu}$. The current and the nematic fields are defined in terms of the composite fermion Ψ by

$$j_i = \frac{\delta S}{\delta a_\mu}, \qquad N_i = \frac{\delta S}{\delta M_i} = \Psi^\dagger T_i \Psi, \tag{4.2}$$

where S is the full action of Eq. (2.13) supplemented by the additional term of Eq. (2.14).

We now compute the current-current correlators including the mixing with the nematic fields to lowest orders in the quadrupolar coupling F_2. To this end, we first calculate the polarization tensor $\Pi_{\mu\nu}$ to include the effects of the nematic fluctuations to lowest order in the quadrupolar interaction F_2. This calculation involves summing over all one-particle-reducible diagrams, i.e., an infinite series of bubble diagrams with two external gauge fields and arbitrary number of quadrupolar insertions connecting the bubbles pairwise. The result of this RPA-type computation is

$$\begin{aligned}\Pi_{ij}(\boldsymbol{q},\omega) &= \Pi^0_{ij} + 2F_2 m_e^2 \sum_{a,b}\langle j_i N_a\rangle\langle N_b j_j\rangle \\ &\quad + (2F_2 m_e^2)^2 \sum_{a,b}\langle j_i N_a\rangle\langle N_a N_b\rangle\langle N_b j_j\rangle + \cdots \\ &= \Pi^0_{ij} + \frac{2F_2 m_e^2 \sum_{a,b}\langle j_i N_a\rangle\langle N_b j_j\rangle}{1-(2F_2 m_e^2)\sum_{a,b}\langle N_a N_b\rangle}\end{aligned} \tag{4.3}$$

(where we have set $\kappa = 0$). Here, Π^0_{ij} is the polarization tensor for the statistical gauge field of the composite fermions with $\nu = 1$, $\langle N_a N_b\rangle$ is the correlator matrix of the nematic order parameters, and $\langle j_\mu N_a\rangle$ is the mixed correlator of a current and a nematic field, both of which were calculated in the previous section. To simplify the notation, in Eq. (4.3) we drop the explicit momentum and frequency dependence of the correlators.

Since we are interested in the low-energy and long-wavelength limit, we expand $\langle N_a N_b\rangle$ in the leading order for both the momentum $\boldsymbol{q}$ and the frequency ω and obtain

$$\begin{aligned}\langle N_1 N_1\rangle = \langle N_2 N_2\rangle &= 2i\frac{1}{Z_F}\frac{\delta Z_F}{\delta M_1 \delta M_1} \\ &= \frac{4\bar{\omega}_c{}^3}{\bar{l}_b{}^2\pi(\omega^2 - 4\bar{\omega}_c{}^2)(1+4\alpha\bar{\omega}_c^2)^2}, \\ \langle N_1 N_2\rangle = -\langle N_2 N_1\rangle &= 2i\frac{1}{Z_F}\frac{\delta Z_F}{\delta M_1 \delta M_2} \\ &= \frac{2\omega\bar{\omega}_c{}^2}{i\pi\bar{l}_b{}^2(\omega^2 - 4\bar{\omega}_c{}^2)(1+4\alpha\bar{\omega}_c^2)^2}.\end{aligned} \tag{4.4}$$

Using the results for the polarization tensor Π_{ij} of Eq. (4.3), we find the following effective Lagrangian for the gauge fields:

$$\begin{aligned}\mathcal{L}_a &= -\frac{1}{2}\Pi_{\mu\nu}(\delta a_\mu + \delta A_\mu)(\delta a_\nu + \delta A_\nu) \\ &\quad + \frac{1}{8\pi}\epsilon^{\mu\nu\rho}\delta a_\mu \partial_\nu \delta a_\rho - \int d^2x' \frac{V(x-x')}{32\pi^2}\delta b(x)\delta b(x').\end{aligned} \tag{4.5}$$

Integrating out the statistical gauge field δa_μ, we finally obtain the full response function $K_{\mu\nu}$ for the external electromagnetic fields,

$$
\begin{aligned}
K_{00} &= q^2 K_0,\\
K_{0i} &= \omega q_i K_0 + i\epsilon_{ik} q_k K_1,\\
K_{i0} &= \omega q_i K_0 - i\epsilon_{ik} q_k K_1,\\
K_{ij} &= \omega^2 \delta_{ij} K_0 - i\epsilon_{ij}\omega K_1 + (q^2\delta_{ij} - q_i q_j) K_2,\\
\mathcal{L}_A &= K_{\mu\nu}\delta A_\mu \delta A_\nu,
\end{aligned} \tag{4.6}
$$

where K_μ is given by

$$
\begin{aligned}
K_0 &= -\frac{\Pi_0}{16\pi^2 D},\\
K_1 &= \frac{1}{4\pi} + \frac{\Pi_1 + \frac{1}{4\pi}}{16\pi^2 D} + \frac{V(\boldsymbol{q})\Pi_0 \boldsymbol{q}^2}{64\pi^3 D},\\
K_2 &= \frac{\Pi_2}{16\pi^2 D} + \frac{V(\boldsymbol{q})(\omega^2\Pi_0^2 - \Pi_1^2)}{D} + \frac{V(q)\Pi_0\Pi_2 \boldsymbol{q}^2}{D},
\end{aligned} \tag{4.7}
$$

and D is

$$
D = \Pi_0^2\omega^2 - \left(\Pi_1 + \frac{1}{4\pi}\right)^2 + \Pi_0\left(\Pi_2 - \frac{V(\boldsymbol{q})}{16\pi^2}\right)\boldsymbol{q}^2. \tag{4.8}
$$

In the above expressions, Π_i are frequency and momentum-dependent functions whose explicit form can be found in Ref. [52].

The poles in $K_{\mu\nu}$ give the spectrum of the collective excitations of the FQHE of Ref. [52], generalized to include both the quadrupolar and the density-density interactions. At long wavelengths, this correlator has a pole at $3\bar{\omega}_c$ (with residue $\sim\boldsymbol{q}^2$), the cyclotron frequency of the electron (recall that $\bar{\omega}_c$ is the effective cyclotron frequency of the composite fermion). This pole is identified as the (cyclotron resonance) Kohn mode [32], slightly shifted here by the extra term we add to the kinetic energy [Eq. (2.14)].

On the other hand, we find that the attractive quadrupolar interaction pushes down to lower energies the lowest collective excitation, the Girvin-MacDonald-Platzman mode [36] (which has residue $\sim\boldsymbol{q}^4$). This mode has the dispersion

$$
\omega^2 = \omega_1^2 + \left(\alpha_{1,2}\bar{\omega}_c{}^3 - \frac{F_2 m_e^2 \bar{\omega}_c{}^3 \kappa}{\bar{l}_b{}^4}\right)(q\bar{l}_b)^2, \tag{4.9}
$$

where we set

$$
\begin{aligned}
\omega_1 &= \frac{4\tilde{F}_2}{\pi} + 2\bar{\omega}_c(1 + 4\alpha\bar{\omega}_c^2),\\
\alpha_1 &= \frac{\omega_1^2 - \bar{\omega}_c'^2}{\bar{\omega}_c'^2 - \bar{\omega}_c'\omega_1 + 2(\omega_1^2 - \bar{\omega}_c'^2)}\frac{1}{(c_1\omega_1 - c_2)t^2},\\
\alpha_2 &= -\frac{\omega_1^2 - \bar{\omega}_c'^2}{\bar{\omega}_c'^2 + \bar{\omega}_c'\omega_1 + 2(\omega_1^2 - \bar{\omega}_c'^2)}\frac{1}{(c_1\omega_1 + c_2)t^2}
\end{aligned} \tag{4.10}
$$

and where we use the notation

$$
\begin{aligned}
c_1 &= \frac{-\tilde{F}_2\bar{\omega}_d}{2(4\bar{\omega}_d^2 - \omega_1^2)}\left(1 - \frac{8\tilde{F}_2\bar{\omega}_d}{\pi(\omega_1^2 - \bar{\omega}_d^2)}\right),\\
c_2 &= \frac{\tilde{F}_2{}^2\bar{\omega}_d\omega_1^2}{(4\bar{\omega}_d^2 - \omega_1^2)^2},\\
\tilde{F}_2 &= \frac{F_2 m_e^2\bar{\omega}_c^2}{\bar{l}_b^2},\\
\bar{\omega}_c' &= \bar{\omega}_c(1 + \alpha\bar{\omega}_c^2),\\
\bar{\omega}_d &= \bar{\omega}_c(1 + 4\alpha\bar{\omega}_c^2),\\
t &= \frac{2}{1 + \alpha\bar{\omega}_c^2} - \frac{2}{2 + 8\alpha\bar{\omega}_c^2}.
\end{aligned} \tag{4.11}
$$

