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THE EFFECTS OF TENSION AND QUALITY OF THE METAL UPON THE CHANGES IN LENGTH PRODUCED IN IRON WIRES BY MAGNETIZATION.

By Byron B. Brackett.

Historical.

FIFTY years ago a machinist of Manchester imagined that he could see the volume of a mass of iron increase when it was magnetized and decrease when the magnetizing force was removed. Hoping to be able to use this principle in the construction of an electro-magnetic engine, he appealed to Dr. Joule[1] to investigate the phenomenon and determine the amount of the change. By immersing the iron to be magnetized in a closed vessel, filled with water, in which stood a fine capillary tube Joule could not detect any change of volume, though it has since been shown[2] that had he used either stronger or weaker fields he probably would have done so. But by a system of compound levers of great multiplying power he proved that an iron bar did change it length when magnetized longitudinally. He observed an increase in length of 1–200,000. As a result of his investigations he proposed the following laws:

1. When soft iron rods are magnetized their length increases

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[1] Joule, Phil. Mag., (3), vol. 30, pp. 76, 225.

[2] Bidwell, Proc. Roy. Soc., vol. 56, p. 94.

and the elongation is approximately proportional to the square of the magnetizing force.

2. Tension applied to the rod diminishes the elongating effect.

3. The elongation is greater for the same intensity of magnetism in proportion to the softness of the metal.

That the two first laws are correct for the fields that Joule used cannot be doubted; but as to the third law there is much uncertainty. The investigations of Shelford Bidwell[1] indicate that not only hardening, but also annealing an iron rod diminishes the elongating effect, and at the best the relation between the softness of the iron and its change of length is to-day very much confused.

It was nearly twenty-five years after Joule's investigations before the question was taken up again experimentally by Barrett[2] and nearly at the same time by Mayer.[3] Barrett employed the tilting mirror, which is described under the apparatus used in this investigation, a device suggested to Barrett by Professor Rowland. He experimented not only upon iron, but also upon nickel and cobalt.[4] He observed an elongation of 1–260,000 for iron and 1–425,000 for cobalt; and a contraction of 1–130,000 for nickel.

Mayer found an elongation of 1–277,000 for iron. Some of his observations upon the action of hard steel seemed to be at variance with Joule's results. But Bidwell has since shown that this apparent difference was due solely to their different methods of experimenting.

In 1885 Bidwell[5] reported the first of a very extensive series of experiments upon the distortions caused by magnetization in iron, nickel and cobalt. He carried his investigations up to fields many times stronger than those used by the earlier investigators. He has worked upon the effects of tension, tempering and annealing. He has experimented with both rods and rings. He found that, at least with his apparatus, rods did not continue to elongate, but reached a minimum length, then gradually shortened until they had less than

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[1] Bidwell, Proc. Roy. Soc., vol. 55, p. 228.

[2] Barrett, Phil. Mag., 1874, vol. 47, p. 51.

[3] Mayer, Phil. Mag., 1873, vol. 45, p. 350; Mayer, Phil. Mag., 1873, vol. 46, p. 177.

[4] Barrett, Nature, 1882, vol. 26, p. 585.

[5] Bidwell, Proc. Roy. Soc., 1885, vol. 38, p. 265; 1886, vol. 40, pp. 109, 257; 1888, vol. 43, p. 407; 1890, vol. 47, p. 469; 1892, vol. 51, p. 495; 1894, vol. 55, p. 228; 1894, vol. 56, p. 94; Phil. Trans. Roy. Soc., 1888, vol. 179 (A), p. 205.

their initial length, apparently approaching a limiting value asymptotically. Investigations upon this subject have also been made by Berget,[1] Bock,[2] Jones,[3] Knott,[4] Lochner,[5] Nagaoka;[6] and two years ago investigations in this line were begun here, in the Johns Hopkins laboratory, by Dr. L. T. More.[7]

At Professor Rowland's suggestion, Dr. More determined the intensity of magnetization in his wires for each change of length observed, and also sought to take into consideration the secondary actions that might affect the length of the wire.

Professor Rowland defends his position on the subject as follows:

The change in length may be partly due to other causes than the magnetization of the metal. Among these one can put the stresses due to magnetization. It is not at all evident that these stresses can be exactly identified with the Maxwellian stresses. If we think of the long wire that More used as composed of a bundle of small elementary magnets tied together at points well inside the poles, the magnets would seem to have no tendency in their central parts to separate from each other; and in that case there would be no pressure at right angles to the lines of induction, unless it can be shown to result from a squeezing outwards of some kind of matter caused by a longitudinal compression.

At the same time, the compressive force which, it is known, will tend to close up a very thin air-gap in a divided magnet must also exist in any magnet, for, according to all our ideas of matter, there is no real difference in the case where the air-gap exists and where it does not; because we still must consider the gaps between the molecules. If we now think of the long elementary magnets as composed of short elementary pieces with their ends so near together that the effects of their poles are neutralized in all action

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[1] Berget, Comp. Rend., tom. 65, p. 722.

[2] Bock, Wied. Ann., 1895, vol. 54, p. 442.

[3] Jones, Phil. Mag., 1895, vol. 39, p. 254.

[4] Knot, Phil. Mag., 1894, vol. 37, p. 141; Proc. Roy. Soc. Edinburg, vol. 18, p. 315; vol. 20, p. 290; vol. 20, p. 334.

[5] Lochner, Phil. Mag., 1893, vol. 36, p. 504.

[6] Nagaoka, Wied. Ann., 53, pp. 481, 487; Phil. Mag., 1894, vol. 37, p. 131; 1896, vol. 41, p. 454.

[7] More, Phil. Mag., 1895, vol. 40, p. 345; Phys. Rev., 1895, vol. 3, p. 210.

upon external bodies and yet allow a space between their ends for a compressible medium to entirely surround them, then this longitudinal pressure would cause both longitudinal shortening and a pressure outwards perpendicular to the induction. It would seem that such a case might represent both the strain in the medium and the strain in the ether.

The value of this compressive force is probably $\frac{B^2}{8\pi}$. It may possibly be $\frac{(B-H)^2}{8\pi}$, but in most cases the two are so nearly equal in value that it would not seriously alter the results to take either force. Taking this as equivalent in its action to a simple mechanical pressure and considering, with it, the ordinary elasticity of the wire, the shortening due to this cause can be computed for each observation and corrections may be made accordingly. Or this force might be considered to neutralize a portion of the tension on the wire equal to it in value.

For reasons like these, Professor Rowland advised Dr. More to correct his observed readings of elongation for a shortening caused by the force $\frac{B^2}{8\pi}$; and these corrected readings were plotted to a basis of induction in the iron instead of being given on a basis of the magnetizing force, as all previous curves in this subject had been given, except those of Nagaoka.

Last year Dr. E. F. Gallaudet[1] made an investigation on this subject in this laboratory. He used the apparatus employed by Dr. More, and his curves were in all respects except one similar to those obtained by Dr. More, Bidwell and others. He did not correct them for a contraction caused by a force of $\frac{B^2}{8\pi}$.

