THE REFRACTIVE INDEX AND REFLECTING POWER OF WATER AND ALCOHOL FOR ELECTRICAL WAVES.[1]

BY A. D. COLE.

THE electromagnetic theory of light as originally given by Maxwell included only the optics of a single wave-length, and required extension to justify and harmonize the facts of selective absorption and dispersion. This extension was made by Helmholtz. According to him a change in the refractive index with the wave-length is to be expected throughout the complete spectrum; but beyond a certain wave-length these values approximate rapidly to a fixed limiting value, so that the index may be regarded as practically constant for longer waves. The period of this characteristic wave-length is theoretically of the same order of magnitude as that of the slowest possible vibration of the molecule, but it is practically unknown. Yet recent researches render it probable that this close approximation to a constant limiting value is exhibited by wave-lengths much smaller than all electrical waves thus far produced. H. Rubens has shown[2] that several substances which fail to satisfy the fundamental relation of Maxwell, $\mu^2 = K$ (where μ is the refractive index and K the dielectric constant of the substance), for light waves, do fulfil it for radiations in the infra-red only a little below the visible spectrum. For some other substances, — resin oil, olive oil, and certain kinds of glass, — Arons and Rubens[3] have shown that this relation is satisfied with electrical oscillations of 6 meters' wave-length, although not with light waves. E. Cohn[4] has further shown that even with water, whose refractive index for light differs so much from the square root of the dielectric constant, the index for long electrical

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[1] Author's abstract from Wiedemann's Annalen der Physik und Chemie. Bd. 57, p. 290, 1896.

[2] Wied. Ann. 45, p. 238, 1892.

[3] Arons u. Rubens, Wied. Ann. 42, p. 581, 1891.

[4] E. Cohn, Wied. Ann. 45, p. 370, 1892.

waves conforms to the law of Maxwell. By entirely different methods Ellinger[1] and Udny Yule[2] have reached similar results. It seems probable that the great change in the value of the refractive index shown by these substances occurs in a part of the spectrum much nearer light-waves than the Hertz oscillations. Still, the researches of Grätz and Fomm[3] and of Drude[4] indicate a certain amount of dispersion even for long electrical waves. It seemed desirable, therefore, to investigate the refractive index of these substances, particularly of water and alcohol, to see if it is the same for very short electrical waves as for long ones.

It seemed first advisable to repeat the measurements of the index for long waves by the method of Cohn with modified apparatus, and to extend the method to alcohol. It will be unnecessary to give more than a very brief account of these experiments, since P. Drude has published the results of a similar research since our work was finished.

I. Velocity of Long Electrical Waves in Water and Alcohol by the Method of E. Cohn.

The apparatus used was very similar to that used by Cohn. An induction coil developed oscillations in two zinc plates 40 cm. square, with attached rods separated by a spark gap. Each of two smaller plates, in front of these, was attached to one end of a long horizontal wire. These wires were stretched parallel, 2.5 cm. apart. They extended several meters through air and then passed through rubber stoppers in the end of a glass trough 68 cm. long, and through the whole length of the trough. The inside of this glass trough was coated with pure silver by chemical deposition. At the inner trough end, and at least two positions outside, bridges of wire were placed across the parallel wires, the proper location being determined when a maximum effect was noticed in two little "Leyden jars" halfway between adjacent bridges, as shown by a Paalzow-Rubens dynamo-bolometer[5] in connection with a re-

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[1] Ellinger, Wied. Ann. 46, p. 513, 1892.

[2] Udny Yule, Wied. Ann. 50, p. 742, 1893.

[3] Grätz u. Fomm, Wied. Ann. 54, p. 626, 1895.

[4] P. Drude, Wied. Ann. 54, p. 352, 1895.

