Volume IV. *September–October, 1896.* *Number 2.*

THE

PHYSICAL REVIEW.

THE VELOCITY OF ELECTRIC WAVES.

By Clarence A. Saunders.

THE velocity of propagation of the front of electric disturbances along metallic conductors is, according to present acceptance, equal to the velocity of light. This was indicated by the old theories; it is a direct conclusion from Maxwell's theory, and on the supposition that k, in the expression for the electrodynamic potential of two current elements upon each other, is equal to unity, is derivable from the equations of Helmholtz.[1] Nearly all of the early experiments made to determine "the velocity of electricity," so called, dealt with the phenomena of current diffusion. The trail left by the electric perturbation in its propagation greatly affected the results, so that they were widely divergent. A great impetus was given to the subject by the publication of Maxwell's electro-magnetic theory, as the demonstration of the equality of light-wave and electric-wave velocities seemed a strong verification of that theory.

The methods that have been employed for the determination of the velocity of electric perturbations may be divided into two classes.

(1) Direct methods, where the time taken by the disturbance to traverse a certain distance is observed, or where the two factors λ and T in the equation $\lambda = vT$ are directly measured.

[1] Helmholtz, Wiss. Abh., Vol. I., pp. 539–544; Blondin, La Lumière Électrique, Sept. 29, 1894; No. 3, p. 121.

(2) Indirect methods, where the velocity depends in any way on a factor that is not observed, but calculated.

With only one exception all the methods employing standing waves on wires have belonged to the latter class, for although the wave-length has been experimentally determined, the period has been calculated from Lord Kelvin's well-known formula, $T = 2\pi\sqrt{LK}$. The values of L and K inserted in this formula have usually been those obtained under ordinary conditions, and not under the actual conditions of the experiment.

The following investigation, undertaken at the suggestion of Professor Arthur G. Webster of Clark University, is also based on the phenomena of resonance. It is, however, a direct method, since both λ and T are experimentally determined. Professor Trowbridge and Mr. Duane[1] have recently published an article on the same subject, in which the same principle is employed. The present piece of work had been commenced, however, some time before, and was carried on without any knowledge of their being occupied in a similar line.

Historical.

In the course of the last fifty years numerous attempts have been made to directly determine the velocity of propagation of electric disturbances. Only a few such attempts, however, require to be mentioned here.

In most cases telegraph wires were used, and as the phenomena dealt with were those of diffusion, the high permeability and specific resistance of the iron exercised a considerable retarding influence. In several instances, as, for example, in the case of submarine cables, the circuit possessed very great capacity. In such cases the velocity, being a function of the capacity, had no fixed value. For the detection of current, instruments with magnets were frequently employed, but the time required for these magnets to act could not be properly eliminated. The same objection holds against the use of galvanometers. Owing to the lack of sensitiveness of the instruments, the arrival of the maximum, or

[1] American Journal of Science, XLIX., p. 297. April, 1895.

of a neighboring part of the wave of diffusion, and *not* of its front, was indicated.

The earliest attempt to measure the speed of propagation was that of Wheatstone[1] in 1834. The circuit consisted of a copper wire 731.5 m. long, and extended backward and forward so as to form twenty parallel lines 15 cm. apart. Three spark gaps were inserted, one at each end, and one in the center. These were arranged vertically one above another in front of a rotating mirror.

Upon discharging a condenser through the circuit, the image of the central spark was found to be displaced with reference to the other two. The amount of this displacement, together with the speed of the mirror, furnished the data for computing the speed of propagation. The value found by Wheatstone was 463,000 Km. per second, or more than one and one-half times the velocity of light. It seems probable that this high value is to be accounted for by the form given to the circuit and the uncertainty in the measurements of displacement.

In 1850 Fizeau,[2] in conjunction with Gounelle, made a series of experiments with wires, both of copper and of iron, the method employed being somewhat similar in principle to that used by him in the determination of the velocity of light. The circuits used were the telegraph lines between Paris and Amiens (314 Km.) and between Paris and Rouen (288 Km.). Fizeau's measurements give a velocity of 101,700 Km. per second for iron wires, and 172,200 Km. per second for copper.

In 1875 W. Siemens[3] made a determination of the velocity by a method far superior to any hitherto used. His apparatus consisted of two Leyden jars, whose external armatures were united through a short wire. One internal armature was connected by a short wire to a metallic point placed opposite a rotating steel cylinder covered with smoke-black; the other was connected with a long telegraph wire, which ended in a point opposite the cylinder near the first. The jars were charged, then discharged by joining

[1] Phil. Trans., Part II., p. 583, 1834; Pogg. Ann., 34, p. 464.

[2] Pogg. Ann., 80, p. 158; Comptes Rendus, 30, p. 437, 1850.

[3] Gesammelte Abhandlungen, p. 365; Ann. d Phys. und Chem., 157, p. 309.

the external armatures. The electricity of the internal armatures suddenly became free, and sparks took place — with an interval of time between their formation — between the points and the cylinder. Knowing the angular distance separating the sparks and the velocity of rotation of the cylinder, the time necessary for the discharge to traverse the length of the long telegraph wire could be calculated. Siemens's experiments were made with iron wires, and gave values of v ranging from 202,600 to 250,600.

The method employed by M. Blondlot[1] in 1893 was practically that of Siemens. Refinement in details, especially in the method of determining the interval of time between the two sparks, constitutes the chief difference.

With a copper wire 1029 m. long the mean of the results obtained gave a velocity equal to 296,400 Km. per second. In another series of experiments on a line 1821.4 m. long the mean velocity was 298,000 Km. per second.