An initial contraction was observed by him that no one has announced before, and he also gave observations that seemed to indicate a very great change in the values of Young's modulus as the magnetization increased. That there could be no real change in the elasticity so large as these, can be shown from Dr. Gallaudet's own tables; for such changes in value of Young's modulus under

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[1] Gallaudet, J. H. U. Thesis, June, 1896.

the larger tensions used would have caused changes of length in the wire many times greater than the changes observed. Moreover, Bock[1] found as a result of his experiments that the changes in elasticity due to magnetization could not exceed one-half per cent.; and Miss Noyes,[2] in a series of experiments upon the changes of Young's modulus with temperature, did not find any evidence of a change of the elasticity from magnetization.

Object of this Investigation.

It was hoped that by carefully observing the behavior of quite a number of different wires under varied physical conditions data might be obtained that would explain the relation of the elasticity of the metal to its change in length under magnetization. It was also hoped that incidentally the question of an initial contraction preceding elongation, as observed by Dr. Gallaudet, might be settled; and that likewise some additional information on the relation of the temper of the metal to its change of length might be obtained.

Apparatus.

I employed the apparatus used by Dr. More and by Dr. Gallaudet. It is very fully described by Dr. More, who[3] gives all the dimensions essential to its construction. The feature that is essentially characteristic of this apparatus, in addition to Professor Rowland's tilting mirror that both Barrett and Bidwell used, is the cylindrical jacket which was suggested by Dr. Ames. The figure here given will show its use. About the part of the wire where the changes in length are to be observed is placed a brass tube having a free internal diameter of over one centimeter. At the bottom end *b* the tube is clamped firmly to the wire while the upper end has a loosely fitting cork merely to keep the axis of the cylinder and the wire coincident. To the top of the cylinder is attached a projecting arm that carries two raised supports *a* and *c*. Above this arm is placed a lever *d*, resting by a knife-edge on the support *a*. This lever also has an inverted knife-edge but a very short

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[1] Bock, *loc. cit.*

[2] Mary C. Noyes, Phys. Rev., 1896, vol. 3, p. 432.

[3] More, *loc. cit.*

distance back of the one that supports its weight, and over the ininverted knife-edge is placed a hook that is firmly clasped to the wire at *g*. Thus any change in the length of the wire between the points *b* and *g* will cause the outer end of the lever to rise or fall. Standing partly on the end of this lever and partly on the support *c* is a little table having three needle-point legs and carrying a vertical plane mirror. Now any movement of the lever arm relative to the support *c* will cause the mirror to tilt; and by observing with a telescope at some distance the image in the mirror of a ver-

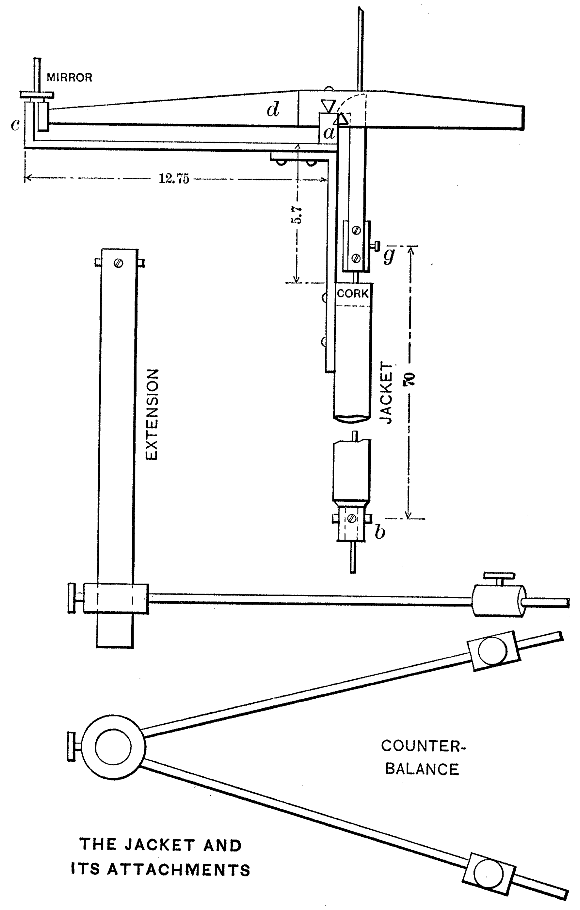

Fig. 1.

tical scale the movement of the lever arm will be greatly multiplied.

If,

$L =$ length of the long arm of the lever,

$l =$ length of the short arm of the lever,

$d =$ distance from one leg of the brass table to the line joining the other two legs, resting on lever end,

$D =$ scale distance from the mirror, then the multiplying power of the apparatus is

$$\frac{L}{l} \times \frac{D \times 2}{a}.$$

$L = 11.672$

$l = 0.4776$

$d = 0.3335.$

D differed slightly for different tests, but was nearly always as great as 170. Thus the multiplying power of the apparatus was always approximately 25,000.

To determine the elongation caused by the tensions used in most of the tests, and to check the values of the modulus of elasticity determined from adding small weights, the mirror at *c* was removed, and another mirror was attached to the lever *d* just above the point *a*, so that the apparatus acted as a simple multiplying lever. This arrangement gave a multiplying power of only $2\frac{D}{l}$, or about 700, and so was capable of being used for much greater elongations.

The wire to be tested is always suspended from a point some distance above the apparatus in such a way that the jacket has its ends well within the magnetizing solenoid. The solenoid rests upon a support entirely independent from that of the wire and those parts of the apparatus attached to the wire; and the two parts are so adjusted that there is no contact between them.

I made but two changes in the apparatus itself that could be supposed to affect the results. In the apparatus as used by Dr. More and by Dr. Gallaudet, the jacket and its attachments were not perfectly balanced. The lever *d* was as nearly balanced as was desirable to have it. But all the weight of the lever came on one side of the wire; and that together with the weight of the arm under it, gave quite a tendency to the jacket to tip sidewise and slightly bend the wire. Then the amount of this bending would doubtless be changed by the magnetic stresses; and especially when no weight was attached to the lower end of the wire, it seemed that this might possibly have quite an effect upon changes so small as those to be observed. An adjustable extension was therefore made to attach to the lower ends of the jacket, extend well down below the magnetizing solenoid, and by means of sliding weights on its two arms exactly counterbalance the moment given the jacket by the arm at the top, together with the lever and mirror. Again, the jacket had been clamped, at its bottom, to the wire by a single set-screw. This it seemed might cause a bend in the wire. The brass plug *b* was therefore bored out quite a little larger than the wire. Two extra radial set-screws were put into *b*, making three equally spaced ones about the circumference. Then a little cylinder of brass, slit lengthwise on one side, bored along its axis to fit the wire and just large enough to slip into the hole in *b*, was put about the wire at the point of attachment. In this way by the use of the three screws the wire could be accurately centred in the jacket.

These precautions seemed necessary, for a slight, almost imperceptible bend in the wire was found by actual experiment to greatly modify the readings obtained. These adjustments were so delicate that it was scarcely possible to judge when they had been properly made except by actual trial. After Young's modulus had been carefully determined, it was considered a fair test of the adjustment of the apparatus to put on and off a weight that ought to cause changes in length approximately equal to those expected to follow from magnetization. If the computed readings were obtained, the adjustment was assumed to be correct.

