ON THE THEORY OF MAGNETO-OPTIC PHENOMENA. II.[1]

BY C. H. WIND.

§ 7. *Reflection and Refraction at the interface of two media, one of which is transparent and shows no Hall-effect, while the Coefficient of Absorption and the Hall-constant of the other possess sensible values; a Magnetic Force is applied parallel to the Plane of Incidence.*

TWO extreme cases are separately examined, namely those of polar and of equatorial reflection. The first problem solved for the two cases is the following: What must be the constitution of the incident light in order that the refracted ray may be circularly polarized as either of the rays considered in paragraph 6?

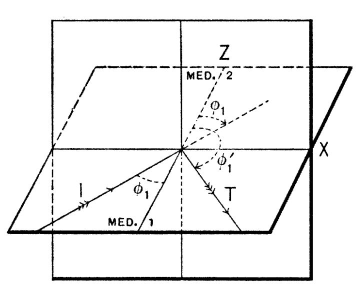

Fig. 2.

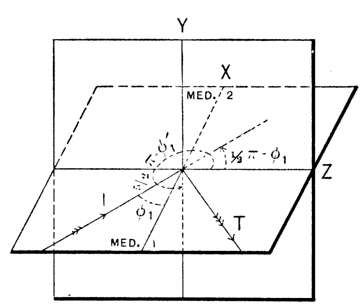

Fig. 3.

The system of axes has different positions for the two cases as shown in diagrams 2 and 3, in which I and T are the incident and

[1]Continued from p. 51.

reflected rays respectively, φ_1 is the angle of incidence, and φ_1' the supplement of the angle of reflection.

Our conditions of continuity (II),(IV), (V) and (VI) become :

POLAR.		EQUATORIAL.	
$(w)_1 = (w)_2$;	$(\alpha)_1 = (\alpha)_2$;	$(u)_1 = (u)_2$;	$(\alpha)_1 = (\alpha)_2$,
$(X)_1 = (X)_2$,	$(\beta)_1 = (\beta)_2$,	$(Y)_1 = (Y)_2$,	$(\beta)_1 = (\beta)_2$,
$(Y)_1 = (Y)_2$;	$(\gamma)_1 = (\gamma)_2$;	$(Z)_1 = (Z)_2$;	$(\gamma)_1 = (\gamma)_2$.

The incident light may be characterized by

$$\left.\begin{aligned} u &= \sigma_1 \cos\varphi_1 . \mathrm{P}_1 \\ v &= s_1 . \mathrm{P}_1 \\ w &= -\sigma_1 \sin\varphi_1 . \mathrm{P}_1 \end{aligned}\right\}, \qquad \left.\begin{aligned} u &= \sigma_1 \sin\varphi_1 . \mathrm{P}_1 \\ v &= s_1 . \mathrm{P}_1 \\ w &= -\sigma_1 \cos\varphi_1 . \mathrm{P}_1 \end{aligned}\right\},$$

where

$$\mathrm{P}_1 = e^{\delta[t - R_1(x\sin\phi_1 + z\cos\phi_1)]}, \qquad \mathrm{P}_1 = e^{\delta[t - R_1(x\cos\phi_1 + z\sin\phi_1)]},$$

$$R_1^2 = \frac{4\pi p_1}{\delta},$$

and the corresponding expressions for the components of $\mathfrak{F}$ and $\mathfrak{H}$, the reflected light on the other hand by the same expressions with σ_1' s_1', φ_1' and P_1' instead of σ_1, s_1, φ_1 and P_1, and the refracted light by

$$\left.\begin{aligned} u &= a_2 \cos\varphi_2 . \mathrm{P}_2 \\ v &= \pm i a_2 . \mathrm{P}_2 \\ w &= -a_2 \sin\varphi_2 . \mathrm{P}_2 \end{aligned}\right\}, \qquad \left.\begin{aligned} u &= a_2 \sin\varphi_2 . \mathrm{P}_2 \\ v &= \pm i a_2 . \mathrm{P}_2 \\ w &= -a_2 \cos\varphi_2 . \mathrm{P}_2 \end{aligned}\right\},$$

