

# Observation of Unconventional Charge Density Wave without Acoustic Phonon Anomaly in Kagome Superconductors $AV_3Sb_5$ ( $A = \text{Rb}, \text{Cs}$ )

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The combination of nontrivial band topology and symmetry-breaking phases gives rise to novel quantum states and phenomena such as topological superconductivity, quantum anomalous Hall effect, and axion electrodynamics. Evidence of intertwined charge density wave (CDW) and superconducting order parameters has recently been observed in a novel kagome material  $AV_3Sb_5$  ( $A = \text{K}, \text{Rb}, \text{Cs}$ ) that features a  $\mathbb{Z}_2$  topological invariant in the electronic structure. However, the origin of the CDW and its intricate interplay with the topological state has yet to be determined. Here, using hard-x-ray scattering, we demonstrate a three-dimensional CDW with  $2 \times 2 \times 2$  superstructure in  $(\text{Rb}, \text{Cs})V_3Sb_5$ . Unexpectedly, we find that the CDW fails to induce acoustic phonon anomalies at the CDW wave vector but yields a novel Raman mode that quickly damps into a broad continuum below the CDW transition temperature. Our observations exclude strong electron-phonon-coupling-driven CDW in  $AV_3Sb_5$  and support an unconventional CDW that was proposed in the kagome lattice at van Hove filling.

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## I. INTRODUCTION

The charge density wave (CDW), a translational symmetry-breaking electronic fluid, plays a crucial role in unconventional superconductors and intertwined electronic orders [1–5]. While CDWs have been isolated from topological excitations, recently experimental evidence of a topological CDW with chiral flux has been observed in a new kagome metal  $AV_3Sb_5$  ( $A = \text{K}, \text{Rb}, \text{Cs}$ ) [6], whose crystal structure and 3D Brillouin zone are shown in Figs. 1(a) and 1(b), respectively. Density-functional-theory (DFT) calculations of the electronic structure find a  $\mathbb{Z}_2$  topological invariant and an electronic filling close to van Hove singularity (saddle point) near the  $M$  point [7,8]. Because of the frustrated lattice structure and sublattice

interference effect, unconventional  $p$ -wave CDW and chiral flux phase have been theoretically predicted near the van Hove filling [4,9–11]. Moreover, the CDW in  $AV_3Sb_5$  may intertwine with the superconductivity at low temperature and give rise to the possible roton pair-density wave and Majorana zero mode [12,13]. Despite the intimate correlations between the CDW, superconductivity, and topological band structure [6–8,11–18], the nature of the CDW, in particular, its interplay with the lattice degree of freedom, remains largely unexplored. This knowledge gap hinders the understanding of unconventional Fermi-surface instabilities in kagome metals near van Hove filling.

Theoretically, the CDW mechanisms are broadly separated into two categories: weak-coupling scenarios based on Fermi-surface instabilities [19–21] and strong-coupling theories derived from local electron-electron or electron-phonon interactions [22–25]. As we show in Figs. 1(c) and 1(d), due to the finite electron-phonon coupling (EPC), the formation of CDWs distorts the underlying lattice and typically results in acoustic phonon anomalies. For a weak-coupling mechanism, the phonon softening is sharply confined near the CDW wave vector  $\mathbf{Q}_{\text{CDW}}$  and is known as the Kohn anomaly [21] [illustrated in Fig. 1(e)]. In

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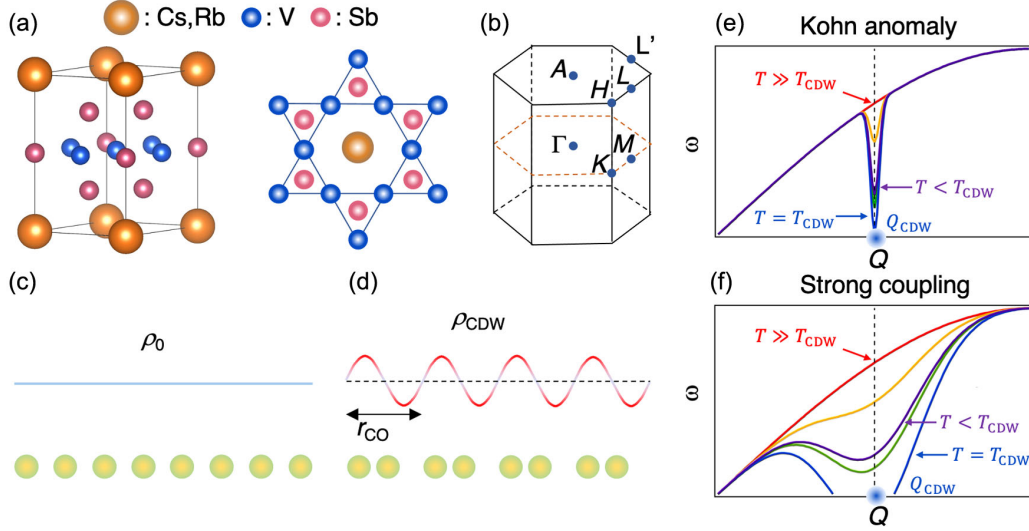