It is easy to check that at the nematic transition of Eq. (3.5), where the "nematic mass" $r \to 0$ at the critical value of the quadrupolar interaction F_2^c [given in Eq. (3.9)], the gap of the GMP mode vanishes, $\omega_1 \to 0$. It is also easy to see that near the phase transition, $\alpha_1 < 0$, $\alpha_2 < 0$, and $F_2\kappa < 0$. Now, provided $\alpha_{1,2} - (F_2 m_e^2\kappa/\bar{l}_b^4) > 0$ near and at the transition, the GMP mode will condense at zero momentum. This results in a nematic phase and the FQH fluid will spontaneously break the rotational symmetry. This condition can be achieved provided the range of the quadrupolar interaction, controlled by κ, is large enough. On the other hand, if $\alpha_{1,2} - (F_2 m_e^2\kappa/\bar{l}_b^4) < 0$, the GMP mode will condense at a finite momentum. This would result a crystalline phase in which electrons break spontaneously the translational and rotational symmetries of the two-dimensional plane. In both cases, the Kohn mode remains gapped at and near the transition, and thus, the liquid crystalline phases are incompressible electronic liquid states and have the quantized Hall response.

Early numerical results by Scarola, Park, and Jain [50] predicted that for certain types of interactions the FQH fluid would become unstable to a uniform exciton condensate associated with the GMP mode. Our results show that their exciton condensate is equivalent to a quantum phase transition to a nematic state.

V. GOLDSTONE MODE AND DISCLINATIONS IN NEMATIC PHASE

We now discuss the properties of the nematic phase. There are several particlelike excitations in the phase. First of all, the Kohn mode and the Laughlin quasiparticles remain massive. The only change is that their propagation is anisotropic. In addition to these excitations, there are two more excitations that are absent in the isotropic phase.

A. Goldstone modes

The nematic order parameter breaks the (continuous) rotational symmetry of the two-dimensional plane and, thus, there is an associated Goldstone mode and an amplitude mode (which is strongly mixed with the GMP mode). The spectrum of the nematic Goldstone mode can be obtained straightforwardly from $\mathcal{L}_M$. In the low-energy regime and deep enough in the nematic phase, $r = -|r| < 0$, we can consider the effective Lagrangian of the phase fluctuations (the Goldstone mode). Similarly to the effective Lagrangian in the Bogoliubov theory of superfluidity (or in the composite boson theory of the FQHE), in the nematic phase the amplitude fluctuations yield an effective Lagrangian for the Goldstone boson (the phase field θ) of the form

$$\mathcal{L}_M = \frac{1}{2}\frac{\rho_n}{v_n}(\partial_0\theta)^2 - \frac{1}{2}\rho_n v_n|\nabla\theta|^2 + \cdots, \tag{5.1}$$

where the nematic stiffness ρ_n and the velocity of the Goldstone modes v_n are given by

$$\rho_n = \frac{\sqrt{|r|\bar{\kappa}}}{2u(1+4\alpha\bar{\omega}_c^2)}, \qquad v_n = 4\sqrt{|r|\bar{\kappa}}(1+4\alpha\bar{\omega}_c^2), \tag{5.2}$$

where the parameters r, $\bar{\kappa}$, and u are given in Eq. (3.8). In the nematic phase, the Goldstone bosons are massless and have a linear dispersion

$$\omega(\boldsymbol{q}) = v_n|\boldsymbol{q}|. \tag{5.3}$$

On the other hand, the nematic Goldstone mode will be gapped if there is an explicit weak symmetry-breaking term. For instance, if the underlying lattice has tetragonal symmetry, the point group symmetry of the 2DEG is C_4. For the nematic order parameter, which is invariant under rotations by π, this term reduces the symmetry to a $\mathbb{Z}_2$ (Ising) symmetry. The appropriate symmetry-breaking term in the nematic Lagrangian has the form

$$\begin{aligned}\mathcal{L}_{\mathrm{SB}} &= -\gamma_1(M_1^2 - M_2^2) - \gamma_2 2M_1M_2 \\ &= -\gamma_1 M^2\cos 4\theta - \gamma_2 M^2 \sin 4\theta,\end{aligned} \tag{5.4}$$

where γ_1 and γ_2 are two coupling constants. In this case, the mass gap of the Goldstone mode is linearly proportional to the strength of these weak symmetry-breaking terms.

B. Disclinations

In $D = 2$ space dimensions, nematic order parameters have topological singularities (or defects) called disclinations [58]. Because of the presence of the Goldstone mode, the disclinations experience logarithmic interaction between them. In 2D, disclinations are half-vortices of the order parameter director field $\boldsymbol{M}$. When two disclinations are separated by a distance R, the energy cost for the configuration is

$$E = \int_{R>|\boldsymbol{x}|>l_0} d^2x\kappa\boldsymbol{M}^2\frac{1}{\boldsymbol{x}^2} = \kappa\boldsymbol{M}^2\ln(R/l_0), \tag{5.5}$$

where l_0 is an ultraviolet cutoff for the integral, which can be taken to be the correlation length of the nematic order parameter in the nematic phase. Hence, at zero temperature, disclinations and antidisclinations are bound in (neutral) pairs, but above a critical temperature T_c, they proliferate

We now show that in the nematic FQH state, the disclination carries electric charge due to the Wen-Zee coupling between the spin connection defined by the nematic fields ω_μ^Q and the gauge fields in the effective action $\mathcal{L}$ [Eq. (3.18)]. To investigate the charge accumulated at the disclination, we first integrate out the statistical gauge field δa_μ in the effective action to find a Wen-Zee term in the effective action,

$$\mathcal{L}_{\omega^Q,\delta A} = \frac{1}{12\pi}\epsilon^{\mu\nu\rho}\omega_\mu^Q\partial_\nu\delta A_\rho, \tag{5.6}$$

where A_μ is just an external weak electromagnetic probe. This term allows us to compute the electric charge of the disclination. Maciejko *et al.* [35] use the term "nematic gauge field" to refer to what we call the nematic spin connection ω_μ^Q.

Let us consider the case in which there exists a disclination (i.e., a π nematic vortex) centered at $\boldsymbol{x} = 0$. (Recall that since the nematic fields are directors, their orientation is defined mod π.) We can calculate the electric charge accumulated at the disclination. We find

$$\delta\rho(\boldsymbol{x}) = \frac{1}{12\pi}\left(-\frac{\boldsymbol{M}^2\delta(\boldsymbol{x})}{(1+4\alpha\bar{\omega}_c^2)^2} - t\frac{|\boldsymbol{M}|\cos 2\theta}{\boldsymbol{x}^2}\right). \tag{5.7}$$

The first term indicates that the spin connection of the nematic disclination acts as the gauge field of a single flux at the disclination core. Thus, a disclination of the nematic field serves as a particle source that changes the local charge density. The second term indicates that the charge density gets redistributed as a result of the nonzero quadrupole moment. In classical electrodynamics, the nonuniform charge density could give rise to an electron quadrupole moment $Q_{ij} = \int d^2x\rho(r)(2x_ix_j - \delta_{ij}|\boldsymbol{x}|^2)$ and vice versa. Since our nematic field couples to the stress tensor, a nematic order with a disclination configuration leads to a new charge density distribution, shown in the second term in Eq. (5.7). The charge of the disclination depends on the strength of the order parameter $|\boldsymbol{M}|$ and is not quantized. This implies that the disclinations will generally have irrational mutual statistics with quasiparticles and irrational statistics with other disclinations. Most of these results were anticipated on phenomenological and symmetry grounds in the work of Maciejko *et al.* [35].