where

$$\mathrm{P}_2 = e^{\delta[t - R_2(x\sin\phi_2 + z\cos\phi_2)]}, \qquad \mathrm{P}_2 = e^{\delta[t - R_2(x\cos\phi_2 + z\sin\phi_2)]},$$
$$R_2 = R_{20}(1 \mp \mu i \cos\varphi_2), \qquad R_2 = R_{20}(1 \mp \mu i \sin\varphi_2),$$

$$R_{20}^2 = \frac{4\pi p_2}{\delta}, \; \mu = \tfrac{1}{2} p_2 q N,$$

and the corresponding formulæ for the components of $\mathfrak{F}$ and $\mathfrak{H}$— finally for the whole set of formulæ

$$\delta = -\frac{2\pi i}{T}.$$

The above conditions of continuity give us then, besides the relation

$$\varphi_1' = \pi - \varphi_1, \tag{63}$$

a set of formulæ, by which the constants σ_1, s_1, σ_1' and s_1' are expressed in terms of a_2. In these formulæ, which we shall not write down here, there occur some double signs $\pm$ and $\mp$ according to the two senses of circular polarization, which may be assumed for the refracted ray.

Let us now assume in the second (metallic) medium *two* circularly polarized rays propagating at the same time, one being right-handed and the other left-handed. One of these will correspond to the above equations with the upper of the double signs, and will be farther determined by the value a_{2+} for the constant a_2; the other corresponds to the same equations with the lower signs, and is farther determined by the value a_{2-} for the constant a_2. In agreement with this notation we shall distinguish the corresponding values of σ_1 etc., similarly by a suffix $+$ or $-$.

On the other hand, we shall suppose the incident light to be plane polarized. Let us now direct our attention especially to the constitution of the reflected ray, leaving the refracted ray for the present out of consideration. I shall distinguish two cases, which we shall refer to as *principal cases*.

I. The plane of polarization of the incident ray is perpendicular to the plane of incidence. In this case $s_{1+} + s_{1-} = 0$ and we have to determine the value b_s of $s_1'{}_+ + s_1'{}_-$ and the value a_p of $\sigma_1'{}_+ + \sigma_1'{}_-$ when $s_{1+} + s_{1-}$ is put equal to unity.

II. The plane of polarization of the incident ray is parallel to the plane of incidence. Therefore $\sigma_{1+} + \sigma_{1-} = 0$, and the question to be answered is: What will be the value a_s of $s_{1+} + s_{1-}$ and the value b_p of $\sigma_1'{}_+ + \sigma_1'{}_-$ when $s_{1+} + s_{1-}$ is put equal to unity?

As a solution to the questions as stated we find the formulæ

POLAR. EQUATORIAL.

$$a_p = \frac{\bar{p}_2 \bar{p}_3}{\bar{p}_4 \bar{p}_1} \cdot e^{i(\delta_2 + \delta_3 - \delta_4 - \delta_1)}$$

$$b_s = \frac{\sigma^4}{\bar{p}_1^2 \bar{p}_4 \cos^3\varphi_1} \cdot \frac{\rho}{2AV} \cdot e^{i(4\tau - 2\delta_1 - \delta_4 + \frac{1}{2}\pi + S)} \qquad \Bigg| \qquad b_s = \frac{\sigma^3 \sin\varphi_1}{\rho \bar{p}_1^2 \bar{p}_4 \cos^3\varphi_1} \cdot \frac{\rho}{2AV} \cdot e^{i(3\tau - \omega - 2\delta_1 - \delta_4 + \frac{1}{2}\pi + S)} \qquad (84)$$

POLAR. EQUATORIAL.

$$a_s = \frac{\overline{p}_2}{\overline{p}_1} \cdot e^{i(\delta_2 - \delta_1 + \pi)}$$

$$b_p = \frac{\sigma^4}{\overline{p}_1^2 \overline{p}_4 \cos^3\varphi_1} \cdot \frac{\rho}{2\Lambda V} \cdot e^{i(4\tau - 2\delta_1 - \delta_4 + \frac{1}{2}\pi + S)} \qquad b_p = \frac{\sigma^3 \sin\varphi_1}{\rho \overline{p}_1^2 \overline{p}_4 \cos^3\varphi_1} \cdot \frac{\rho}{2\Lambda V} \cdot e^{i(3\tau - \omega - 2\delta_1 - \delta_4 - \frac{1}{2}\pi + S)} \tag{85}$$

