ON THE USE OF THE INTERFEROMETER IN THE STUDY OF ELECTRIC WAVES.

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Outline.

1. Introduction.—Use of the interferometer in the study of light.
2. Historical statement.—Analogy between light and electric radiation not complete. Work of Sarasin and de la Rive, . . . for Hertzian apparatus proving that the wave-length measured depends on the receiver. Is the radiation from a Hertzian vibrator simple or complex? Experiments and theory dealing with this point. Bearing of this theory in our experiments.
3. Apparatus.—Interferometer; conditions governing choice of vibrator and receiver; sensitiveness; absence of diffracted and scattered radiation; preliminary observations.
4. Measurement of λ.—Degree of accuracy.
5. Estimation of "δ'" from the interference curve.—Proof that this does not give the logarithmic decrement of the vibrator, as has been supposed. Comparison of experiment with theory.
6. Influence of receiver upon the interference curve.—Inference as to the period and damping coefficient of our receiver.
7. Dependence of λ upon vibrator.—No agreement between values of λ found by other observers for vibrators similar to our own.
8. Determination of μ.—Degree of accuracy. Conclusion.

1. Use of the Interferometer in the Study of Light.

The interferometer, as a special form of refractometer has been called by Professor Michelson, has been found to be a very powerful instrument in the study of light. By its means the nature and wave-length of the radiation from a source and the index of refraction of a transparent medium have been found with the greatest accuracy. It is true that a grating or prism spectroscope must be used for the study of complex light, but for comparatively simple radiation the interferometer possesses a far greater power of analysis. It was with the object of constructing an interferometer for electric waves and of using it in their study that this research was undertaken.

2. Here it may be well to point out that electrical and light oscillations differ in an important respect. The breadth of a spectral line of a homogeneous gas has been accounted for upon various assumptions among which is the one that the oscillations of the molecules of the gas gradually die down owing to its communicating energy to the surrounding medium, or to other causes. Experiments, however, have failed to verify this assumption.[1] In the case of electrical radiation, on the other hand, theory indicates and experiment proves that the oscillations are always damped.

Another difference is in the apparatus used for detecting the radiation in the two cases. For light, the eye is the usual detector. Though it is sensitive for radiation lying within a very small range, it has no period of its own and does not possess the power of influencing the measured wave-length. That electrical receivers, as a rule, have this power has been proved by the experiments of Sarasin and de la Rive,[2] Klemencic and Czarmak,[3] and quite recently by those of Wiedeburg.[4] In fact, for the different forms of Hertzian vibrators and receivers used by these physicists, the wave-length measured depended almost entirely upon the receiver.

The explanation of this fact led to the important question whether the radiation from a Hertzian vibrator is simple or complex. Upon this point opinions have differed. From the phenomenon of multiple-resonance as discovered by Sarasin and de la Rive[2] and Klemencic and Czarmak[3] many physicists were led to the conclusion that the radiation is complex. The experiment of Zehnder[5] showing that the rays of electric force are analyzed by a grating into a spectrum pointed in the same direction, while Garbasso and Aschkinass[6] found that rays, in passing through a prism made up of

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[1] A. A. Michelson, Astrophysical Journal, p. 251, November, 1895.

[2] Sarasin and de la Rive, Compt. Rend., 115, p. 439, 1892.

[3] Klemencic and Czarmak, Wied. Ann., Vol. 50, 1893.

[4] Wiedeburg, Wied. Ann., Vol. 59, p. 496, 1896.

[5] Zehnder, Wied. Ann., Vol. 53, p. 172, 1894.