In 1895 Professor Trowbridge and Mr. Duane[2] employed a method depending upon the principle of resonance. A condenser with its armatures connected by a short wire with an inserted spark-gap formed the primary. Opposite the primary condenser plates, and separated from them by plates of hard rubber, were two secondary plates, to which were attached two wires. These extended for a considerable distance parallel to one another, and ended in a secondary spark-gap. The primary and secondary were "tuned" to resonance, and the nodes and loops of the stationary waves set up in the secondary were measured by means of a bolometer. The period of these waves was obtained by photographing the secondary spark after reflection from a rotating concave mirror. The wave-length and the period being thus known, the velocity of propagation was directly determined. The average of the results gave a velocity of 2.816×10^{10} cm. per second. Later experiments by Messrs. Trowbridge and Duane,[3] in which the method employed was the same, give the value 3.0024×10^{10} cm. per second for the velocity of propagation.

[1] Comptes Rendus, 117, p. 543. October, 1893.
[2] American Journal of Science, XLIX., p. 297. April, 1895.
[3] *Ibid.*, Vol. L., p. 104. August, 1895.

For the benefit of those who wish to enter somewhat fully into the history of the general subject of electric propagation an extended list of references is appended.

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Method and Apparatus.

Resonance is employed in the present experiments in the way indicated by Professor Oliver Lodge[1] in his experiments on the "recoil kick." It was thought that, by refinement in the details and by the exercise of great care, this method of obtaining resonance could be made to furnish good results, and the sequel has confirmed the expectation.

From the condenser K_0 (Fig. 1) two wires are stretched parallel to each other with spark-gaps at *a* and *b*, thus forming a primary circuit *A* and a secondary *B*. If oscillatory sparks are formed at *a* by means of a Holtz machine, or an induction coil, connected to the armatures of the condenser, oscillations are set up in the secondary *B*. The period of the oscillations in the primary is governed by the self-induction and the capacity of the circuit, and can therefore be varied by moving *a* back and forward. When the period of the primary approaches the natural period of the secondary, we have the phenomena of resonance, and violent oscillations are set up in *B* so that the *b* gap can be widened greatly before the sparks cease there. Maximum resonance is obtained when these periods are equal. If the circuit *B* act like an organ-pipe open at both ends, $l_2 = \frac{\lambda}{2}$ (Oliver Lodge). The current is zero at the ends, and the potential a maximum. If, however, the circuit act like a pipe closed at one end, $l_2 = \frac{\lambda}{4}$. The current at one end is a maxi-

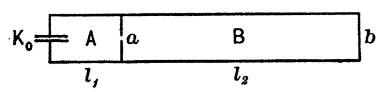


Fig. 1.

[1] Proc. Roy. Soc., 50, p. 2, 1891.

mum, and at the other zero. The same is true of the potential. This latter case is the one my experiment deals with. The wave formed is the fundamental.

A series of preliminary experiments was carried out with short wires. A spark micrometer was used at b, and the number of turns of the screw between the lengths of the b spark, when the discharge indifferently chose b and a, and when the sparks ceased at b altogether, gave a measure of the resonance. The wave-length can be calculated from Lord Kelvin's formula $\lambda = 2\pi\sqrt{LK}$, where L is measured in electromagnetic, and K in electrostatic units. The value of L can be obtained for parallel wires from Lord Rayleigh's formula for very rapid oscillations: $L = 4\,l \log \frac{b}{a}$, where b is the distance separating the wires, and a is their radius. For our rectangular primary circuit, L can approximately be considered to consist of two parts: one for two wires of length l_1 and distant apart b say, and the other for two wires of length b, and distant l_1 from one another. The capacity of the condensers is probably somewhat different in the circumstances of the experiment from that measured by ordinary methods. Its value here is that under very rapid oscillations, while its determination is made under relatively slow alternations. The capacities of our different condensers — gallon Leyden jars of best Bohemian glass — were roughly obtained by comparison with a one-half microfarad condenser, and also very carefully in electro-magnetic measure by the method described in Maxwell's "Electricity and Magnetism," § 776, and somewhat modified by J. J. Thomson.[1] When a condenser is discharged n times per second through a galvanometer, the relation between its capacity and the resistance that will produce the same galvanometer deflection when inserted in the circuit instead of the condenser, is given by $nK = \frac{1}{R}$. By the use of Wheatstone's bridge we can obtain this relation at once. The pairs of opposite points AB and BC of the bridge are respectively connected through the galvanometer and the battery. The arm AC is not closed, but the points AC are connected with two poles R and S,

[1] Phil. Trans., 1883, Part III., p. 707.

between which a vibrator P oscillates rapidly. The plates of the condenser are joined with A and P, so that when P is in contact with S, the condenser is charged — a momentary current through the galvanometer resulting — and when with R it is short-circuited. When the resistances are so adjusted that the deflection of the galvanometer due to the momentary current is just balanced by the deflection due to the steady current, the above formula very approximately holds. For the vibrator P a commutator with a lever was used, each of whose ends oscillated between contact points on two steel springs. The lever was worked by an arm fixed to its center that pressed against an eccentric attached to the end of the axle of the self-regulating motors to be described below. Through the contacts made by the lever, the condenser was charged and discharged once every revolution of the motor. Twenty-five storage cells were employed with the bridge.

The following data were obtained in the resonance experiments with short wires. The numbers under the heading "recoil" represent the number of turns of micrometer screw; the other column gives the length of the primary wires l_1. Much better resonance was obtained with the jars of large capacity. The influence of the condition of the surface of the balls, between which the sparks passed, was very great; the surface had to be always kept as clean as possible. Far sharper resonance was obtained with the Toepler-Holtz machine used, than with the induction coil. The distance between the wires is denoted by d.