Λ and V representing wave-length and speed of the light considered in the first (transparent) of the two media. The other magnitudes that occur here for the first time are defined by the equations

$$\rho e^{iS} = qN \tag{82}$$

(ρe^{iS} or qN is the quantity which we shall afterwards refer to as the Hall-constant),

$$\sigma e^{i\tau} = \frac{R_{20}}{R_1}, \tag{80}$$

and the following equations

$$\overline{\rho e^{i\omega}} = \sqrt{1 - \sigma^{-2} e^{-2i\tau} \sin^2\varphi_1},$$

$$m = \frac{\sigma \rho}{\cos\varphi_1},$$

$$tg\delta_1 = \frac{m \sin(\tau + \omega)}{1 + m \cos(\tau + \omega)}, \qquad \overline{p}_1^2 = 1 + m^2 + 2m \cos(\tau + \omega),$$

$$tg\delta_2 = \frac{m \sin(\tau + \omega)}{-1 + m \cos(\tau + \omega)}, \qquad \overline{p}_2^2 = 1 + m^2 - 2m \cos(\tau + \omega),$$

$$tg\delta_3 = \frac{m \sin(\tau + \omega)}{-tg^2\varphi_1 + m \cos(\tau + \omega)}, \qquad \overline{p}_3^2 = tg^4\varphi_1 + m^2 - 2m\, tg^2\varphi_1 \cos(\tau + \omega),$$

$$tg\delta_4 = \frac{m \sin(\tau + \omega)}{tg^2\varphi_1 + m \cos(\tau + \omega)}, \qquad \overline{p}_4^2 = tg^4\varphi_1 + m^2 + 2m\, tg^2\varphi_1 \cos(\tau + \omega).$$

From formulæ (84) and (85) we see that as a consequence of the magnetization there will appear in addition to the ordinary reflected light (a_p, a_s), which, in both the principal cases, has its plane of polarization the same as that of the incident light itself and remains unaltered, another component (b_s, b_p) with a plane of polarization perpendicular to that of the ordinary reflected light. This new component is called the *magneto-optic component*. Its appearance constitutes the phenomenon known as the *Kerr-effect.* If we represent the ratio of the amplitude of the magneto-optic component to the amplitude of the incident ray in the two cases by μ_p and μ_i, respectively, we find as a consequence of the above equations

POLAR.	EQUATORIAL.
$\mu_p = \mu_i = \frac{\sigma^4}{p_1^2 p_4 \cos^3 \varphi_1} \cdot \frac{\rho}{2AV} \cdot$ (90)	$\mu_p = \mu_i = \frac{\sigma^3 \sin \varphi_1}{\rho p_1^2 p_4 \cos^3 \varphi_1} \cdot \frac{\rho}{2AV} \cdot$ (90)

Denoting the phase differences of the magneto-optic component, relatively to the parallel polarized ordinary reflected ray, for the first and the second principal cases by m_p and m_i respectively, we find similarly:

	POLAR.	EQUATORIAL.	(13)
(91)	$m_p = m_i - \pi = 4\tau - \delta_1 - \delta_2 - \delta_4 - \frac{1}{2}\pi + S.$	$m_p = m_i = 3\tau - \omega - \delta_1 - \delta_2 - \delta_4 - \frac{1}{2}\pi + S.$	(91)

The quantities σ and τ depend by the equations

$$tg\, 2(\tau - H) = -\cos 2I\, tg 2H \tag{88}$$

and

$$\sigma = tgI \sqrt{\frac{\cos 2H}{\cos 2(\tau - H)}} \tag{89}$$

upon the optical constants I and H known as principal angle of incidence and principal azimuth.

The expressions (90) and (91) differ from those derived from Van Loghem's dissertation[1] as given by Sissingh[2] and Zeeman[3] for the same quantities. This difference arises on one hand from the fact that our point of departure is Maxwell's system of equations and not that of Helmholtz, as we have in this way gotten rid of a certain quantity, A, which is not completely measurable and which is found in Van Loghem's expressions for μ_i and μ_p. On the other hand, a new quantity, S, not occurring in Van Loghem's equations, has made its appearance in ours. This S is the argument of the Hall-constant, the latter being by our fundamental hypothesis assumed to be a complex quantity.

[1] Van Loghem, Doctor dissertation, Leyden, 1883.
[2] Sissingh, Arch. néerl. 27, p. 236, 1893.
[3] Zeeman, Arch. néerl. 27, p. 274, 1894.