[6] Garbasso and Aschkinass, Wied. Ann., Vol. 53, p. 534, 1894.

glass plates upon which were pasted strips of tinfoil, are dispersed and concluded that rays of electric force may be considered not necessarily as monochromatic but, with the same justification as in the case of light, as composite. On the other hand Bjerknes[1] using Lecher's arrangement[2] attempted to prove that the radiation is due to a simple damped oscillation of the form $Ae^{-\gamma^{t}}\sin(at+a')$. The theory of his method, which differed from the later work of Klemencic and Czarmak chiefly in this that he used waves along wires, while they used waves in air is as follows: Assuming that the oscillations are of the form $Ae^{-\alpha t}\sin at$, he finds the effect upon the receiver of two infinite trains of waves, one of which is direct from the vibrator and the other reflected from the ends of the wires. This effect (using notation and limits of integration to suit our experiment) is given by

$$J=\int_0^{\frac{2x}{v}} Ae^{-2\alpha t}\sin^2 at\,dt+\int_0^{\infty} dt\left[Ae^{-\alpha t}\sin at+Ae^{-\alpha\left(t+\frac{2x}{v}\right)}\sin a\left(t+\frac{2x}{v}\right)\right]^2$$
$$=\frac{A^2}{2\alpha}\left(1+e^{\frac{2\alpha x}{v}}\cos 2a\frac{x}{v}\right).$$

In Bjerknes' experiment x was the distance of the receiver from the end of the wires; in our experiments it is the distance of a mirror from the "zero position"; in both cases $2x$ is the total difference in path of the two trains of waves. Thus the curve whose ordinates are proportional to the intensity of the electrical radiation for a point whose abscissa is x, is leaving out a constant, a damped cosine curve. It was found that the experimental agreed fairly well with the theoretical curve. It was therefore concluded that the oscillations followed the law assumed.

This conclusion is open to two possible criticisms which are suggested by the questions: first, may not the wires with their terminal plates or the receiver exercise a selective action on the radiation, and second, may not the interference curve be obtained assuming another law for the oscillations. Indeed Bjerknes found that the interference curve was influenced by the change of the distances between the plates of the vibrator and the terminal plates of the

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[1] Bjerknes, Wied. Ann., Vol. 44, p. 513, 1891.

[2] Lecher, Wied. Ann., Vol. 41, p. 850, 1890.

wires—that as this distance increased the waves were not so rapidly damped but were fainter—a result which agrees with the theory that the more feeble the radiation the less rapid is its decay. Probably the wires were too long to affect the result but that is a point which in general should be considered.

The results obtained by Sarasin and de la Rive and the other physicists named above, indicating that the radiation is complex, may be accounted for by assuming that the oscillations are simple and damped and that the receiver has a period of its own and is comparatively undamped. On the other hand, neglecting the criticisms brought forward, it would be difficult to account for the regularity of the interference curve obtained by Bjerknes on the assumption that the radiation is complex. On the whole the question has not been satisfactorily settled. We are justified in expecting that the problem may be approximately solved by means of the interferometer.

The question which concerns us is this—even if electric radiation is, in general, sufficiently simple to be analyzed by the interferometer, how shall we interpret the interference curve—is its form influenced by the receiver? The latter point is a matter for experiment to decide in the case of the receiver used.

Note.—The theory which applies here is rather unsatisfactory. Assuming that the receiver, when no outside forces act, executes damped pendulum motions and that the oscillations of the vibrator are of the same nature we obtain the equation:

$$\frac{d^2\varphi}{dt^2}+\beta\frac{d\varphi}{dt}+(\beta^2+b^2)\varphi=Ae^{-\alpha t}\sin(at+e)$$

where φ represents the potential difference between the parts of the receiver. Let us impose the conditions $\varphi=\frac{d\varphi}{dt}=0$ when $t=0$. The solution may be expressed in the form—

$$\varphi=C\sin\left(\frac{a+b}{2}t+c'\right) \text{ where}$$

$$C=\frac{A^2}{(a^2-b^2)^2+(a^2-b^2)(\alpha^2-\beta^2)}e^{-(\alpha+\beta)t}\Big[e^{-(\alpha-\beta)t}+e^{(\alpha-\beta)t}-2\cos(a-b)t+$$

$$2\frac{1+\cos\varepsilon}{a+b}((a-b)e^{\alpha-\beta}-(a-b)\cos(a-b)t-(\alpha-\beta)\sin(a-b)t)+$$

$$2\frac{\sin\varepsilon}{a+b}((\alpha-\beta)e^{(\alpha-\beta)t}-(\alpha-\beta)\cos(a-b)t-(a-b)\sin(a-b)t)\Big].$$

The action on the receiver due to the two trains of waves is

$$\int_0^{\frac{2x}{v}}\Big(\varphi(t)\Big)^2dt+\int_0^\infty\left[\varphi(t)+\varphi\left(t+\frac{2x}{v}\right)\right]^2dt=2K+2\int_0^\infty\varphi(t)\varphi\left(t+\frac{2x}{v}\right)dt.$$

This is the equation of the interference curve. But in view of the uncertainty of the hypothesis and the complexity of the result, it seems needless to expect assistance from this theory.

