MINOR CONTRIBUTIONS.

On a Possible Development of the Idiostatic Electrometer.

By C. Barus.

1. The idiostatic electrometer has not hitherto been developed to an extent comparable with the quadrant electrometer, at least in so far as measurement of small potential differences is concerned. Yet the idiostatic method is apparently the more reasonable, seeing that it introduces no foreign potentials to coöperate with those under investigation. It might seem possible, moreover, to secure an advantage by dispensing with the viscous liquid in which the swinging needle of the quadrant electrometer is damped. I purpose, therefore, in the following paper to describe certain results obtained in an endeavor to perfect the idiostatic electrometer. The experiments proved to be delicate; indeed, one needs to cope with the trying difficulties which I have met with in these attempts to really appreciate the admirable efficiency of the quadrant electrometer.

2. The two forms of apparatus available are the gold-leaf type and the condenser type, but only the latter gave me results which seem worth describing. The pattern adopted is identical with Lord Kelvin's[1] absolute electrometer, and the conditions of sensitiveness are therefore given. To fix the ideas, let the considerations refer to the apparatus annexed, as this is a diagrammatic view of the one used below. Here *A* is the fixed, and *D* the movable, plate of the condenser, *c* the guard ring, Fig. 2 being a

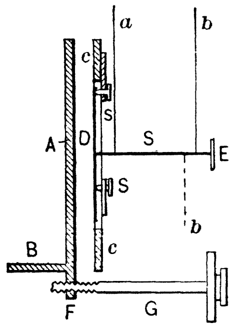

Fig. 1.

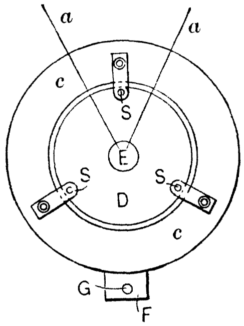

Fig. 2.

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[1] See papers on Electrostatics and Magnetism, p. 281 *et seq.*

front view, Fig. 1 a section at right angles to it. If the disk is to be movable parallel[1] to itself, it may be suspended from parallel silk threads a and b, each of which is broadly bifilar above (see Fig. 1) to prevent lateral oscillation. The disk D is made very light, and the guard ring closed behind in form of a shallow box, with a small central hole at d to admit of free motion of the stem S. Three small screws s serve as stops for the disk D. Finally, the plate A is free to slide on B parallel to itself, and this motion is controlled by the micrometer screw G working in the lug F. E is a small, thin-plate mirror rigidly connected with the movable disk D by the stem S. If the system DSE, with the broad disk D, be made very light, the air damping is excellent.

Suppose, now, that the difference of potential between A and D in this apparatus is 1 volt, and their distance apart d. Then, if A be the area of D in square centimeters, and F the total force on the disk in dynes,

$$F = A/(8\pi \cdot 10^4 \cdot 9d^2).$$

If $A = 8\pi$, or if the area of A be made 25.133 cm^2. (diameter 5.65 cm., a convenient size in practice), we have, simply, $F = (300d)^{-2}$; or

$d = 1.00$ cm. $F = 0.000011$ dynes
$= 0.10$ $= 0.001100$
$= 0.01$ $= 0.110000$

Thus the forces to be measured are excessively small even for two disks as near together as $\frac{1}{10}$ mm.

In the above apparatus, forces to equilibrate these values are, for small deflections, $mg \cdot \phi$, where m is the mass of the system DSE, and ϕ the deflection of the pendulum. If l be the length of the pendulum, and x its displacement, $\phi = x/l$; and therefore $x = Fl/mg$.

The value for l being 22.5 cm., the mass of the disk 0.5 g., and its appurtenances 0.71 g., the excursions x for different distances apart d of the condenser plates A and D are:

$d = 1.00$ cm. $x = 0.0000003$ cm.
$= 0.10$ $= 0.0000352$
$= 0.01$ $= 0.0035200$

The excursions x to be measured are thus correspondingly small, and, for $d > 0.1$ cm., small even as compared with the wave length of light.

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[1] If rotation of the disk around the horizontal is to accompany the motion, threads a and b' may replace a and b. The mirror E may then be observed with scale and telescope.

3. Having recently had occasion to work with Michelson's refractometer,[1] it seemed worth investigating whether the method could be used for a movable mirror *E* under the above conditions. The excellent damping secured for the disk *D* had made this probable. The adjustments are given in the diagram (Fig. 3), where *E'* is the stationary plane mirror corresponding to the mirror *E* of the system *DSE*, *P* the thick refractometer plate, and *T* the telescope. The sodium flame and appurtenances are placed beyond *L*. The optical conditions are very fully discussed in Michelson's great memoir (*l.c.*).

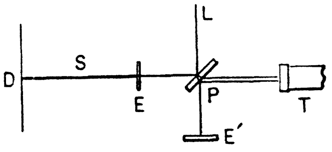

Fig. 3.

I found it convenient to use the large, flat plate of an old Kirchhoff spectrometer, the above electrometer *ADE* replacing one of the telescopes, and the mirror *E'* the other. The glass plate *P* on leveling screws was moved on the plate. *E'* being revoluble on the axis of the spectrometer, it required fully four complete rotations of the micrometer tangent screw (about 0.4°) to pass the fringes from the initial to the final vanishing point through maximum visibility.

