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ON ELECTROMAGNETIC INDUCTION AND RELATIVE MOTION.

By S. J. Barnett.

THIS investigation originated in 1902 with an attempt to develop a method for the solution of the old problem of unipolar induction. In its best known form this problem may be stated as follows: A magnet symmetrical about its axis and rotating about this axis uniformly is touched in two points A and B not in the same equatorial plane by the ends A and B of a wire ACB. A steady current traverses the circuit thus formed. Which is the seat of the electromotive force, the wire or the magnet? That is, do the lines of magnetic induction rotate with the magnet and cut the external conductor ACB; or do the lines, like the external conductor, remain fixed while the magnet rotates through them? Faraday thought that his experiments proved the second alternative, and Pluecker shared his views, while Weber disagreed. It was first shown by S. Tolver Preston[1] that the experimental results could, as is now well known, be explained equally well on either hypothesis.

Preston went further and adduced cogent arguments in support of the first alternative, which, in a later paper,[2] he quotes Lord Rayleigh as also favoring. In the first place, Faraday had admitted that a magnet in translation carried its lines of induction with it; and Preston called attention to the fact that translation is involved in all rotation (rotation, according to his statement, being a particular case of translation); thus a large magnet in rotation is made up of many small magnets in translation, though the translation is in curved lines. In the second place if, instead of rotating the magnet, we keep the magnet fixed and rotate the conductor ACB in the opposite direction with the same speed, we obtain the same electromotive force *and its seat is in the conductor* ACB. Now

[1] S. Tolver Preston, Phil. Mag. (5), 19, 1885, p. 131.
[2] S. Tolver Preston, Phil. Mag. (5), 19, 1885, p. 215.

the *relative motion* between the parts of the system of magnet and wire is exactly the same in the two experiments, hence we should expect ACB to be the seat of the electromotive force when the magnet rotates and the wire remains fixed. A third argument, relating to the similarity between an electric coil and a magnet, cannot be accepted as valid, inasmuch as we know no more about the phenomena of unipolar induction in the case of an electric coil rotating without iron than we do in the case of a magnet. Both sets of phenomena can be explained equally well in the same way on either hypothesis.

Lecher[1] was not satisfied that Preston's argument was conclusive and published some highly interesting experiments, also made with closed circuits, in 1895. Although he regarded these experiments as favoring Faraday's view, they cannot be considered as establishing it. Lecher also attempted to solve the problem by electrostatic methods, but without success.

In 1902 it occurred to me that the problem could be solved by the following method. A cylindrical condenser is placed in an approximately uniform magnetic field parallel to its axis, and the magnets producing the field are rotated while the condenser is short-circuited by a wire at rest like itself. While the magnets are still rotating, the connection between the armatures of the condenser is broken; and the condenser is tested for charge after the field is annulled, or the rotation stopped, or the condenser removed. It was argued that if the lines of induction moved with the magnets the condenser would receive charges which could be computed, and that it would remain uncharged in case the lines remained fixed and the magnets moved through them. It was found later that a somewhat similar idea, discussed below, had previously occurred to Preston;[2] and in 1908 Mr. Tracy D. Waring[3] proposed an experiment essentially the same in principle as that which had occurred to me. As shown below, however, our reasoning was erroneous. Since what precedes was written, there has appeared an experimental paper on the subject by Mr. E. H. Kennard,[4] which also is based on incorrect theory.

[1] E. Lecher, Wied. Ann., 54, 1895, p. 276.

[2] S. Tolver Preston, Phil. Mag. (5), 31, 1891, p. 100.

[3] Tracy D. Waring, Trans. Am. Inst. Elec. Eng., 27, 1908, p. 1366.

[4] E. H. Kennard, Phil. Mag. (6), 23, 1912, p. 937. The inconclusive character of Mr. Kennard's work will be apparent after reading the theoretical part of this article. Using a method resembling my own, but with the iron core of the electromagnet rotating alone while the magnetizing coil remained fixed like the condenser, he thought he had proved that the lines of induction did not move with the iron, because the electrometer received no charge in his experiments. This conclusion, as shown here, does not follow. It should be pointed out also that Mr. Kennard's calculation of the effect to be expected on the moving line hypothesis assumes that on this hypothesis all the lines in his experiment would move with the iron;

Preston imagined a magnet *NS*, Fig. 1, rotating about its axis near a flat circular metal plate *K*; and concluded that if the lines of induction remained fixed the magnet would become charged by the motional electromotive forces within it, and the plate *K* by electrostatic induction, as indicated in (*a*). And he concluded that on the other hypothesis the plate would be charged as in (*b*), the magnet being uncharged except to a slight extent by induction. Preston consulted Hertz with reference to the matter; and Hertz "laid stress on the great interest attaching to

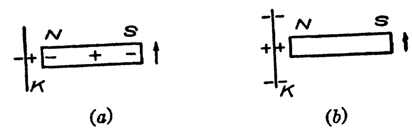

Fig. 1.

the inquiry, and agreed that the conditions of test as proposed should be capable of deciding this question"; but he informed Preston that "with a normally sensitive electrometer a considerable velocity of rotation with a magnet of large size would be calculably required to produce a distinct deflection under either hypothesis." And Preston stated that "facts of this kind discouraged from trying the experiment." At the same time he expressed hope for the production in the future of a much more sensitive electrometer, with a quartz suspension.