The work so far described was performed with the Hertzian vibrator and receivers. I am not aware that any corresponding work has been done with spheres unless it is that of Lang[1] and of Bose.[2] Lang obtained interference effects by a method analogous to that of Quinke in sound. But it seems probable that reflections from the tubes to which the radiation was led would obscure the effect. Bose states that the radiation from a sphere under the conditions existing in his experiment gives a *line* spectrum from which the inference is to be drawn that the vibrations are not only simple but very slightly damped. He also states that the wave-lengths measured were independent of the periodicity of the receiver. If this were true it must have been on account of the dead beat character of the receiver—not on account of the method used.

The experiments to be described in this paper will show (1) that the oscillation due to electrical disturbances on spherical conductors are for the most part, rather highly damped sine functions, (2) that the interference curve is influenced by the receiver—sometimes to such an extent as to change the apparent wave-length.

3. APPARATUS.

The interference device shown in Fig. 1 was patterned after Michelson's simple but effective interferometer. Radiation coming from the parabolic mirror P falls at an incidence of 45° on a *separating surface* S which reflects part to M and transmits the rest

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[1] Victor von Lang. Phil. Mag., Feb. 1896.

[2] Bose. Proc. Roy. Soc., Oct. 16, 1896.

which goes to *M'*. Of the radiation reflected from *M* and *M'* the parts transmitted and reflected by *S* fall together upon *R* (See Fig. 1). Difference of path is obtained by the motion of one or both mirrors. To obtain as large an effect as possible in *R* the separating surface should transmit and reflect one-half of the radiation falling on it. For this purpose there were pasted on a piece of cardboard, half a metre square, strips of tin foil 1x40 cms. At first the adjacent edges of two neighboring strips were one centimeter apart. Later it was found necessary to remove every alternate strip, making the adjacent edges 3 cms. apart.[1] The reflection and transmis-

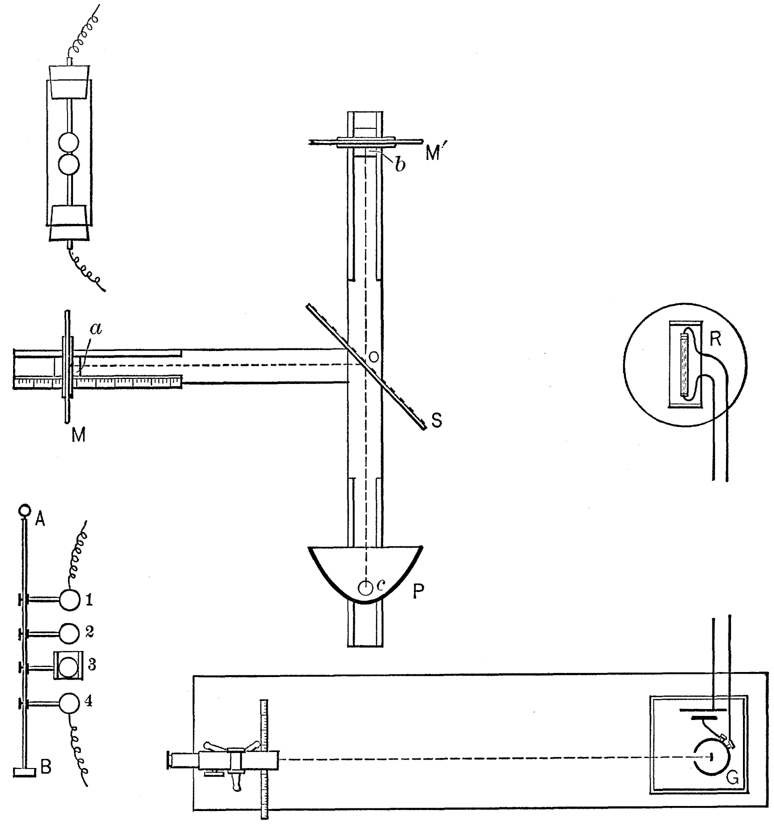

Fig. 1.