FIG. 1. The crystal structure of  $AV_3Sb_5$  and CDW-induced phonon anomalies. (a) shows the crystal structure of  $AV_3Sb_5$  (space group  $P6/mmm$ , no. 191), which consists of V-Sb slabs that are separated by alkali elements. The V-Sb slab contains a V-kagome lattice and two Sb sites laying in the kagome plane and above the V triangle. (b) shows the 3D BZ and high-symmetry points of  $AV_3Sb_5$ . (c) and (d) show the valence and core electron density in the normal and CDW phase, respectively. Because of EPC, the core electrons will follow the CDW period and yield acoustic phonon anomalies. (e) Nesting-induced Kohn anomalies. (f) Strong-coupling-induced phonon softening in an extended momentum space near  $Q_{CDW}$ . Red, yellow, green, blue, and purple colors represent temperatures from  $T \gg T_{CDW}$  to  $T < T_{CDW}$ . The phonon hardening for  $T < T_{CDW}$  is due to the CDW amplitude mode.

contrast, strong-coupling mechanisms usually give rise to phonon anomalies in an extended  $Q$  range near  $Q_{CDW}$  [26–29], as shown in Fig. 1(f). Comparing with previously explored CDW systems, the kagome structure has a unique sublattice interference effect, which may give rise to unconventional Fermi-surface instabilities that carry finite angular momentum [11,30]. How lattice excitations interact with the finite-angular-momentum CDW remains unresolved. Here we combine inelastic x-ray scattering (IXS), Raman spectroscopy, and angle-resolved photoemission spectroscopy (ARPES) to show evidence of a novel electronic driven CDW in  $AV_3Sb_5$  ( $A = Cs, Rb$ ). We demonstrate that the bulk CDW has three-dimensional (3D)  $2 \times 2 \times 2$  superstructure, which fails to induce longitudinal-acoustic (LA) and transverse-acoustic (TA) phonon anomalies. The absence of acoustic phonon anomaly firmly excludes strong electron-phonon-driven CDW. Interestingly, we discover a novel Raman mode that emerges at the CDW temperature  $T_{CDW}$  and is quickly damped below  $T_{CDW}$  into a continuum that broadly peaks at 20 meV. Our results uncover the intriguing nature of the CDW phase in  $AV_3Sb_5$  and shed light on the intertwined electronic and lattice instabilities in kagome metal near the van Hove filling.

## II. THREE-DIMENSIONAL $2 \times 2 \times 2$ SUPERSTRUCTURE

We first determine the bulk CDW superlattice peaks using meV-resolution XRD. Figures 2(a) and 2(b) show

the temperature-dependent  $L$  scan of  $RbV_3Sb_5$  at  $Q = (3.5, 0, 0)$  and  $(3.5, 0, 0.5)$  in reciprocal lattice units (r.l.u.). The onset temperature at  $T = 103 \pm 0.5$  K is consistent with the CDW transition temperature in  $RbV_3Sb_5$  (see the Appendix A). The absence of hysteresis, as shown in Figs. 2(c) and 2(d), supports a second-order or a weak first-order phase transition. The observation of a CDW peak at half integer  $L$  in Fig. 2(b) proves that the CDW in  $RbV_3Sb_5$  forms a 3D  $2 \times 2 \times 2$  superstructure rather than a 2D  $2 \times 2$  superstructure as suggested by early STM and XRD measurements [6,8,13,16] and consistent with a recent STM work that shows a  $\pi$  phase shift of the in-plane CDW between stacking layers [12]. Interestingly, as shown in Fig. 2(a) ( $L = 0$ ) and Fig. 2(b) ( $L = 0.5$ ), the up-turn slopes of the CDW peak intensity are different, suggesting the coexistence of different electronic orders [17] or different stacking schemes [31]. It should be noted that CDW peaks at  $L = 0$  and 0.5 have different sensitivity to atomic distortions, which may also be related to different up-turn slopes. By fitting the CDW peak with a Lorentzian-squared function [32,33], we find that the CDW correlation length defined as the inverse half width at half maximum ( $1/\text{HWHM}$ ) is larger than 500 Å along the  $L$  direction, establishing a long-range-ordered 3D CDW. As shown in Figs. 2(e) and 2(f), the same 3D CDW superlattice structure is also observed in  $CsV_3Sb_5$ , suggesting a ubiquitous CDW mechanism for all  $AV_3Sb_5$ . We note that the CDW peaks in  $CsV_3Sb_5$  are over twice broader than those of  $RbV_3Sb_5$ , which might be due to the larger stacking fault in  $CsV_3Sb_5$  (see Supplemental Material [31]). Interestingly, we find that