§ 8. *Comparison between theory and experiment in the case of the Kerr-effect.*

Upon comparing the results of observations upon the Kerr-effect, by Sissingh, Zeeman and myself, with the predictions of Lorentz's theory in its original form, a discrepancy has been found[1] in the case of iron, cobalt, and nickel between the values of the phase difference observed and those computed. The difference between the observed and computed values is nearly independent of the angle of incidence and is often referred to as *Sissingh's phase difference.* This quantity S' was found to be 85°, 40°.5 and 36°.5 in the cases of iron, cobalt, and nickel, respectively. Bearing in mind the difference between Van Loghem's formulæ for m_i and m_p and ours, as characterized at the end of the last section, it is clear that these observations mentioned will completely agree with our theory, if we attribute to the argument S of our complex Hall-constant a value equal to the Sissingh phase difference S'.

The values of $\rho \cos S$ calculated from the experimentally determined ratio of amplitudes and phase difference are usually much greater than those obtained for the Hall-constant by experiments on the common Hall-effect itself. But this discrepancy does not afford any evidence against the exactness of the theory; for there is no reason whatever to suppose that the quantity $\rho \cos S$ should be the same in the case of oscillating currents of arbitrary period as in the case of steady currents.

§ 9. *Difference between the results of theory and experiment.*

The ratio of amplitudes as determined by experiment has not always been found exactly proportional to the same quantity as calculated from the theory. On the other hand, certain measurements by Zeeman with light reflected from iron and cobalt mirrors at nearly normal incidence have furnished values of S' differing by several degrees from the values previously found for other angles of incidence. The author is of the opinion that neither of these two classes of deviation is of much consequence in its bearing upon the theory. On one hand we do not know exactly the relations existing between the Hall-constant and the magnetic quantities involved, nor do we know

[1] Cf. f. i. Wind, Verh. d. Physik. Gesellsch. Berlin 13, p. 84, 1894.

everything about the magnetic properties of the metals, especially nickel, which shows the greatest difference between the observed and computed ratio of amplitudes; and there also occur large differences in the magnetic and other properties of different samples of the same metal. On the other hand, the reflecting surfaces are in most cases covered with a thin layer of foreign matter, which, perhaps, may affect to some extent the phenomenon of reflection. These facts might certainly be found sufficient to account for the discrepancies mentioned between experiment and theory.

§ 10. *Propagation of light in any medium and reflection at an interface, as in § 7; a magnetic force being applied in a direction perpendicular to the plane of incidence. A new magneto-optic phenomenon.*

Many authors have asserted that as a consequence of the symmetry of the conditions no magneto-optic effect is to be expected when the magnetic field is perpendicular to the plane of incidence; and, as a matter of fact, no such effect has hitherto been discovered. Yet upon considering this question I have not been able to see the force of the reasons which have led to the denial of any magneto-optic effect produced by such magnetization. Although it is easy to derive from principles of symmetry that a true Kerr-effect can only be caused by *parallel* magnetization, yet the same principle does not by any means exclude *any* effect whatever of perpendicular magnetization upon the reflected light. I have therefore examined what the above theory teaches in relation to this problem, by applying it to the case of perpendicular magnetization.

It appears that our theory indicates that no difference of velocity or absorption between circularly polarized rays of different sense will be caused by this kind of magnetization, nor will there be produced in either of the two principal cases a new component in the reflected light with its plane of polarization perpendicular to that of the ordinary reflected ray. But the theory also indicates that the perpendicular magnetization *should* produce an optical effect of another kind. It should affect in a certain degree the apparent values of the quantities known in the theory of ordinary metallic reflection as the *phase difference* φ and the *reëstablished azimuth* h.

The formulæ by which the alteration in these quantities can be expressed in terms of the Hall-constant and the angle of incidence (φ_1) are

$$h - h_0 = \tfrac{1}{2} \sin 2h_0 \,.\, D_1 \cos D_2, \qquad (102)$$

$$\varphi - \varphi_0 = D_1 \sin D_2, \qquad (103)$$

where

$$D_1 e^{iD_2} = - 2 \mu \lambda^2 \frac{\sin 2 \varphi_1}{\cos^2 \varphi_1 - \lambda^2 \cos^2 \varphi_2}, \qquad (101)$$

while

$$\lambda = \sigma^{-1} e^{-i\tau}.$$

As μ can be expressed in the magneto-optic constants ρ and S, known by observations on the Kerr-effect, and other quantities known by the ordinary metallic reflection, we can make use of (102) and (103) to calculate the values of $\varphi - \varphi_0$ and $h - h_0$ for every angle of incidence.