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[1] This agrees with the conclusion of Lord Layleigh deduced from theory, viz.—a narrow aperture parallel to the electric vibrations transmits very much less than is reflected by a conductor elongated in the same direction. Phil. Mag., April, 1897, p. 272.

sion powers of the grating were determined by well-known means and it was found that when the strips were parallel to the electric oscillations, this grating reflected and transmitted about half of the radiation falling on it.

The wooden arms *oa* and *bc* were grooved so that the supports carrying the mirrors *M*, *M'* and *P* were movable along *oa*, *ob*, *oc*. The angle *aob* was approximately a right angle and the mirrors were placed each normal to its arm by optical means. The mirrors *M* and *M'* were of plate glass with the silvered surface towards o. The parabolic zinc mirror *P* had a focal length of 2.5 cm., a height of 40 cms. and an aperture of 30x40 cm. As it was intended to use waves not longer than 10 cm. the dimensions of *M* and *M'* and of the aperture of *P* were large enough to obviate any serious diffraction effects. The focal length of *P* was chosen about one-fourth of the wave-length in order that the direct and reflected waves might be in the same phase. But as the radiation was too strong (unless the galvanometer was short circuited) this precaution was unnecessary.

The arrangement for the vibrator usually consisted of two spheres connected by fine wires to the secondary terminals of the induction coil and sparking to each other in oil. Each sphere was screwed on the end of a hard rubber rod which passed through a rubber cork and this was inserted in a glass tube about 10 cm. long and 2.5 to 3 cm. in diameter. This tube was partially filled with oil—usually vaseline oil, sometimes paraffine oil. A small hole blown in the side of the tube allowed one, by rubbing paper between the spheres, to partially clean them without removing the vibrator from its position. Righi's arrangement[1] was also used. Four hard rubber rods each carrying a sphere, were attached to a vertical rod in the mirror *P*. All distances were adjustable. The third sphere was fastened by means of melted wax in a small glass cylinder of such a size that it held sufficient oil for the spark gap and yet allowed the spheres 2 and 3 to approach until their surfaces touched. The advantage of this arrangement was that all the spark gaps could be fairly well cleaned by paper without removing the spheres. The rod *AB* could be rotated so that the spheres could be brought out towards the aperture of *P* or moved back to the focal line.

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[1] Righi, Mem. del R. Acad. dei Sc. del Inst. di Bologna, T. IV., 1894.

The receiver chosen for this experiment must fulfill certain conditions. It must be sufficiently sensitive to respond to radiation reaching it after reflection and diffraction, sufficiently constant to give quantitative results, and as we desire to determine the nature of the radiation it should respond equally well to waves of all periods sent out by the vibrator. Now it has been shown that Hertzian receivers do not fulfill this last condition. It was thought that a receiver which responds to electric waves of periods extending over a wide range would measure accurately the radiation falling on it. Having had some experience in the use of the *coherer* (as named by Lodge, though the name seems misleading) we thought that if it could be made constant it would answer our purpose. Consequently we used a Branley or Lodge receiver consisting of small nails or pieces of wire about 1 cm. long, in a glass tube 15 or 20 cm. long and filled with a lubricating oil. This was placed in the focal line of a semi-circular zinc reflector of aperature 10x20 cm. A sheet of tin foil over part of the aperture served to shut out, if necessary, part of the radiation. Lead covered wires connected the receiver with the galvanometer and battery which were enclosed in a tin box. Thus, with the exception of the aperture of the mirror, the whole circuit was enclosed in metal.