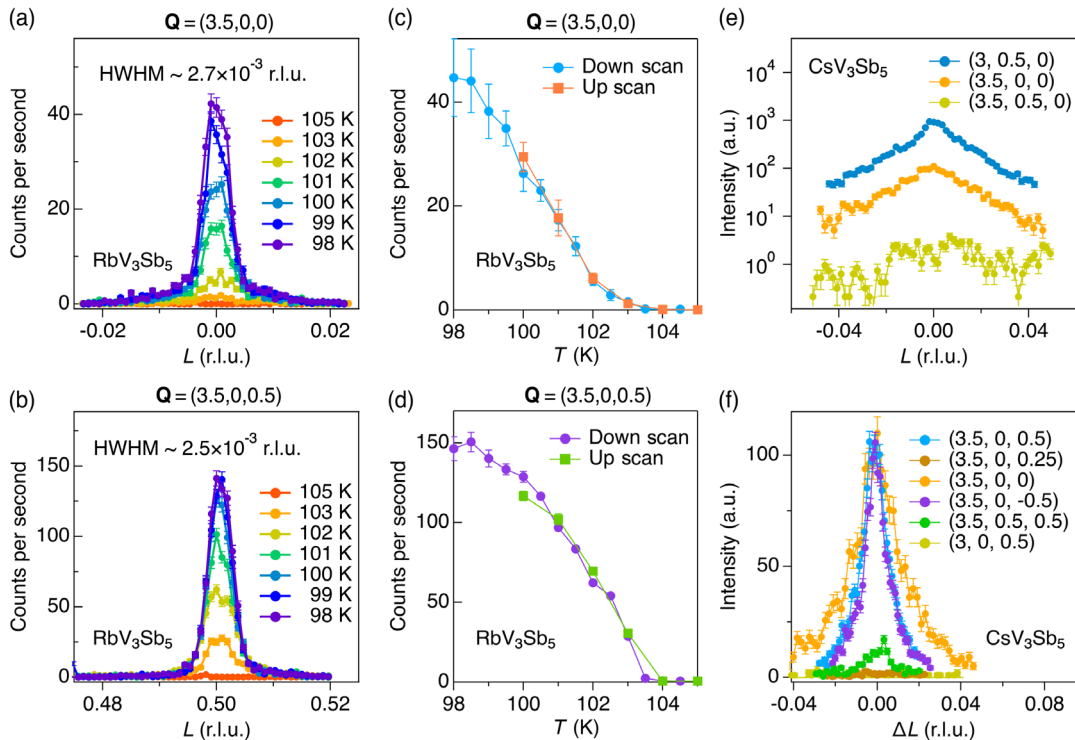


FIG. 2. 3D CDW determined by meV-resolution XRD. (a),(b) The temperature-dependent CDW peaks of  $\text{RbV}_3\text{Sb}_5$  at  $\mathbf{Q} = (3.5, 0, 0)$  and  $(3.5, 0, 0.5)$ . The CDW peak at half-integer  $L$  demonstrates a  $2 \times 2 \times 2$  superstructure. The CDW peaks are extremely sharp along  $L$ , with HWHM of approximately  $0.0025$  r.l.u. at  $T = 98$  K. (c) and (d) show the extracted temperature-dependent CDW peak intensity at  $\mathbf{Q} = (3.5, 0, 0)$  and  $(3.5, 0, 0.5)$ , respectively. The peak intensity takes a sharp upturn at the CDW onset temperature of  $103$  K for  $\text{RbV}_3\text{Sb}_5$ , which is consistent with the specific heat measurement (see Appendix A). The absence of hysteresis with a second-order phase transition. CDW peaks in  $\text{CsV}_3\text{Sb}_5$  are shown in (e) and (f). The peak intensity at  $\mathbf{Q} = (3, 0.5, 0)$  is over 10 times larger than the one at  $\mathbf{Q} = (3.5, 0, 0)$  [panel (e)], consistent with larger structure factors at  $\mathbf{Q} = (H, K + 0.5, 0)$ ,  $H = \text{odd integer}$  [8]. The absence of peaks at  $\mathbf{Q} = (3, 0, 0.5)$ ,  $(3.5, 0.5, 0)$ ,  $(3.5, 0, 0.25)$  confirms the same  $2 \times 2 \times 2$  superstructure in  $\text{CsV}_3\text{Sb}_5$ . The error bars in panels (a),(b),(e),(f) represent 1 standard deviation assuming Poisson counting statistics. The error bars in panels (c) and (d) denote the  $2\sigma$  returned from the pseudo-Voigt fittings that extract the peak intensity.

while the CDW is long-range ordered, the integrated CDW intensity that is proportional to the square of the lattice distortion is extremely small. Comparing with fundamental Bragg peaks, the CDW peak intensity is 3–5 orders of magnitude weaker, demonstrating small lattice distortions. An important consequence of the 3D  $2 \times 2 \times 2$  superstructure is that it breaks the rotational symmetry from  $C_6$  to  $C_2$  [34]. This result thus indicates that the STM-observed  $C_6$  symmetry breaking is likely a structural effect rather than electronic nematicity.

### III. ABSENCE OF ACOUSTIC PHONON ANOMALY

With the 3D CDW established, we explore the origin of the CDW in  $\text{AV}_3\text{Sb}_5$  using IXS and Raman spectroscopy. As we show in Fig. 1, the formation of CDWs is always accompanied by acoustic phonon anomalies because of EPC. Indeed, even in the cuprates, where CDWs are short ranged and possibly dynamical, a 15% LA phonon softening has been observed [26,27], demonstrating the acoustic phonon is a sensitive probe of CDW correlations. DFT calculation of the phonon dispersion of  $\text{AV}_3\text{Sb}_5$

shows large negative frequency near the  $M$  and  $L$  points at zero temperature [15,31], indicating strong electron-phonon-coupling-induced CDW in  $\text{AV}_3\text{Sb}_5$  [28,29,35,36]. To test this scenario, we perform IXS measurement along  $(3, 0, 0) \rightarrow (3.5, 0, 0)$  and  $(3, 3, 0) \rightarrow (3.5, 2.5, 0)$  to selectively enhance LA and TA phonon modes in our experimental geometry (see Ref. [31]). Unexpectedly, as we show in Figs. 3(a)–3(e), CDWs in  $\text{AV}_3\text{Sb}_5$  fail to induce phonon anomalies on both TA and LA modes from  $T = 300$  K  $> T_{\text{CDW}}$  to  $50$  K  $< T_{\text{CDW}}$ . We note that in Fig. 3(d), beside the LA peaks at  $12$  meV, there is a low-energy peak at  $7$  meV in the spectra. This low-energy peak is the leakage signal of the TA mode as it locates at the same energy of the TA peak shown in Fig. 3(e). This 7-meV mode also presents an intensity enhancement at the low temperature similar to the TA peak. The intensity change below  $T_{\text{CDW}}$  is likely due to the CDW-induced lattice distortion that modifies the dynamic structure factor [37]. For the same reason, the 13-meV phonon mode shown in Fig. 3(e) also changes below  $T_{\text{CDW}}$ . The absence of phonon softening thus excludes strong EPC as the driving force of the CDW in  $\text{AV}_3\text{Sb}_5$ .