In this manner I have computed the values given in the accompanying table as the maximum values of the effect to be expected from certain very strong magnetic fields perpendicular to the plane of incidence.

Metal	Maximal *diminution* of *phase difference*			Maximal *augmentation* of *re-established azimuth*		
	Angle of inc. being	Amount		Angle of inc. being	Amount	
		In radians:	In ¼ λ:		In radians:	In min.:
Fe	79°	0.00609	0.0039	71°	0.00582. $\frac{1}{2}$ sin $2h_0$	8.3
Co	77°	255	16	66°	109 "	1.70
Ni	75°	218	14	62°	064 "	1.03

A glance at the numbers in this table will show that even the maximum effect to be expected is in all cases very small. Only in a few cases can it fall within the reach of the common method of observation by polarizer, compensator, and analyzer. In my original paper I have therefore proposed a more sensitive method of observation, by which the effect predicted may perhaps be measured with some degree of exactness. The conclusions of this section have thus indicated a new method by which our theory may be tested.

The author having found no opportunity to undertake any experiments himself in this direction, Dr. Zeeman has kindly tried to detect the new effect on iron mirrors[1] with the apparatus of the Leyden Laboratory, mounted especially for magneto-optical work. In these experiments he employed the old method of observation, which is scarcely sensitive enough for the purpose. Yet Dr. Zeeman has found a slight variation of φ and h; moreover he has found that the effect observed is in complete agreement with theory, not only as to its direction, but even, within the limits of observation, as to its numerical value.[2]

§ 11. *The Faraday effect.*

The rotation of the plane of polarization by thin films of magnetized iron, cobalt, or nickel (discovered by Kundt) may be left out of account for the present, since the theory of such phenomena, although completely contained in our general theory, presents some additional complications. But the same effect in dielectric media, one of Faraday's grand discoveries, is very easily understood and calculated by our theory. The latter gives for the quantity commonly called Verdet's constant the value

$$\omega = \frac{n_0^3}{\Lambda^2} \cdot \frac{q}{V} \cdot 5400, \tag{121}$$

where n_0 is the refractive index of the medium, Λ the wave-length and V the velocity of the light considered, while q is the Hall-constant (here a real quantity) per unit of magnetic force. For carbon disulphide, for example, this formula gives for q, as calculated from the known value of Verdet's constant for this medium and other data, the value :

$$q = 1.92 \times 10^{-4} \; C.\,G.\,S. \text{ units.}$$

§ 12. *Drude's theory of magneto-optical phenomena.*[3]

Between Drude's theory and the one here presented there is only one essential point of difference, Drude introducing into his theory

[1] Zeeman, Versl. Kon. Akad. v. Wetensch. Amst. 5, p. 183, 1896; Communications fr. th. Lab. of Physics, Leyden, No. 29.

[2] We ought to state that other theories of magnetno-optic phenomena, like Goldhammer's and Drude's, might as well as ours have led to the prediction of a magneto-optic effect of the above kind.

[3] Drude: Wied. Ann. 46, p. 353, 1892.

a real quantity as a magneto-optic constant, which should be replaced by a complex quantity in order to cause his theory to agree with ours and with experiment. Drude himself has stated in his more recent papers that his real constant is not quite sufficient to characterize magneto-optic phenomena.

For better comparing our theory with Drude's we write our fundamental equations (*A*), (*B*) and (*C*), after having eliminated $\mathfrak{C}$, in this form :

$$\left.\begin{array}{l}\text{Rot}\ \mathfrak{F} = -\dot{\mathfrak{H}} \\ \text{Rot}\ \mathfrak{H} = 4\pi(p\mathfrak{F} + p^2q[\mathfrak{N}\,.\,\mathfrak{F}])\end{array}\right\} \qquad (122)$$

Then our theory appears to depend formally on an extension of the *second* of Maxwell's equations, written in this form :

$$\left.\begin{array}{l}\text{Rot}\ \mathfrak{F} = -\dot{\mathfrak{H}} \\ \text{Rot}\ \mathfrak{H} = 4\pi p\mathfrak{F}\end{array}\right\}, \qquad (123)$$

this extension being of no consequence at all, when the magnetic force $\mathfrak{N}$ is zero.