In order to make the deflections small enough to be read, the galvanometer, a simple D'Arsonval, was short circuited by a resistance varying from one-fourth to one-half of an ohm. It is hardly necessary to state that in finding the proper grating the ordinary phenomena of polarized radiation were observed. For example, if the receiver, grating and vibrator were placed in a line, with the grating immediately in front of the receiver and its strips horizontal (perpendicular to the vibrator), almost all the radiation passed through. If the strips were vertical only one-half passed through. A block of wood with its fibres vertical transmitted much less than it did when its fibres were horizontal.

It was found that the receiver responded nearly as well when the electric oscillations were vertical as when they were horizontal. In this respect, therefore, it differs from the Hertzian receiver. In order that the diffracted and scattered radiation be as small as possible, the axis of the mirror, and therefore, also the vibrator, were vertical, for in this position it was not necessary to have metallic ends in the mirror. In our experiments the total scattered radiation was only three per cent. of the effect when there was zero difference of path in the two beams. Considering the fact that the receiver was sensitive to radiation reaching it in all directions, whether from the air or from the wires leading to the galvanometer, and that it responded slightly to the long waves of the induction coil, which was not enclosed in metal, this result was felt to be satisfactory. A small induction coil with an ordinary automatic interruptor, operated by one storage cell, completed the apparatus. A number of interruptors were tried, but they were not found to be more constant than the one belonging to the coil, and were more complicated. Usually the key was closed for about one second. Closing it for a longer time did not increase the deflection, but only served to destroy the sparking surfaces.

4. MEASUREMENT OF λ. FIG. 2, CURVE 1.

This figure was found by taking the mean of four deflections for every position of the mirror which was moved always in one direction and through 5 mm. each time. After 80 or 100 readings the vibrator became nearly useless, showing a deterioration with time. So in all observations after this the mirror was moved forward and back: *e. g.*, to determine the relative readings for three positions, *a*, *b*, *c*, the series *a b c b a* was taken. If we are concerned with the wave-length and not with the interference curve it is necessary to find only the maxima and minima positions. But the accuracy with which one of these positions can be determined depends on the *sharpness* of the curve at that point. Consequently the first step in the determination of λ (*or even of* μ) is to roughly plot the interference curve, observe the sharpness of the maxima and minima and choose the sharper of these from which is to be estimated the desired quantity.

In order to see to what degree of accuracy λ may be found, the positions of four successive minima were observed. The following results were obtained, the numbers referring to the position of the mirror on the scale

	Second behind.	First behind.	First before.	Second before.	Mean for λ|2.
Expt. 1. . .	6.45	11.	15.55	20.2	4.58 cm.
Expt. 2. . .	6.55	10.95	15.5	20.1	4.53 cm.
Expt. 3. . .	6.5	10.97	15.53	20.17	4.55 cm.

and the mean of these gives $\lambda/2=4.56$ cm. to within a fraction of one per cent. The vibrator consisted of two spheres 1.93 cm. in diameter arranged as in Fig. 1.

5. Estimation of "δ'."

If we accept the theory given on page 5 as correct we can from the curve find the logarithmic decrement of the vibrator. Let a_1, a_2, a_3, be readings for $x=0=\lambda/4$ and $\lambda/2$ respectively, then

$$aT=\delta=2\log\frac{a_1-a_2}{a_3-a_2}$$

Now the first minimum and maximum of the curve (1) correspond nearly with the values of $x=\lambda/4$ and $\lambda/2$ respectively. Using these values in the curve (1) we get, since $a_1=14.5$, $a_2=3.3$, $a_3=9.1$,