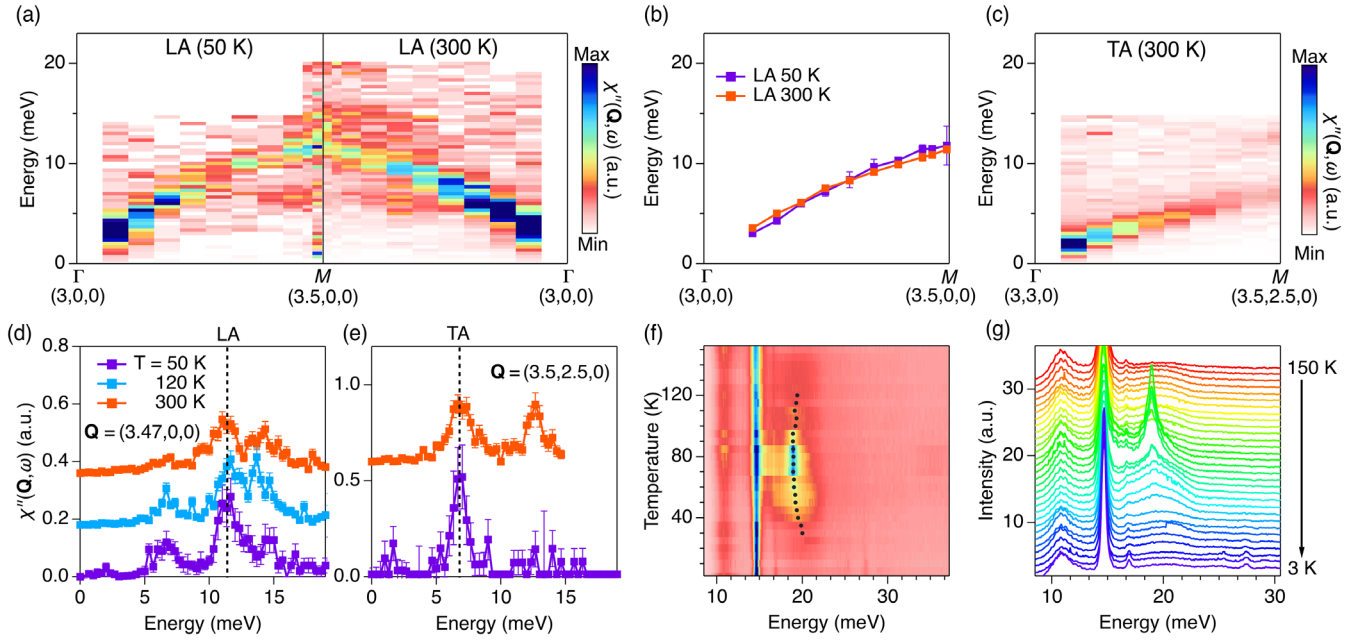


FIG. 3. Evidence of unconventional electronically driven CDW. (a) compares the IXS determined LA of  $\text{RbV}_3\text{Sb}_5$  along  $\Gamma(3,0,0)$  to  $M(3.5,0,0)$  at temperatures of  $T = 300$  K and  $50$  K ( $T_{\text{CDW}} = 103$  K). (b) Extracted-temperature-dependent phonon dispersions from (a). (c) TA in  $\text{RbV}_3\text{Sb}_5$  measured along  $\Gamma(3,3,0)$  to  $M(3.5,2.5,0)$  at room temperature ( $300$  K). (d) and (e) compare temperature-dependent IXS spectra at  $\mathbf{Q} = (3.47, 0, 0)$  and  $(3.5, 2.5, 0)$ . The dashed lines in (d) and (e) highlight LA and TA, respectively. All IXS data are Bose-factor corrected, and the elastic contribution at  $\omega = 0$  is subtracted by fitting the IXS raw data (see Supplemental Material [31]). (f),(g) Temperature-dependent Raman spectra on  $\text{CsV}_3\text{Sb}_5$  from  $150$  to  $3$  K. Beside the sharp optical phonon peak at  $11$ ,  $15$ , and  $17$  meV, there is a continuum that is broadly centered near  $19$  meV (the  $11$ -meV peak may actually be at lower energy but appears to be at  $11$  meV due to the cutoff of the longpass filter at  $11$  meV). The black dots in (f) mark the peak positions of the continuum near  $19$  meV. When cooling toward  $T_{\text{CDW}}$ , the center of this continuum gradually shifts from  $19.5$  to  $19$  meV and starts to build a strong, sharp, and asymmetric peak at  $T_{\text{CDW}} = 94$  K. Moving to lower temperature, this peak is quickly damped and hardened to  $20$  meV at  $30$  K. Below  $30$  K, two new phonon modes at  $25.4$  and  $27.5$  meV are observed, which may correspond to the stripe phase observed by a recent STM study [16]. The error bars in panels (d),(e) represent 1 standard deviation assuming Poisson counting statistics. The error bars in panel (b) denote the  $2\sigma$  returned from the fitting that extracts the spectral peak position.