A most evident source of error in this work is the change of length caused by temperature. The greatest value of $\frac{dl}{l}$ observed in my investigation which seemed to be due to magnetization was 33.92×10^{-7}. A change of one degree centigrade in the temperature of the wire would give for $\frac{dl}{l}$ about $120. \times 10^{-7}$. To prevent temperature changes the magnetizing solenoid was made originally from two coaxial brass cylinders in such a way as to leave a space for water between them. In all the experiments of this investigation water was being continually forced into this space from the bottom and was overflowing at the top. Thus there was a jacket of continuously changing water between the magnetizing coils and the jacket attached to the wire. Yet if the larger currents were left on even for a short time the effects of temperature changes were quickly visible. If the conditions have become steady and all the apparatus has attained a constant temperature, and then a current is put on and allowed to remain, the resulting higher temperature first affects and expands the jacket on the wire, causing an apparent *contraction* of the wire itself, and as the higher temperature reaches the wire *expansion* will be observed until at last, in the steady state resulting, the wire will have apparently expanded or contracted according to the relative coefficients of heat expansion for the wire and for the jacket. Fortunately, these changes are comparatively slow and can be separated with a limited degree of accuracy from readings that can be taken quickly. But if one were to take a series of readings from the modulus of elasticity, using small changes of weight while the magnetizing current remained on, it might be foreseen that the results would apparently vary first in one direction and then in the other, due to the unequal changes in the length of the jacket and the wire caused by the heating.

Incident to the great multiplying power of the apparatus, the slightest mechanical vibration of the wire under test made it absolutely impossible to read the changes of length accurately, and, in many cases, to read the figures of the scale at all. To get a support for the wire as free from vibrations as possible, a weight of some seventy-five or eighty pounds was made by filling a wooden box with bricks and old storage battery plates, and this was suspended by a single coiled brass spring to an arm projecting from a side-wall of the room some ten feet above the apparatus. The spring was so designed that, with the weight it carried, its period of vibration was very slow, and when the wire to be tested was suspended from the bottom of the weight very little trace of the ordinary vibrations of the building ever reached the wire. But to have the system work perfectly, it was necessary that the wire and the weight which it carried be attached exactly under the center of mass of the large weight above. To facilitate this adjustment four projecting arms were attached to the bottom of the box forming the weight and on these arms were hung small movable baskets of shot. When all was properly adjusted the scale image in the telescope would be blurred for a half-minute by a wagon passing on the cobble-stone pavement some twenty or thirty feet from the apparatus, but otherwise all was undisturbed.

General Method of Working.

It was decided that, for each kind of iron tested, series of elongation readings should be taken for several different tensions, that the elongations should be observed at each point both with the magnetizing field on and with the field off, and that for all observations of elongation the corresponding inductions in the iron should be obtained.

When the wire had been properly adjusted in the apparatus, with a weight attached to the lower end of the wire to give the desired tension and after the test with the small weight already mentioned had been made, the first elongation reading was taken by sending a very weak current through the magnetizing solenoid, quickly observing the scale reading in the telescope and then breaking the current and reading the telescope again. This procedure was repeated with gradually increasing values of the current until the maximum current was reached. The elongation readings obtained with the field on, I shall call the "total" elongation, since it represents both the temporary and the permanent or residual change in length. The elongation observed after a certain magnetizing field has been put on and then taken off, will be called the "residual" elongation for that definite field.

All the currents used in magnetizing were left on only long enough to get the required readings accurately. A very careful watch was kept for temperature changes and when they began to appear, the readings were stopped until all had cooled down to the steady state again. The currents used were measured by carefully tested Weston instruments of different capacity according to the currents to be read.[1]

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[1] See end of article for notation used in tables and plates.

The elongation data given to this investigation were all taken the first time the wire was magnetized after it had been put into the apparatus. No method seemed efficient in completely demagnetizing the wire while it was in place in the apparatus. Alternating currents would not do it, probably because the ends of the wire extended so far beyond the solenoid. After using the alternating current, the ends of the wire would still show a decided polarity of the same kind that it showed before, and the effects upon the elongation curve were very marked. If the magnetizing field was applied again in the same direction as before the curve was very similar to the one obtained at first though the amount of the elongation was somewhat reduced. But if the field was applied in the opposite direction the first effect with weak fields was a contraction, followed by a decrease of the contraction, then by elongation and the rest of the curve was like other curves for the same wire.

The inductions corresponding to the elongations were obtained from a separate series of readings by means of the ballistic galvanometer, using the method of increasing reversals and breaking the current to get the difference between the total and residual inductions. It was exactly the method used by Professor Rowland[1] in his well known ring experiments. Though actual test had shown the field to be practically uniform along that part of the solenoid where the elongation of the wire was measured, yet, as a precaution against irregularities in the induction of the wire, the test coil was distributed in four equal parts equally spaced along the portion of the wire under observation. For the lower inductions seven hundred turns were used, while for the higher inductions this number was reduced to four hundred turns. The galvanometer with its circuit was calibrated for each connection; and, moreover, the ratio of the deflections obtained by the two different arrangements was always taken each time the change was made.

With soft iron wires the alternate current was applied after the elongation readings had been taken and the induction readings were then made from the same wire. While the elongation curve would have been quite different under the second magnetization, yet actual test showed that the first induction curve taken for any wire did not

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[1] Rowland, Phil. Mag. (4), vol. 46, p. 140.

differ from subsequent ones. For the hard wires tested the induction curves were taken from different wires apparently exactly like the ones used for the elongation tests.

WORK DONE.

During my investigation I made not less than thirty-five elongation tests. All of the more important series of readings were repeated, with different wires, several times, in order that none of the curves should represent chance results. No series given in the tables rests upon the evidence of a single experiment. Tests are reported upon piano-wire in its natural condition under two different stresses, annealed piano-wire under three different stresses and soft annealed iron wire under four different stresses, making nine series in all.

The lower tension used for natural piano wire was the smallest that would apparently free the wire from bends, for it could not be obtained in a perfectly straight form. The larger tension was a little more than half way to the elastic limit. These wires have a diameter of 1.25 mm. The same wires used in these tests were then heated to a bright red by passing a current through them and were allowed to cool slowly in the air. The slightly burned outside of the wires was then carefully removed by the use of fine flint-paper, after which they were tested under the same tension as before. Since the wires came out of the annealing process perfectly straight it was possible with these wires to make a test under the tension caused by the apparatus alone.

But neither with the natural nor annealed piano wire were the changes caused by magnetization very great, or the effects of a permanent strain especially marked. The most extensive investigation was therefore made upon very soft annealed iron wire. The different pieces were cut from one continuous piece, and parts taken out at different places were analyzed by Mr. Nakaseko of this university, and were pronounced by him to be very pure and quite free from carbon as well as from all other impurities. All of these wires were heated to the bright red state just as in the case of the annealed piano wire, but here it was only to free them from kinks. They gave the same magnetization curve before and after this process; and while the same elongation curve could not be obtained in the two cases, all of my work would lead me to think that the difference was due solely to the bends in the wire before the heating process.