$$\delta=0.74.$$

But it is certain that this theory, though it may apply when receivers are used which are dead beat compared with the vibrators, does not apply in the case of Klemencic and Czarmak's experiments nor in our own; for we have found that the interference curve is influenced by the receiver—a fact which this theory neglects. However, it is interesting to compare the values of $2\log\frac{a_1-a_2}{a_3-a_2}$ (which we will call δ' and which has some connection with the damping of the vibrator or receiver or both), found by Klemencic and Czarmak and others, for Hertzian vibrators and receivers, with the corresponding value for the spheres and coherer. Their values varied from 0.3 to 0.5 while ours varied from 0.5 to 1. This indicates either a more complex radiation or a higher damping coefficient in the case of our apparatus than in that of the earlier experiment. But these values (0.5 to 1) are considerably lower than that suggested by theory. For the amplitude of the oscillations produced by the disturbance of a distribution of electricity on a sphere is where a=radius of the sphere, $=e^{-\frac{v}{2a}t} \quad =e^{-\frac{\delta}{T}t} \quad \therefore \quad \delta=\frac{v}{2a}T=\lambda/2a$. In the case of this vibrator $a=1$ nearly and $\lambda=9.+$, hence $\delta=\underline{4.5.}$ This, however, is the case of an isolated sphere, a condition which does not exist in our experiment. But in later work we used one sphere and a small knob sparking to it and the value of did not exceed the value 1.0. Hence it appears either that δ' is not equal to δ or that Thomson's theory does not agree with experiment.[1]

6. Influence of Receiver upon the Interference Curve.

So far nothing has been said regarding the wave-length measured as to whether it is that of the vibrator or is due to a property of the receiver. It seemed that the period of the latter might depend: on (1) the dimensions and arrangement of the nails; (2) the dimensions of the tube. In order to vary these quantities three other receivers were used. In receiver II there were nails like those in receiver I, but the tube and terminals were different. Receiver III had in place of nails, copper wire cut up into lengths varying from 2 mm. to 2 cm. In receiver IV were used steel spheres about 4 mm. in diameter. Two vibrators were used, No. 1 already referred to and No. 2, which differed from it only in having spheres .93 cm. in diameter. The interference curves are given in Fig. 2. If the receiver exercised no influence on the interference curve the curves 1, 3, 5, 7, should be similar, as also should 2, 4, 6, 8. We reach, therefore, a very definite conclusion, the interference curve depends partly on the receiver.

In the curves 1, 3, 5, 7, we notice that λ has approximately the value 9.2 cm. This value then we take to be the wave-length of the radiation from vibrator I.

The curve (No. 2) for vibrator 2 and receiver 1 was observed many times and it always assumed forms between the dotted and continuous lines of No. 2. It seems due to two components, one of wave-length 8.6 cm. and the other of half that wave-length. In

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[1] J. J. Thompson, "Recent Researches," p. 370.