#### IV. EVIDENCE OF GAPPED CDW PHASE MODE

Raman spectroscopy provides further insight into the origin of CDW. Figures 3(f) and 3(g) show temperature-dependent Raman spectra of  $\text{CsV}_3\text{Sb}_5$  with  $T_{\text{CDW}} = 94$  K. According to DFT calculations (see Supplemental Material in [31]), four Raman active modes at  $8.6$ ,  $15.1$ ,  $16.2$ , and  $17.4$  meV corresponding to the  $E_{1g}$ ,  $E_{2g}$ ,  $A_{1g}$ , and  $B_{2g}$  modes, respectively, are allowed for the normal-state structure. At  $150$  K, we observe three optical phonon modes at  $11$ ,  $15$ , and  $17$  meV, which we attribute to the  $E_{1g}$ ,  $E_{2g}$ , and  $B_{2g}$  modes (the  $11$ -meV peak may actually be at lower energy but appears to be at  $11$  meV due to the cutoff of the longpass filter at  $11$  meV). Interestingly, we observe a continuum that is broadly centered around  $19.5$  meV. As we cool down toward the  $T_{\text{CDW}}$ , the center of this continuum gradually shifts to lower energy and eventually develops into a strong, sharp, and asymmetric peak at  $19$  meV, demonstrating its intimate correlation with the CDW. Unexpectedly, moving to a lower temperature, the intensity of this peak is suppressed. Remarkably, this unconventional mode shows clear softening above  $T_{\text{CDW}}$

and hardening below  $T_{\text{CDW}}$ , reminiscent of the CDW phason and amplitudon excitations near the  $\mathbf{Q}_{\text{CDW}}$  [28,29,31,38–40]. A similar effect near the Brillouin zone center has been observed in CDW material  $\text{TiSe}_2$  [29], which also has a  $3\text{D } 2 \times 2 \times 2$  superstructure. The temperature-dependent Raman mode together with the absence of an acoustic phonon anomaly suggest that the CDW in  $\text{AV}_3\text{Sb}_5$  might be a novel realization of particle-hole condensation in a metallic system, where the particle-hole excitation is highly damped by itinerant electrons. In this scenario, the absence of phonon softening might be a signature of a large CDW phason gap that is induced by the commensurability effect [41]. As we discuss in more detail in Appendix B, for commensurate CDW, the phason gap can be expressed as [41]

$$\omega_\phi \sim \Delta \left( \frac{\Delta}{D} \right)^{(M-2)/2}, \quad (1)$$

where  $D$  is the electronic bandwidth,  $\Delta$  is the CDW gap, and  $M$  is the CDW period relative to the lattice constant  $a_0$ . Usually, the bandwidth  $D \gg \Delta$  and therefore,  $\omega_\phi \sim 0$ .

However, for CDW period  $M = 2$ , which is the case for  $AV_3Sb_5$ ,  $\omega_\phi \sim \Delta$ . Besides the commensurability effect, a weak first-order phase transition as suggested by recent nuclear-magnetic- and quadruple-resonance measurements [42,43] may also be responsible for the absence of the acoustic phonon anomaly. DFT calculations suggest that the inverse start-of-David distortion corresponding to an unstable optical phonon is the energetically favored structure in the CDW phase [15]. While the softening of the optical phonon is expected to induce LA phonon anomalies (see Supplemental Material [31]), the weak first-order phase transition in  $AV_3Sb_5$  may yield a discontinuous and small modification of LA that is not resolved by our measurement.

Below 30 K, we observe additional Raman active modes at 25.4 and 27.5 meV, demonstrating an additional symmetry-breaking phase at low temperature. Comparing with a previous STM study [16], this new low-temperature phase is possibly the unidirectional CDW. Interestingly, this new symmetry-breaking phase correlates strongly with the CDW-related continuum, where the intensity of the continuum significantly suppresses when the new Raman modes emerge.

## V. SADDLE POINT AND QUASI-NESTED ELECTRON AND HOLE POCKETS

To disclose the particle-hole scattering channel that is connected by the 3D CDW, in Fig. 4, we show the low-temperature ( $T = 15$  K) electronic structure of  $RbV_3Sb_5$  determined by ARPES measurements and DFT calculations. As shown in Figs. 4(a) and 4(b), the overall Fermi-surface topology is similar to previous ARPES studies of  $KV_3Sb_5$  and  $CsV_3Sb_5$  [8,44], hence, confirming the ubiquitous van Hove filling of  $AV_3Sb_5$ . Photon energies of  $h\nu = 100$  and  $90$  eV correspond to  $k_z = \pi$  and  $0.15\pi$  around the  $\bar{M}$  point, respectively (see Supplemental Material [31] for the  $k_z$  map). The most important observations are shown in Figs. 4(c) and 4(d) corresponding to the blue cut in Figs. 4(a) and 4(b), respectively. At  $k_z = \pi$ , we observe a shallow electron band with a band bottom of approximately 50 meV, while at  $k_z = 0.15\pi$ , we observe a steep holelike band dispersing toward the Fermi level. The shallow electron and hole band can be further revealed by the stacking momentum-distribution curves (MDCs) displayed in Figs. 4(e) and 4(f). Similar electronic structure is also observed in  $CsV_3Sb_5$  (see Supplemental Material