[5] A. Paalzow u. H. Rubens, Wied. Ann. 37, p. 769, 1889.

flecting galvanometer. The "Leyden jars," as in the researches of Rubens,[1] consisted of short pieces of thick-walled glass tube threaded on the wires and surrounded by a single turn of fine wire as "outer coating." A second pair of "Leyden jars" was placed within the liquid in the trough, and served to show resonance when the wire bridges were brought into proper positions. These had threads of mercury as outer coatings, insulated by glass from the liquid in the trough. The research differed from that of Cohn in the following respects:

(1) Metallic containing vessels — one of zinc and one of silvered glass — were used. These were provided with metallic covers, as Rubens and Arons have shown that this is important with some substances. For water and alcohol, however, it seems to make little difference, as measurements with the glass trough unsilvered gave about the same result.

(2) The containing vessels were much smaller — from 3.5 to 10 liters capacity, instead of 50. This was made possible by having the parallel wires only 2.5 cm. apart, instead of 7 cm.

(3) The use of at least three bridges outside, and as many inside, gave opportunity to eliminate the influence of the trough end. Cohn assumed that the correction of the wave-length in air due to this cause was the same as for that in the liquid. But this was proved untrue experimentally for our apparatus, and so the values of the waves next the trough end were not used in the calculations.

(4) Four bridge positions were found within the trough of water instead of only two.

The results for distilled water are presented in tabular form. The letters λ_a and λ_w denote half-wave-lengths in air and water respectively, $\frac{\lambda_a}{\lambda_w}$ = ratio of velocities $= \mu$, the refractive index.

The experiments with the silvered trough were much more carefully performed than the others, and the value 8.95 for 19° C. cannot be far wrong. P. Drude has obtained 8.7 for waves 60 cm. long, and Cohn and Zeemann, 8.93 to 8.99.

In similar measurements with commercial absolute alcohol more

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[1] H. Rubens, Wied. Ann. 41, p. 154, 1890.

difficulty was experienced in getting good resonance and well-defined maxima. Only two bridge positions within the liquid could be obtained. The alcohol seemed to have higher absorption for waves of the length used than even quite impure water. Drude has noted the same difficulties. For waves whose half-length in air was 129.4 the refractive index 5.24 at 18° was obtained. Drude found a smaller number, 4.74.

Trough used.	λa.	λw.	$\mu = \frac{\lambda a}{\lambda w}$.	**Temp. C.**
Glass 68 × 7.5 × 7 cm. . . .	261.3	27.8	8.5	18°
Zinc 100 × 10 × 10	299.7	33.5	8.9	16–18°
Glass silvered 68 × 7.5 × 7 cm.	155.7	$\left.\begin{array}{l}17.1\\17.6\\17.4\end{array}\right\} 17.4$	8.96	18–20°

II. REFRACTIVE INDEX FOR SHORT ELECTRICAL OSCILLATIONS.

The experiments just described demonstrate that the refractive index of water for waves of 300 cm. to 600 cm. is practically constant, and those of Drude carry the limit of constancy down to 60 cm. Our results for alcohol give a higher value for alcohol than his, as we have seen. This may be because we used waves four or five times as long as his. The difference, however, is too small, and the method too uncertain, to justify the inference that the index varies with the wave-length. Later experiments with the short-period oscillations first produced by Righi[1] form a surer basis for such a conclusion.

I first tried to measure the deviation of Righi waves 5 cm. long, produced by a hollow prism, with sides of thin plate glass 15 × 25 cm. and 10° refracting angle. Although measurements could be made in this way with resin oil, the method failed for water and alcohol. The loss of energy by absorption and reflection was so

[1] A. Righi, Rendiconte della R. Accad. de Lincei 2. 1 Sem (11), p. 505.

great that no sparks could be seen in a Righi resonator on the other side.