The original form of Lorent's theory is declared by Drudenot to be wholly satisfactory,[1] since according to his opinion there are more grounds for explaining the effect of magnetization by extending the first of the equations (123) (as was tried by that author himself), than by extending the second of those equations. As by our extension of Lorentz's theory (our hypothesis as to the constant q being complex) the general character of that theory remains quite unaltered, it may be of some importance to state here that our equations are transformed into those of Drude merely by introducing a new vector $\mathfrak{F}_D$ satisfying the equation

$$\mathfrak{F}_D = \mathfrak{F} + pq\,[\mathfrak{N}\,.\,\mathfrak{F}], \qquad (124)$$

and eliminating the vector $\mathfrak{F}$. By doing so (123) becomes

$$\left.\begin{array}{l}\text{Rot}\,(\mathfrak{F}_D - pq\,[\mathfrak{N}\,.\,\mathfrak{F}_D]) = -\dot{\mathfrak{H}} \\ \text{Rot}\,\mathfrak{H} = 4\pi p\mathfrak{F}_D\end{array}\right\} \qquad (125)$$

and Drude's objection does not hold any longer, while we may as well write $\mathfrak{F}_D$ as $\mathfrak{F}$ in Maxwell's equations (123), these two vectors being identically the same as long as $\mathfrak{N} = 0$. Which of the two

[1] Drude, *l. c.*, p. 376, 377.

vectors $\mathfrak{F}$ or $\mathfrak{F}_D$ is in a magnetized medium considered as the electric force depends entirely upon the way in which we feel inclined to represent to our minds the mechanism of electromagnetic phenomena. In the author's original paper it is shown that to consider $\mathfrak{F}$ as the electric force corresponds to assuming a new system of material points to act upon what we call electricity in consequence of the act of magnetization; while if we consider $\mathfrak{F}_D$ as the electric force this corresponds to assuming that the act of magnetization brings about a change in the *manner* in which the system of material points, which is considered as being endowed with the electromagnetic energy, acts upon electricity.

Drude's theory is less general than ours, since it introduces Maxwell's equations in such a form as to include the hypothesis that the electric current consists of two different parts, one of which obeys Ohm's law, while the other is equal to the time variation of the electric displacement (displacement current). This special hypothesis is not introduced in our theory.

§ 13. *Goldhammer's theory*.[1]

It is not difficult to show that our fundamental equations may be readily transformed into those of Goldhammer and that in this respect our theory is equivalent to his. Yet the theory of the latter, like that of Drude, involves the special hypothesis referred to at the end of the last section.

It is true that Goldhammer denies any direct relation between the Kerr-effect and the Hall-effect,[2] but this is entirely due to the fact that he neglects the probability of the Hall-constant being largely variable with the period of vibration T.

§ 14. *Application of the principle of symmetry to reflection on a metallic mirror.*

We shall assume as evident the following principle: If any system can be considered as its own reflected image with reference to a certain plane V, not only as regards the position of the material points of the system, but also as regards the laws of their mutual

[1] Goldhammer, Wied. Ann., 46, p. 71, 1892.
[2] Goldhammer: *l. c.*, p. 96.

action, then corresponding to every possible state of motion in the system there is another possible state of motion which behaves towards the first as its reflected image with reference to the plane V. From this principle the following theorems are derived, relating to the reflection of plane polarized light either with its plane perpendicular to the plane of incidence or parallel to it (the two principal cases formerly mentioned):

I. If the mirror is not magnetized the plane of polarization of the reflected ray must bear the same relation to the plane of incidence as does that of the incident ray.

II. The same is true when *perpendicular* magnetization is applied; hence this kind of magnetization is only able to produce in the reflected ray a magneto-optic component that is polarized in the same way as the ordinary reflected ray (see paragraph 10).

III. Reflected light which is polarized in the same way as the incident ray cannot be itself modified by *parallel* magnetization; hence this kind of magnetization is able to produce merely a new component with its plane perpendicular to the ordinary reflected ray. As shown by the Kerr-effect it actually does produce such a component.

§ 15. *Applications of the law of reciprocity.*

From this law, which may be extended to magneto-optic phenomena, a fourth theorem may be readily derived.