VI. RESPONSE OF THE NEMATIC FQH FLUID TO CHANGES IN THE GEOMETRY

In this section, we explore the response of nematic FQH fluid to a long-wavelength change in the geometry of the underlying surface (i.e., the crystal) on which the 2DEG is defined, such as a shear distortion. Changes in the geometry can be described in terms of a background spatial metric g_{ij}

$$g_{ij} = \begin{pmatrix} 1-2e_1 & -2e_2 \\ -2e_2 & 1+2e_1 \end{pmatrix}, \tag{6.1}$$

which modifies the form of the action of Eq. (2.13) to the following expression:

$$\begin{aligned}\mathcal{S} = &\int d^2xdt\Big[\Psi^\dagger(x)D_0\Psi(x) - \frac{1}{2m_e}(\boldsymbol{D}\Psi(x))^\dagger\cdot(\boldsymbol{D}\Psi(x))\Big] - \int d^2xdtV\frac{\delta b(x)^2}{32\pi^2} + \int d^2xdt\frac{\varepsilon^{\mu\nu\lambda}}{8\pi}\delta a_\mu\partial_\nu\delta a_\lambda \\ &+\int d^2xdt\Big[\frac{1}{4F_2m_e^2}\boldsymbol{M}^2 + \frac{\kappa}{4F_2m_e^2}\sum_{i=1,2}\nabla M_i\cdot\nabla M_i + \frac{\kappa}{4F_2m_e^2}[2e_1(\partial_x^2-\partial_y^2)+2e_22\partial_x\partial_y]\boldsymbol{M}^2 \\ &+\frac{M_1+e_1}{m_e}\Psi^\dagger(D_x^2-D_y^2)\Psi + \frac{M_2+e_2}{m_e}\Psi^\dagger(D_xD_y+D_yD_x)\Psi - \alpha\Psi^\dagger\Big(-\frac{\boldsymbol{D}^2}{2m_e}-\frac{\bar\rho\pi}{m_e}\Big)^3\Psi + \frac{2\boldsymbol{e}\cdot\boldsymbol{M}}{m_e}\Psi^\dagger\boldsymbol{D}^2\Psi\Big],\end{aligned} \tag{6.2}$$

where the covariant derivative is now

$$D_\mu = \partial_\mu + i(A_\mu + a_\mu + \omega_\mu^b), \tag{6.3}$$

where ω^b is the spin connection of the background metric. Here, we use the result from our recent work [47] that upon attaching two Chern-Simons flux to the fermion, the composite particle effectively carries spin 1 and couples to the spin connection of the background geometry. Notice also that the quadrupolar interaction is modified by a change of the geometry. To simplify matters, we consider only a contact density-density coupling (parametrized by the interaction strength V).

We now make use of the results of the preceding sections (and of the results of Ref. [47]) to derive the effective Lagrangian for the nematic fields $\boldsymbol{M}$ at low energies and long distances. It is given by

$$\begin{aligned}\mathcal{L}_M = &\epsilon^{ij}\frac{\bar\rho}{2}\Big(\frac{M_i}{1+4\alpha\bar\omega_c^2}+e_i\Big)\partial_0\Big(\frac{M_j}{1+4\alpha\bar\omega_c^2}+e_j\Big) \\ &+\frac{\bar\omega_c}{2\bar\pi\bar l_b^2}|\boldsymbol{e}|^2 + \epsilon^{ij}\bar\rho e_i\partial_0 e_j \\ &+\frac{1}{12\pi}\epsilon^{\mu\nu\lambda}\Big(A_\mu+\omega_\mu^b+\frac{1}{2}\omega_\mu^m\Big)\partial_\nu\Big(A_\lambda+\omega_\lambda^b+\frac{1}{2}\omega_\lambda^m\Big) \\ &-r|\boldsymbol{M}|^2 - \frac{1}{48\pi}\epsilon^{\mu\nu\lambda}\omega_\mu^m\partial_\nu\omega_\lambda^m - \frac{u}{4}(\boldsymbol{M}^2)^2.\end{aligned} \tag{6.4}$$

Here, $\bar\rho$ is the average electron density, and we denote by ω^m the spin connection for the sum of the background and nematic spin connections,

$$\begin{aligned}\omega_0^m &= \epsilon^{ij}\Big(\frac{M_i}{1+4\alpha\bar\omega_c^2}+e_i\Big)\partial_0\Big(\frac{M_j}{1+4\alpha\bar\omega_c^2}+e_j\Big), \\ \omega_x^m &= \epsilon^{ij}\Big(\frac{M_i}{1+4\alpha\bar\omega_c^2}+e_i\Big)\partial_x\Big(\frac{M_j}{1+4\alpha\bar\omega_c^2}+e_j\Big) - [\partial_x(tM_2+e_2)-\partial_y(tM_1+e_1)], \\ \omega_y^m &= \epsilon^{ij}\Big(\frac{M_i}{1+4\alpha\bar\omega_c^2}+e_i\Big)\partial_y\Big(\frac{M_j}{1+4\alpha\bar\omega_c^2}+e_j\Big) + [\partial_x(tM_1+e_1)+\partial_y(tM_2+e_2)].\end{aligned} \tag{6.5}$$

To better illustrate the effect of nematic fluctuations, we rewrite the action in terms of a separate dependence on the background metric and on the metric defined by the nematic order parameter, and obtain

$$\begin{aligned}\mathcal{L}_M = &\epsilon^{ij}\frac{\bar\rho}{2}\Big(\frac{M_i}{1+4\alpha\bar\omega_c^2}+e_i\Big)\partial_0\Big(\frac{M_j}{1+4\alpha\bar\omega_c^2}+e_j\Big) - r|\boldsymbol{M}|^2 + \frac{\bar\omega_c}{2\pi\bar l_b^2}|\boldsymbol{e}|^2 + \epsilon^{ij}\bar\rho e_i\partial_0 e_j - \frac{u}{4}(\boldsymbol{M}^2)^2 \\ &+\frac{1}{12\pi}\epsilon^{\mu\nu\lambda}\Big(A_\mu+\frac{3}{2}\omega_\mu^b\Big)\partial_\nu\Big(A_\lambda+\frac{3}{2}\omega_\lambda^b\Big) - \frac{1}{12\pi}\epsilon^{\mu\nu\lambda}\omega_\mu^Q\partial_\nu A_\lambda - \frac{1}{48\pi}\epsilon^{\mu\nu\lambda}\omega_\mu^b\partial_\nu\omega_\lambda^b \\ &+\frac{1}{12\pi(1+4\alpha\bar\omega_c^2)}\epsilon^{\mu\nu\rho}(M_1\partial_\mu e_2 - M_2\partial_\mu e_1 + e_1\partial_\mu M_2 - e_2\partial_\mu M_1)\partial_\nu A_\rho \\ &+\frac{1}{12\pi(1+4\alpha\bar\omega_c^2)}\epsilon^{\mu\nu\rho}[M_1\partial_\mu e_2 - M_2\partial_\mu e_1 + e_1\partial_\mu M_2 - e_2\partial_\mu M_1 + (1+4\alpha\bar\omega_c^2)\omega_\mu^Q]\partial_\nu\omega_\rho^b.\end{aligned} \tag{6.6}$$

In the isotropic phase, the nematic field is massive, so it can be integrated out. This generates operators that are higher order in derivatives and irrelevant in the low-energy and long-distance regime of our theory. Finally, we obtain the following simple expression for the effective theory of background metric in the symmetric phase:

$$\mathcal{L}_M = \epsilon^{ij}\frac{3\bar{\rho}}{2}e_i\partial_0 e_j + \frac{1}{12\pi}\epsilon^{\mu\nu\lambda}\left(A_\mu + \frac{3}{2}\omega_\mu^b\right)\partial_\nu\left(A_\lambda + \frac{3}{2}\omega_\lambda^b\right) - \frac{1}{48\pi}\epsilon^{\mu\nu\lambda}\omega_\mu^b\partial_\nu\omega_\lambda^b, \tag{6.7}$$

which is consistent with our recent results [47].