$d = 60.$

Jar I.				Jar II.		Jar III.		Jar IV.	
Cm.	Recoil.	Cm.	Recoil.	Cm.	Recoil.	Cm.	Recoil.	Cm.	Recoil.
38	20	58	35.5	48	37	48	14	118	6.0
48	29	59	35.5	53	44	58	23	128	10.0
58	36	60	36.0	55	46	64	32	138	13.0
63	40	61	38.0	56	48	66	38	142	14.0
68	36	62	39.0	57	53	67	39	143	14.0+
78	15	63	41.0	58	54	68	40	144	14.5
88	5	64	42.0	59	51	69	42	145	15.0+
		+65	42.5	61	45	70	44	146	15.5
		66	42.0−	63	40	+71	49	147	16.0
		67	42.0−	68	19	72	48	148	16.5
		68	39.0			73	47	+149	17.0+
						74	47	150	16.5
						75	46	151	16.0−
						76	43	152	15.5
						77	37	153	15.0+
						78	33	154	15.0
						88	6	158	15.0−
								168	6.0

$d = 60.$

Jars I. and II.		Jars I. and III.		Jars I. and IV.		Jars II. and III.		Jars II. and IV.		Jars III. and IV.	
Cm.	Recoil.	Cm.	Recoil.	Cm.	Recoil.	Cm.	Recoil.	Cm.	Recoil.	Cm.	Recoil.
148	7.0	138	6.0	228	9.0	138	3.5	204	0.0	208	4.0
155	10.5	148	9.0	238	11.0	148	8.5	219	5.0	218	5.0
156	15.0	158	10.0	240	11.5	158	9.5	224	6.0	228	7.0
+157	16.0	162	11.5	242	12.0	161	10.0	229	6.5	238	8.0
158	15.5	164	12.0−	244	12.0+	162	11.0	234	8.0−	248	9.0
159	15.0	166	12.0−	246	13.0	163	12.0	239	8.0+	250	9.5
160	13.5	170	12.0+	+248	13.5	164	12.0	244	10.0	252	9.75
161	12.5	+172	12.5+	250	12.0	165	11.0	245	10.0	254	10.0
163	11.5	174	12.5−	252	11.5	166	14.0	246	10.0	256	10.0+
		176	12.0	258	10.0	+167	14.0+	247	11.0	258	10.5
		178	11.5	268	6.0	168	13.0	+248	11.5	−260	11.0
		188	7.5			169	12.0	249	11.0	+262	11.0
		198	3.0			178	6.5	250	10.0	264	8.0
								254	5.0	266	7.0
								259	3.0	268	6.0
										278	4.0

$d = 30.$

Jar I.		Jar II.		Jar III.	
Cm.	Recoil.	Cm.	Recoil.	Cm.	Recoil.
68	5.0	63	5.5	78	5.5
73	6.5	68	8.0	88	9.0
78	8.0	73	8.5	93	10.5
83	10.5	78	11.0	98	11.5
88	11.5	81	13.0	99	11.5
90	13.5	82	14.0	100	12.0
92	15.0	83	15.0	101	12.0+
94	15.5	84	15.0+	102	13.0
+96	15.75	85	15.5	103	14.0
98	15.0	86	16.5	+104	14.5
100	13.0	+87	17.0	105	14.0
102	12.0	88	16.0	106	12.5
108	5.5	89	14.0	108	11.0
		90	12.5	113	7.0
		92	10.0	118	1.5
		93	8.5		

The curves plotted from these data for the separate jars are given in Fig. 2. The ordinates represent the recoil; the abscissæ

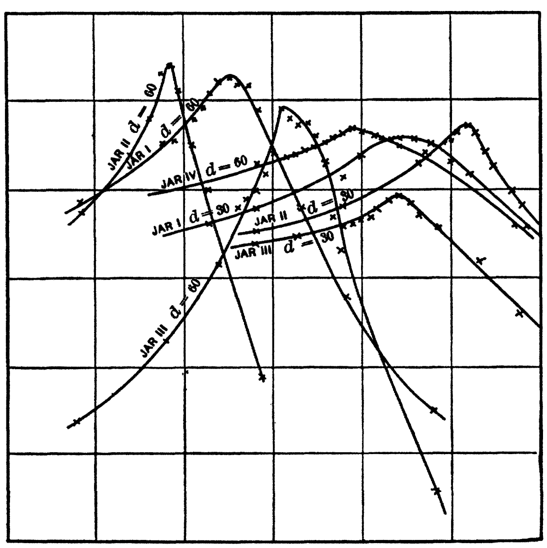


Fig. 2.

the length of the A circuit. It is seen that the curves rise somewhat less rapidly on the left than they fall on the right. In general, where there is less capacity there is less marked resonance; hence the curves plotted for the pairs of jars joined in series would be more flattened.

If the wave-lengths be calculated from the self-induction and capacity in the way indicated above, we obtain the following results. The radius of the wires is equal to 0.079 cm. Brass rods were used for the cross-connections with radius equal to 0.28 cm.

Jar.	b	l_1	l_2	L for A circuit.	K for A circuit.	λ calculated.	$4l_2$
I.	60	65	5097	3048	3492	20,498	20,388
	30	96	5078	2977	3492	20,258	20,312
II.	60	58	5104	2834	3767	20,530	20,416
	30	87	5087	2753	3767	20,230	20,348
III.	60	71	5091	3262	3255	20,470	20,364
	30	104	5070	3178	3255	19,980	20,280
IV.	60	149	5013	5518	1906	20,329	20,052
I. and II.	60	157	5005	5676	1812	20,151	20,020
I. " III.	60	172	4990	6093	1685	20,130	19,960
I. " IV.	60	248	4914	8198	1230	19,954	19,656
II. " III.	60	167	4995	5956	1746	20,260	19,980
II. " IV.	60	248	4914	8177	1265	20,214	19,656
III. " IV.	60	261	4901	8346	1202	19,902	19,604

The last two columns give the wave-lengths as calculated from the capacity and self-induction of the primary circuit, and also as directly obtained from the length of the secondary.