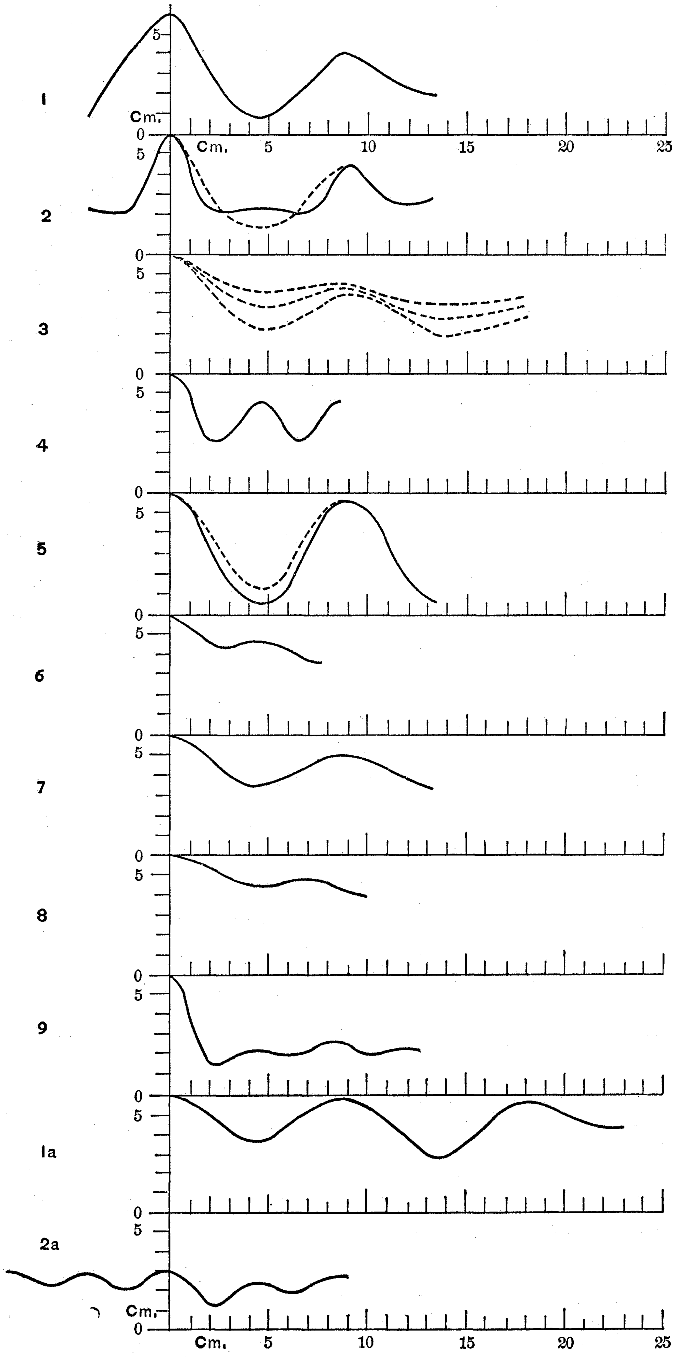

Fig. 2.

view of the fact that receivers II and III for this vibrator gave a wave-length of 4.2 to 4.4 cm., we are led to believe that the component of the shorter wave-length is due to the vibrator and that the other component, which is present probably to a slight extent in the radiator, owes its prominence to a resonance action of the receiver. The interpretation that receiver I has a period corresponding to the long wave receives confirmation in the fact that the curve for vibrator I has such well defined maxima and minima. But, whatever interpretation be placed on the interference curve, it is evident that it cannot always be taken to accurately represent the radiation. On the other hand the wave-length for either vibrator is maintained by all the receivers (leaving out curevs 6, 7, 8, for which the wave-lengths are very indefinite). This fact shows that our receiver has far less influence on the interference curve than has the Hertzian receiver. In other words, the damping coefficient of the former is far less greater than that of the latter. Our expectation, therefore, that the coherer used is highly damped has been fulfilled, but it appears from our work that its damping coefficient is of the same order of magnitude as that of the vibrator and consequently it influences the interference curve.

In view of this action of the receiver we are not able to accurately analyze the radiation, but using a number of receivers we are able to arrive at a fair estimate of its nature. We are thus led to state that the chief component of the radiation from spheres is due to a damped oscillation.

7. DEPENDENCE OF λ UPON VIBRATOR.

Besides the vibrators 1 and 2 we used the following arrangements: Vibrator No. 3, consisting of one sphere of No. 1, to which sparked a small platinum bead. The interference curve is given in No. 1_a. The wave-length is the same as that of No. 1, but the maxima and minima are less pronounced. Vibrator 4 differed from No. 3 only in having one of the spheres of vibrator 2 in place of one from No. 1. The interference curve is that of 2_a. Here again the wave-length is the same as when the two spheres are used. We also used two steel spheres 0.79 cm. in diameter together and singly. Their curves, which are similar to those already found, are not given. The wave-length was about 4 c.m. Righi's form of vibrator, with the two spheres of No. 1 sparking in oil and those of No. 2 sparking in air, was used. The wave-length was again approximately equal to that of No. 1.

It is here seen that the wave-length is nearly proportional to the diameter of the sphere (the law is expressed more nearly by $\lambda = m(r+a)$ where m and a are constants and $r=$ radius of the sphere), and that it is independent of the arrangement by means of which the discharge is brought about. I was not able to observe any change of wave-length due to a change of sparkling distance. It is interesting to compare the values of λ obtained by other observers with those which we have found. It will be seen that no agreement exists between the results of any two observers.