In the nematic phase, the nematic order couples to the electrons (and the composite fermions) as an effect mass m^{ab} tensor. In the isotropic phase, the Hall viscosity [37,39,44,47,61,63,64], defined by $\eta^H = \eta_{xy}^{xx} = -\eta_{xy}^{yy}$, is isotropic, and for the $\nu = 1/3$ Laughlin FQH state it is found to be given by $\eta^H = (3/2)\bar{\rho}$, in agreement with earlier results. In the nematic phase, provided the nematic order is uniform in space, the Hall viscosity $\eta^H = (\eta_{xy}^{xx} - \eta_{xy}^{yy})/2 = 3\bar{\rho}/2$ remains the same as in the isotropic FQH fluid phase. However, since the system is spatially anisotropic, we can define the combination of the components of the viscosity $\eta^D = (\eta_{xy}^{xx} + \eta_{xy}^{yy})/2 \propto \bar{\boldsymbol{M}}\eta^H$, which indicates that there is a viscosity response for a mixed shear and a dilation deformation, which, however, is not universal. In particular, when the nematic order is uniform in space, geometric quantities such as the Hall viscosity, orbital spin, central charge remains unchanged at the universal value.

On the other hand, in the presence of a disclination in the nematic phase, from Eq. (6.6) we see that the Hall viscosity is modified. If $\boldsymbol{M}(\boldsymbol{x})$ is a configuration of the nematic order parameter with a disclination at $\boldsymbol{x} = \boldsymbol{x}_v$ with winding number n_v, the Hall viscosity of the fluid is now

$$\eta(\boldsymbol{x}) = \eta_0^H + \frac{1}{12\pi}\frac{|\boldsymbol{M}|^2}{(1+4\alpha\bar{\omega}_c^2)^2}n_v\delta(\boldsymbol{x}-\boldsymbol{x}_v) + 2t[(\partial_x^2 - \partial_y^2)M_1 + 4\partial_x\partial_y M_2]. \tag{6.8}$$

The first term is equal to the Hall viscosity of the isotropic phase. The second term shows the change of the Hall viscosity due to the nematic disclination. Here, n_v is the winding number of the disclination and $\boldsymbol{x}_v$ is the coordinate of the disclination core. The third term indicates the charge density redistribution results from the nematic order as a quadrupole moment, which affects the value of the Hall viscosity. However, the orbital spin and the gravitational Chern-Simons term remain the same in both phases.

In the recent work of Maciejko *et al.* [35], the authors considered an effective description of the nematic FQH state and the transition between an isotropic state and the anisotropic state. They used a composite boson theory and wrote down the symmetry-allowed terms in the effective Lagrangian. Interestingly, they identified the coupling between the nematic order parameter and the statistical gauge field by making an analogy to the case of the magnetization of the quantum Hall ferromagnet. Furthermore, they found that the critical theory has the dynamical scaling exponent $z = 2$ due to the Berry phase term for the nematic order parameter, which we also find here. However, in their description, the coefficient of the Berry phase term is the full Hall viscosity of the FQH fluid. Instead, here we show that the Berry phase term is not exactly equal to the Hall viscosity of the FQH fluid, but it is equal to the Hall viscosity of the integer quantum Hall mean-field state of the composite fermions.

VII. CONCLUSIONS

In this work, we study the nematic quantum phase transition inside a FQH state in a 2DEG with an attractive quadrupolar interaction between electrons. We use the Chern-Simons theory of composite fermions. Since the FQH state is gapped, a critical attractive quadrupolar coupling is needed for the system to develop a finite quadrupole density and to break rotational symmetry spontaneously. The quantum phase transition has dynamical exponent $z = 2$ and it is in the universality class of the quantum phase transition in the dilute Bose gas. The $z = 2$ quantum criticality is a consequence of a Berry phase term present in the effective action for the nematic fields related to (but not the same as) the Hall viscosity. We show that the coefficient of the Berry phase term is the Hall viscosity of the mean-field theory of the composite fermions in a background nematic field. The actual Hall viscosity of the FQH fluid (both in the isotropic and in the nematic phase) is obtained as a response to a shear distortion of the geometry in which the electrons move. Furthermore, we uncover the existence of a geometric Chern-Simons term between the nematic order parameters and the gauge fields. The term is of the same form as the Wen-Zee term, and the "spin connection" is interpreted in terms of the order parameter instead of the background metric. Then the flux of the "spin connection" is proportional to the disclination density in the nematic phase and, as a consequence, the disclination carries nonquantized gauge charge and statistics.

After the identification of the criticality and the phases, we investigate the excitations near the quantum phase transition. As the the nematic quantum phase transition is approached, the mass gap of the GMP mode of the FQH fluid is shown to vanish continuously. On the other hand, the Laughlin quasiparticles and the Kohn mode remain gapped at the transition and, thus, the Kohn theorem is not violated at or near the transition. Depending on the microscopic details of the interactions, we show that the GMP mode can close its gap either at finite or at zero momentum, giving rise to either a nematic or a crystal (or stripe) phase. Both liquid crystalline phases obtained through the softening of the GMP mode are incompressible electronic liquid crystals and, thus, are expected to have a fractionally quantized Hall response. It is notable that the mechanism of the isotropic-nematic transition described here is special for FQH states. For the integer quantum Hall states, the lowest excitation is the Kohn mode, inter-Landau-level excitation, and it cannot close its gap at zero momentum without a large amount of Landau-level mixing and a strong violation of Galilean invariance.

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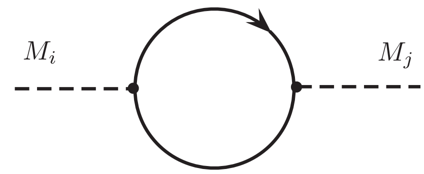

FIG. 1. Correlator of the nematic order parameters.

APPENDIX A: CALCULATION OF THE NEMATIC CORRELATORS

In this Appendix, we summarize the calculation of the correlators of the nematic order parameters that we use extensively in the main text.

First, we focus on the symmetric part of the correlators, which is the mass term for the order parameters. Here, for simplicity, we set the magnetic length $\bar{l}_b$ and electron bare mass m_e to be 1. At the end, we restore the magnetic length and mass in the expression by dimensional analysis. For the filling fraction $\nu = p/2p+1$, the correlator shown in Fig. 1 is

$$\begin{aligned}\langle N_1(\boldsymbol{r}_1,t_1)N_1(\boldsymbol{r}_2,t_2)\rangle &= 2i\frac{\delta Z}{\delta M_1\delta M_1} = -i\langle \mathcal{T}\Psi^\dagger(\boldsymbol{r}_1,t_1)(D_x^2-D_y^2)\Psi(\boldsymbol{r}_1,t_1)\Psi^\dagger(\boldsymbol{r}_2,t_2)(D_x^2-D_y^2)\Psi(\boldsymbol{r}_2,t_2)\rangle\\ &= -i\sum_{m>p}^{\infty}\sum_{l=0}^{p-1}\sum_{k_1,k_2}[e^{i(\omega_m-\omega_l)(t_2-t_1)}\Theta(t_1-t_2)\phi^\dagger_{l,k_1}(\boldsymbol{r}_1)(D_x^2-D_y^2)\phi_{m.k_2}(\boldsymbol{r}_1)\phi^\dagger_{m,k_2}(\boldsymbol{r}_2)(D_x^2-D_y^2)\phi_{l,k_1}(\boldsymbol{r}_2)\\ &\quad + e^{-i(\omega_m-\omega_l)(t_2-t_1)}\Theta(t_2-t_1)\phi^\dagger_{m,k_2}(\boldsymbol{r}_1)(D_x^2-D_y^2)\phi_{l.k_1}(\boldsymbol{r}_1)\phi^\dagger_{l,k_1}(\boldsymbol{r}_2)(D_x^2-D_y^2)\phi_{m,k_2}(\boldsymbol{r}_2)], \qquad \text{(A1)}\end{aligned}$$