This wire as tested had a diameter at 1.31 mm. Its behavior was investigated under five different tensions, ranging from that due to the apparatus only up to the elastic limit of the wire. Under the largest tension the wire would elongate the usual amount upon the addition of a very small weight, but would require quite a little time to recover when this weight was removed. Possibly because the elastic changes were so slow under this tension, it was impossible to take both the total and residual elongation curves. The best that could be done was to keep the current on all the time and correct as well as could be done for the temperature changes to obtain the curve for total elongation.

Not only were more tests made on this kind of iron, but the readings obtained have been worked out, somewhat more fully. Elongation curves from these results have been plotted both to H and to I, and in both cases the contractions that Professor Rowland believes will be caused by the force $\frac{B^2}{8\pi}$ have been plotted in a separate curve and the elongation curve as modified by this contraction has also been given. For the residual curves this supposed contraction is computed from $\frac{(4\pi I)^2}{8\pi}$, the residual value of I, of course, being taken.

The value of Young's modulus used in computing the contraction curves for this iron was 2.12, and was determined both from small changes of tension, using the higher multiplying power and from larger changes of tension using the lower multiplying power. The latter method was considered the more reliable, especially in determining the elongations due to large tensions. But this method could not be used to investigate the possibility of a change in modulus caused by magnetization, since it required so much time to adjust the weights that temperature effects from the magnetizing current would surely modify the results. By using the smaller changes of tension, no variation in the modulus could be detected at any stage of magnetization, except such as were within the limits of error of the method.

It was desired to take the induction readings for exactly the same values of the magnetizing current that had been used in the elongation tests; but on account of the bad condition of the storage batteries that I had to use during most of my work, I could only make the currents approximately equal in the two cases. The points on each curve, especially where there could be any doubt as to the exact location of that curve, were taken very close together; and then through interpolation on the curves, the corresponding values of elongation and induction could be found with as great accuracy as the curves could be plotted.

GENERAL RESULTS.

Speaking very generally, all the curves of elongation when plotted to H show either no change of length up to nearly the field at which the magnetic saturation occurs or up to that point their elongation curve is similar in shape to the induction curve. Not far beyond this the residual curves become practically horizontal and show no further change in length of any consequence; while the curve of total elongation begins at nearly the same point to descend along an approximately straight line. The residual curves for soft iron seem to fall quite a little after the higher fields have been applied. But while there seems to be no doubt that a slight shortening of the wire occurs, due probably to a rearrangement of the molecules under the strain of the higher fields, yet, I am sure that this shortening is exaggerated in my readings by temperature effects that could not be avoided. All the changes of length due to any change in the magnetic conditions are very quick; and the almost instantaneous throw of the lever and mirror from one position to another, when quite large, as it was here, caused vibrations that required a little extra time to die out, and consequently allowed some heating of the jacket to occur. All attempts to damp this vibration involved the possibility of displacing parts that must move so freely as the lever and the mirror; and to put the current on and off gradually involved about the same possibility of heating as did the time required for the vibrations to cease. It thus seemed best to make these readings as well as could be done and then give them with the above caution. In some cases I have drawn the ends of the curves somewhat above the plotted points, thinking them much more nearly correct there than if drawn through the points. For the same reason I think it probable that the ends of the total elongation curves should show a marked tendency to curve upwards from the straight line, as has been found by Bidwell.[1]

Effect of the Field.

In any case the residual curve shows that for quite a distance beyond the point of saturation no permanent change of length occurs; and the curve of total elongation shows, through these same field values, a contraction directly proportional to the field strength. Thus these two curves, at least along this portion, seem to show clearly that there is an elongation due directly to the induction in the iron and a contraction caused by the field, directly proportional in value to the field strength, as has been suggested by Dr. Galludet.[2] Every set of curves given in this investigation seems to show a contraction due to the magnetizing field. Where plotted to I, the elongation, in any one test, is always greater for the same value of induction when the field is off than when it is on; and the curves modified by the supposed contraction due to $\frac{B^2}{8\pi}$ and $\frac{(4\pi I)^2}{8\pi}$, both when plotted to H, and when plotted to I, indicate the same thing.

Increased tension on the wire apparently causes the contraction to begin at a lower field and at a lower induction in the iron; but when the elongation reaches the straight line portion of the total elongation curve plotted to H, it continues on the same slope without any regard to the tension upon the wire. In other words the straight portions of all these curves for the same kind of wire are parallel. For another kind of wire the slope of this portion of the curve will be entirely different; but here again the curves for this wire, taken under different tensions, will have parallel parts. This would seem to indicate that the contraction caused by the field is not only proportional to the field, but that it depends also upon some definite constant for each kind of iron. For the three different kinds of iron that I have studied these constants would have to be about in the ratio of 12.5, 18 and 29 for the natural piano wire, an-

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[1] Bidwell, *loc. cit.*

[2] Gallaudet, *loc. cit.*

nealed piano wire, and the soft iron respectively. It does not seem possible to identify these values directly with either the elasticity or the magnetic permeability of the wires. The curves as modified by the $\frac{B^2}{8\pi}$ correction would have a slightly smaller absolute value for these constants, but the ratio of their values would be very nearly the same.

Looking at the total elongation curves plotted to I, it will be seen that the retraction of length does not appear until the value of μ becomes very much reduced, or until a relatively large increase of the field is necessary for a small increase of induction in the wire. As the field necessary to give a certain increase of induction becomes larger and larger, the curves show a more and more rapid contraction. Since this part of the curve gradually becomes more and more nearly vertical it seems evident that it must approach asymptotically the vertical line drawn through the maximum or limiting value of I, which for this soft iron would doubtless be about 1700.[1] This would indicate that there is no limit to the contraction which is caused by the field, but the contractions resulting from a definite amount of field increase might finally become less and less, as Bidwell[2] has observed.

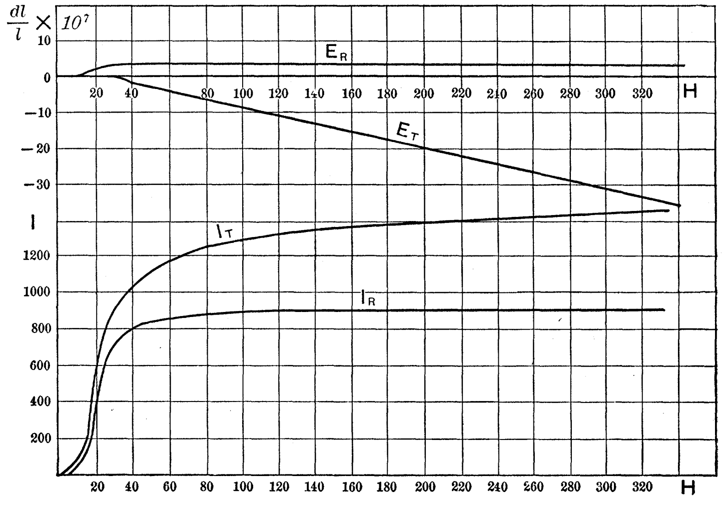

Plate 1.
Natural Piano Wire: Tension 658. (See Tables 1 and 2.)