Next an attempt was made to produce a more sensitive receiving apparatus. Klemenčič[1] had used a thermopile resonator for long waves. This arrangement on a greatly reduced scale gave most excellent results in our work. In Fig. 1, *A* and *B* are bits of copper foil 10 mm. long by 1.5 mm. wide and 4 mm. apart. To each is soldered a fine wire 4 mm. long and bent to an L shape. One is of iron, 0.02 mm. in diameter, the other of German silver, 0.08 mm. in diameter. The wires cross in the middle and are attached to weak copper springs, which maintain a gentle pressure at the point of contact. These springs are connected to a low-resistance reflecting galvanometer. This resonator is mounted on a light wooden frame at the focus of a concave parabolic mirror made of cardboard, covered with three rows of tinfoil strips, 24 mm. long, placed parallel to the focal line of the mirror. As the thermo-element proved very sensitive to heat radiation, it and its mirror were permanently enclosed in a cardboard box, a material which experiment had shown was perfectly transparent to the electrical waves.

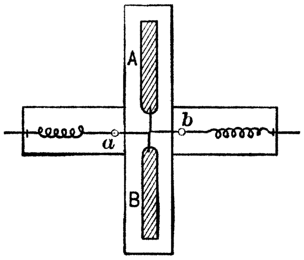

Fig. 1.

A Righi exciter 1 meter away produced galvanometer deflections of 100 to 200 scale divisions. Experiment proved, as theory requires, that deflections were proportional to the energy of the oscillations. Thus the deflection was four times as great when the resonator was 25 cm. distant from the exciter as when it was 50 cm. away. Thus quantitative results were obtainable.

But again the prism experiments failed. A deflection of 100 vanished completely when a prism of water or alcohol was introduced. A flat cell with a layer of water 11 mm. thick allowed only 2 per cent to 3 per cent of the energy to pass. Another with a water film only 0.4 mm. thick showed these results:

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[1] Klemenčič, Wied. Ann. 45, p. 62, 1891.

Empty cell, 65 per cent; filled with alcohol, 57 per cent; with water, 32 per cent.

As the reflection of 5 cm. waves on a film 0.4 mm. thick can be only very slight, even for a liquid with refractive index 9, the large energy loss must be due, directly or indirectly, to absorption.

This experience led to the abandonment of the prism experiments and an effort to measure the refractive index for electrical oscillations by an indirect method. The possession of a receiver capable of giving quantitative results suggested the use of Fresnel's formulas connecting reflected energy with refractive index. Their applicability involves the fulfillment of the condition that the square of the absorption coefficient of the substance be small compared to $(\mu - 1)^2$, where μ is the index, a condition which is certainly met by water and alcohol. Further, for water the independence of the reflecting power and the absorption coefficient was established by experiments in which the latter was made to vary through wide limits without affecting the former materially.

It was not convenient to compare directly the amounts of direct and reflected energy, but it was practically simpler to compare the reflecting power of the substance studied with that of a metal surface, and this amount with that received directly.

Reflecting Power of a Metal for Electrical Waves.

The exciter used by Righi for developing electrical oscillations of short period can be used only in a vertical position. This made another form necessary, and that in Fig. 2 was finally adopted. *A* and *A′* are glass tubes 5 cm. long. Their inner ends support two bits of brass wire 8 mm. long, with ends rounded. These glass tubes are held by corks in a wider T-tube, as shown, and allow the distance between the bits of brass to be varied. Two copper wires of 1 mm. diameter, their ends bent into a round loop, are held by the friction of little springs centrally in the glass tubes. The outer ends of these wires are joined to the terminals of an induction coil, whose discharge thus has three spark gaps. The T-tube is filled with heavy petroleum oil, so that the middle spark gap is in oil, the others in air. The tube may be turned about the axis *ab*. A bit of capillary tube *c* allows the gases formed by the decomposition of the oil to escape. For other positions the gases escape through *E*, which is an elbow, connected by a ring of rubber tubing with the third limb of the T-tube. The whole is mounted on a wooden support attached to a spherical concave mirror, so that the middle spark gap remains at the focal point of the mirror when the latter is inclined. The energy radiated was found to vary only slightly when the lengths of the air-gaps were changed, but was much influenced by the length of the oil-gap.