in which $\omega_m = \bar{\omega}_c m + \alpha(m\bar{\omega}_c)^2, m \in \mathbb{Z}$ is the cyclotron energy of the composite fermion at the mth Landau level. For $\nu = 1/3$ filling, $p = 1$. Thus, the sum over m simply becomes the sum over $m > 0$ and we can set $l = 0$ at the end of the calculation. To proceed, we perform the Fourier transformation of the correlator. Here, we denote by

$$\phi_{l,k_1^x}(\boldsymbol{r}_1) = e^{ik_1^x x_1}\sqrt{\frac{1}{\sqrt{\pi}2^l l!}}e^{-(y_1+k_1^x)^2/2}H_l(y_1+k_1^x) \qquad \text{(A2)}$$

the Landau wave functions in the $A_y = 0$ gauge. Here $H_l(x)$ is the lth Hermite polynomial. In Fourier space, we find

$$\begin{aligned}\langle N_1N_1(\boldsymbol{q},\omega)\rangle = C_{lm}\sum_m\int dk_i d^2x_i dy_i\Bigg[&\frac{e^{i(x_2-x_1)(k_1^x-k_2^x+q_x)+iq_y(y_2-y_1)}}{\omega-(\omega_m-\omega_l)+i\epsilon}e^{(-1/2)[(y_i+k_1^x)^2+(y_i+k_2^x)^2]}\\ &\times H_l(y_1+k_1^x)(D_x^2-D_y^2)H_m(y_1+k_2^x)H_m(y_2+k_2^x)(D_x^2-D_y^2)H_l(y_2+k_1^x)\\ &-\frac{e^{i(x_2-x_1)(-k_1^x+k_2^x+q_x)+iq_y(y_2-y_1)}}{\omega+(\omega_m-\omega_l)-i\epsilon}e^{(-1/2)[(y_i+k_1^x)^2+(y_i+k_2^x)^2]}\\ &\times H_m(y_1+k_2^x)(D_x^2-D_y^2)H_l(y_1+k_1^x)H_l(y_2+k_1^x)(D_x^2-D_y^2)H_m(y_2+k_2^x)\Bigg], \qquad \text{(A3)}\end{aligned}$$

where

$$C_{lm} = \frac{1}{2^{l+m} l! m! 2\pi^2}. \tag{A4}$$

We now change the variables to

$$\begin{aligned} \tilde{u}_i &= y_i + \frac{k_1^x + k_2^x}{2}, \qquad \tilde{v} = \frac{k_1^x - k_2^x}{2}, \qquad u_1 = y_1 + \frac{k_1^x + k_2^x}{2} + iq_y/2, \\ u_2 &= y_2 + \frac{k_1^x + k_2^x}{2} - iq_y/2, \qquad v = \tilde{v} - iq_y/2, \qquad v^* = \tilde{v} + iq_y/2, \end{aligned} \tag{A5}$$

and we integrate out the x_i's to obtain

$$\begin{aligned} \langle N_1 N_1(\boldsymbol{q},\omega)\rangle = C_{lm} \sum_m \int du_i dv \\ \times \Bigg[\frac{\delta(\tilde{v} + q_x/2)}{\omega - (\omega_m - \omega_l) + i\epsilon} e^{-u_i^2 - 2vv^*} H_l(u_1 + v)(D_x^2 - D_y^2) H_m(u_1 - v^*) H_m(u_2 - v)(D_x^2 - D_y^2) H_l(u_2 + v^*) \\ - \frac{\delta(\tilde{v} - q_x/2)}{\omega + (\omega_m - \omega_l) - i\epsilon} e^{-u_i^2 - 2vv^*} H_m(u_1 - v^*)(D_x^2 - D_y^2) H_l(u_1 + v) H_l(u_2 + v^*)(D_x^2 - D_y^2) H_m(u_2 - v) \Bigg]. \end{aligned} \tag{A6}$$

Since $-iD_x H_m(u_1 - v^*) = (u_1 - v^*) H_m(u_1 - v^*)$ (we choose the Landau gauge $D_x = \partial_x + i\bar{b}y, D_y = \partial_y$), we have the following:

$$\begin{aligned} -iD_x H_n &= 1/2 H_{n+1} + n H_{N-1}, \\ -iD_y H_m(u_1 - v^*) &= i[-1/2 H_{m+1}(u_1 - v^*) + m H_{m-1}(u_1 - v^*)]. \end{aligned} \tag{A7}$$

In this way, the correlator can be simplified to the following expression:

$$\begin{aligned} \langle N_1 N_1(\boldsymbol{q},\omega)\rangle = C_{lm} \sum_m \int du_i dv \Bigg[\frac{\delta(\tilde{v} + q_x/2)}{\omega - (\omega_m - \omega_l) + i\epsilon} e^{-u_i^2 - 2vv^*} m^2 H_{l+1}(u_1 + v) H_{m-1}(u_1 - v^*) H_{m-1}(u_2 - v) H_{l-1}(u_2 + v^*) \\ - \frac{\delta(\tilde{v} - q_x/2)}{\omega + (\omega_m - \omega_l) - i\epsilon} e^{-u_i^2 - 2vv^*} m^2 H_{l+1}(u_1 + v) H_{m-1}(u_1 - v^*) H_{m-1}(u_2 - v) H_{l-1}(u_2 + v^*) \Bigg]. \end{aligned} \tag{A8}$$

We can use the expression for the inner product of the two Hermite polynomials, which is written in terms of the Laguerre polynomials,

$$\begin{aligned} \int du_1 e^{-u_i^2} H_l(u_1 + v) H_m(u_1 - v^*) \\ = 2^m \sqrt{\pi} l! (v^*)^{m-l} L_l^{m-l}(-2vv^*), \end{aligned} \tag{A9}$$

if l is not larger than m. Here, v is related to q_x, q_y after we integrate over $\tilde{v}$. $L_l^{m-l}(-2vv^*)$ is the polynomial of q^2 whose leading order is always a constant piece. We can always express the result by expanding it in terms of ω and q by order. The leading order in q and ω of $\langle N_1 N_1(q,\omega)\rangle$ (coming from $l = 0, m = 2$) includes a constant piece:

$$\begin{aligned} \langle N_1 N_1(q,\omega)\rangle &= \frac{1}{\pi(\omega - 2\bar{\omega}_c)(1 + 4\alpha\bar{\omega}_c^2)} \\ &\quad - \frac{1}{\pi(\omega + 2\bar{\omega}_c)(1 + 4\alpha\bar{\omega}_c^2)} + O(q^2) \\ &= \frac{4\bar{\omega}_c}{\pi(\omega^2 - 4\bar{\omega}_c^2)(1 + 4\alpha\bar{\omega}_c^2)} + O(q^2). \end{aligned} \tag{A10}$$

By dimensional analysis, we need to multiply $\bar{\omega}_c^2/\bar{l}_b^2$ to the correlator to restore the coefficients by setting the magnetic length to be unity in the calculation:

$$\begin{aligned}\langle N_1 N_1(q,\omega)\rangle &= \frac{4\bar{\omega}_c^3}{\bar{l}_b^2\pi(\omega^2-4\bar{\omega}_c^2)(1+4\alpha\bar{\omega}_c^2)} + O(q^2)\\ &= -\frac{\bar{\omega}_c}{\bar{l}_b^2\pi(1+4\alpha\bar{\omega}_c^2)} + O(q^2) + O(\omega^2).\end{aligned} \tag{A11}$$

The first term contributes to the mass term of the nematic order parameters.