IV. If polar or equatorial magnetization produces a magneto-optical component with its plane perpendicular to that of the ordinary reflected ray (according to III), the ratio of its amplitude to that of the incident ray must be the same in both the principal cases. In the case of polar reflection the phase differences are also the same in both the principal cases; in the case of equatorial reflection they will on the contrary show a difference equal to π.

This theorem is in perfect agreement with the formulæ previously derived from theory [see (90) and (91)].

Theorems II, III and IV enable us to predict some of the main features of the effect of magnetization on reflected light. Moreover, the direction of the reflected ray is completely determined by the peculiar periodical character of the electromagnetic vibrations.

together with the condition of continuity of the normal component of current. Therefore a special theory is wanted only in order to furnish the values of the ratio of amplitudes and of the phase difference for either of the two principal cases.

§ 16. *Interpretation of the complex nature of the Hall-constant.*

In a short note upon this subject[1] the author has already shown that the interpretation of a complex Hall-constant may be as follows: If it be assumed that the total current $\mathfrak{C}$ consists of two parts,[2] namely, a conductive current $\mathfrak{C}_1$ and a displacement current $\mathfrak{C}_2$, each of which obeys its own law as to the relation between current and electric force, we might attribute a Hall-effect to each of them; but there would be more reason to ascribe a different rotatory effect to the different parts of the current than to ascribe an equal one. We might denote the Hall-constant for the current $\mathfrak{C}_1$ provisionally by h, and for the current $\mathfrak{C}_2$ by k; we then get our equation (C) by putting

$$h = \frac{\rho}{N} \cos S + \frac{2\pi}{NT} \frac{p_2}{p_1} \rho \sin S \tag{135}$$

and

$$k = \frac{\rho}{N} \cos S - \frac{T}{2\pi N} \frac{p_1}{p_2} \rho \sin S, \tag{136}$$

and considering that ρe^{iS} stands for qN (compare equation (82)). A simple discussion of these formulæ shows that the theory which *ab initio* makes h equal to k (corresponding to Lorentz's original theory) will follow from our theory, if we assume that S is equal to zero. It shows further (see the original paper) that Drude's assumption of a real magneto-optic constant corresponds to putting the constant h equal to zero; *i. e.*, to the assumption that the conductive current in the case of optical phenomena shows *no* Hall-effect. J. J. Thomson,[3] upon comparing the Kerr-phenomenon with the Hall-effect, concludes that in the case of light vibrations no Hall-effect is to be

[1] Wind, Versl. Kon. Akad. v. Wetensch. 3, p. 82, 1894; Verhand. d. Physik. Gesellsch. Berlin 13, S. 84, 1894.

[2] Cf. the end of section 12.

[3] J. J. Thomson, Rec. Res., i. El. a. Magn., p, 486, 1893.

ascribed to the conductive current. This is in agreement with Drude's theory, but contradicts the theory here presented and also experience.

§ 17. *A physical interpretation of the fundamental equations for the Hall-effect and the Kerr-effect founded upon Lorentz's theory of the motion of electricity by ions.*

In agreement with the views of Lorentz, we shall now assume that ions exist in the ether without disturbing the contiguity of this medium. These ions may be acted upon and sometimes set into motion by forces depending upon local disturbances in the ether. As to what may be the real nature of these disturbances we do not venture to make any suggestion. We only assume that they may be described by the aid of two vectors called respectively the *dielectric displacement* $\mathfrak{d}$ and *magnetic force* $\mathfrak{H}$.

These fundamental ideas lead to a conception of electric current as the sum of the time variation of dielectric displacement in the ether and of a displacement of electricity.

Lorentz's considerations furnish at once our equations (A), (B), (I) to (VI); equation (C) is thus the only one lacking, and requires further deduction. According to Lorentz we must consider in equations (A), (B), (I) to (VI) the vector $\mathfrak{F}$ to be the force (per positive unit of charge) which, in consequence of the dielectric displacement, is acting upon an ion at rest. This vector is determined by the equation