After these preliminary experiments, long wires were stretched from the physical to the chemical laboratory and back three times, a distance in all of about 580 m. A very careful measurement of their length was made beforehand in the following way. A portion was subjected to a tension approximately equal to that the wire would bear when finally suspended, and was left under this tension for some time. It was then carefully measured with a steel tape. Another portion was treated in the same way without severing the wire, and the process was continued for the whole length. Very great care had to be taken in insulating the wires;

in fact, it was only after long, glazed porcelain knobs were adopted that anything like good resonance could be obtained. The wires were extended at a distance of about six meters from the ground. Four Leyden jars with a total capacity of 17,552 cm. were used as the condenser. Resonance was first obtained with a Toepler-Holtz machine, and then the machine was replaced by a Ruhmkorff coil. The resonance on this long circuit was naturally not so sharp as on a shorter one; the maximum was determined to one part in 150 of the length of the primary. In the case of the two shorter circuits used in obtaining the final results, the maximum was determined to one part in 400. As the maximum obtained affects almost entirely the factor L, in the expression for the wave-length, and as this occurs under the square root, the wave-length is determined to one part in 800. Again, it is to be noted that an error in the period of the primary does not carry along with it an error of equal magnitude in the period of the secondary. The period of the oscillations in the secondary will be determined by the dimensions of its circuit, along which the wave is actually traveling, and these oscillations natural to the secondary will be set up when they approximate in period to those of the primary.

In order to obtain the period of the oscillations, photographs were taken of the secondary spark by means of a revolving plane mirror and lens. The mirror (see Plate opposite) was constructed by J. A. Brashear, of Allegheny, for Professor Michelson after the latter's experiments on the velocity of light. The mirror is made from a single piece of steel of a thickness sufficient to guard against lateral distortion. It has four circular polished plane surfaces at right angles to one another, each 38 mm. in diameter. The amount of light that can be employed is thus doubled, and the air-vortex caused by the oblique motion of the reflecting surfaces lessened. The top and bottom of the mirror terminate in pivots which are slightly conical. The bottom of the box, in which the mirror rests, is pierced by a screw-hole, through which passes a hollow cylinder (R) having both its internal and external surfaces cut into a screw. This screw can be adjusted in height so as to bring the revolving mirror vertically into the proper position. Its opening widens conically at the top, so that the conical surface of the lower pivot of the mirror fits accurately into the top of the opening. An interior screw, *T*, turned by the head, passes through the opening. This screw terminates on the top in a diamond plane. When the screw *R* has been properly adjusted, *T* is screwed in till the diamond touches the end of the steel pivot of the revolving mirror, so as just to support the weight of the latter. The proper position is indicated by the mirror's freedom of motion. A binding-screw clamps the screws when adjusted. An identically similar arrangement is provided for the top of the mirror. The mirror is placed in a strong frame of iron tubes, through which air is driven. The air issues from orifices opposite to turbine grooves, with which the axle of the mirror is provided above and below, and sets the mirror in rapid rotation. To control the mirror and to render its motion uniform, a valve turned by the hand, and a large wind-chest, 60 cm. × 80 cm., provided with heavy weights, were respectively used. The blast of air was furnished by an 18-inch Sturtevant fan blower, driven by a three horse-power motor.

In the plate the central figure represents one of the two boxes between which the mirror rotates. The air blast enters by the four cylinder terminals. To the right of the box is *T* and at the extreme right *R*. *T* screws into *R*, *R* into the central hole in the box, and both *T* and *R* are screwed in from above and from below until they properly touch the conical bearing of the mirror.

The determination of the velocity of the mirror when regulated was a matter of very great importance. Optical comparison with some fixed standard appears to be the readiest and most accurate method. Professor Michelson compared the rotation-period of the mirror with the vibration-period of a standard tuning fork, but the smallness of the amplitude of the fork's vibration renders the observation somewhat difficult. The standard used in the present case was a motor (*e*, Fig. 3) accurately regulated, as already described by Professor A. G. Webster,[1] by a synchronous alternating motor (*g*), rotating on the same axle. To this axle was attached a disk (*y*) with holes symmetrically situated on the circumference. The light from a lamp (*k*) passed through a large condensing lens

[1] Electrical World, Vol. XXII., No. 10. Sept. 2, 1893.

(*h*), and after passing through the holes in the disk (*y*) fall on the rotating mirror (*l*). The holes in the disk through which the light passes move vertically, whilst the rotating mirror moves horizontally. The number of bands of light seen in the mirror gave the ratio of the rotation velocities of the motor and the mirror. If the mirror were regulated so as to run at a fixed ratio, the bands remained stationary in the field. Fluctuations in the velocity were most delicately indicated by motion of the bands up or down. The field of the governing motor was independently excited. The current through the armature was made intermittent by means of an electrically maintained tuning fork (*u*) provided with a mercury break. The mercury cup was kept cool and the waste products

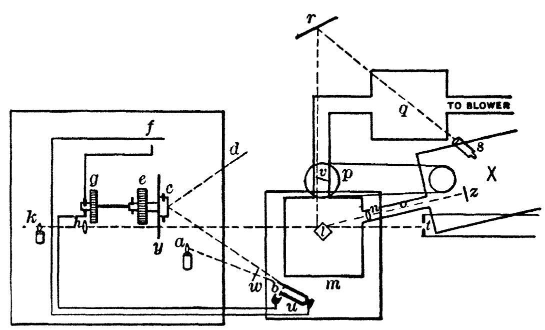


Fig. 3.

were carried away by causing a current of water to continually pass over. The fork made 25 vibrations per second. The light from a lamp (*a*) passed through a small rectangular slit (*w*), was reflected from a mirror attached to the electrically driven tuning fork (*u*), and was then reflected from a mirror placed at the end of the axle of the motors, and somewhat inclined to the axle. The eye at (*d*) would see, when the motor was rotating and the fork at rest, a continuous circle of light, and, when the fork vibrated, a series of bright arcs. These arcs reduced in number to two when there was synchronism between fork and motor. The speed of the driving motor was varied by means of a fluid resistance in the circuit of its field magnets until this synchronism was attained The controlling current through the alternator was then thrown on by turning the switch at (*f*). The motors would then continue to run with uniform speed. If then the absolute frequency of the fork be determined, we know the absolute velocity of rotation of the motor, and hence that of the mirror.