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[1] Ewing, Magnetic Ind., p. 145.

[2] Bidwell, *loc. cit.*

TABLE I.

Elongation data for piano wire in natural state and under tension of 658 *kg. per square cm.*

$\frac{dl}{l} \times 10^7$ due to this tension = 2933.

H	E_T	E_R	H	E_T	E_R
.914	.00	.00	59.49	−3.92	3.36
1.83	.00	.00	69.50	−5.04	3.36
2.74	.00	.00	79.50	−6.16	3.36
4.11	.00	.00	94.15	− 7.28	3.36
5.49	.00	.00	104.2	− 8.67	3.36
8.23	.00	.00	116.1	−10.09	3.36
9.14	.00	.00	132.7	−11.78	3.36
13.09	.00	.56	143.1	−13.15	3 36
15.90	.00	1.12	155.5	−15.12	3.36
19.68	.00	1.68	167.3	−16.24	3.36
22.41	.00	2.24	183.0	−17.92	3.36
26.85	.00	2.80	203.5	−20.73	3.36
29.95	−0.56	3.36	219.6	−21.85	3.36
33.15	−0.84	3.36	240.0	−24.66	3.36
38.10	−1.40	3.36	260.6	−26.90	3.36
45.08	−2.52	3.36	278.8	−29.68	3.36
48.72	−3.08	3.36	301.8	−31.39	3.36
53.50	−3.47	3.36	338.5	−35.30	3.36

EFFECTS OF TENSION.

As to the effects of tension, it will be seen that up to the limit of my magnetizing field, the numerical sum of the elongation and contraction is nearly the same for all tests of the same kind of wire regardless of the tension used. But the greater the tension the less will be the elongation and the greater the contraction obtained. Whether we examine the curves as plotted to H or to I, we shall see that after the contraction begins, the length of the wire as compared to its length before magnetization, becomes less as the tension is greater. That is, if we disregard, for the moment, the elongations caused by the tensions used, and consider the wires to all have equal lengths when magnetization begins, then we may say that on the parts of the curves which we are considering, for equal fields or for equal inductions, the greater the tension the shorter the wire. For example, if we examine the total elongation curves, as

Table II.

Magnetization data for wire in Table I.

H	B	μ	I_T	I_R
1.83	98	53.6	7.7	.0
2.75	169	61.5	13.2	.75
6.40	459	71.7	36.1	6.4
8.09	707	87.4	55.6	14.1
10.75	982	91.4	77.3	25.1
12.71	1374	108.1	108.3	36.1
15.50	2394	154.6	189.2	99.2
19.45	5814	299.5	460.5	339.5
21.98	7747	353.0	615.2	488.0
26.65	10167	381.5	807.5	652.5
29.52	11020	374.0	874.5	709.0
32.82	11843	361.2	939.5	751.0
37.65	12578	334.0	999.9	794.0
40.73	13091	322.0	1038.	805.
44.45	13464	303.0	1069.	830.
48.00	13798	287.8	1095.	839.
52.75	14133	268.1	1120.	861.
58.35	14608	250.5	1159.	864.5
66.65	15117	227.0	1199.	876.
76.50	15637	204.5	1238	883.
91.42	16141	177.0	1238	890
101.2	16351	161.5	1294	896
112.5	16663	148.2	1318	898
128.4	16848	131.2	1331	900
139.2	16969	121.9	1338	900
151.0	17271	114.4	1363	902
165.0	17345	105.2	1368	902
178.2	17528	98.4	1381	904
198.3	17598	88.8	1385	904
217.4	17937	82.4	1412	906
238.0	18188	76.4	1428	908
256.2	18206	71.1	1428	908
274.5	18375	66.9	1440	910
297.3	18447	62.2	1445	910
331.8	18582	56.0	1453	913

plotted to H, for soft wire under least and under greatest tension we shall see that, after the curves begin to descend, all points on the curve of greatest tension are about 40 units below those of least tension. But before any change of length due to magnetization began the elongation caused by the tension, expressed in the

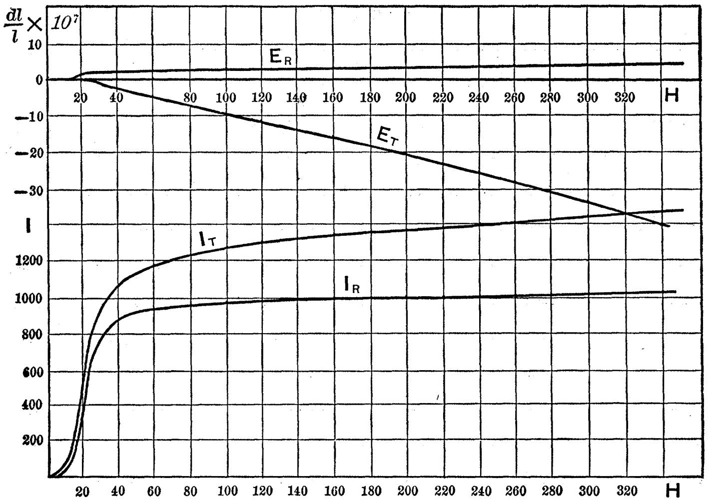

Plate 2.

Natural Piano Wire: Tension 1949.

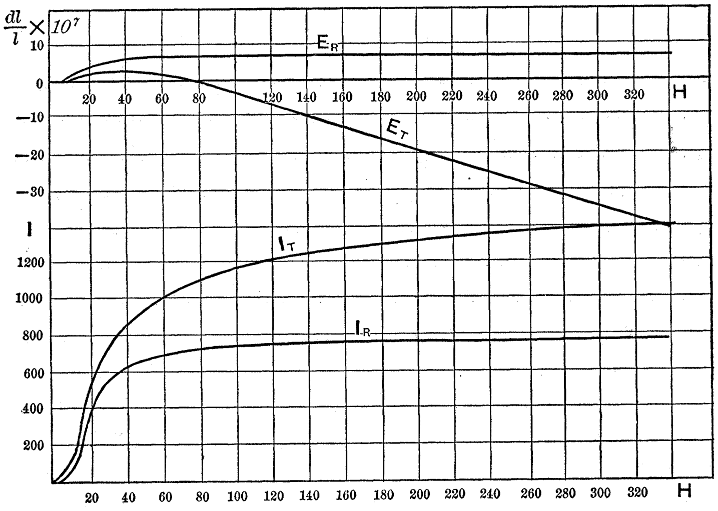

Plate 3.

Annealed Piano Wire: Tension 62.

TABLE VII.

Elongation data for annealed piano wire under tension of 699 *kg. per square cm.*

$\frac{dl}{l} \times 10^7$ due to this tension $= 3178$.