Thus far I have not obtained results with a *free* disk *D*. The fringes so observed were fleeting; they shifted, rotated, and changed form continually when visible, and reappeared too rarely. But I have not given the matter up, as I think that, with better provision against air currents and earth tremors, the attempt will be feasible. For the present, I added stop screws as shown in the figure at *s*. These kept the fringes in the field, but did not quite wipe out their slow and regular motion. With a rather heavy disk (3.5 g.) of tin plate, and a distance apart *d* of about 0.25 cm., I obtained a deflection of one fringe per 20 volts of potential difference between *A* and *D*. For a free disk, the excursion *x* should have corresponded to about fifteen fringes for 20 volts; or we would have obtained fifteen times as much displacement had it not been necessary to lift the disk off the stops.

4. The plan for a feasible apparatus has thus been suggested; viz. to allow the disk to recline very lightly on stops (*s*, *s*, *s*, Fig. 1), and to move the plate *A* forward by the micrometer screw until the disk is just lifted off by a fraction of a wave length. Then the difference of potential is simply proportional to the distance *d* between the plates.

Before deciding to make a definite piece of apparatus with a micrometer slide, I wished to test the present arrangement further with the object of ascertaining how far the sensitiveness could be pushed. Accordingly in

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[1] Michelson, "Valeur du Mètre en Longueurs d'Ondes Lumineuse," Tome xi., Trav. et Mém. du Bureau internat. des Poids et Mesures, 1894.

the following experiments the face F is still reckoned in terms of the number of wave lengths which pass the cross-hair in the telescope when the condenser is charged and discharged alternately.

To obtain a sensitive system, the disk D was made of thin plate mica, silvered on both sides. The mass of the movable system DSE was thus reduced to 0.71 g. It was charged through the stop screws s. In operating the apparatus I encounted a formidable difficulty, inasmuch as with a very light disk all but free, there was *no contact* between the disk and the stops, and it was now, as a rule, found impossible to charge the disk with small potentials. Liquid contacts, aside from other reasons, are evidently out of the question because of the relatively immense capillary forces involved. After many trials I finally found that a sure contact could be maintained between the guard ring and the disk by connecting the two permanently with a long narrow pendent strip of gold leaf (not shown in Figs. 1 and 2). The ends of this were attached with mucilage to the guard ring C, and the disk D, in such a way as not to hamper the motion. Again, because of the difficulty of obtaining contacts[1] with small voltages it was now possible to move the disk D almost indefinitely near A, and thus obtain relatively great sensitiveness, without the risk of discharging the condenser. Finally, in view of the close proximity in question, the motion of the disk was so slow, that there was no difficulty in counting the fringes on charging or discharging.

A trial of the apparatus in the new form gave me much better results, the fringes passing for a single volt being about 10. Since in an undisturbed apparatus less than $\frac{1}{10}$ fringe may be estimated, the sensitiveness has thus become about $\frac{1}{100}$ volt. Had the disk been about 0.05 cm. from the opposed condenser plate, about 5 fringes would have passed. Hence the disk must have been very near the plate, and the stop screws in good adjustment. Many experiments of this kind were made, but they yielded no additional results. Most of the work had to be done at night.

5. In conclusion it is interesting to inquire into the construction and sensitiveness of divers apparatus of the present kind under favorable conditions. It would be best to make the plate A and guard ring C of plate glass with rounded outer edges and silvered on both sides. The disk D, if made of microscope cover glass, silvered, would not weigh above 0.035 g. per square centimeter, or about 0.88 g. for the area 8π cm^2., which is within one gram for the movable system of disk and mirror. In such a case, parallelism between the silvered plates could be tested in the usual optic way by reflection, and the adjustment of the set screws controlled by

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[1] The usual absence of contact here recalls the case of two soap bubbles which fail to adhere unless pressed together for some time, or otherwise brought in contact. Cf. E. Kaiser, Wied. Ann., LIII., p. 667, 1894.

their own reflected images. If with better protection against air currents, etc., it is possible to work with a free disc *D*, the electrometer would remain absolute in its registry. The sensitiveness of such an apparatus would probably be of the order of 0.001 volt for a minimum of dielectric thickness.

If the absolute character of the instrument be sacrificed in favor of greater sensitiveness, one may profit by the above experience, and replace the movable system *DSE* by a sheet of gold leaf made to cover the whole space on the inner face of the guard ring smoothly. The mirror *E* could then be reduced in size so as to weigh not much above a milligram. In such a system the counter force would be in part flexure and in part gravity. The former may be evaluated by the aid of the equation

$$5x = (r^4/s^3)(p/E),$$

where x is the central displacement due to the pressure of p dynes per square centimeter, r the radius of the disk (2.8 cm.), and s its thickness (say 0.0001 cm.). The pendulum forces are found as above. The results show that in this case $x = F/4$ nearly, where F is the total force on the disk, as given in § 2. Hence such an instrument should in the extreme case register as small a potential as 100 microvolts, supposing $d = 0.01$ cm., and the motion of one-tenth of an interference fringe discernible.

Finally, the best conditions are secured by measuring the counterforces not by gravity, but in the usual way, by torsion. I have made an estimate based on the above experiments, supposing that two identical disks like *D*, in Fig. 1, are symmetrically joined by a light rigid rod, 20 cm. long, and suspended from a fine wire, 0.01 cm. in diameter, attached to the middle of the rod, with the ends fixed 10 cm. above and below it. In such a case, if $d = 0.01$ cm., the motion of one-tenth interference fringe would correspond to less than 20 microvolts. Whether this sensitiveness can be attained or not will depend very largely on quiet surroundings. Beyond this there does not seem to be more serious difficulty than was encountered in the above experiments carried on with very ordinary facilities.

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EMPIRICAL FORMULÆ FOR VISCOSITY AS A FUNCTION OF TEMPERATURE.

BY A. WILMER DUFF.

SEVERAL formulæ for the viscosity of liquids at different temperatures have been proposed. The writer has recently required the use of such a formula for a wide range of variation of viscosity. The following