The large iron frame which held the mirror, the regulating motors, and the smaller pieces of apparatus were placed on heavy stone piers, which rested on a deeply laid brick foundation. Disturbing vibrations due to external causes were thus carefully guarded against.

The spark-gap, in which the long wires end, was placed at (*t*). The light was reflected from the mirror (*l*) down the dark chamber (*o*), passing through the long-focus Alvan Clark lens (*n*). The sensitive plate was exposed at (*z*). (*x*) is a small dark room, where the observer was seated. The valve (*v*) was put under the control of the observer by means of a wheel attached to the valve, and by means of a cord with pulleys. The light from the lamp (*k*), after traversing the holes in the disk, was reflected from the rotating mirror, and then from the fixed mirror (*r*) into the telescope (*s*), placed at the eye of the observer. A pasteboard hood was placed over the frame of the mirror, with small rectangular slits inserted, so that the light falling on the mirror from the front could not pass down the dark chamber at the back. A large induction coil was used to produce the sparks. Sharp breaks were obtained by having the primary current pass through two brushes, which pressed against the circumference of a wheel (*c*) 10 cm. in diameter attached to the end of the axle of the regulating motors. Part of the circumference of the wheel was copper, and part hard rubber. Where the current was broken a strip of mica was inserted. This arrangement for obtaining sharp sparks worked very satisfactorily. A switch in the chamber (*x*) permitted the throwing on of the primary current and the production of sparks at will.

The modus operandi of the apparatus is then as follows: The fork (*u*) and the current of water through the mercury cup are set going. The motors are brought into synchronism with the fork. The blower is started. The observer takes his place in the dark chamber. By means of the valve and the bands of light seen in the telescope (*s*) the mirror is regulated. When the lines of light on the face of the mirror stand perfectly stationary, the switch is thrown on to produce the sparks, and the sensitive plate is exposed.

The rate of vibration of the tuning fork was determined by the method of comparing with a clock introduced by Lord Rayleigh.[1] The principle employed was to accurately govern an electro-magnetic engine by an interrupter fork. The accompanying figure exhibits the arrangement. To a horizontal shaft revolving on steel points were attached two hard rubber disks which carried a number of soft iron bars disposed symmetrically on the circumference. Directly beneath these bars was placed an electro-magnet excited

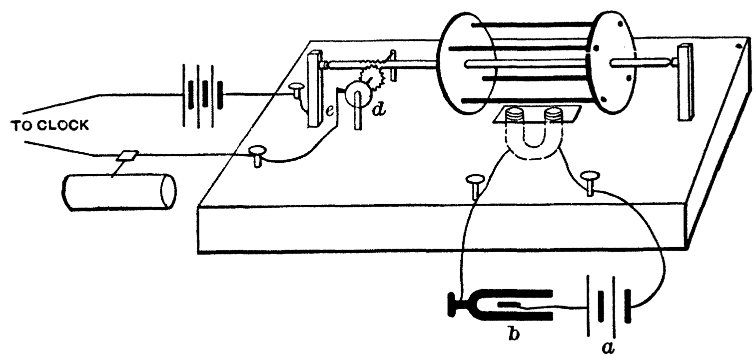


Fig. 4.

intermittently by means of a battery (*a*) and a fork (*b*). If the magnet is excited just as one of the bars approaches, the latter is attracted; so that if the passages of the bars exactly synchronize with the excitations, and therefore with the vibrations of the fork, the wheel is maintained in rotation. To check the oscillations which set in when any disturbance is communicated to the wheel, one of the disks had a channel hollowed out on the inside near the circumference, and this groove was filled with mercury. When the wheel rotates rapidly, the mercury acts as a rigid body, and tends to check any moderate variations of speed. For the 25-fork four bars were used. The wheel was set in rotation by the hand, and the proper speed was indicated by the apparent standing still

[1] Phil. Trans., 1883, Part I., p. 316.

of the bars when viewed in a mirror on the end of the fork. Engagement can also be very readily detected by the ear.

A screw-gear, carrying a small wheel (*d*), was attached to the shaft. The clock, a chronograph, and the wheel (*d*) with a small piece of hard rubber inserted in its circumference for a break, were put in series with a battery. Contact was made with the wheel (*d*) by a small spring (*e*). The circuit was broken every second by the clock, and once in every revolution by the small wheel. The time of revolution of (*d*) can thus be, with great accuracy, objectively determined by comparison with the seconds' record on the chronograph sheet. The screw on the axle gives the ratio between (*d*) and the phonic wheel, and the ratio between the phonic wheel and the fork is known from the number of the bars. If $n =$ number of seconds in the time of rotation of (*d*); $a =$ ratio between (*d*) and the shaft; $c =$ ratio between shaft and fork $=$ number of bars.

Wheel (*d*) makes $\frac{1}{n}$ revolutions per second.

Shaft makes $\frac{a}{n}$ revolutions per second.

Fork makes $\frac{ac}{n}$ vibrations per second.

In our case $a = 68$, $c = 4$, $n = 11$, approximately.