H	E_T	E_R	H	E_T	E_R
.914	.00	.00	52.18	−0.567	5.39
1.830	.00	.00	58.10	− 1.42	5.39
2.74	.00	.00	69.45	− 3.42	5.39
3.66	.00	.00	78.22	− 4.54	5.39
4.57	.00	.00	93.20	− 7.37	5.39
5.49	.00	.00	103.3	− 9.06	5.39
8.14	.00	.17	114.7	−10.78	5.39
9.23	.17	.567	131.2	−14.17	5.39
11.95	.567	.850	141.6	−15.88	5.39
12.81	.850	1.42	157.9	−18.70	5·39
15.54	1.70	2.27	166.4	−21.00	5.39
19.69	2.55	4.25	182.4	−23.25	5.39
21.99	2.84	4.82	203.2	−27.20	5.39
27.00	2.84	5.11	219.2	−30.61	5.39
29.72	2.27	5.11	237.9	−32.88	5.39
32.90	1.99	5.39	258.2	−36.84	5.39
37.92	1.70	5.39	274.2	−40.26	5.39
40.98	1.13	5.39	304.1	−44.22	5.39
44.38	.567	5.39	340.8	−49.02	5.39
48.00	.00	5.39			

same units used in the curves was 7950 in one case and only 223 in the other case. What probably occurs is a reduction of the original elongation of 7950 units by 40 units, and beyond this all the phenomenon is very nearly as in a case of no tension. Suppose, then, the wire to become less elastic with magnetization, and that the value of Young's modulus has increased by about one-half per cent.[1] when the turning point in the magnetization curve is reached; then the elongation that originally existed will be diminished by one-half per cent., and we have explained the greater contraction resulting from magnetization when a wire is under tension than when it has no tension. This change in the modulus of elasticity will explain the reduced elongations of all other curves in the soft iron series; and a similar explanation will apply to all curves

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[1] Bock, *loc. cit.*

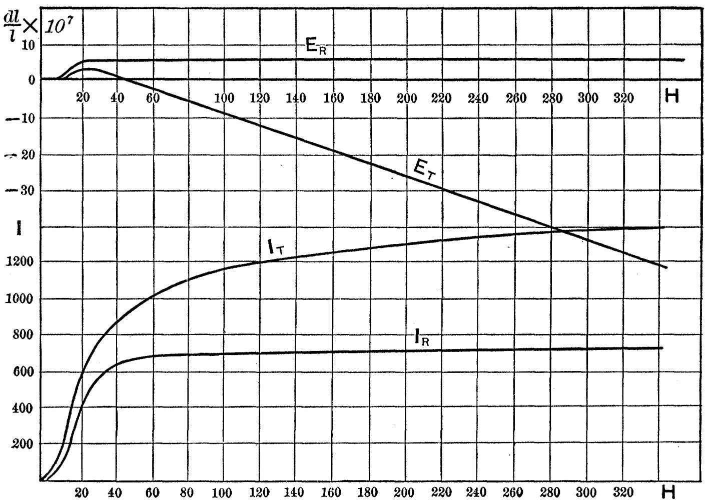

Plate 4.

Annealed Piano Wire: Tension 699. (See Tables 7 and 8.)

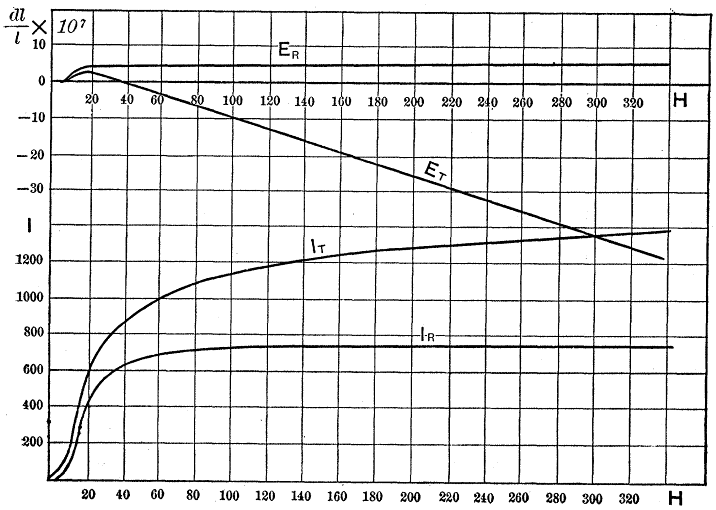

Plate 5.

Annealed Piano Wire: Tension 1949.

TABLE VIII.

Magnetization data for wire in Table VII.

	B	μ	I_T	I_R
1.83	99	55.	7.7	0
2.65	176	66.4	13.8	2.5
3.66	269	73.5	21.1	3.0
4.67	362	77.6	28.4	3.1
5.49	433	79.0	33.9	6.7
8.09	837	103.6	65.9	25.7
9.15	1024	112.0	80.8	27.7
10.74	1399	130.5	110.6	47.2
12.66	2077	164.5	164.2	88.3
15.18	3581	236.0	284.0	182.0
16.91	4352	258.0	345.0	227.0
19.19	5689	295.6	450.5	324.0
20.80	6306	303.8	500.0	361.8
22.41	6952	310.0	551.0	409.2
30.49	8990	294.0	715.5	553.5
36.80	10327	281.0	819.0	608.8
39.75	10740	271.0	852.5	629.0
43.25	11143	258.0	883.0	651.0
47.18	11577	245.5	917.5	656.0
51.45	11972	233.0	949.0	668.0
56.95	12407	218.3	984.0	679.0
69.01	13369	193.8	1060.	690.
77.00	13707	178.0	1085.	695.
91.45	14482	158.3	1142.	700.
101.6	14592	143.7	1152.	706
112.9	15043	133.0	1190	712
129.0	15389	119.2	1214	712
139.3	15629	112.3	1232	714
151.4	15871	104.6	1251	716
166.2	16066	96.4	1266	717
178.2	16428	92.2	1295	718
198.1	16558	83.6	1301	720
215.4	16915	78.5	1329	722
235.4	17055	72.6	1339	724
251.5	17282	68.7	1356	726
274.5	17375	63.2	1361	726
297.2	17497	58.9	1369	730
327.0	17917	54.8	1399	733

in this investigation, though the change of Young's modulus must be considerably less than one-half per cent. for the annealed piano wire and still less for the natural piano wire.

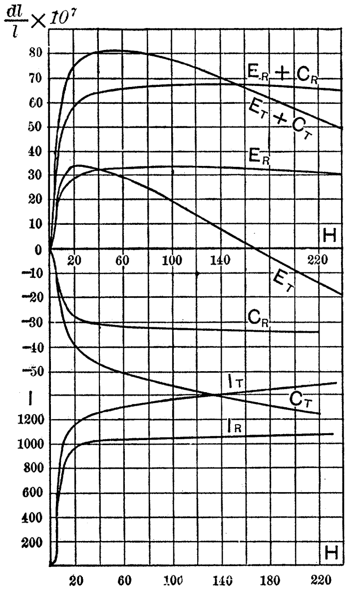

Plate 6.
Soft Iron: Tension 48.3.
(See Tables 11 and 12.)

