A METHOD FOR ENERGY MEASUREMENTS IN THE INFRA-RED SPECTRUM AND THE PROPERTIES OF THE ORDINARY RAY IN QUARTZ FOR WAVES OF GREAT WAVE LENGTH.[1]

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IN previous determinations of the distribution of energy in the infra-red spectrum, either the principle of the thermoelement, or of the more modern bolometer, has been almost without exception the basis of measurement.[2]

In the present study, however, a modified form of the Crookes radiometer has proved so efficient that I take the liberty of describing briefly the apparatus and method by which the observations on quartz, given in the second half of the paper, were made.

I. APPARATUS AND METHOD.

1. *The Spectrometer.*

A diagram showing the relation of parts of the reflecting spectrometer used is given in Fig. 1. Rays enter the instrument through the bilateral slit s_1, are brought to parallelism by the concave mirror S_1, and, after reflection on the plane mirror S_2, traverse the prism P, are again collected by the concave mirror S_3, and brought to focus at s_2 in the tube F. The only unusual feature of the arrangement is the introduction of the plane mirror S_2 rigidly attached to the prism, a modification due to Wadsworth,[3] in which, contrary to the usual process, the telescope remains fixed, and the spectrum is made to travel across the field of view by rotating the combined prism and mirror. This adjustment is

[1] Read before the Berlin Academy, Nov. 5, 1896.

[2] Pringsheim [Wied. Ann., 18, p. 32 (1883)], in the study of certain wave lengths in the infra-red solar spectrum, used a Crookes torsion radiometer combined with a grating, but the form of apparatus differed, in many respects, from the arrangement here to be described.

[3] Phil. Mag., 38, p. 337, 1894.

necessary to the present method, because, unlike the bolometer, the radiometer is not conveniently portable.

The eye-piece in the tube F was removed, and a second bilateral slit s_2 was brought into the focal plane of the concave mirror S_3. Rays emerging from this slit were collected by the rock-salt lens

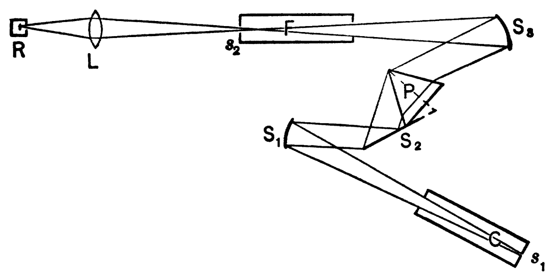


Fig. 1.

L, and brought to focus on one vane of the radiometer at R. The large fluorite prism $P\,(\phi = 60°\ 0')$ was calibrated in accordance with Paschen's[1] indices of refraction.

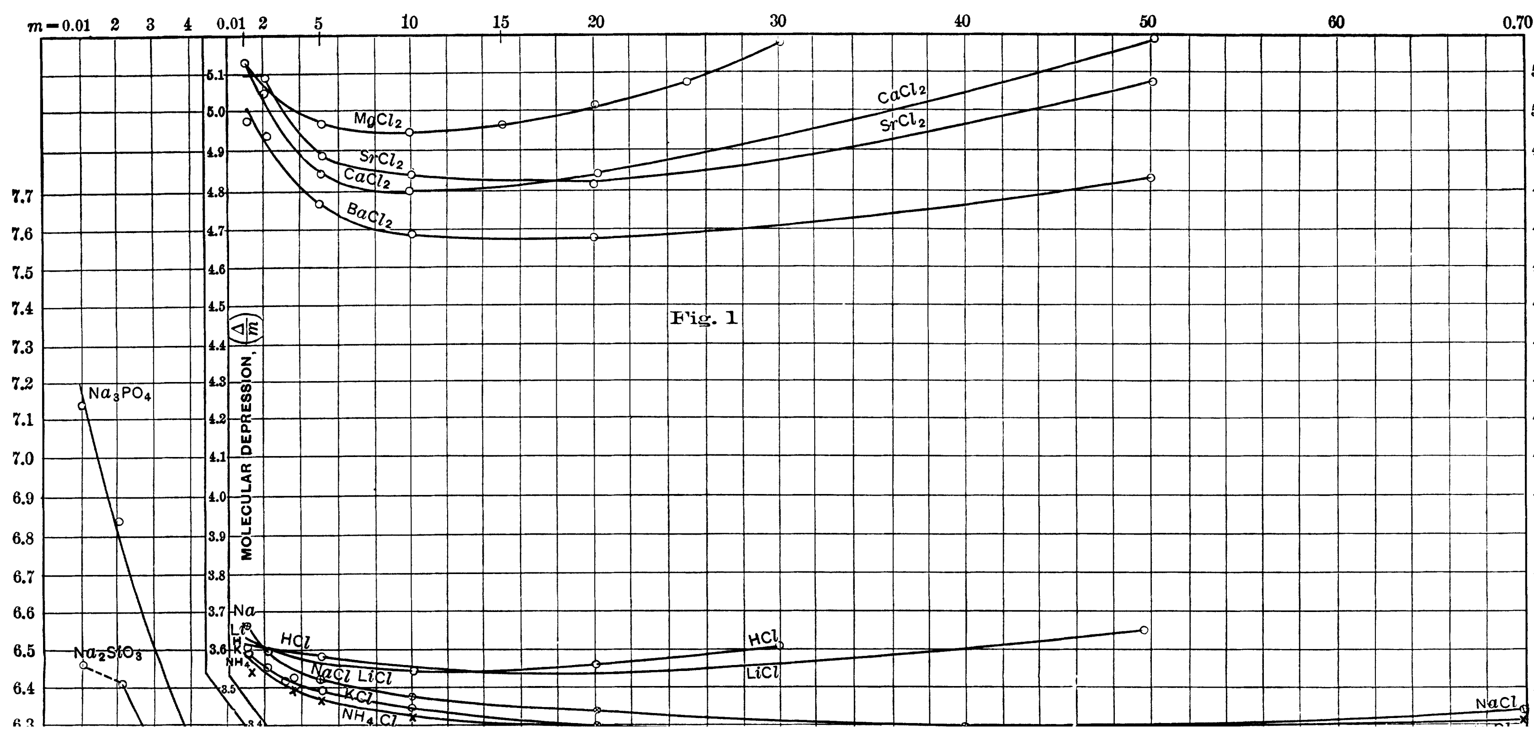
The calibration curve was tested in the visible spectrum, and found correct to 2 units in the fourth decimal place. In the infra-red the curve was again tested for the middle of the absorption band in sylvite, given at $\lambda = 7.08\ \mu$ by Rubens,[2] and here observed at $\lambda = 7.088\ \mu$ according to the curve. At this point a sylvite plate 4 mm. thick transmitted only 6 per cent of the incident energy. This may be accepted as evidence of the purity of the spectrum obtained, and of the absence in this region of more than a small percentage of stray rays of shorter wave lengths.