Dia. of Spheres.	Value of λ.	Arrangem't of Vibrator.	Receiver.	Method of Meas'g λ.	Observer.	Reference.
mm.	mm.		Nail			
39.7	88	Righi's	Coherer.	Interference	Lang.	
24.4	82	"	"	through	"	Wied. Ann., 3,
10.6	80	"	"	two tubes.	"	1896.
80.	200	Righi's	Silver on	Interference	Righi.	L'eclairage Elec-
37.5	106	"	Glass. Spark	"	"	trique. No. 3,
8.	26	"	Observed.	"	"	p. 360. 1895.
7.8	18.4	Lodge's one	Spiral spring	Grating.	Bose.	Proc. Roy. Soc.,
		Sphere.	Coherer.			LX, 361, 1896.
19.3	91	Righi's and	Coherers.	Interferom-		See pages 13–16.
9.3	43	Lodge's.		eter.		
7.9	40					
Length of						Physical Review.
cylinder.			Thermo-			1896.
8x1 mm.	50	Righi's	element.	?	Cole.	Wied. Ann.
1.3x0.5 "	6	"	"	Interference	Lebedew.	No. 9, 1895.

Surely but one interpretation can be placed upon the values of λ as found by Lang. It is that the wave-length measured is due chiefly to the receiver.

8. Determination of μ.

In order to determine accurately the index of refraction of a medium transparent to electric waves by the interferometer it is necessary that we have an interference curve with well defined maxima and minima. The influence of the receiver on this curve is, of course, a matter of no concern except that it should assist in giving this definition. Neither does the question of the nature of the radiation concern us unless (what is exceedingly improbable) the medium in question exercises selective absorption for these long waves.

To prevent the radiation which is reflected from the surfaces of the plate from reaching the receiver, the plate should be inclined at an angle to the rays. If the index of refraction is not much greater than unity, this precaution is unnecessary. If, however, the plate is kept normal to the rays, its thickness should be varied gradually through at least a wave-length and the mean of all the results taken.[1]

In order to find the degree of accuracy with which the value of μ may be determined by this method, observations were made with a block of wood 8.2 cm. thick. The displacements of the first and second minima on both sides of the zero position, and with the block before either mirror were found. Using the formula $\mu=(d+\delta)/d$ the following six values were obtained: 1.87, 1.80, 1.86, 1.83, 1.88, 1.83; giving a mean of 1.845 with a mean square error of 0.027. In these observations the fibres of the wood were horizontal. When the fibres were vertical the values 2.03, 2.01 were found. These results are sufficient to show (1) that the error in determining μ need not exceed one per cent., (2) that the wood used is doubly refracting.

Conclusion.—The points of difference between light and electric radiation which have been noticed in § 2 tend to make the problem of the analysis of the radiation more difficult in the case of electricity than in that of light. For there are added to the variables of the vibrator those of the receiver. However, when we find that there is present in all the interference curves for one vibrator a certain

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[1] The only theory which appears to us to apply is complicated and will not assist us. The action on the receiver is given by

$$\int_0^{\frac{2x}{v}}[\phi(t)]^2\,dt+\int_0^{\infty}[\phi(t)+F(t)]^2dt \text{ where } F(t) \text{ is found}$$

by taking account of the radiation reflected successively from the surfaces of the plate. This itself is a complicated expression.

element we are safe in saying that that element is due to an oscillation of a definite period on the vibrator. We are not justified at present in going further than this.

The following are the conclusions at which we arrive:

1. The interference curves depend on both the vibrator and receiver.

2. The influence of each of these varies.

3. The logarithmic decrement of the receiver is of the same order of magnitude as that of the vibrator.

4. The chief component of the radiation and the period of the vibrator may be determined from a number of interference curves.

5. This receiver could be used to analyze the radiation where the oscillations are but slightly damped.

6. The error in determining the index of refraction need not exceed one per cent.

7. Progress is to be looked for in obtaining: (*a*) a more constant source; (*b*) a more constant, dead beat, yet sensitive receiver.

This research was carried on at the Ryerson Physical Laboratory under the direction of Head Professor A. A. Michelson, to whom as also to associate Professor Stratton, I am indebted for many suggestions and much encouragement.

RYERSON PHYSICAL LABORATORY,
UNIVERSITY OF CHICAGO, May 1, 1897.