Equally, however, with those who have tried to solve the problem by experiments on closed circuits, all who have proposed electrostatic tests to decide between the two hypotheses have, if we admit the validity of current electromagnetic theory, fallen into error; for it is not difficult to show that the total electric intensity and the electric density are in all cases the same on the two hypotheses. The error has arisen through failure to take proper account of the motional intensity in the dielectric.

For the special cylindrical condenser system considered here and imagined to be placed in a uniform magnetic field, whose direction is parallel to its axis and which is produced by a coil or magnet system in rotation, the electric field may be investigated as follows. The length of each cylinder will be considered great in comparison with the distance between

whereas it is impossible to say what fraction, on this hypothesis, would adhere to the iron and move, and what fraction would adhere to the coil and remain at rest. On account of the fact that a part of his field-producing system was at rest, instead of all being in motion, the experiments are also inconclusive as to the important matter of relative motion discussed below. To make them of value it would be necessary to make experiments on the charge taken by a short-circuited condenser, or the motional electromotive force developed in a Faraday disc, rigidly attached to the magnetizing coil of an electro-magnet whose core remains fixed while the coil rotates with the disc. The condenser experiments would be difficult; successful experiments with the disc would, apparently, be impossible.

the two cylinders, so that end effects will be negligible, and the outer cylinder may be supposed longer than the inner and closed by conductors at the ends. A fine wire may be supposed to connect the two armatures.

On the hypothesis of fixed lines it is evident that there will be no intensity, either field or motional, anywhere within the condenser system.

To obtain the field on the hypothesis of moving lines, imagine first that no motional intensity is produced in the dielectric. Then if ψ denotes the electromotive force produced in the wire by the motion of the lines of induction across it, the condenser will be charged to the voltage $V = -\psi$; and if r' and r'' denote the radii of the opposing faces of the internal and external armatures, the field intensity E, now the total intensity, at any point within the condenser and distant r from the axis will be

$$E = \frac{V}{r \log r''/r'} = -\frac{\psi}{r \log r''/r'}.$$

Now imagine that no motional intensity is produced in the wire, and consider the intensity produced by the motion of the lines of induction through the dielectric. The total intensity will be the same as if the field and wire remained fixed and the *whole* dielectric rotated in the opposite direction, and will be given by the equation[1]

$$f = e + E = \frac{\omega B}{r}\left(\frac{r''^2 - r'^2}{2 \log r''/r'}\right) = \frac{\psi}{r \log r''/r'}.$$

In the actual case the two effects will be superposed, so that the total intensity will be zero at every point. The same thing is true of the total intensity within the inner armature.

On both hypotheses, therefore, the condenser system remains uncharged.[2] The result is entirely independent of the magnitude of the dielectric constant.[3]

To establish the general proposition,[4] consider a magnet NS and any system of conductors such as a metal disc K, arranged according to Preston's dea, Fig. 2. First imagine the magnet to start into rotation

[1] S. J. Barnett, PHYS. REV., 27, 1908, p. 432. The above result follows at once by adding the motional intensity ωBr to the field intensity given by equation (11), $(K - 1)/K$ being appropriately replaced by unity.

[2] That the charge is zero also follows at once from the consideration that the dielectric and short-circuiting wire together form a closed circuit, so that the electromotive force induced in the wire is just equal and opposite to that induced in the dielectric.

[3] See S. J. Barnett, Ann. der Phys., 30, 1909, p. 416.

[4] This subject, as I found after independently developing the theory for my own experiments several years ago, had been previously treated by Poincaré (L'Éclairage Électrique, 23, 1900, p. 41) and by M. Abraham (Theorie der Elektrizitaet, Vol. I., 1907, p. 418); but neither treatment seems to me satisfactory.

while the lines of induction remain fixed. Then at any point P within the magnet there is a motional electric intensity $e = [vB]$, where v denotes the velocity and B the induction at P. Owing to this intensity a transient current exists in the magnet and there is developed in and about it an electrical field which becomes steady as the motion becomes uniform. In the steady state the field intensity E at P is equal and opposite to the motional intensity e, the total intensity $f = e + E$ being zero. In the conductor K (at rest)[1] each intensity is zero. In the dielectric the field intensity E exists alone. The potential difference V, equal to the line integral of the intensity E, from any point B to any point A on the surface of the magnet along any line whatever connecting the two points is equal to the line integral ψ of e along any line ADB from A to B within the magnet. The electric density, which is proportional to the divergence of the total intensity f, is zero everywhere except at the interfaces between the conductors and the dielectric.