The antisymmetric part, proportional to $M_1\partial_0 M_2$, of the correlator can be calculated in the same way. The full result for the correlator is

$$\begin{aligned}\langle N_1 N_2(\boldsymbol{q},\omega)\rangle &= C_{lm}\sum_m \int du_i dv\\ &\quad\times\bigg[\frac{\delta(\tilde{v}+q_x/2)}{\omega-(\omega_m-\omega_l)+i\epsilon}e^{-u_i^2-2vv^*}H_l(u_1+v)(D_x^2-D_y^2)H_m(u_1-v^*)H_m(u_2-v)(D_x^2-D_y^2)H_l(u_2+v^*)\\ &\quad-\frac{\delta(\tilde{v}-q_x/2)}{\omega+(\omega_m-\omega_l)-i\epsilon}e^{-u_i^2-2vv^*}H_m(u_1-v^*)(D_x^2-D_y^2)H_l(u_1+v)H_l(u_2+v^*)(D_x^2-D_y^2)H_m(u_2-v)\bigg]\\ &= C_{lm}\sum_m\int du_i dv\bigg[\frac{\delta(\tilde{v}+q_x/2)}{\omega-(\omega_m-\omega_l)+i\epsilon}e^{-u_i^2-2vv^*}\frac{m^2}{i}H_{l+1}(u_1+v)H_{m-1}(u_1-v^*)H_{m-1}(u_2-v)H_{l-1}(u_2+v^*)\\ &\quad+\frac{\delta(\tilde{v}-q_x/2)}{\omega+(\omega_m-\omega_l)-i\epsilon}e^{-u_i^2-2vv^*}\frac{m^2}{i}H_{l+1}(u_1+v)H_{m-1}(u_1-v^*)H_{m-1}(u_2-v)H_{l-1}(u_2+v^*)\bigg].\end{aligned} \tag{A12}$$

The leading-order behavior in q and ω comes from the term with $l=0$ and $m=2$. Within this approximation, the leading low-frequency and low-momenta behavior of the correlator is

$$\langle N_1 N_2(q,\omega)\rangle = \frac{2\omega}{i\pi(\omega^2-4\bar{\omega}_c^2)(1+4\alpha\bar{\omega}_c^2)^2} + O(q^2). \tag{A13}$$

Again, we now multiply the factor $\bar{\omega}_c^2/\bar{l}_b^2$ to restore the units properly to find the result:

$$\langle N_1 N_2(q,\omega)\rangle = i\frac{\omega}{2\bar{l}_b^2\pi(1+4\alpha\bar{\omega}_c^2)^2} + O(q^2) + O(\omega^2). \tag{A14}$$

The coefficient of the leading term is the Hall viscosity of the integer quantum Hall state.

APPENDIX B: CALCULATION OF THE MIXED CORRELATORS OF NEMATIC AND GAUGE FIELDS

The nematic-gauge coupling term could be obtained in a similar way:

$$\begin{aligned}\langle N_1(\boldsymbol{r_1},t_1)j_0(\boldsymbol{r_2},t_2)\rangle &= -i\langle\mathcal{T}\Psi^\dagger(\boldsymbol{r}_1,t_1)(D_x^2-D_y^2)\Psi(\boldsymbol{r}_1,t_1)\Psi^\dagger(\boldsymbol{r}_2,t_2)\Psi(\boldsymbol{r}_2,t_2)\rangle\\ &= C_{lm}\sum_m\int du_i dv\\ &\quad\times\bigg[\frac{\delta(\tilde{v}+q_x/2)}{\omega-(\omega_m-\omega_l)+i\epsilon}e^{-u_i^2-2vv^*}H_l(u_1+v)(D_x^2-D_y^2)H_m(u_1-v^*)H_m(u_2-v)H_l(u_2+v^*)\\ &\quad-\frac{\delta(\tilde{v}-q_x/2)}{\omega+(\omega_m-\omega_l)-i\epsilon}e^{-u_i^2-2vv^*}H_m(u_1-v^*)(D_x^2-D_y^2)H_l(u_1+v)H_l(u_2+v^*)H_m(u_2-v)\bigg]\\ &= C_{lm}\sum_m\int du_i dv\bigg[\frac{\delta(\tilde{v}+q_x/2)}{\omega-(\omega_m-\omega_l)+i\epsilon}e^{-u_i^2-2vv^*}mH_{l+1}(u_1+v)H_{m-1}(u_1-v^*)H_m(u_2-v)H_l(u_2+v^*)\\ &\quad-\frac{\delta(\tilde{v}-q_x/2)}{\omega+(\omega_m-\omega_l)-i\epsilon}e^{-u_i^2-2vv^*}mH_{l+1}(u_1+v)H_{m-1}(u_1-v^*)H_m(u_2-v)H_l(u_2+v^*)\bigg].\end{aligned} \tag{B1}$$

The leading order term is $\propto p^2$, which comes from the contribution of $(l=0, m=2), (l=0, m=1)$, and yields the expression

$$\langle N_1(p)j_0(-p)\rangle = \frac{1}{2\pi}\left(\frac{2}{1+\alpha\bar{\omega}_c^2} - \frac{2}{2+8\alpha\bar{\omega}_c^2}\right)(p_x^2 - p_y^2). \tag{B2}$$

Following the above calculation, we can obtain other linear coupling terms between the nematic field and the gauge field:

$$\begin{aligned}
\langle N_2(p)j_0(-p)\rangle &= \frac{1}{2\pi}\left(\frac{2}{1+\alpha\bar{\omega}_c^2} - \frac{2}{2+8\alpha\bar{\omega}_c^2}\right)2p_xp_y,\\
\langle N_1(p)j_x(-p)\rangle &= \frac{1}{2\pi}\left(\frac{2}{1+\alpha\bar{\omega}_c^2} - \frac{2}{2+8\alpha\bar{\omega}_c^2}\right)\omega p_x,\\
\langle N_1(p)j_y(-p)\rangle &= \frac{1}{2\pi}\left(\frac{2}{1+\alpha\bar{\omega}_c^2} - \frac{2}{2+8\alpha\bar{\omega}_c^2}\right)(-\omega p_y),\\
\langle N_2(p)j_x(-p)\rangle &= \frac{1}{2\pi}\left(\frac{2}{1+\alpha\bar{\omega}_c^2} - \frac{2}{2+8\alpha\bar{\omega}_c^2}\right)\omega p_y,\\
\langle N_2(p)j_y(-p)\rangle &= \frac{1}{2\pi}\left(\frac{2}{1+\alpha\bar{\omega}_c^2} - \frac{2}{2+8\alpha\bar{\omega}_c^2}\right)\omega p_x.
\end{aligned} \tag{B3}$$

These terms contribute partly to the Wen-Zee coupling. For a complete expression of the Wen-Zee term, we also need to evaluate the correlator between two nematic fields and one gauge field (see the Feynman diagrams of Fig. 2):

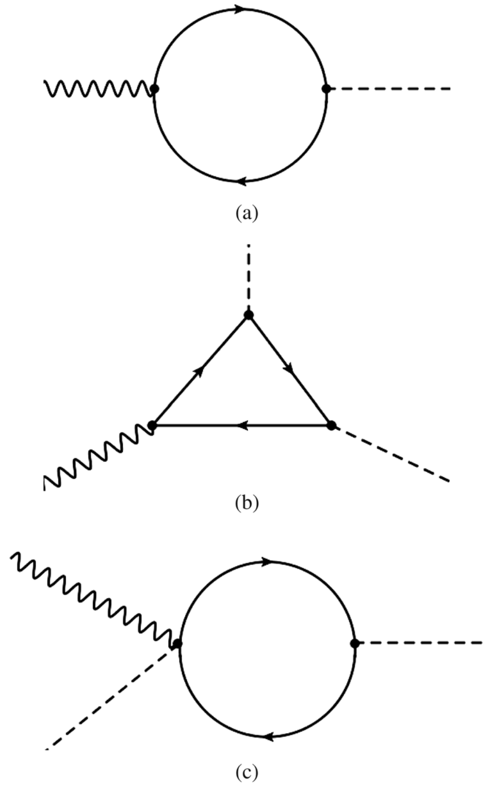

FIG. 2. Diagrams contributing to the Wen-Zee term. Here, the wiggly line represents the gauge field a_μ and the dotted line represents the nematic field M_i. The thick line represents the composite fermion propagator.

$$\langle N_1(\boldsymbol{r_1},t_1)N_2(\boldsymbol{r_2},t_2)j_0(\boldsymbol{r_3},t_3)\rangle = -\langle \mathcal{T}\Psi^\dagger(\boldsymbol{r}_1,t_1)(D_x^2-D_y^2)\Psi(\boldsymbol{r}_1,t_1)\Psi^\dagger(\boldsymbol{r}_2,t_2)(D_xD_y+D_yD_x)\Psi(\boldsymbol{r}_2,t_2)\Psi^\dagger(\boldsymbol{r}_3,t_3)\Psi(\boldsymbol{r}_3,t_3)\rangle, \tag{B4}$$

$$\langle N_1N_2j_0\rangle(q,p) = \langle N_1(\boldsymbol{r_1},t_1)N_2(\boldsymbol{r_2},t_2)j_0(\boldsymbol{r_3},t_3)\rangle \exp[-i\omega(t_2-t_1)-i\omega_0(t_3-t_2)]\exp[iq(r_2-r_1)+ip(r_3-r_2)]. \tag{B5}$$