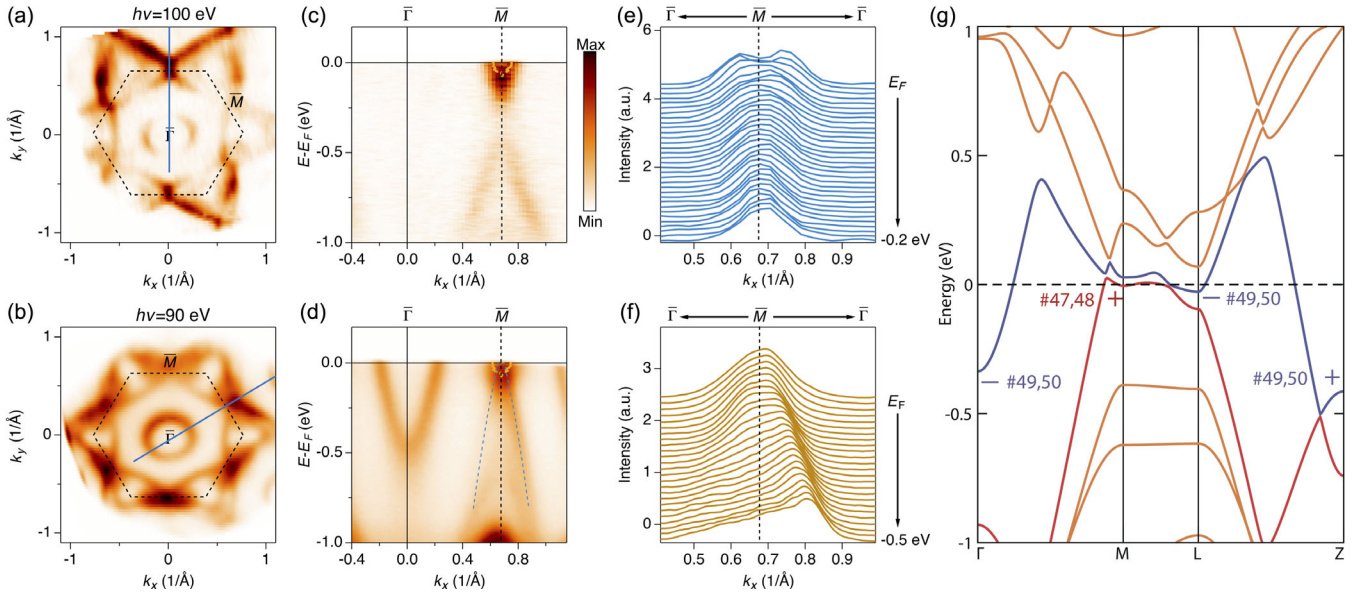


FIG. 4. 3D-CDW-induced band inversion. (a) and (b) show Fermi-surface maps measured at photon energies of  $h\nu = 100$  and  $90$  eV corresponding to  $k_z = \pi$  and  $0.15\pi$  around the  $\bar{M}$  point, respectively. Data are taken at  $T = 15$  K. The blue lines in (a) and (b) mark the high-symmetry cuts shown in (c) and (d), respectively. The inner potential is extracted to be around 8.2 eV (see Supplemental Material [31]). (c) and (d) selectively enhance the electron band near the unfolded  $L$  point and the hole band near the unfolded  $M$  point. The yellow dots in (c) and (d) are extracted from the MDC peaks shown in (e). The blue dashed line in (d) is a guide to the eye. The blurry spectral weight near the Fermi level in (d) likely comes from the bulk projected surface state [8]. (e) and (f) show stacking plots of MDCs to further reveal the electron and hole bands in panels (c) and (d). (g) shows the electronic band structure of  $RbV_3Sb_5$  calculated by VASP [45–47] after a full relaxation on atomic positions with an atom pairwise-correction method (DFT-D3) [48] since the van der Waals forces play an important role along the  $c$ -layer stacking direction. The Fermi energy is shifted by 100 meV in order to match the experimental results. There is a hole pocket in the vicinity of the  $M$  point ( $\bar{\Gamma}$ - $M$  direction) carrying a parity of “+” and an electron pocket in the vicinity of the  $L$  point carrying a parity of “-” contributed by different bands. Our DFT result confirms the parity distribution of the band structure that is reported to possess a nontrivial  $\mathbb{Z}_2$  topological invariant and induce topological surface states around the  $M$  point [8].

[31]). Our observations establish two particle-hole channels that are possibly relevant to the CDW: (i) the saddle points at the  $M$  point [4,5,11,15] and (ii) the quasinested electron and hole pockets at the  $M$  and  $L$  points [see Fig. 1(b)]. Because of the high density of state of the saddle point, the leading Fermi-surface instability, if any, should be driven by the saddle point [5,11,30,42], while the electron-hole nesting may help to stabilize the  $L$  component of the 3D CDW. We note, however, that the electronic structure near the  $M$  point [Fig. 4(d)] is not fully captured by the DFT calculations of the ideal kagome structure shown in Fig. 4(g), which might be due to surface effects [8] or a number of stacking faults in  $AV_3Sb_5$  (see Supplemental Material [31]).