2. *The Radiometer—Construction.*

The details of construction of the radiometer will be doubtless most easily understood from Fig. 2, which shows a section of the instrument facing the spectrometer.

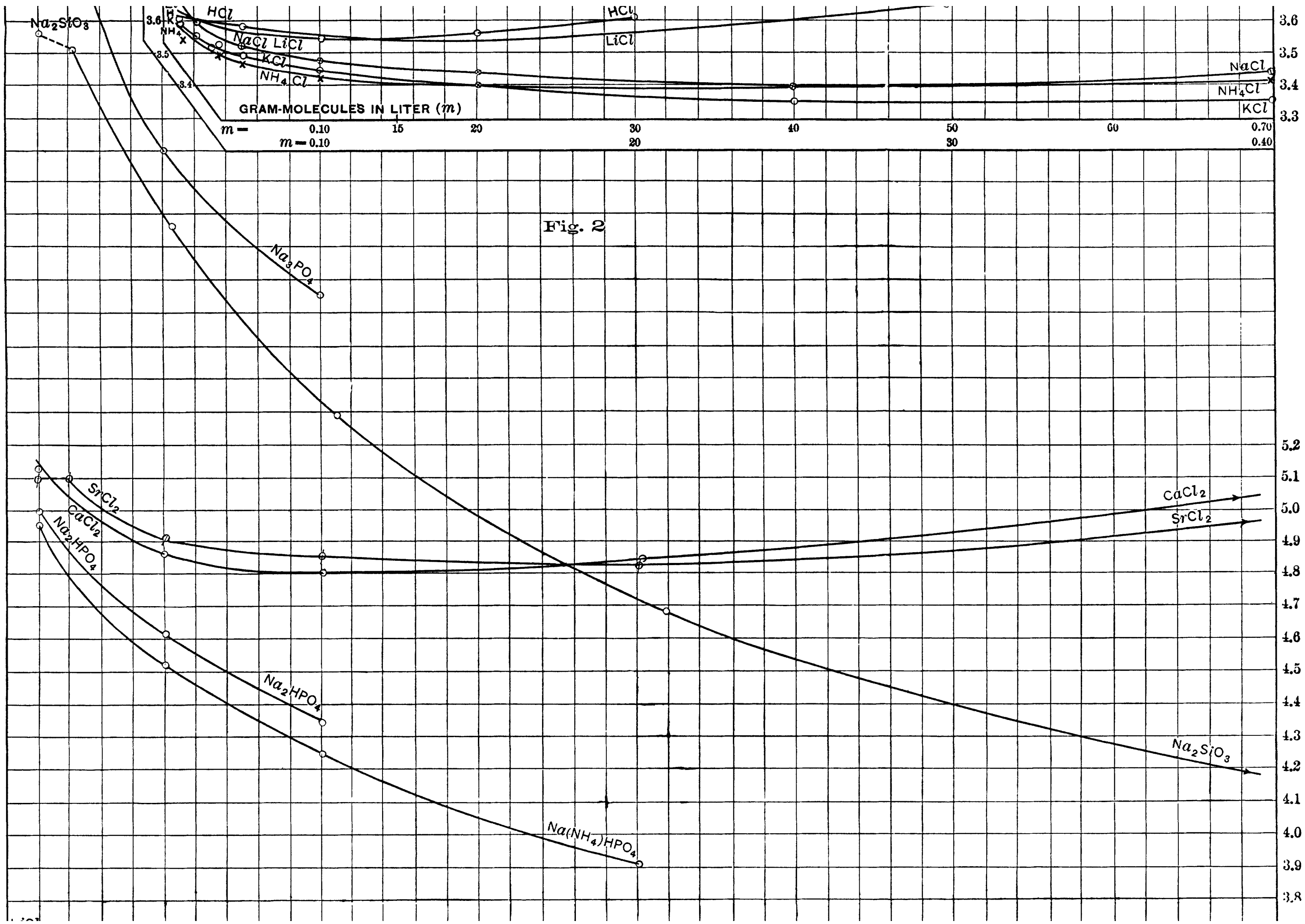
The figure is drawn to scale, and represents the instrument at half its natural size. The outer case A, a block of bronze with an axial boring from the top to within 5 mm. of the base, is supported upon three leveling screws, and is closed at the top by a bell of

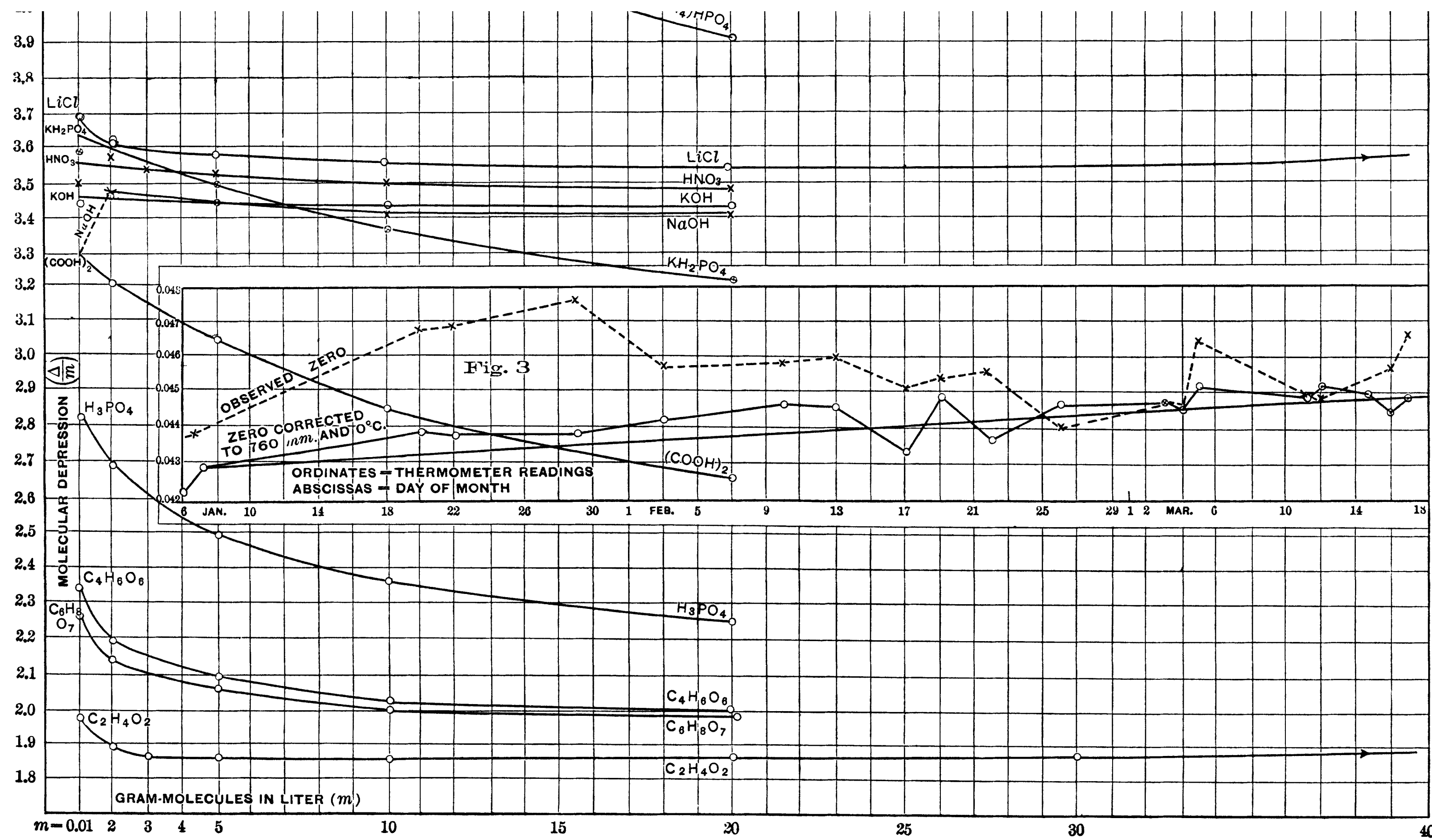
[1] Wied. Ann., 53, p. 301, 1894. [2] Wied. Ann., 54, p. 481 (footnote), 1895.