In order to secure as great regularity in the seconds' breaks of the clock as possible, the circuit was made through a mercury drop, and was broken by a strip of mica attached to the lower end of the pendulum. The mercury drop formed the horizontal connection between two small glass tubes filled with mercury. When by careful adjustment the drop was not brushed away by the mica strip in its passage, sharp regular breaks were obtained. The clock employed was constructed by Howard & Co., Boston.

To determine the absolute pitch of the fork, the daily rate of gain or loss of the clock was obtained by astronomical observations. A series of observations with the transit instrument, extending over the period of the experiments, was made, and the clock was found to gain 6.5 seconds per day.

The accuracy of the method can be seen from the following series of results:

Rotations of wheel (d).	No. of seconds.	Rate of fork.
24	263.1929	24.8031
29	318.0273	24.8029
27	296.0932	24.8030
27	296.0908	24.8032
31	339.9575	24.8031
29	318.0286	24.8028
25	274.1549	24.8035
27	296.0944	24.8029

Average = 24.8031.

The relation between the displacement on the photographic plate and the velocity of propagation can be found in the following way. We will suppose, preliminarily, that the axis of rotation of the mirror lies in its face. Let s (Fig. 5) be the spark-gap, and xx' the two positions of the mirror corresponding to the beginning and end of the displacement. While the mirror turns through the angle α, the image of s turns from a to b through the angle 2α. The photographic arrangement for obtaining an image of ab, taken as an object, is to be considered as entirely separate. The greater the distance of the spark-gap from the mirror, the greater is ab. With a given distance of spark-gap from mirror, the shorter the focal length of the lens that can be used the better, because the shorter-focus lens can be placed nearer the mirror, and will therefore gather more of the light. The sum of the distances of the mirror from the lens and from the spark-gap was made equal to twice the focal length of the lens. Then the displacement on the plate $a'b'$ was equal to ab, and there was no magnification.

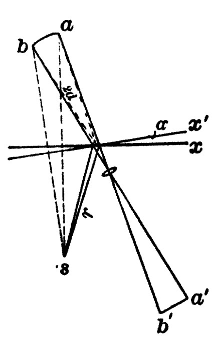


Fig. 5.

If n = number of turns of mirror per second the angular velocity $= 2\pi n$.

Then in the time $T = \frac{l}{V}$ the mirror turns through the angle $2\pi n\frac{l}{V} = \alpha$ say.

But $ab = r \cdot 2\alpha$.

$$\therefore\ a'b' = ab = 4\pi nr\frac{l}{V}$$

Suppose now the mirror, as in our experiment, does not rotate about a point on its surface. It will be readily seen from Fig. 6 that the locus of the image of a fixed point in the rotating mirror is an epitrochoid (limaçon). S is the spark-gap. M is one position of the mirror, O being the point about which it rotates. The angle α is measured from the fixed line OS. The angle $SOO' =$ the angle $S'O'O$. O', the center of a circle C' equal to the circle C, moves to its different positions as C' rolls upon C. The locus therefore of S', the image of S, is an epitrochoid, and its center of curvature is the center of the mirror Q, the point of contact of the two circles. The displacement $S'S''$ of the image, due to the rotation of the mirror through the angle $\delta\alpha$, subtends at the center of the mirror the angle $2\,\delta\alpha$. Consequently the formula obtained above still holds good so long as $\delta\alpha$ is sufficiently small. In our experiment $\delta\alpha$ was approximately equal to only one-half a minute and therefore the same law of the double angle holds as in the previous case.

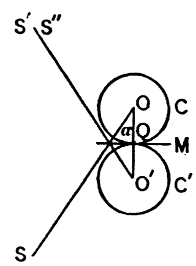


Fig. 6.

The spark-gap was placed at a distance of 186 cm. from the mirror in our experiments. All the distances were carefully measured by means of two sliding rods. Great care was exercised in the determination of the focal length of the lens, which was equal to 100.66 cm. A transparent scale divided into mm. was used, and the method followed was that indicated in Kohlrausch's *Physical Measurements*, No. 5.

Effect of the Resistance of the Wires.

If the wires were perfect conductors, the velocity of propagation would be the same as that of a wave in the dielectric. The question may be asked whether the resistance influences the velocity.

The equation for the propagation of the current or potential is, as first shown by Heaviside (*Phil. Mag.*, 1876, "Electrical Papers," Vol. I., p. 54),

$$L\frac{\partial^2 V}{\partial t^2} + 2R\frac{\partial V}{\partial t} = \frac{1}{K}\frac{\partial^2 V}{\partial x^2}$$

where L is the self-induction of the pair of parallel wires per unit of length, K the capacity per unit of length, and R the resistance of a single wire per unit of length. If $R = 0$, this becomes

$$\frac{\partial^2 V}{\partial t^2} = \frac{1}{KL}\frac{\partial^2 V}{\partial x^2},$$

which is the equation of propagation with the velocity

$$v = \frac{1}{\sqrt{KL}}.$$

If R is not zero, put $V = we^{-\frac{R}{L}t}$ and the equation becomes

$$\frac{\partial^2 w}{\partial t^2} = v^2\frac{\partial^2 w}{\partial x^2} + \frac{R^2}{L^2}w.$$

This is satisfied by the periodic solution

$$w = \sin\frac{2\pi}{\lambda}(x - v't)$$

of wave-length λ and velocity of propagation v', where

$$v'^2 = v^2 - \frac{R^2}{L^2}\frac{\lambda^2}{4\pi^2}.$$

For R we must put the resistance as affected by the imperfect penetration of the periodic current. According to Lord Rayleigh, when n, the frequency of the oscillations is very great

$$R' = \sqrt{\pi n \mu R}.$$

In the case of our longest circuit $n = 125{,}000$, roughly,

$$R\frac{\rho}{\pi r^2} = \frac{1620}{\pi(0.079)^2} \quad R' = \sqrt{\frac{n\rho}{r^2}} = 1.8 \times 10^5,$$

$$\therefore\ v'^2 = v^2 - \frac{R^2\lambda^2}{L^2 4\pi^2} = 9.10^{20} - 6.10^{16}.$$

The error, therefore, in the square of the velocity due to the neglect of the second term is only one part in 15,000, or in the velocity itself, one part in 30,000. This correction is too small to be taken into account.