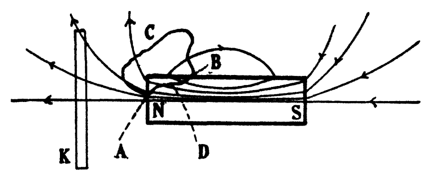

Fig. 2.

If, on the other hand, the lines of induction move as if rigidly attached to the magnet, e, E, and $f = e + E$ are all zero within the magnet; in the conductor K the total intensity f is zero, so that $E = -e$; while in the dielectric the total intensity is $f = e + E$. And this total intensity at any point is precisely the same as on the other hypothesis; for the integral of f along the line BCA is equal to the integral of e along the same line, since the integral of E vanishes along BDA and therefore along BCA, and this integral of e is equal to ψ. As A and B are any points on the magnet and ACB any line of any shape whatever connecting them, it follows that the total intensity f at any point is the same on the two hypotheses. Hence, if the *whole* dielectric is cut by the moving lines, so that the motional intensity, as well as the field intensity, acts on the whole dielectric, the electric density everywhere is exactly the same as before.

If the lines of induction neither remain at rest nor move with the full angular velocity of the magnet, the result, so far as f and the charges are concerned, is precisely the same. Though e and E will differ according to the hypothesis made, we shall always have $E = -e$, or $E + e = 0$, in the conductors, and $f = e + E$ in the dielectric, $E + e$ at any point being independent of the hypothesis.

[1] The proposition is easily demonstrated, by the application of the same method, for the case in which one or more conductors of the system K, the material dielectric, or the æther, are in motion.

Moreover, if we reject the hypothesis of the æther, as existing independently of the electromagnetic field, the result is the same. For to the electric displacement produced by the motional intensity acting on the material dielectric must be added that which would, on current theory, accompany the lines of magnetic induction if they moved in space otherwise free or filled with radiation.

Hence all attempts to solve the problem of unipolar induction by the measurement of charges or total intensities or electromotive forces are vain. For the same reason the problem cannot be solved by measuring the force upon a charged body in the field of a rotating magnet, as in an experiment proposed by Sir Oliver Lodge,[1] and suggested by Mr. Waring[2] as a means of solving this problem. It appears that the only way in which the problem can be solved electrically is to measure the two intensities separately, and how this can be done is not evident.

Much importance still attaches, however, to the proper execution of the experiment described above. For there can be no doubt, from the experiments of Faraday and many others, that if the short-circuited condenser is rotated about its axis in the fixed magnetic field it becomes charged by the motional electromotive force in the short-circuiting wire; while if the condenser and its short-circuiting wire remain fixed and the magnet or electric coil producing the field is rotated with the same speed in the opposite direction, the relative motion between the two systems is exactly the same as before, and one would expect the same charges to be developed. Indeed, the fact that the electromotive force induced in a closed circuit by relative motion between it and a magnet is independent of the one that moves is one of the considerations that led Einstein to the principle of relativity. Furthermore, a positive result would both reveal a flaw in current electro-magnetic theory and confirm Preston's idea on the seat of the electromotive force in unipolar induction. I have therefore carried to its completion the investigation begun with a different idea long ago.

After a few preliminary experiments in 1904, in which the whole condenser was short-circuited in the magnetic field and moved out after insulation to the electrometer connections, improved apparatus, in which only the outer armature moved, was tried several years later, but with inconclusive results. Work on the problem was resumed this year, and with the apparatus described below it has been possible to obtain precise and conclusive results.

Although the phenomena of unipolar induction hitherto known are

[1] Oliver Lodge, Modern Views of Electricity, 1907, Sec. 73. See also Phil. Mag. (5), 27, 1889, p. 469.

[2] Tracy D. Waring, loc. cit.

essentially the same whether a rotating magnet or a rotating electric coil produces the magnetic field,[1] it was thought worth while to investigate the behavior of both. The first experiments in the recent work were on fields without iron.