m = 0.01 2 3 4 0.01 2 5 10 15 20 30 40 50 60 0.70
MOLECULAR DEPRESSION, (Δ/m)
MgCl₂
SrCl₂
CaCl₂
BaCl₂
Na₃PO₄
Na₂SiO₃
Na
Li
K
NH₄
HCl
NaCl LiCl
KCl
NH₄Cl
NaCl

Fig. 1


GRAM-MOLECULES IN LITER (m)
m — 0.10 15 20 30 40 50 60 0.70
m — 0.10 20 30 0.40
HCl
LiCl
NaCl
KCl
NH4Cl
Na2SiO3
Na3PO4
SrCl2
CaCl2
Na2HPO4
Na(NH4)HPO4
6.5 6.4 6.3 6.2 6.1 6.0 5.9 5.8 5.7 5.6 5.5 5.4 5.3 5.2 5.1 5.0 4.9 4.8 4.7 4.6 4.5 4.4 4.3 4.2 4.1 4.0 3.9 3.8
3.6 3.5 3.4 3.3
5.2 5.1 5.0 4.9 4.8 4.7 4.6 4.5 4.4 4.3 4.2 4.1 4.0 3.9 3.8
Fig. 2



LOOMIS: FREEZING-POINTS OF SOLUTIONS.

glass *B* ground on the upper surface of the case. *H* is a stop-cock to cut off the connection to a Raps' mercury pump used to produce the vacuum. The case *A* contains two lateral borings: *C*, closed by a plate of mirror glass through which the deflections of the suspension are read, and *F*, closed by a fluorite plate through which the rays to be measured enter the radiometer. The arrangement of parts in the window *F* is seen in Fig. 2 *a*, which shows a sectional plane perpendicular to the former one.

The circular fluorite plate *P*, 5 mm. thick, is held in a screw fastening with rubber packing at *d*, and the whole mounted on a brass plate cemented to the case. In the boring *F* a short brass tube *r* is soldered, which extends to within 2 mm. of the axis of *A*, and carries a thin plate of mica *k* on its inner end.

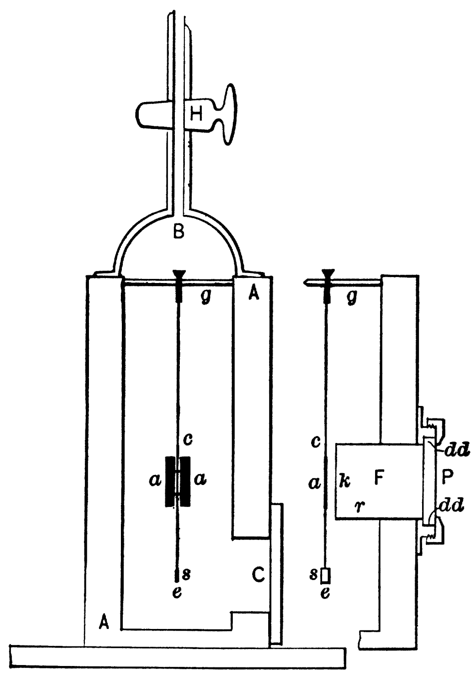


Fig. 2. Fig. 2 *a*

Directly behind the mica plate hang the two equal mica vanes *aa*, both blackened on the same side, which form the important part of the torsion suspension. The vanes are held together by thin whips of drawn glass, and on the line midway between them is fastened the glass fiber *ce*, which forms the axis of rotation and carries near its lower end the mirror *s*. A very fine quartz fiber attached at *c* is suspended from the bridge *g*, and gives the directive force to the system.

The total weight of the suspension was 7 *mg*.

3. *The Radiometer as a Measuring Instrument.*

The sensitiveness of the radiometer was shown by Crookes[1] to be closely connected with the pressure of the gas in the inclosing vessel. A study at comparatively high pressures begun by Stoney and Moss[2] showed that the sensitiveness for the same pressure changed with the distance of the blackened vane from the opposite wall, increasing as the distance decreased.

Crookes[1] also observed that in the region of pressures where the radiometric sensitiveness was high, the logarithmic decrement of a torsion pendulum began to decrease rapidly with the pressure. These are the principal considerations that affect the present instrument, which, when the vanes were 2½ mm. from the mica plate and the pressure[3] in the vessel 0.05 mm., had a full period of vibration of twelve seconds, and was at its maximum sensitiveness. With further pumping the period of vibration could be brought down to eight seconds, but the shortening was accompanied by a decrease in sensitiveness. With greater pressure the period increased until it became aperiodic, and at atmospheric pressure if the upper end of the fiber was given 90° twist fully a minute elapsed before the suspension followed to its new position.