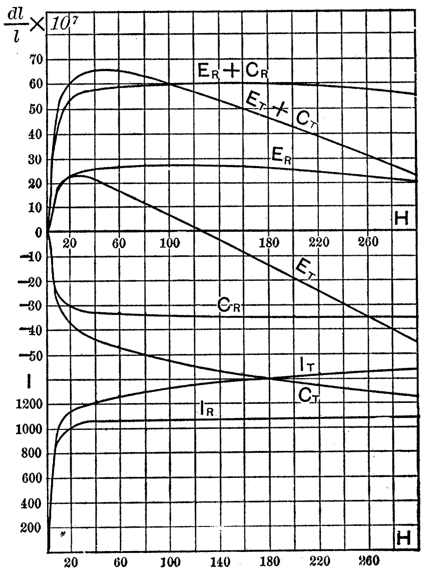

Plate 7.
Soft Iron: Tension 430. (See Tables 13 and 14.)

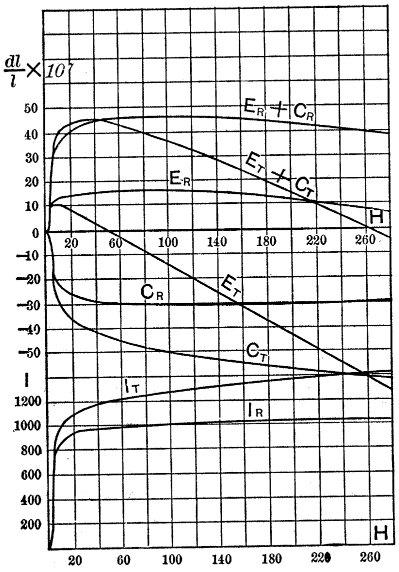

Plate 8.
Soft Iron: Tension 752.
(See Tables 15 and 16.)

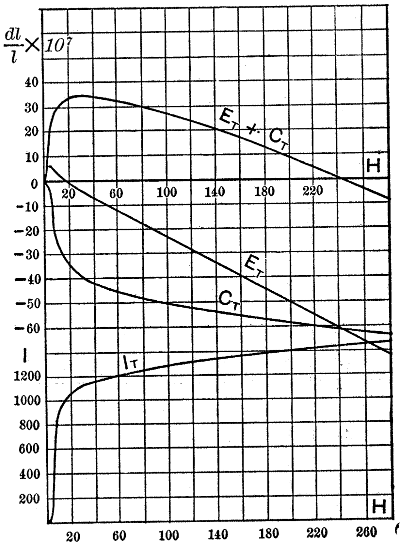

Plate 9.
Soft Iron: Tension 1720.

Table XIII.

Elongation data for soft, annealed iron wire under tension of 430 *kg. per square cm.*

$\frac{dl}{l} \times 10^7$ due to this tension $= 1988$.

H	E_T	E_R	H	E_T	E_R
.457	0.29	0.28	42.99	20.92	25.92
.915	0.853	0.68	46.48	20.38	26.11
1.51	1.71	1.42	50.68	19.62	26.19
1.82	1.99	1.82	54.82	18.76	26.19
2.42	3.41	2.84	60.08	17.61	26.19
2.88	4.55	4.09	67.00	15.92	26.22
3.43	6.27	5.69	76.50	13.41	26.40
3.93	7.97	7.40	91.95	9.96	26.72
4.62	10.52	9.68	101.4	6.95	26.72
5.26	12.39	11.37	113.2	3.99	26.72
6.12	14.51	13.64	129.5	− 0.57	26.60
7.31	17.06	15.91	139.8	− 3.13	26.60
9.14	19.21	18.18	152.4	− 6.27	26.40
10.40	20.19	19.33	163.7	− 9.68	25.60
12.21	21.32	20.44	178.9	−13.93	25.01
14.41	21.71	21.18	198.6	−19.21	24.42
17.54	22.78	22.77	228.5	−26.16	24.15
22.18	23.20	23.78	246.6	−31.00	23.02
25.32	23.20	24.20	262.5	−36.12	21.00
30.18	22.78	24.78	283.2	−42.99	18.77
33.48	22.20	25.01	308.3	−54.50	15.32
37.39	21.61	25.60			

Conclusions.

The investigation seems to have established the following laws for magnetization under conditions like those used in my work:

1. Any increase in the magnetic induction tends to lengthen the iron wire. As shown by the curves of elongation plotted to I, the relation between I and elongation is nearly the inverse of the relation between H and I shown in the ordinary induction curve, until well beyond the maximum value of μ, when it curves in an opposite direction on account of the field effect.

2. The magnetizing field tends to shorten the wire and the shortening due to this cause apparently has no limit. Up to a field of two or three hundred the shortening seems to be directly proportional to the field strength and then it seems to begin to approach asymptotically some limiting value.

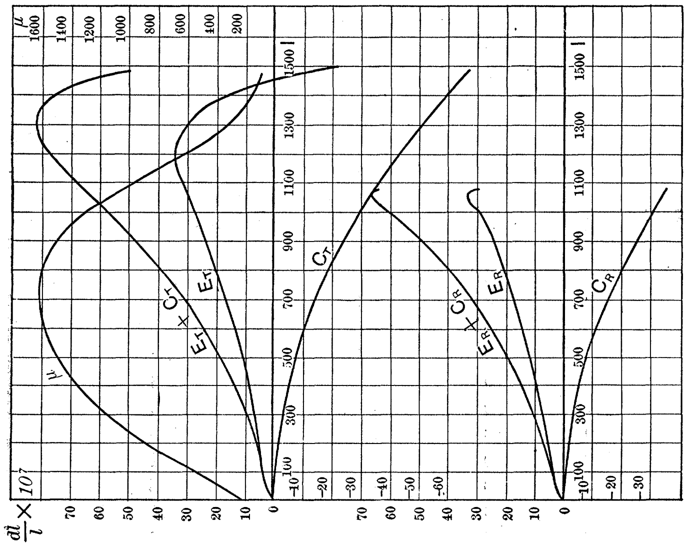

Plate 10.

Soft Iron: Tension 48.3. (See Tables 11 and 12.)

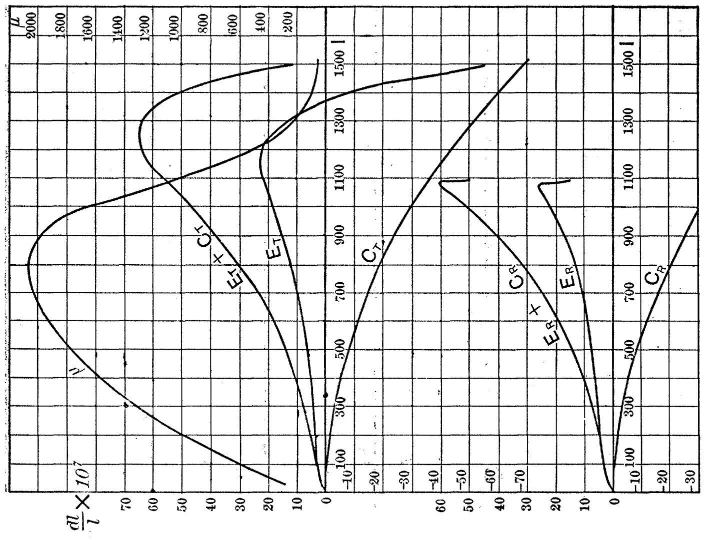

Plate 11.