$$\mathfrak{F} = 4\pi V^2 \mathfrak{d} \tag{138}$$

An ion in motion is, however, acted upon not only by this force $\mathfrak{F}$, but also by an additional force, which we may call the electrodynamic force, and which is represented by the vector product of the velocity and the magnetic force $\mathfrak{H}$ at the point. This additional force is undoubtedly insignificant as long as we deal with ordinary velocities of ions, and with those magnetic intensities which result from the electromagnetic disturbances constituting light. But it acquires a sensible value when the ions are moving, a very strong magnetic field being superposed. If the magnetic force in such a field is called $\mathfrak{N}$, we have for the total force $\mathfrak{E}$ acting upon each positive unit of charge moving with the velocity $\mathfrak{v}$

$$\mathfrak{E} = \mathfrak{F} + [\mathfrak{v}.\mathfrak{N}] \tag{139}$$

Now it is not difficult to show that equation (C) which contains the mathematical description of the Hall-effect, may be deduced from the following simple hypothesis, viz.: *The mean velocity of the positively charged ions (kations) is not exactly opposite to that of the negatively charged ions (anions).*

We have already shown that the Kerr-effect is included in the same equation (C). It is only necessary to make q a complex quantity in order to get a theory in complete accordance with observation. Now this complex nature of the Hall-constant q can be shown to follow from this new hypothesis, namely, *that the ions existing in a metallic medium are of two different classes.*

Indeed, we may imagine, I think, as existing in the metallic medium in addition to those ions to which special attention is paid in Lorentz's paper, and which I shall call *dielectric ions*, another class of ions, which I shall call by the name of *conductive ions*. These are distinguished from the first kind by the fact that in consequence of their motion they give rise to a resisting force proportional to their velocity. We shall call e the charge of an ion, and n the number of ions in unit volume, and write ε' for $\frac{1}{n}\frac{\overline{\mathfrak{v}}}{\overline{e\mathfrak{v}}}$[1] in the case of conductive ions, and ε'' for $\frac{1}{n}\frac{\overline{\mathfrak{v}}}{\overline{e\mathfrak{v}}}$ in the case of dielectric ions, where the horizontal line is to indicate a certain mean value of the quantity considered; again we shall call $\mathfrak{M}'$ the displacement of electricity ("elektrisches Moment") due to the motion of the first class of ions, and $\mathfrak{M}''$ that due to the motion of the second class. We thus distinguish quantities relating especially to one or the other class of ions by the single or the double accent. We then have

$$\mathfrak{E}' = \mathfrak{F} + \varepsilon' [\mathfrak{M}' . \mathfrak{N}], \tag{141}$$

$$\mathfrak{E}' = \mathfrak{F} + \varepsilon'' [\mathfrak{M}''. \mathfrak{N}]. \tag{142}$$

We cannot give a rigorous deduction of the equation of motion for the separate ions. Yet by a plausible train of reasoning we are led to assume them to be of this form

[1] By a mistake the number n of ions is left out of consideration in the original (Dutch) paper.

$$\mathfrak{E}' = (\xi + \eta\partial)\,\mathfrak{M}', \tag{145}$$

$$\mathfrak{E}'' = \varsigma\,\mathfrak{M}'', \tag{144}$$

where ξ, η and ς are constant quantities relating to the medium considered.

As according to our above conception of electric current

$$\mathfrak{C} = \dot{\mathfrak{d}} + \dot{\mathfrak{M}}' + \dot{\mathfrak{M}}'', \tag{146}$$

we find by (145), (144), (141), (142) and 138)

$$\mathfrak{C} = p\,\mathfrak{F} + r\,[\mathfrak{N}\,.\,\mathfrak{F}], \tag{C'}$$

where is put

$$\partial\left[\frac{1}{4\pi V^2} + \frac{1}{\xi + \eta\partial} + \frac{1}{\varsigma}\right] = p \tag{150}$$

and

$$-\,\partial^2\left[\frac{\varepsilon'}{(\xi + \eta\partial)^2} + \frac{\varepsilon''}{\varsigma^2}\right] = r. \tag{151}$$

By this way we see our considerations lead to the equation (C′), which is entirely equivalent to our fundamental equation (C).

Although in the original paper other conclusions will be found mentioned, I shall mention only one of them here. The considerations of the last few pages may be immediately applied to the electrical phenomena in electrolytic solutions They furnish the following expression for the Hall constant in such a medium :

$$q = \frac{1}{n\,[e]}\,\frac{v - u}{v + u},$$

if $[e]$ denote the ionic charge apart from sign, and u and v the "Wanderungsgeschwindigkeiten" of kations and anions.