Near the maximum point the sensitiveness changes very slowly, so that the period of vibration is here the better test for the pressure sought.

If the vanes be brought within a millimeter of the mica window, for pressures greater than 0.1 mm., the sensitiveness is much increased, but the disadvantage is twofold, as the change involves an increased period, and the throws for small quantities of energy are greater in proportion than for large quantities.

For pressures near, or below, the maximum sensitiveness, small changes in the distance have no noticeable effect either on the sensitiveness or on the period.

Under the conditions which prevailed throughout the measurements the throws of the radiometer were proved proportional to the energy in several different ways; one of which was the meas-

[1] Proc. Roy. Soc., Nov. 16, 1876. [2] Proc. Roy. Soc., Feb. 22, 1877.
[3] Measured with the MacLeod gauge.

urement of the reflection percentage as dependent upon the wave length in the case of a fluorite surface under an incidence angle of 25°.[1]

The results obtained are compared in Table I. with reflection computed by the Fresnel formula, using Paschen's[2] observed indices of refraction. Wave lengths appear in the first column, observed reflections in the second, computed reflections in the third, and the differences in the fourth.

TABLE I.

REFLECTION OF FLUORITE.

$r = 25°. \quad R = \frac{R_\perp + R_\parallel}{2}.$

λ	R obs.	R comp.	Δ	λ	R obs.	R comp.	Δ
0.59	0.031	0.032	−1	4.0	0.030	0.030	0
1.00	0.031	0.032	−1	4.5	0.029	0.029	0
1.50	0.031	0.031	0	5.0	0.029	0.028	+1
2.00	0.031	0.031	0	5.5	0.029	0.028	+1
2.36	0.030	0.031	−1	6.0	0.028	0.027	+1
2.50	0.031	0.031	0	6.5	0.028	0.026	+2
3.00	0.031	0.031	0	7.0	0.027	0.025	+2
3.50	0.031	0.030	+1				

The sensitiveness at which the radiometer was held during the measurements which follow, was such that the rays from a candle at a distance of 6 m., when allowed to fall on one of the vanes, gave a deflection of 60 divisions on a millimeter scale at a distance of 1 m. from the instrument. This corresponds to a throw of more than 2100 scale divisions for a candle 1 m. away.

The compensating action of the two vanes was so nearly perfect, that rays from a source directly in front of the radiometer, which, when allowed to fall upon either vane alone gave a throw

[1] These observations were made by comparing the reflection of fluorite with that of silver as a standard, as in the case of quartz to be described later. As the reflection of fluorite is only about 3 per cent of the corresponding reflection of silver, throws — the larger of which was from thirty to forty times the smaller — were directly compared, and the test of proportionality was consequently a very severe one.

[2] Wied. Ann., 53, p. 301, 1894.

of 60 scale divisions, gave less than a millimeter when both were exposed. Thus it was possible to have an extremely sensitive instrument which at the same time held steadily to a fixed zero point, and gave throws trustworthy to the tenth of a millimeter.

As compared with the linear bolometer or the thermopile, the present form of radiometer has the following advantages: —

1. It is uninfluenced by all magnetic and thermo-electric disturbances, which render work with a very sensitive galvanometer tedious and unsatisfactory.

2. It can be more accurately compensated against rays which do not come from the source to be measured, and is consequently more free from "drift."

3. The radiometer is free from any disturbance corresponding to the air currents that arise about the warmed bolometer-wire.

It has, however, the following disadvantages: —

1. It is not as easily portable as either the bolometer or the thermopile.

2. All the rays to be measured must traverse the window of the radiometer, and be subject to its selective reflection and absorption.

3. In its present form the radiometer cannot be separated from a mercury air pump for more than a week at a time and remain sensitive.

II. Observations on Quartz.

Our knowledge of the optical properties of quartz in the visible spectrum has been extended in the one direction by observations of Sarasin[1] in the ultra-violet, and in the other, by observations of Mouton,[2] and later of Rubens,[3] in the infra-red. Rubens' observations extend to a wave length $\lambda = 4.2\,\mu$. Beyond this point direct measurements of refractive indices are impossible because of the heavy absorption which in quartz begins in this region.

The present study begins here, and consists of observations of the reflection and transmission of quartz between the wave lengths

[1] Archiv. des Sc. Phys., 3, 10, p. 303, 1883. [2] Comptes Rend., 88, p. 1190, 1879.
[3] Wied. Ann., 53, p. 277, 1894, and 54, p. 480, 1895.

$\lambda = 4\mu$ and $\lambda = 9\mu$, where for another reason — the absorption of the fluorite prism used for dispersion — these observations come to an end.

1. *The Observation of Reflection.*

The reflection of quartz was not compared directly with the intensity of the incident beam, but for convenience with the reflection of silver, which had been previously determined.

The study of silver was made by comparing the beam reflected under an angle of 15° with the direct, by means of the "swinging arm" apparatus used earlier by Rubens.[1] The mirror studied was a silver deposit on glass.

The results are given in Table II., which contains also the values for the reflection percentages of silver observed by Rubens[1] and by Langley.[2] The agreement is seen to be very close. That the values here given are however slightly higher, in the visible spectrum, than those of either Rubens or Langley, can doubtless be

TABLE II.

λ	Nichols.	Rubens.	Langley.	λ	Nichols.	Rubens.
0.35	—	—	61	0.90	96.0	95.8
0.38	—	—	73	1.00	—	96.5
0.40	—	—	79	1.15	—	97.0
0.43	82.7	—	—	1.40	—	97.4
0.44	86.4	—	—	1.65	—	97.7
0.45	—	87.0	85	2.00	97.2	97.3
0.49	90.1	—	—	2.50	96.5	97.0
0.50	–	88.3	89	3.00	97.3	98.3
0.54	91.5	—	—	3.50	98.3	—
0.55	—	90.3	91	4.00	100.0	—
0.59	91.6	—	—	6.00	99.8	—
0.60	—	92.7	92	7.00	99.6	—
0.64	93.6	—	—	8.00	99.0	—
0.65	—	93.3	93	8.40	99.5	—
0.70	—	94.5	93	8.65	99.2	—
0.75	95.0	—	94	9.00	100.0	—
0.80	—	95.2	—	—	—	—

[1] Wied. Ann., 37, p. 249, 1889. [2] Phil. Mag., 27, p. 10, 1889.

explained by differences in polish, upon which the reflection in this region in great measure depends.

In the infra-red, on the contrary, the reflection is but little affected by ordinary differences in polish, and a silver mirror grown perceptibly yellow with age gave the same percentage of reflection at wave lengths 6μ and 8μ as a perfectly fresh one. For waves longer than 4μ observations show the reflection to be so nearly total, that in the comparison with quartz the intensity of the beam from the silver mirror was taken as unity.