Soft Iron: Tension 430. (See Tables 13 and 14.)

TABLE XIV.

Magnetization data for wire in Table XIII.

H	B	μ	I_T	I_R	C_T	C_R
1.46	464	318	36.8	8.09	.04	.0
1.92	752	392	59.7	23.5	.10	.02
2.29	1496	654	119.0	65.3	.42	.13
2.75	3235	1178	255.2	182.0	1.97	.98
3.39	5658	1672	442.5	347.5	6.00	3.57
4.58	9405	2060	748.5	634.5	16.6	11.9
5.22	10695	2060	852.5	733.8	21.4	15.9
6.08	11616	1910	924.2	820.0	25.3	19.9
7.32	12487	1710	994	880.0	29.3	22.9
9.04	12909	1430	1025	915.0	31.3	24.8
10.29	13270	1290	1055	937.5	33.1	26.0
12.09	13572	1120	1079	976	34.6	28.2
14.24	13864	973	1102	987.5	36.1	28.9
17.20	14197	826	1128	1008	37.8	30.1
21.60	14532	673	1154	1026	39.7	31.1
24.63	14775	600	1173	1042	41.0	32.1
29.28	14959	511	1187	1053	42.0	32.8
32.28	15082	468	1198	1053	42.7	32.8
35.81	15296	427	1213	1053	43.9	32.8
38.60	15399	399	1221	1053	44.5	32.8
41.00	15441	377	1225	1055	44.8	32.9
45.00	15605	347	1237	1056	45.8	33.0
52.63	15793	300	1252	1058	46.9	33 1
58.12	15938	274	1263	1058	47.7	33.1
68.60	16229	236	1285	1058	49.5	33.1
78.25	16408	210	1298	1059	50.6	33.1
93.15	16733	179	1324	1070	52.6	33.9
102.9	16903	165	1336	1070	53.6	33.9
114.3	17064	149	1348	1070	54.6	33.9
130.6	17411	133	1374	1074	56.8	34.2
140.0	17462	123	1380	1076	57.3	34.3
154.3	17674	115	1393	1076	58.5	34.3
166.5	17907	107.5	1411	1076	60.3	34.3
183.1	17983	98.0	1416	1078	60.7	34.4
203.8	18244	89.0	1436	1078	62.5	34.4
226.3	18366	81.3	1443	1078	63.3	34.4
249.2	18429	73.9	1446	1080	63.7	34.5
270.0	18790	69.5	1473	1080	66.3	34.5
297.5	18947	63.6	1483	1081	67.4	34.5
341.6	19422	56.8	1516	1092	70.8	35.3

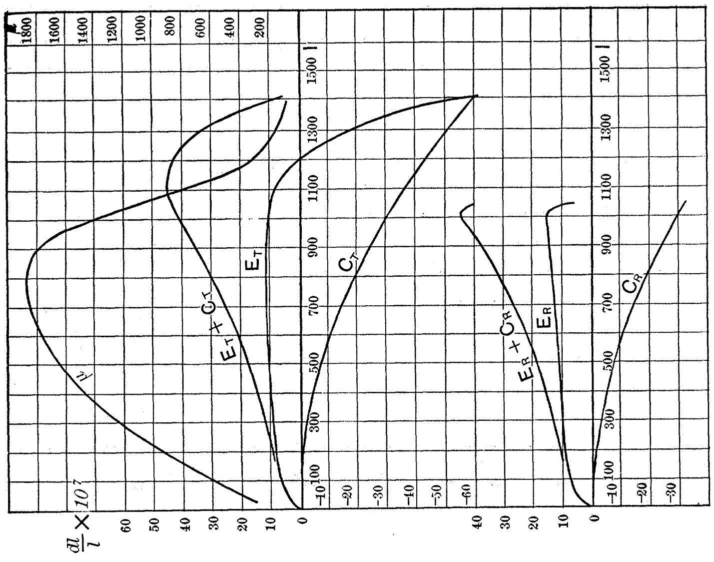

Plate 12.

Soft Iron : Tension 752.

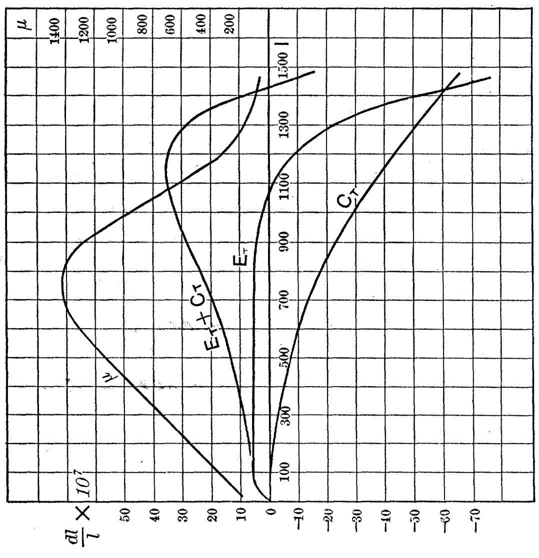

Plate 13.

Soft Iron : Tension 1720. (See Tables 17 and 18.)

3. The elasticity changes with the induction, the modulus being in some cases one-half per cent. greater at the highest magnetization than when the magnetization began. But the law of the change is unknown further than that the elasticity changes only as the induction changes.

In closing, I desire to acknowledge my indebtedness to Professor Rowland, not only for suggesting this investigation to me, but also for help and kind consideration at all stages of the work.

I am also greatly indebted to Dr. Ames, to Dr. Duncan and to Mr. H. S. Hering for valuable suggestions on many portions of the experimental work and for a very considerate interest in the whole investigation.

Explanation of the Tables and the Notation used in the Tables and Plates.

$H \equiv$ the magnetizing field.

$E_T \equiv \frac{\text{change of length}}{\text{length}} \times 10^7$ while H is on.

$E_R \equiv \frac{\text{change of length}}{\text{length}} \times 10^7$ after H is removed.

$B \equiv$ total induction or $4\pi I + H$.

$I_T \equiv$ magnetization of specimen with H on.

$I_R \equiv$ magnetization of specimen with H off.

$\mu \equiv$ magnetic permeability or $\frac{B}{H}$.

$C_T \equiv \frac{\text{change of length}}{\text{length}} \times 10^7$ that would be caused by a mechanical pressure $= \frac{B^2}{8\pi}$.

$C \equiv \frac{\text{change of length}}{\text{length}} \times 10^7$ that would result from a mechanical pressure $\frac{(4\pi I_R)^2}{8\pi}$.

If these compressive forces actually exist then the values of E_T and E_R computed from the observed changes are too small by C_T and C_R respectively and :

$E_T + C_T \equiv$ the real $\frac{\text{change of length}}{\text{length}} \times 10^7$ with H on.

$E_R + C_R \equiv$ the real $\frac{\text{change of length}}{\text{length}} \times 10^7$ with H off.

JOHNS HOPKINS UNIVERSITY, June 30, 1897.