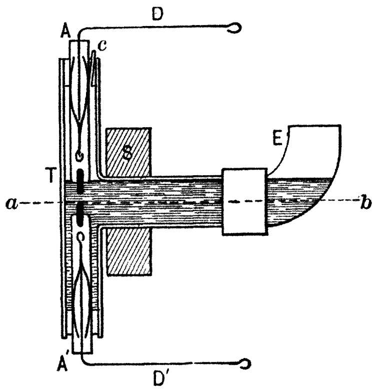

Fig. 2.

Figure 3 shows the arrangement employed to measure the reflecting power of a zinc plate. *C* is the spherical mirror, 50 cm. in diameter, at whose focus *E* the exciter was placed. At a distance of 25 cm. was a pin about which revolved a wooden arm 30 cm. long. At its end the receiving apparatus already described was placed. After four observations of the direct radiation along the axis of the mirror at *R*, the wooden arm was revolved 90° to *R'*, the movable zinc plate *M*, 40 cm. square, placed at an angle of 45° to the axis, and four readings of the reflected energy taken. A series comprised 24 to 32 readings, taken by fours in each position alternately. Three series were taken with the oscillations perpendicular to the incident plane, and eight for those parallel to it. The results were as follows :

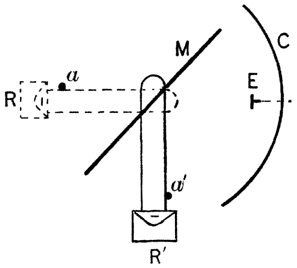

Fig. 3.

Oscillations perpendicular.	**Oscillations parallel.**
99.5	87.9
101.1	93.1
100.1	88.8
	93.2
	92.9
	95.9
	93.8
	93.5
Mean . 100.2	Mean . 92.4

Thus a metal surface at 45° reflected the radiation without loss if the oscillations were perpendicular to the incident plane, but with about 8 per cent loss if the oscillations were in the incident plane.

Reflection of Electrical Oscillations by Liquid Surfaces.

The axis of the exciter with its mirror was inclined 45° downward, and that of the receiver 45° in the opposite direction. At the intersection of these axes the surface of the liquid to be studied was placed, the liquid being contained in a flat developing tray, 44 × 36 cm. On account of the strong absorption of water and alcohol for these short waves, no interference with reflections from the bottom of the tray was to be feared. A screen of metal, 40 cm. square, was placed between exciter and receiver to cut off any direct radiation. Supports were provided to hold a plate of sheet zinc just above the liquid surface. Three observations of the energy reflected from the metal surface were made, then the metal removed, and an equal number made for the radiation reflected from the liquid. Twenty to thirty readings constituted a series. Five series for oscillations perpendicular, and four for those parallel to the incident plane, were made. An example shows the kind of results obtained. The numbers are galvanometer readings in scale divisions.

Oscillations perpendicular.		Oscillations parallel.	
Water surface.	Metal surface.	Water surface.	Metal surface.
31.0	41.0	18.5	33.0
31.0	41.0	18.5	32.0
30.0	43.0	19.5	30.0
31.0	42.0	18.5	32.5
33.0	43.0	19.5	35.0
30.0	45.0	21.0	31.0
30.0	40.0	17.5	31.5
31.0	43.0	18.0	32.0
31.0	43.0	18.0	27.0
29.0	40.0	15.5	
33.0	43.0	19.0	
30.0	43.0	17.5	
Mean 30.8	42.3	18.4	31.6

Whence

$$\text{reflecting power (perpendicular)} = \frac{30.8}{42.3} = 73 \text{ per cent,}$$

and

$$\text{reflecting power (parallel)} = \frac{18.4}{31.6} \times 0.924 = 53.7 \text{ per cent,}$$

0.924 being the proportion of the total radiation falling upon it that was reflected by the metal plate at 45°.