A prism surface cut perpendicular to the optic axis of a negatively rotating quartz crystal was chosen for the study of the reflection of quartz.

To gain the very small incidence angle of 5°, the arrangement of apparatus shown in Fig. 3 was adopted, in which p is the zircon plate of the Linnemann burner which supplies the energy. L_1 is a rock-salt condensing lens, M_1 the device for holding the mirrors to be compared, and M_2 a silver plane mirror to direct the rays into the spectrometer slit s_1. A shutter was introduced at S and a diaphragm at D. The two plates forming the sides of the spectrometer slit were covered with squared paper, and the position of the real image formed by the lens L_1 of a platinum point at c served as a check to control the adjustment in the change from silver to quartz and the reverse.

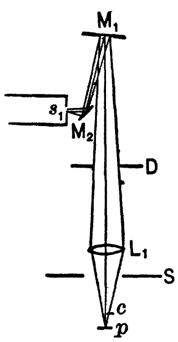


Fig. 3.

The surfaces to be compared were carefully mounted in two exactly equal carriers, each of which fitted into the holder M_1, and were held firmly in place by two springs.

To show the treatment of single observations and at the same time the behavior of the radiometer, the observations at wave lengths 8.72μ and 8.52μ are given entire. Of the fifteen series of observations, made at as many different wave lengths between 8μ and 9μ, the former furnishes the best agreement of single values among themselves, and the latter the worst.

Table III.

TWO SETS OF OBSERVATIONS OF THE REFLECTION OF QUARTZ Q COMPARED WITH THE REFLECTION OF SILVER S.

u^2, the percentage of reflection, is obtained alternately from $\frac{2Q_n}{S_n + S_{n+1}}$ and $\frac{Q_n + Q_{n+1}}{2S_{n+1}}$.

$\lambda = 8.72\,\mu$. Slits 1.5 mm. wide.				$\lambda = 8.52\,\mu$. Slits 1 mm. wide.			
Scale readings.		Throws.	u^2	Scale readings.		Throws.	u^2
S_1	532.2 497.1	35.1	—	S_1	532.9 506.1	26.8	—
Q_1	531.8 509.9	21.9	62.7	Q_1	533.0 514.2	18.8	71.9
S_2	531.9 497.0	34.9	62.9	S_2	532.7 507.0	25.7	72.0
Q_2	531.9 510.0	21.9	62.9	Q_2	532.9 514.6	18.3	70.9
S_3	531.8 496.9	34.9	62.5	S_3	533.1 507.0	26.1	70.9
Q_3	531.7 510.0	21.7	62.5	Q_3	532.6 513.9	18.7	72.9
S_4	531.8 497.0	34.8	62.2	S_4	532.3 506.9	25.4	74.0
Q_4	531.4 509.9	21.5	62.3	Q_4	532.7 513.7	19.0	74.0
S_5	531.3 497.1	34.2	—	S_5	532.1 506.3	25.8	—
Average, 62.6 per cent.				Average, 72.4 per cent.			

All the observed values of the reflection percentages of quartz are collected together in the second column of Table IV., the respective wave lengths being given in column 1. Column 3 gives the wave-length interval, $\lambda_1 - \lambda_2$ in μ, corresponding to the angular width of the slits used, both of which were always opened equally wide. These values thus show in a sense the purity of the spectrum at each of the observed points.

TABLE IV.

λ	u^2 $i=5°$	$\lambda_1-\lambda_2$	T obs.	$\lambda_1-\lambda_2$	T_1 corr.	κ_0^2 10^{-6}
4.20	—	—	90.0	0.07	—	—
4.40	—	—	89.7	0.07	—	—
4.50	3.40	0.07	{ 92.0 93.6	0.07 0.10 }	99.0	0
4.70	—	—	90.1	0.06	—	—
4.80	—	—	89.3	0.07	—	—
4.90	—	—	85.9	0.07	—	—
5.00	3.00	0.07	{ 69.7 71.3	0.07 0.10 }	75.0	41
5.05	—	—	71.0	0.05	—	—
5.10	—	—	74.6	0.05	—	—
5.15	—	—	74.5	0.05	—	—
5.20	—	—	71.1	0.07	—	—
5.25	—	—	62.3	0.05	—	—
5.30	2.73	0.07	54.1	0.05	57.0	174
5.40	—	—	67.8	0.06	—	—
5.50	—	—	72.2	0.10	—	—
5.70	—	—	80.0	0.11	—	—
5.80	2.50	0.09	81.1	0.05	85.4	16
5.90	—	—	75.5	0.11	—	—
6.00	—	—	72.8	0.11	—	—
6.10	—	—	70.6	0.11	—	—
6.20	—	—	60.2	0.11	—	—
6.25	1.80	0.11	{ 59.2 59.4	0.11 0.05 }	61.9	174
6.30	—	—	63.3	0.10	—	—
6.35	—	—	68.1	0.11	—	—
6.40	—	—	{ 77.2 75.9	0.11 0.11 }	—	—
6.45	1.46	0.10	{ 79.8 82.3	0.11 0.05 }	83.5	42
6.50	—	—	77.2	0.10	—	—
6.55	—	—	72.0	0.10	—	—
6.60	—	—	68.1	0.10	—	—
6.65	—	—	68.1	0.05	—	—
6.70	1.19	0.10	67.6	0.10	69.0	121
6.80	—	—	74.5	0.10	—	—
6.90	—	—	80.5	0.10	—	—
7.00	0.65	0.12	81.3 [2]	0.09	82.2	23
7.10	0.46	0.12	[1] 79.3 [2]	0.09	80.1	62
7.20	0.44	0.12	75.3 [2]	0.09	76.2	76
7.30	0.31	0.12	70.4 [2]	0.09	71.2	121
7.40	0.29	0.12	66.7 [2]	0.09	67.4	166

[1] Graphically interpolated.

[2] Double dispersion applied to transmission spectrum in this region.

TABLE IV. (*continued*).