By redefining the variables,

$$\begin{aligned}
u_1 &= y_1 + \frac{k_1^x+k_2^x}{2} + iq^y/2, \qquad v = \frac{k_1^x-k_2^x}{2} - iq^y/2, \qquad \tilde{v} = \frac{k_1^x-k_2^x}{2},\\
u_2 &= y_2 + \frac{k_2^x+k_3^x}{2} - i(q^y-p^y)/2, \qquad v_0 = \frac{k_2^x-k_3^x}{2} + i(q^y-p^y)/2, \qquad \tilde{v}_0 = \frac{k_2^x-k_3^x}{2},\\
u_3 &= y_3 + \frac{k_1^x+k_3^x}{2} - ip^y/2, \qquad v+v_0 = \frac{k_1^x-k_3^x}{2} + ip^y/2, \qquad \tilde{v}+\tilde{v}_0 = \frac{k_1^x-k_3^x}{2},
\end{aligned} \tag{B6}$$

we can write the time-ordered correlator (for $t_1 > t_2 > t_3$) as

$$\langle N_1N_2j_0\rangle(q,p) = \exp[-u_i^2 - vv^* - v_0v_0^* - (v_0+v)(v^*+v_0^*)]\exp(-iq/2\wedge p/2)$$
$$C_{lmn}\sum_{m,n}\int du_i dv dv_0 \frac{\delta(\tilde{v}+q^x/2)\delta(\tilde{v}_0+p^x/2)}{[\omega-(\omega_m-\omega_l)+i\epsilon][\omega_0-(\omega_n-\omega_l)+i\epsilon]}$$
$$H_l(u_1+v)(D_x^2-D_y^2)H_m(u_1-v^*)H_m(u_2+v_0)(D_xD_y+D_yD_x)H_n(u_2-v_0^*)H_n(u_3-v^*-v_0^*)H_l(u_3+v+v_0)$$
$$= \exp[-u_i^2 - vv^* - v_0v_0^* - (v_0+v)(v^*+v_0^*)\exp(-iq/2\wedge p/2)$$
$$C_{lmn}\sum_{m,n}\int du_i dv dv_0 \frac{\delta(\tilde{v}+q^x/2)\delta(\tilde{v}_0+p^x/2)}{[\omega-(\omega_m-\omega_l)+i\epsilon][\omega_0-(\omega_n-\omega_l)+i\epsilon]}$$
$$[H_{l+1}(u_1+v)mH_{m-1}(u_1-v^*)H_{m+1}(u_2+v_0)(in)H_{n-1}(u_2-v_0^*)H_n(u_3-v^*-v_0^*)H_l(u_3+v+v_0)$$
$$+ H_{l+1}(u_1+v)mH_{m-1}(u_1-v^*)H_{m-1}(u_2+v_0)(-im)H_{n+1}(u_2-v_0^*)H_n(u_3-v^*-v_0^*)H_l(u_3+v+v_0)]. \tag{B7}$$

For this three-point correlator, there always exists an antisymmetric phase factor $\exp(-iq/2\wedge p/2)$ (known as the Moyal phase, see, e.g., Ref. [65]), which is responsible for the Wen-Zee response.

The leading-order contribution for the three-point time-ordered correlator (at $t_1 > t_2 > t_3$) comes from the choice of $[l=0, n=0, m=2]$; thus,

$$\langle N_1N_2j_0\rangle(q,p)|_{t_1>t_2>t_3} = \frac{-q_xp_y+q_yp_x}{4\pi[\omega-(\omega_2-\omega_0)](\omega_0)} + \cdots. \tag{B8}$$

In a similar way, we also have

$$\langle N_2N_1j_0\rangle(q,p)|_{t_1>t_2>t_3} = \exp[-u_i^2 - vv^* - v_0v_0^* - (v_0+v)(v^*+v_0^*)]\exp(-iq/2\wedge p/2)$$
$$C_{lmn}\sum_{m,n}\int du_i dv dv_0 \frac{\delta(\tilde{v}+q^x/2)\delta(\tilde{v}_0+p^x/2)}{[\omega-(\omega_m-\omega_l)+i\epsilon][\omega_0-(\omega_n-\omega_l)+i\epsilon]}$$
$$H_l(u_1+v)(D_xD_y+D_yD_x)H_m(u_1-v^*)H_m(u_2+v_0)(D_x^2-D_y^2)H_n(u_2-v_0^*)H_n(u_3-v^*-v_0^*)H_l(u_3+v+v_0)$$
$$= \exp[-u_i^2 - vv^* - v_0v_0^* - (v_0+v)(v^*+v_0^*)]\exp(-iq/2\wedge p/2)$$
$$C_{lmn}\sum_{m,n}\int du_i dv dv_0 \frac{\delta(\tilde{v}+q^x/2)\delta(\tilde{v}_0+p^x/2)}{[\omega-(\omega_m-\omega_l)+i\epsilon][\omega_0-(\omega_n-\omega_l)+i\epsilon]}$$
$$[H_{l+1}(u_1+v)(im)H_{m-1}(u_1-v^*)H_{m+1}(u_2+v_0)(n)H_{n-1}(u_2-v_0^*)H_n(u_3-v^*-v_0^*)H_l(u_3+v+v_0)$$
$$+ H_{l+1}(u_1+v)(im)H_{m-1}(u_1-v^*)H_{m-1}(u_2+v_0)(m)H_{n+1}(u_2-v_0^*)H_n(u_3-v^*-v_0^*)H_l(u_3+v+v_0)]. \tag{B9}$$

In leading order,

$$\langle N_2N_1j_0\rangle(q,q_0)|_{t_1>t_2>t_3} = \frac{q_xp_y-q_yp_x}{4\pi[\omega-(\omega_2-\omega_0)](\omega_0)} + \cdots. \tag{B10}$$

The other time-ordered correlator can be obtained in a similar way, which finally gives Wen-Zee coupling. Finally, we have

$$\langle N_iN_jj_\mu\rangle(q,p) = \frac{\epsilon^{ij}\epsilon^{\lambda\nu\mu}p_\lambda q_\nu}{(1+4\alpha\bar{\omega}_c^2)^2 4\pi}. \tag{B11}$$

APPENDIX C: PRROF OF GAUGE INVARIANCE AT THE RPA LEVEL

To calculate the collective excitation of the nematic FQH state, we first treat the quadrupolar interaction perturbatively in the RPA level and then integrate out the gauge fluctuation. During the RPA procedure, we keep only the reducible diagrams for the infinite geometric series. Here, we proof that the polarization tensor of such RPA level correction is gauge invariant. The polarization at RPA level has the form

$$\Pi^{\mathrm{RPA}}_{ij} = \Pi^0_{ij} + (2F_2 m_e^2)\sum_{a,b}\langle j_i N_a\rangle\langle N_b j_j\rangle + (2F_2 m_e^2)^2\sum_{a,b}\langle j_i N_a\rangle\langle N_a N_b\rangle\langle N_b j_j\rangle + \cdots = \Pi^0_{ij} + \frac{(2F_2 m_e^2)\langle j_i N_a\rangle\langle N_b j_j\rangle}{1-(2F_2 m_e^2)\langle N_a N_b\rangle}. \tag{C1}$$