Final Results.

After a determination of the velocity for the long circuit had been made, the wires were cut and results obtained for new lengths. The speed of the mirror was varied in each case. The primary spark took place between balls 9 mm. in diameter; the secondary spark was formed between magnesium points which were separated beyond the equivalent sparking distance so as to have true resonance. The rate of the fork was taken daily, but it was found to be practically constant: the room was kept at a uniform temperature, so that no temperature coefficient had to be applied. There were no special peculiarities in the photographs, except that a large number of plates were obtained with relatively very great distances between the oscillations. These distances too were variable, being different for different plates, with no apparent law existing. It may have been that these long oscillations were due to the direct action of the primary with its large capacity. The oscillations were measured on a dividing engine with a glass of low magnifying power.

The following tables exhibit the results.

Circuit I.

Resonance: $l_1 = 2701$ cm.; $l_2 = 55{,}592$ cm.; $\lambda = 222{,}368$ cm.

Plate.	No. of oscillations.	Total displacement.	Plate.	No. of oscillations.	Total displacement.
		centimeters			centimeters
1	3	2.653	7	3	2.648
2	4	3.531	8	4	3.521
3	4	3.544	9	4	3.538
4	4	3.531	10	5	4.410
5	5	4.428	11	5	4.421
6	3	2.661	12	4	3.526

Speed of mirror = 49.7420
Total number of oscillations = 48.0000
Sum of displacements = 42.4120
Displacement per oscillation = 0.8836

$V = 2.934 \times 10^{10}$ cm.

Plate.	No of oscillations.	Total displacement.	Plate.	No. of oscillations.	Total displacement.
		centimeters			centimeters
1	2	2.640	7	3	3.949
2	2	2.636	8	3	3.959
3	3	3.949	9	4	5.270
4	3	3.941	10	2	2.626
5	4	5.256	11	3	3.945
6	2	2.637			

Speed of mirror = 74.6130
Total number of oscillations = 31.0000
Sum of displacements = 40.8080
Displacement per oscillation = 1.3164
$V = 2.954 \times 10^{10}$ cm.

Circuit II.

Length of wire removed = 15,240 cm.
Length of wire left = 43,053 cm.
Resonance: $l_1 = 1484$ cm.; $l_2 = 41,569$ cm.; $\lambda = 166,276$ cm.

Plate.	No. of oscillations.	Total displacement.	Plate.	No. of oscillations.	Total displacement.
		centimeters			centimeters
1	4	2.582	9	5	3.224
2	4	2.579	10	4	2.580
3	3	1.933	11	4	2.581
4	3	1.936	12	4	2.579
5	6	3.871	13	5	3.224
6	5	3.228	14	3	1.935
7	5	3.223	15	5	3.227
8	4	2.580			

Speed of mirror = 49.552
Total number of oscillations = 64.000
Sum of displacements = 41.282
Displacement per oscillation = 0.645
$V = 2.994 \times 10^{10}$ cm.

Plate.	No. of oscillations.	Total displacement.	Plate.	No. of oscillations.	Total displacement.
		centimeters			centimeters
1	3	2.896	8	4	3.868
2	3	2.899	9	4	3.866
3	3	2.901	10	4	3.870
4	4	3.868	11	3	2.899
5	4	3.865	12	3	2.900
6	3	2.896	13	4	3.865
7	4	3.865	14	4	3.864

Speed of mirror = 74.328
Total number of oscillations = 50.000
Sum of displacements = 48.322
Displacement per oscillation = 0.966
$V = 2.998 \times 10^{10}$ cm.

Circuit III.

Length of wire removed = 14,909 cm.
Length of wire left = 28,144 cm.
Resonance: $l_1 = 614$ cm.; $l_2 = 27,530$ cm.; $\lambda = 110,120$ cm.

Plate.	No. of oscillations.	Total displacement.	Plate.	No. of oscillations.	Total displacement.
		centimeters			centimeters
1	6	2.562	8	5	2.134
2	6	2.558	9	4	1.708
3	4	1.705	10	4	1.706
4	3	1.279	11	5	2.131
5	5	2.134	12	5	2.134
6	5	2.130	13	6	2.559
7	5	2.133			

Speed of mirror = 49.5520
Total number of oscillations = 63.0000
Sum of displacements = 26.8730
Displacement per oscillation = 0.4266
$V = 2.998 \times 10^{10}$ cm.

Plate.	No. of oscillations.	Total displacement.	Plate.	No. of oscillations.	Total displacement.
		centimeters			centimeters
1	5	3.204	9	5	3.201
2	5	3.202	10	4	2.561
3	5	3.203	11	4	2.562
4	4	2.564	12	5	3.201
5	5	3.200	13	5	3.200
6	6	3.844	14	6	3.845
7	6	3.840	15	5	3.202
8	5	3.205			

Speed of mirror = 74.3280
Total number of oscillations = 75.0000
Sum of displacements = 48.0340
Displacement per oscillation = 0.6405

$$V = 2.995 \times 10^{10} \text{ cm.}$$

Plate.	No. of oscillations.	Total displacement.	Plate.	No. of oscillations.	Total displacement.
		centimeters			centimeters
1	3	2.556	4	3	2.558
2	4	3.410	5	4	3.414
3	3	2.559			