The results of the several series are collected in the following table: —

Oscillations perpendicular.	Oscillations parallel.
73.6 %	45.6 %
73.9	53.7
69.8	57.1
73.0	54.6
71.8	
Mean . 71.8 %	Mean . 52.7 %

These values were inserted in the two Fresnel formulas for rays polarized in the two planes,

$$(u_{\text{par}})^2 = \left(\frac{\sqrt{\mu^2 - \sin^2 i} - \cos i}{\sqrt{\mu^2 - \sin^2 i} + \cos i}\right)^2, \tag{1}$$

and

$$(u_{\text{perp}})^2 = \left(\frac{\mu^2 \cos i - \sqrt{\mu^2 - \sin^2 i}}{\mu^2 \cos i + \sqrt{\mu^2 - \sin^2 i}}\right)^2, \qquad (2)$$

where i is the incident angle 45°, μ the refractive index, and u the amplitude of the oscillations. If the electrical oscillations are perpendicular to the plane of polarization,

R parallel $= (u_{\text{perp}})^2$,
and R perpendicular $= (u_{\text{par}})^2$.

From equation (1) $\mu = 8.8$
(2) $\mu = 8.9$
Mean 8.85

These results were obtained with distilled water, but the addition of conducting substances, even 50 cc. of H_2SO_4, did not change the amount of energy reflected, although the absorbing power must have been greatly increased. Hence the absorbing power of pure water cannot be great enough to render the Fresnel formulas inapplicable.

Thus practically the same value was found for the refractive index of water for 5 cm. waves as for those 150 cm. to 300 cc. in length. But this was not true of alcohol. Nine series of observations with it gave these results: —

	Oscillations parallel.		**Oscillations perpendicular.**
	39.7 %		13.6 %
	35.3		17.5
	44.3		14.5
	42.0		16.2
	41.4		
Mean .	40.5 %	Mean .	15.4 %

Inserting these values

Formula (1) gives $(u_{\text{par}})^2 = R_{\text{perp}} = 15.4$; $\mu = 3.25$
$(u_{\text{perp}})^2 = R_{\text{par}} = 40.5$; $\mu = 3.15$
Mean $\mu = 3.2$

Thus the two formulas give the same value for μ, but this value is much lower than that obtained for long waves. This suggests that alcohol possesses strong dispersion for radiation of the wave-lengths used. Apparatus was partially completed for studying oscillations of 1 cm. and of 8 cm. wave-length, but time was lacking to complete the research. We hope to be able to resume the work soon.

The results obtained were so unexpected that the method was tested by applying it to a liquid whose refractive index had been shown to be the same for long electrical waves as for light waves. Refined petroleum (kerosene oil) was taken. A deep vessel was used to avoid disturbing reflections from the bottom. The energy reflected at 45° for oscillations perpendicular was 9.2 per cent, corresponding to $\mu = 1.5$, a value which agrees well with those for light and for long electrical waves.

In conclusion we gather the principal results into one table.

Substance.	Reflecting power.		Refractive index.	
	Oscillations I.	Oscillations II.	Formula (1).	Formula (2).
Metal (zinc) . .	100.0 %	92.4 %	—	—
Water	71.8	52.7	8.8	8.9
Alcohol	40.5	15.4	3.2	3.1
Refined petroleum	9.2	—	1.5	—

Summarizing all the results, we find that

(1) For oscillations of 300 cm. to 600 cm. total wave-length the refractive index for water is 8.95, for alcohol 5.20.

(2) For oscillations of 5 cm. wave-length the two Fresnel formulas give the same value for the refractive index from observations of energy reflected at 45°. For water this value is 8.85, for alcohol 3.2.

(3) The refractive index of alcohol is decidedly greater for long than for short electrical oscillations.

I desire to express my thanks to Dr. Rubens, of the University of Berlin, for numerous suggestions and other help during the whole course of the investigation, and also to Professor Warburg, in whose laboratory the work was done, for his interest and advice.