To prove its gauge invariance, we need to prove only

$$p_\mu \Pi^{\mathrm{RPA}}_{\mu\nu}(\omega, p) = 0. \tag{C2}$$

Or if we write it in real space,

$$\partial_\mu \Pi^{\mathrm{RPA}}_{\mu\nu}(x, y) = 0. \tag{C3}$$

It is obvious that $p_\mu \Pi^0_{\mu\nu} = 0$. Thus, to prove gauge invariance we only need to prove that $p_\mu\langle j_\mu N_a\rangle = 0$:

$$\begin{aligned}
2\partial_{r_i^1}\langle j_i(r_1)N_1(r_2)\rangle &= i\partial_{r_i^1}\langle D_i^\dagger\Psi^\dagger(r_1)\Psi(r_1)\Psi^\dagger(r_2)(D_x^2-D_y^2)\Psi(r_2)\rangle - i\partial_{r_i^1}\langle\Psi^\dagger(r_1)D_i\Psi(r_1)\Psi^\dagger(r_2)(D_x^2-D_y^2)\Psi(r_2)\rangle\\
&= i\langle\partial_{r_i^1}D_i^\dagger\Psi^\dagger(r_1)\Psi(r_1)\Psi^\dagger(r_2)(D_x^2-D_y^2)\Psi(r_2)\rangle + i\langle D_i^\dagger\Psi^\dagger(r_1)\partial_{r_i^1}\Psi(r_1)\Psi^\dagger(r_2)(D_x^2-D_y^2)\Psi(r_2)\rangle\\
&\quad - i\langle\partial_{r_i^1}\Psi^\dagger(r_1)D_i\Psi(r_1)\Psi^\dagger(r_2)(D_x^2-D_y^2)\Psi(r_2)\rangle - i\langle\Psi^\dagger(r_1)\partial_{r_i^1}D_i\Psi(r_1)\Psi^\dagger(r_2)(D_x^2-D_y^2)\Psi(r_2)\rangle\\
&= i\langle D_i^\dagger D_i^\dagger\Psi^\dagger(r_1)\Psi(r_1)\Psi^\dagger(r_2)(D_x^2-D_y^2)\Psi(r_2)\rangle + i\langle D_i^\dagger\Psi^\dagger(r_1)D_i\Psi(r_1)\Psi^\dagger(r_2)(D_x^2-D_y^2)\Psi(r_2)\rangle\\
&\quad - i\langle D_i^\dagger\Psi^\dagger(r_1)D_i\Psi(r_1)\Psi^\dagger(r_2)(D_x^2-D_y^2)\Psi(r_2)\rangle - i\langle\Psi^\dagger(r_1)D_iD_i\Psi(r_1)\Psi^\dagger(r_2)(D_x^2-D_y^2)\Psi(r_2)\rangle\\
&= i\langle D_i^\dagger D_i^\dagger\Psi^\dagger(r_1)\Psi(r_1)\Psi^\dagger(r_2)(D_x^2-D_y^2)\Psi(r_2)\rangle - i\langle\Psi^\dagger(r_1)D_iD_i\Psi(r_1)\Psi^\dagger(r_2)(D_x^2-D_y^2)\Psi(r_2)\rangle\\
\partial_{r_0^1}\langle j_0(r_1)N_1(r_2)\rangle &= -\langle\partial_{r_0^1}\Psi^\dagger(r_1)\Psi(r_1)\Psi^\dagger(r_2)(D_x^2-D_y^2)\Psi(r_2)\rangle - \langle\Psi^\dagger(r_1)\partial_{r_0^1}\Psi(r_1)\Psi^\dagger(r_2)(D_x^2-D_y^2)\Psi(r_2)\rangle.
\end{aligned} \tag{C4}$$

In all, we have

$$\begin{aligned}
-i\partial_{r_\mu^1}\langle j_\mu(r_1)N_1(r_2)\rangle &= \left\langle\Psi(r_1)\Psi^\dagger(r_2)(D_x^2-D_y^2)_{r_2}\Psi(r_2)\Psi^\dagger(r_1)\left(-i\partial_0+\frac{\boldsymbol{D}^{\dagger 2}}{2}+\mu\right)_{r_1}\right\rangle\\
&\quad -\left\langle\left(i\partial_0+\frac{\boldsymbol{D}^2}{2}+\mu\right)_{r_1}\Psi(r_1)\Psi^\dagger(r_2)(D_x^2-D_y^2)_{r_2}\Psi(r_2)\Psi^\dagger(r_1)\right\rangle\\
&= \left\langle G(r_1,r_2)(D_x^2-D_y^2)_{r_2}G(r_2,r_1)\left(-i\partial_0+\frac{\boldsymbol{D}^{\dagger 2}}{2}+\mu\right)_{r_1}\right\rangle - \left\langle\left(i\partial_0+\frac{\boldsymbol{D}^2}{2}+\mu\right)_{r_1}G(r_1,r_2)(D_x^2-D_y^2)_{r_2}G(r_2,r_1)\right\rangle.
\end{aligned} \tag{C5}$$

Recall that the Green function has the property

$$\left(i\partial_0+\frac{\boldsymbol{D}^2}{2}+\mu\right)_{r_1}G(r_1,r_2) = \left(i\partial_0+\frac{\boldsymbol{D}^2}{2}+\mu\right)_{r_1}\langle\Psi(r_1)\Psi^\dagger(r_2)\rangle = \delta(r_1-r_2), \tag{C6}$$

and similarly for the adjoint. Thus, we have

$$-i\partial_{r_\mu^1}\langle j_\mu(r_1)N_1(r_2)\rangle = 0. \tag{C7}$$

Thus, we have shown that the polarization tensor at the RPA level is gauge invariant.

APPENDIX D: NEMATIC COLLECTIVE EXCITATIONS

To obtain the collective excitations of the nematic FQH state, we calculate the polarization tensor K_{ij} for the external electromagnetic gauge field. The poles in K_{ij} give the spectrum of the excitations:

$$\begin{aligned}
K_{00} &= q^2 K_0,\\
K_{0i} &= \omega q_i K_0 + i\epsilon_{ik}q_k K_1,\\
K_{i0} &= \omega q_i K_0 - i\epsilon_{ik}q_k K_1,\\
K_{ij} &= \omega^2\delta_{ij}K_0 - i\epsilon_{ij}\omega K_1 + (q^2\delta_{ij}-q_iq_j)K_2,\\
K_0 &= -\frac{\Pi_0}{16\pi^2 D},\\
K_1 &= \frac{1}{4\pi} + \frac{\Pi_1+\frac{1}{4\pi}}{16\pi^2 D} + \frac{V(q)\Pi_0 q^2}{64\pi^3 D},\\
K_2 &= \frac{\Pi_2}{16\pi^2 D} + \frac{V(q)(\omega^2\Pi_0^2-\Pi_1^2)}{D} + \frac{V(q)\Pi_0\Pi_2 q^2}{D},\\
D &= \Pi_0^2\omega^2 - (\Pi_1+\theta)^2 + \Pi_0\left(\Pi_2 - \frac{V(q)}{16\pi^2}\right)q^2,\\
\theta &= \frac{1}{4\pi}.
\end{aligned} \tag{D1}$$

We solve $D(q,\omega) = 0$ to find the poles of $K_{\mu\nu}$:

$$\Pi_0^2\omega^2 - (\Pi_1 + \theta)^2 + \Pi_0\left(\Pi_2 - \frac{V(q)}{16\pi^2}\right)q^2 = 0. \qquad \text{(D2)}$$

As the left-hand part of Eq. (D2) involves a sum of infinite numbers of polynomials, it is impossible to solve it exactly. What we can try to do instead is to assume dispersion $\omega = \omega_1 + \alpha_n q^{2n}$ and find the solution asymptotically near zero momentum. We keep only the lowest terms in q and ω. As we can see, the first two terms have leading order $O(1)$, while the last term has leading order $O(q^2)$. To solve this equation, we have to subtract the constant piece from the first two terms and set them as zero:

$$\left|\frac{\bar{\omega}_c{}'\omega_1}{\omega_1^2 - \bar{\omega}_c{}'^2} + \frac{c_1 t^2 \omega_1}{\alpha}\right| = \left|\frac{\bar{\omega}_c{}'^2}{\omega_1^2 - \bar{\omega}_c{}'^2} + \frac{t^2 c_2}{\alpha} + 2\pi\theta\right|. \qquad \text{(D3)}$$

This gives us the solution for the lowest excitation in the nematic FQH state, which is the GMP mode.

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