λ	u^2 $i=5°$	$\lambda_1-\lambda_2$	T obs.	$\lambda_1-\lambda_2$	T_1 corr.	κ_0^2 10^{-6}
7.50	0.42	0.12	59.5	0.09	60.1	286
7.60	0.86	0.09	51.5	0.09	52.5	466
7.70	1.34	0.09	35.8	0.09	36.9	1,149
7.80	3.25	0.09	18.0	0.09	19.3	3,215
7.90	[1] 6.00	0.09	12.2 [2]	0.09	13.9	4,747
7.92	6.69	0.09	—	—	—	—
8.00	14.0	0.08	5.6	0.09	7.8	8,136
8.02	17.2	0.08	—	—	—	—
8.05	[1] 23.0	0.08	3.1	0.09	5.8	10,240
8.10	—	—	> 0.5	0.09	—	—
8.12	43.3	0.08	—	—	—	—
8.22	63.3	0.08	—	—	—	—
8.32	70.4	0.08	—	—	—	—
8.42	75.0	0.08	—	—	—	—
8.52	72.4	0.08	—	—	—	—
8.62	51.1	0.08	—	—	—	—
8.65	56.5	0.08	—	—	—	—
8.67	58.4	0.08	—	—	—	—
8.72	62.6	0.12	—	—	—	—
8.80	67.1	0.12	—	—	—	—
8.82	64.0	0.12	—	—	—	—
8.90	60.5	0.08	—	—	—	—
8.92	58.0	0.11	—	—	—	—
9.00	51.5	0.08	—	—	—	—
9.02	49.2	0.11	—	—	—	—

Wave lengths and reflection percentages are shown again in a curve in the plate opposite page 312. The curve falls very gradually from $\lambda = 4.5\ \mu$ to a minimum at $7.4\ \mu$, where the observed reflection was only 0.29 per cent.

From $\lambda = 7.4\ \mu$ to $\lambda = 8.45\ \mu$ the curve rises with surprising rapidity, reaching a maximum value of 75 per cent near the latter wave length, after which it falls to a second minimum of 51 per cent near $\lambda = 8.6\ \mu$ and rises to a second maximum of 66 per cent at $\lambda = 8.8\ \mu$. It then falls regularly to $\lambda = 9\ \mu$, where the observations of reflection end.

The error in the reflection percentages here given is due wholly

[1] Graphically interpolated.

[2] Double dispersion applied to transmission spectrum in this region.

to the presence of a small percentage of rays of shorter wave length, never entirely absent from measurements in this region, and which here has the effect to bring the two maximum points too low, and the second one lower in proportion than the first. The first minimum at $\lambda = 7.4\,\mu$ is probably too high, as will be seen later.

2. *The Observation of Transmission.*

For the study of transmission, the lens L_1 (see Fig. 3) and the lamp were brought into line with the collimator of the spectrometer, and the intensity of the direct ray compared with that of the ray transmitted through a thin plate of quartz placed directly before the slit.

The plate used was a thin scale cut perpendicular to the optic axis of the crystal and mounted in a glass frame with a free aperture 10×5 mm. The spectrum of light reflected from the plate under an angle of $15°$ gave $10+$ interference bands between the Fraunhofer lines C and D, from which the thickness $18\,\mu$ was computed.

Column 4 of Table IV. gives the observed percentages of transmission T for the wave lengths given in column 1. Column 5 contains the wave-length interval $\lambda_1 - \lambda_2$ corresponding to the angular width of the slits; and column 6 gives the values of T_1, *i.e.* the values of T corrected for the reflection at both surfaces, computed by the formula

$$T_1 = \frac{T}{(1 - 2u^2)^2}.$$

The observed transmissions extend from $\lambda = 4.2\,\mu$ to $\lambda = 8.05\,\mu$ and the observations are plotted as a curve in the plate with wave lengths as abscissæ and percentages of transmission as ordinates.

Beyond $\lambda = 7\,\mu$, to make sure of the purity of the spectrum, which later became important, a second fluorite prism and slit were introduced into the path of the rays before they entered the spectrometer, thus giving double dispersion. In the path of the curve between $4.2\,\mu$ and $7\,\mu$ are five maxima and four minima, of which three in each case are sharply defined. From $7\,\mu$ on, the curve sinks so rapidly that it was impossible to follow it beyond 8.1 μ, where, through a thickness only two and one-fourth times the wave length of the incident ray, only a doubtful 0.5 per cent struggled through.

Although the whole region between 8.1 μ and 9 μ was carefully searched, no trace of transmitted energy large enough to measure was discovered.

TABLE V.

λ	ν − Root.	ν + Root.	u^2 corrected.	ν accepted.	ν Rubens.	Δ
4.50	0.692	1.450	—	1.450	1.442	+ 8
5.00	0.706	1.417	—	1.417	1.411	+ 6
5.30	0.719	1.393	—	1.393	1.386	+ 7
5.80	0.730	1.368	—	1.368	1.343	+ 25
6.25	0.765	1.309	—	1.309	1.287	+ 22
6.45	0.785	1.274	—	1.274	1.257	+ 17
6.70	0.803	1.242	—	1.242	1.212	+ 30
7.00	0.853	1.172	—	1.167[1]	1.145	+ 22
7.10	0.873	1.144	—	1.125[1]	1.117	+ 8
7.20	0.877	1.141	0.150	1.080	1.089	− 9
7.30	0.899	1.113	0.025	1.032	1.056	− 24
7.40	0.899	1.112	0.000	1.000	1.020	− 20
7.50	0.880	1.137	0.135	0.930	0.979	− 49
7.60	0.835	1.199	0.568	0.861	0.933	− 72
7.70	0.798	1.256	—	0.798	0.881	− 83
7.80	0.702	1.432	—	0.702	0.819	− 117
7.90	0.611	1.645	—	0.611	0.746	− 135
8.00	0.478	2.172	—	0.478	0.657	− 179
8.05	0.366	—	—	0.366	0.603	− 237

3. *The Dispersion.*

In general, it is possible to compute ν, the index of refraction, for every wave length λ for which u^2 and D_1 are known, by substituting, in the Cauchy formula, the observed values of u^2 and the extinction coefficient κ_1. In reflection at normal incidence, the intensity of the ray $u_{\parallel}^2$ polarized in the plane of incidence and the ray $u_{\perp}^2$ polarized in the plane at right angles are equal, and for 5°, the observed incidence, the equality may, within the limits of error, still be assumed.

[1] Interpolated graphically.

The simpler expression, that for $u_{\parallel}^2$, was consequently used in the computation, and the observed reflection, u^2, put into the formula

$$u^2 = \frac{(\cos i - \sqrt{\nu^2 - \sin^2 i})^2 + \kappa_1^2}{(\cos i + \sqrt{\nu^2 - \sin^2 i})^2 + \kappa_1^2},^1 \qquad (1)$$

which may be expressed as an explicit function of ν of the form

$$\nu^2 = \sqrt{\sin^2 i + \left\{ \cos i \left(\frac{1 + u^2}{1 - u^2} \right) \pm \sqrt{\cos^2 i \left(\frac{1 + u^2}{1 - u^2} \right)^2 - (\kappa_1^2 + \cos^2 i)} \right\}^2}, \qquad (2)$$

in which the extinction coefficient κ_1 for the incident angle i may, in this case, be put equal to κ_0, the extinction coefficient for normal incidence. These values, computed from the formula[2]

$$T_1 = e^{\frac{-4 \pi d \kappa_1}{\lambda}}, \qquad (3)$$

are given in column 6, Table IV. In the formula, $d = 18\,\mu$, the thickness of the plate, and λ is the given wave length.