## VI. CONCLUSION

In summary, we demonstrate an unconventional 3D CDW in the  $\mathbb{Z}_2$  kagome superconductor  $AV_3Sb_5$ , where the formation of CDW fails to induce phonon anomalies near  $\mathbf{Q}_{CDW}$ . Our observations exclude strong EPC-driven CDW in  $AV_3Sb_5$  and point to an unconventional CDW phase in an intertwined electronic state.

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*Note added.*—Recently, the CDW gap  $\Delta_{CDW}$  was observed in ARPES [49,50] and optical studies [51] near the  $M$  point. This new observation does not contradict the absence of phonon anomaly. We note that the CDW gap opening can be a consequence of the  $2 \times 2 \times 2$  superstructure, which induces band folding and gap opening.

## APPENDIX A: METHODS

### 1. Sample

Single crystals of  $AV_3Sb_5$  ( $A = \text{Rb}, \text{Cs}$ ) are grown from Rb ingot (purity 99.9%), Cs ingot (purity 99.9%), V powder (purity 99.9%), and Sb grains (purity 99.999%) using the self-flux method, similar to the growth of  $\text{RbV}_3\text{Sb}_5$  [14]. The mixture is put into an alumina crucible and sealed in a quartz ampoule under partial argon atmosphere. The sealed quartz ampoule is heated to 1273 K for 12 h and soaks there for 24 h. Then it is cooled down to 1173 K at 50 K/h and further to 923 K at a slow rate. Finally, the ampoule is taken out from the furnace and decanted with a centrifuge to separate  $(\text{Cs}, \text{Rb})\text{V}_3\text{Sb}_5$

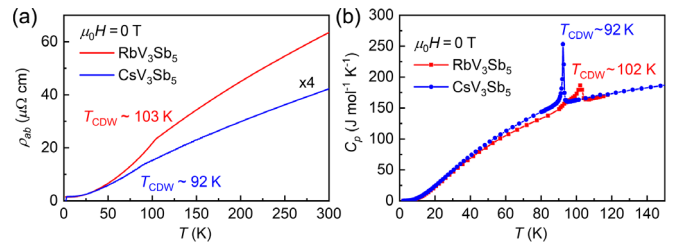


FIG. 5. (a) and (b) show transport and specific heat measurement of  $\text{RbV}_3\text{Sb}_5$  (red) and  $\text{CsV}_3\text{Sb}_5$  (blue).

single crystals from the flux. The obtained crystals have a typical size of  $2 \times 2 \times 0.02 \text{ mm}^3$ . CDW transition is clearly observed in transport and specific heat measurement shown in Figs. 5(a) and 5(b), respectively.

## 2. Inelastic x-ray scattering

The experiments are conducted at beam line 30-ID-C (HERIX) at the Advanced Photon Source. The highly monochromatic x-ray beam of incident energy  $E_i = 23.7 \text{ keV}$  ( $\lambda = 0.5226 \text{ \AA}$ ) is focused on the sample with a beam cross section of  $35 \times 15 \text{ \mu m}^2$  (horizontal vertical). The total energy resolution of the monochromatic x-ray beam and analyzer crystals is  $\Delta E \sim 1.5 \text{ meV}$  (full width at half maximum). The measurements are performed in transmission geometry. Typical counting times are in the range of 30 to 360 sec per point in the energy scans at constant momentum transfer  $\mathbf{Q}$ .  $H, K, L$  are defined in the hexagonal structure with  $a = b = 5.472 \text{ \AA}$ ,  $c = 9.073 \text{ \AA}$  at room temperature for  $\text{RbV}_3\text{Sb}_5$ , and  $a = b = 5.495 \text{ \AA}$ ,  $c = 9.309 \text{ \AA}$  at room temperature for  $\text{CsV}_3\text{Sb}_5$ .

## 3. Hard-x-ray diffraction

High-precision x-ray-scattering measurements are performed at the *in situ* and resonant 4-ID beam line of National Synchrotron Light Source II (NSLS-II). The photon energy, which is selected by a cryogenically cooled Si(111) double-crystal monochromator is  $11.47 \text{ keV}$ . The sample is mounted in a closed-cycle displx cryostat in a vertical scattering geometry, and the sigma-sigma scattering channel is measured using an MgO(440) polarization analyzer and silicon drift detector.

## 4. ARPES experiment

The ARPES experiments are performed on single crystals of  $\text{RbV}_3\text{Sb}_5$  and  $\text{CsV}_3\text{Sb}_5$ . The samples are cleaved *in situ* in a vacuum better than  $5 \times 10^{-11} \text{ torr}$ . The experiment is performed at beam line 21-ID-1 at the

NSLS-II. The measurements are taken with synchrotron light source and a Scienta-Omicron DA30 electron analyzer. The total energy resolution of the ARPES measurement is approximately  $15 \text{ meV}$ . The sample stage is maintained at low temperature ( $T = 15 \text{ K}$ ) throughout the experiment.

## 5. Raman experiment

Raman spectroscopy is performed in a Montana Instruments closed-cycle cryostations100 and utilize a Hubner Photonics 532-nm diode-pumped-laser excitation and an Isoplane SCT-320 imaging spectrograph with a 400B-eXcelon CCD camera and a 2400-groove/mm visible-holographic grating. Semrock dichroic and longpass filters are integrated in the optics train. For all Raman spectra reported here, a 2-mW laser-excitation power is used, and Raman spectra are acquired for 300 s each. The sample is initially cooled to  $3 \text{ K}$  then heated to  $150 \text{ K}$ . All of the reported Raman spectra are then acquired while monotonically recooling the sample from  $150$  to  $3 \text{ K}$ . The optics train is refocused at each temperature after waiting for the sample temperature to stabilize in order to correct for changes due to thermal expansion and to ensure that the sample is in thermal equilibrium before acquiring a spectrum.