Speed of mirror = 99.1040
Total number of oscillations = 17.0000
Sum of displacements = 14.4970
Displacement per oscillation = 0.8528

$$V = 2.999 \times 10^{10} \text{ cm.}$$

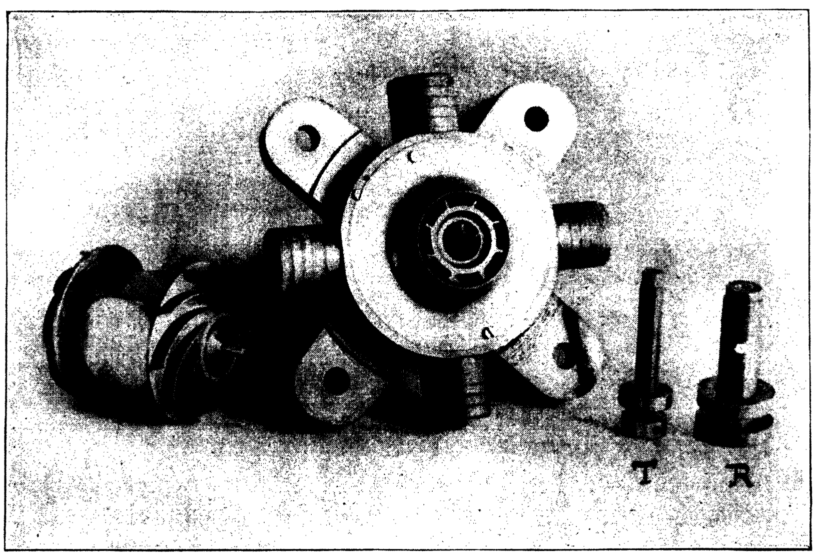
The value of the velocity calculated from the formula is corrected so as to be referred in the above to mean solar time. Why the longer circuit gives a lower velocity than the other two is not quite evident. The difference may probably be accounted for partly by the fact that the resonance was not so sharp in the case of this circuit and the maximum could not be determined to the same degree of precision. The oscillations, it is noticed too, are not so constant in length as in the case of the shorter circuits.

The value of the average velocity for the three circuits is 2.982×10^{10} cm. per second, and for the last two 2.997×10^{10} cm. per second.

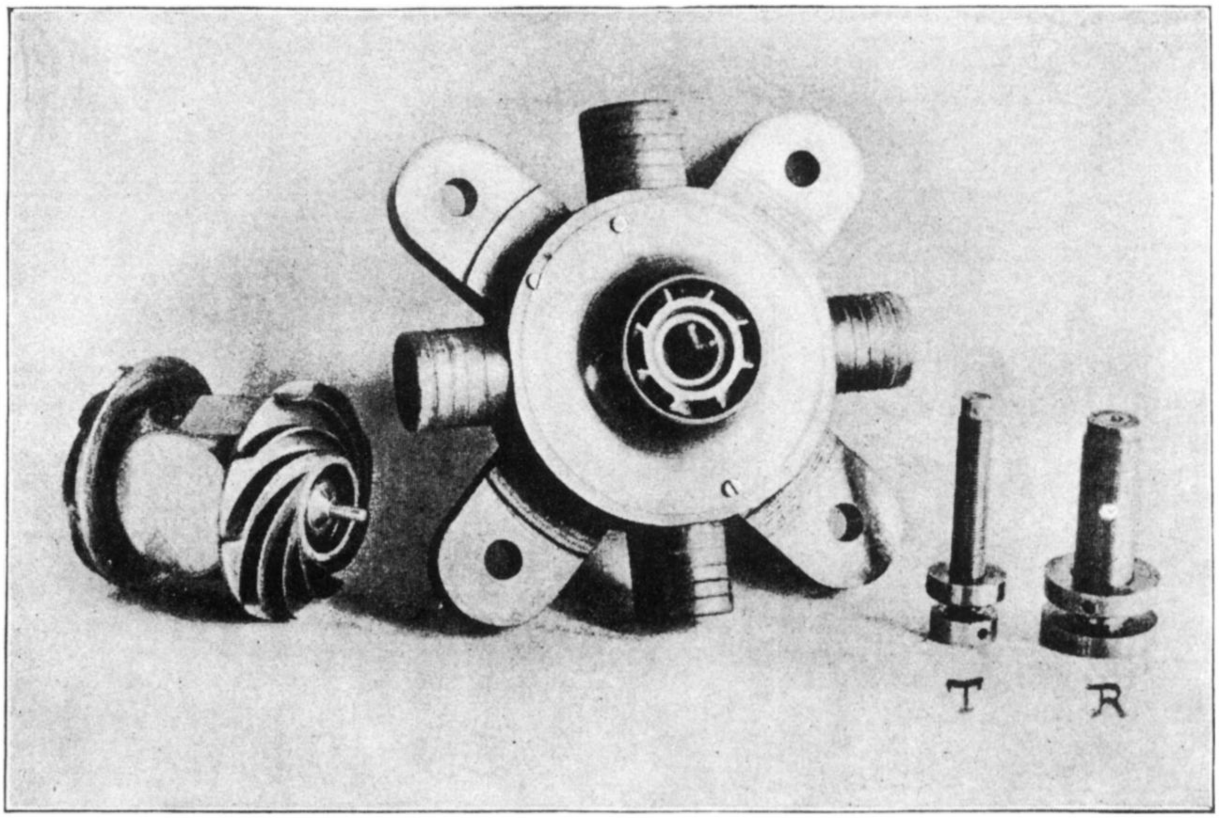
I wish to express special thanks to Prof. A. G. Webster for the kindness shown and the advice given continually throughout the progress of this work.

LABORATORY OF CLARK UNIVERSITY.

[The above research was completed in July, 1895, but various circumstances have prevented its earlier presentation for publication. — A. G. W.]



SAUNDERS: ELECTRIC WAVES.



SAUNDERS: ELECTRIC WAVES.