As the value of κ_0^2 at $8.05\,\mu$, the highest observed, is only 102×10^{-4}, $\kappa_0^2 m$ (2) was put equal to zero, so that the refraction, in this case, depends solely upon the reflection, and the modified Fresnel formula is applicable within the limits of error.

Equation (2) gives two values of ν for every value of u^2, depending on which root of the inner radical is chosen. The index given by the positive root corresponds to a reflection u^2 for a ray striking at an angle of 5° from the rarer into the denser medium, while the negative root gives an index which would give the same reflection for the same incident angle for a ray crossing the boundary in the opposite direction. Both values are given in Table V., and again as curves in the plate. From the beginning out to $\lambda = 7.4\,\mu$, where u^2 reaches its lowest value, there is no question which index is the correct one, but beyond this point the choice is a doubtful one, as will be seen later. The equation as a whole possesses one important peculiarity: whenever u^2 is very small, minute changes in u^2 produce comparatively large changes in ν. At $7.4\,\mu$, for

[1] Wüllner Lehrbuch d. Physik, Bd. II., p. 536, Leipzig, 1883.
[2] Wüllner Lehrbuch d. Physik, Bd. II., pp. 123–126.

example, $u^2 = 0.29$ per cent, and $\nu = 1.11$ or 0.9; while for $u^2 = 0$, $\nu = 1$.

Reflection percentages in the neighborhood of zero must therefore be very carefully treated, and the possibility of false values due to stray light, never entirely absent in such work, must be taken into account.

The wide slits which it was necessary to use, and the nearness in the reflection spectrum to a maximum point of at least two hundred and fifty times the intensity here involved, make the conditions for stray energy most favorable. The importance of using double dispersion, which furnishes the best precaution against such an error, was unfortunately first realized after the measurements had all been taken. The observed reflection percentages in this region are in consequence doubtless too large.

If the reflection of 0.29 per cent observed at $7.4\,\mu$ was due wholly to stray energy from near $8\,\mu$, which is not unlikely under the circumstances, and the reflection percentages from $7.2\,\mu$ to $7.6\,\mu$ be corrected by this amount, and ν again computed for the corrected values, points are obtained which are connected by broken lines on the plate.

The position of these broken lines is suggested by the whole course of the curves outside this doubtful region, and the writer regards these new values of ν as nearer correct than the old ones, in which case the most natural conclusion is that the curves intersect rather than touch at $7.4\,\mu$. Quartz and air are thus believed to change places here, and later, air to play the part of the denser medium; but it is impossible, on the basis of these results alone, to say with certainty which of the two possible dispersion curves is the correct one.

The course of either curve between $7\,\mu$ and $8\,\mu$, together with the heavy absorption and the change in reflection, which at $\lambda = 7.4\,\mu$ is probably less than that of any body known in the visible spectrum, and which at $\lambda = 8.4\,\mu$ rivals that of burnished silver for violet light, show conclusively that quartz in this region passes over completely from a non-metallic to a metallic body.

4. *The Ketteler-Helmholtz Dispersion Formula.*

Rubens,[1] supplementing his own careful observations with the other experimental material in hand at that time, computed for quartz the five constants in a form of the Ketteler-Helmholtz dispersion formula, which is rigorously applicable only where the absorption bands are two in number, and linear. The values of ν given by these constants appear in column 6 of Table V. and again in the plate.

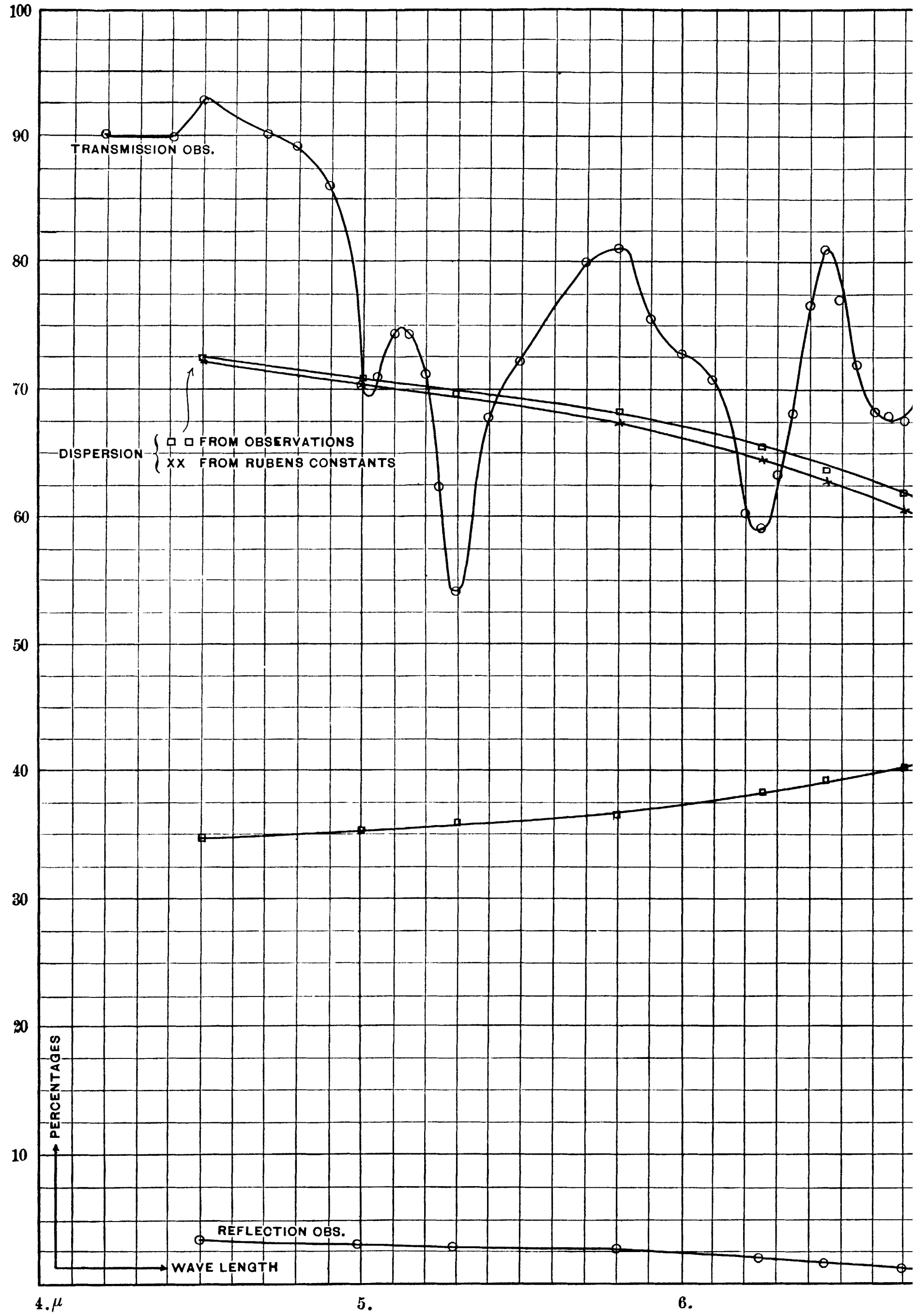
Only the general character of the curve thus obtained and that of the preferred curve computed from the observations can be considered the same. Before $\lambda = 7.2\,\mu$ the latter curve lies higher, afterward lower than the former one.