## 6. DFT calculations

The electronic band structure of  $\text{RbV}_3\text{Sb}_5$  is calculated by VASP [45–47] after a full relaxation on atomic positions with an atom pairwise-correction method (DFT-D3) [48]. The phonon band structure of  $\text{RbV}_3\text{Sb}_5$  is calculated using VASP within the Perdew-Burke-Ernzerhof exchange correlation based on density-functional perturbation theory. An equivalent  $k$ -point mesh of  $9 \times 9 \times 6$  is used in the self-consistent calculation, and the cutoff energy for the plane-wave basis is  $400 \text{ eV}$ . Prior to the phonon spectra calculation, crystal structure is relaxed with the residual force on each atom less than  $0.001 \text{ eV/\AA}$ .

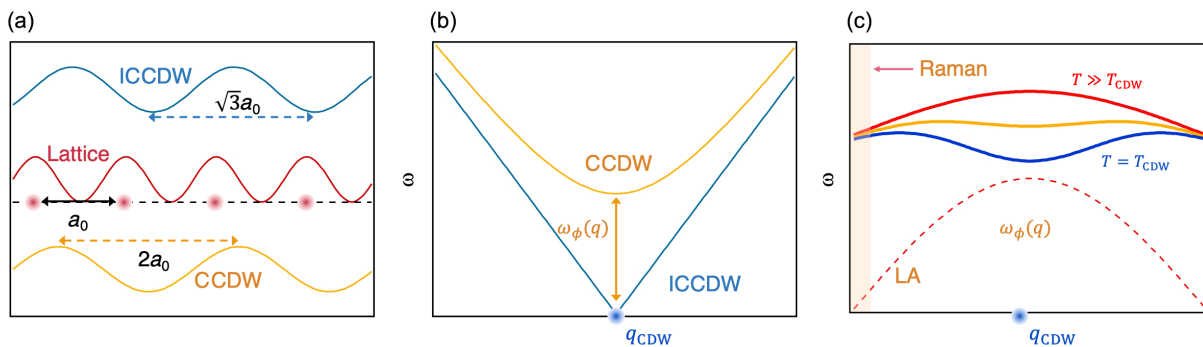


FIG. 6. (a) schematically shows commensurate (yellow) and incommensurate (cyan) CDW periods with respect to the lattice (red). (b) Yellow and cyan curves are the calculated phase modes for CCDW and ICCDW, respectively. (c) shows the CDW mechanism without electron-acoustic phonon interaction. The temperature-dependent particle-hole excitations are shown as red, yellow, and blue curves. The dashed line show the LA mode, whose energy at  $q_{\text{CDW}}$  is less than  $\omega_\phi$ . Shaded pink area indicates the momentum position of the Raman measurement.

## APPENDIX B: COMMENSURABILITY EFFECT

For incommensurate CDW (ICCDW), the condensation energy  $E_{\text{cond}}$  is independent of the phase of CDW,  $\phi$ . For commensurate CDW (CCDW), however, the condensation energy becomes phase dependent,

$$E_{\text{cond}}(\phi) = -\frac{n(\epsilon_F)\Delta_{\text{CDW}}^2}{\lambda} \left(\frac{\Delta}{D}\right)^{M-2} \cos(M\phi), \quad (\text{B1})$$

where  $\lambda$  is the dimensionless EPC constant, and  $n(\epsilon_F)$  and  $\Delta$  are the density of state at  $E_F$  and the CDW gap, respectively.  $M$  is the CDW period relative to the lattice constant  $a_0$ .  $D$  is the electronic bandwidth. This phase-dependent  $E_{\text{cond}}(\phi)$  means that gliding the CDW requires finite energy corresponding to a finite gap in the CDW phase mode

$$\omega_\phi(q) = \left(\frac{\omega_F^2}{M} + v_F^2 \frac{m}{m^*} q^2\right)^{1/2}, \quad (\text{B2})$$

where  $\omega_F^2 = (4M^2/\lambda)(m\Delta^2/m^*)(\Delta/D)^{M-2}m^*$ .  $m^*$  is the effective mass. At  $q = q_{\text{CDW}}$ , Eq. (B2) gives

$$\omega_\phi = \sqrt{\frac{4M}{\lambda} \frac{m}{m^*} \Delta} \left(\frac{\Delta}{D}\right)^{(M-2)/2}. \quad (\text{B3})$$

Usually,  $D \gg \Delta$  and therefore,  $\omega_\phi \sim 0$ . However, for  $M = 2$ , which is the case for  $\text{AV}_3\text{Sb}_5$ ,  $\omega_\phi = \sqrt{(4M/\lambda)(m/m^*)}\Delta$ . Interestingly, a simple parabolic fitting of the ARPES data shown in Fig. 4 gives  $(m/m^*) \sim 3$  for the shallow electron and hole pockets, suggesting large  $\omega_\phi$  in  $\text{AV}_3\text{Sb}_5$ . As we show in Fig. 6(c), due to the large phason gap, the particle-hole condensation will not interact with acoustic phonon modes, as these modes are laying inside the phason gap.

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