While the indirect method here employed for measuring the refractive index is at best a very rough one, still the divergence of the two curves is systematic, and is so great that the discrepancy cannot be entirely accounted for by the inaccuracies of the method. For the same wave length the difference between the refractive indices given by the two curves finally reaches 39 per cent. On the other hand the wave-length difference given by the two curves for the same index of refraction nowhere exceeds $0.16\,\mu$ and the agreement is closer than might have been expected when it is considered that the furthest point observed by Rubens was at $\lambda = 4.2\,\mu$. On the basis of constants computed for the region lying before this wave length, which were introduced into a special form of the general equation, the curve has been continued a full octave farther to $\lambda = 8.05\,\mu$.

It is also possible that there are still other absorption bands beyond $\lambda = 9\,\mu$, not taken into account in the formula, which influence the dispersion in the region studied.

Before closing I wish to thank Professor E. Warburg for the friendly interest he has taken in these experiments, and for the means to further them put at my disposal. I wish to express, also, my deep appreciation of the kindly interest in this study shown by Professor Heinrich Rubens, under whose direction it was made, and to acknowledge my indebtedness to him for many helpful

[1] Wied. Ann., 53, p. 277, 1894, and 54, p. 480, 1895.



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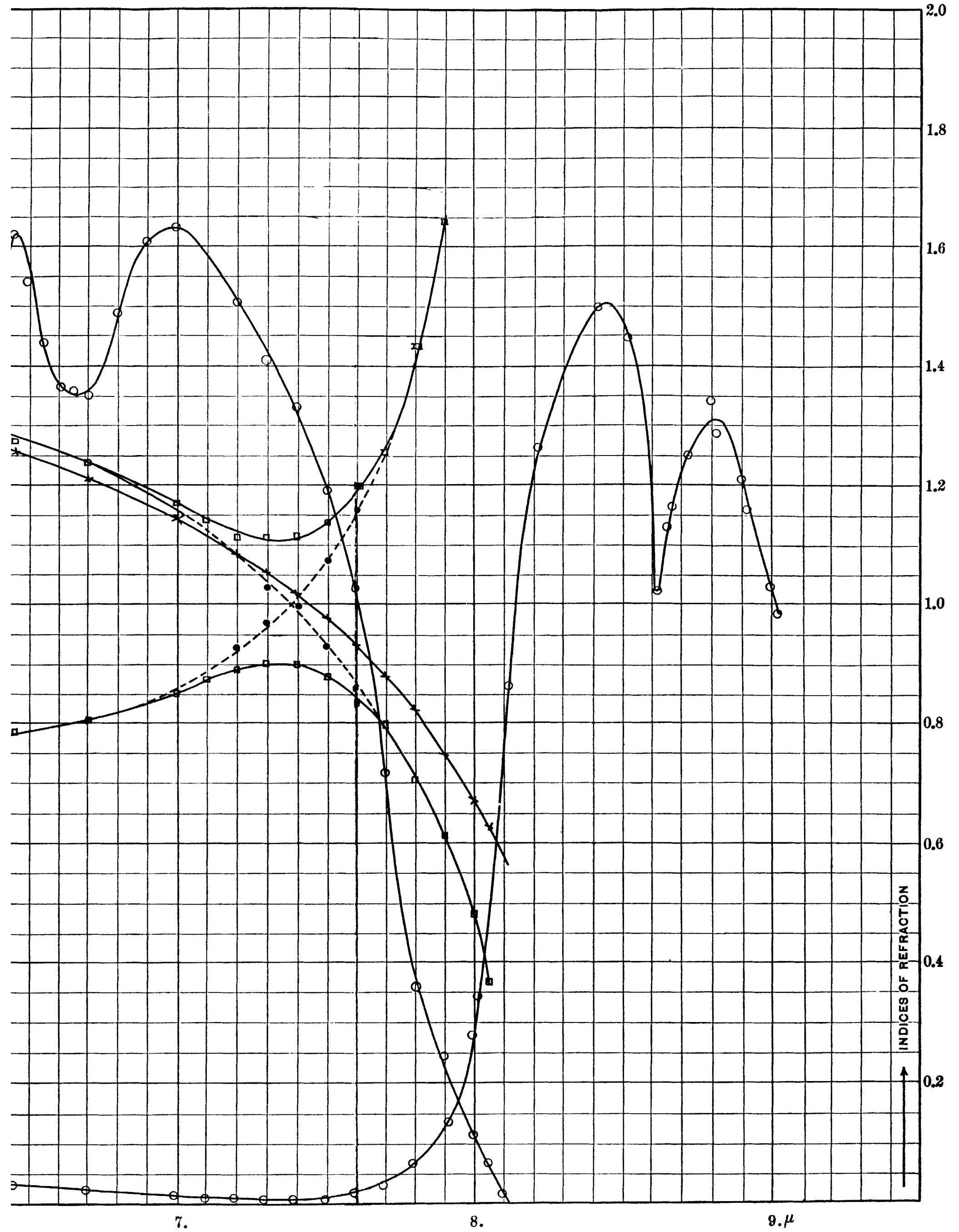

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suggestions. I am further indebted to the kindness of Professor E. Pringsheim, at whose suggestion the work of building the radiometer was undertaken, for friendly advice concerning many details in the construction.

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