A simplified diagram of apparatus is given in Fig. 3. *C*, the coil which produced the magnetic field, was wound from about 5,000 turns of No. 14

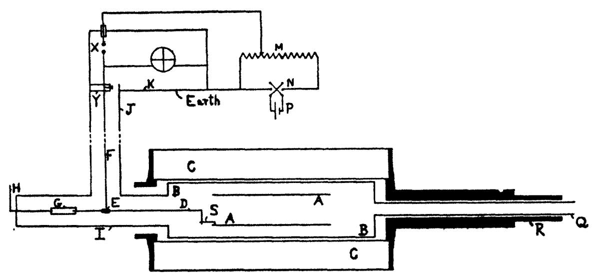

Fig. 3.

DCC copper wire on a substantial brass bobbin, to whose ends were fitted hollow axles of brass and bronze. The outer armature *B* of the (air) condenser was a brass tube 28 cm. long and 6.67 cm. in internal diameter. It was continuous with the brass tubes *Q*, *I*, and *J* and the brass lining of the wooden electrometer and key boxes *K*. The inner armature *A* was a brass tube 14.9 cm. long and 3.97 cm. in external diameter, mounted coaxially with the coil and with *B* at their common center, and insulated from *B* with two small amber blocks. To one end of *A* was attached a straight wire *S*. This wire, together with the bent wire *D*, which passed through a brass sleeve *E* in an amber support and was operated by the bent wire *H*, insulated from it by the amber block *G*, formed a key for connecting *A* by the wire *F* to the electrometer and its key *X*. The construction of this key *X* is illustrated in Fig. 4, except for a small lever

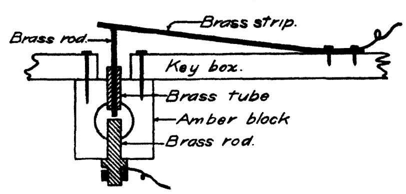

Fig. 4.

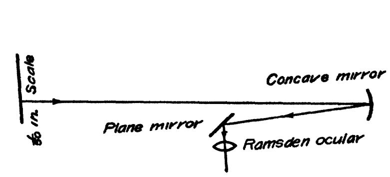

Fig. 5.

by the operation of which the key could be closed. Voltages for standardizing the apparatus were supplied by a dry cell *P* and a 10,000 ohm universal shunt box *M*, to which the cell was connected through the reversing switch *N*. The moving system was mounted in heavy brass bearings

[1] See especially O. Grotrian, Ann. der Physik, 10, 1903, p. 270.

screwed to a heavy wooden board, which was bolted to the cement floor. It was driven by an electric motor at measured speeds in the neighborhood of 20 revolutions per second and supplied with measured currents in the neighborhood of 20 amperes. The method of procedure made it necessary for the current to traverse the coil for only a few seconds in the several minutes required for each complete observation. The fixed system *QBI* was supported on the same base which carried the moving system. The electrometer was a modified Dolezalek instrument mounted on a wall which was absolutely free from appreciable vibration. The optical system made up a sort of Newtonian telescope, as shown in Fig. 5, and made it possible to read tenths of divisions on a one fiftieth inch scale 2¼ meters from the mirror. Sensibilities up to more than 10,000 divisions (double deflection) per volt were used. A slow stream of dried and filtered air was passed almost continuously through the tube *Q*, the condenser system *AB*, the tubes *I* and *J*, and the electrometer and key boxes *K*. To keep the condenser system cool during observations a rapid stream of air was ordinarily passed through the space between *QBI* and the coil *C*.

If the space between the tubes *A* and *B* were traversed by a uniform magnetic flux ϕ, and the short-circuited condenser system instead of the coil were rotated at a speed of n revolutions per second, the condenser would become charged to an electromotive force $n\phi$. The charge taken by the wire *DEF* would be very small in comparison with that taken by *AA* both in this case and in the actual case in which ϕ is not uniform. This charge does not enter into the calculations for experiments made with the key system described above, and is negligible in the later experiments made with the arrangement of keys described below.

To determine experimentally the electromotive force E to which the condenser is charged when it and the key system remain fixed while the coil rotates at the speed n revolutions per second, the procedure is as follows: The electrometer key *X* being closed and *N* open, the key *DS* is closed, thus connecting *A* and *B* together while the coil is in motion and the flux ϕ traverses the region between *A* and *B*. Then, in succession, contact between *D* and *S* is broken, the field is annulled and the motor switch opened while *X* is opened[1] and finally *DS* again closed.

[1] If the electrometer needle is deflected by the magnetic field of the coil or magnets, it is important that *X* be left closed until the needle has come to rest after the field is annulled. Otherwise a portion of the charge bound by the needle in its deflected position will be released to the insulated system as the needle approaches its zero, and there will remain a deflection which changes sign with the current. This effect, being independent of the rotation, is easily eliminated. This was done in some of the earlier work with iron when sufficient precision had been attained to detect the effect; but in all the final work, both with and without iron, the needle was allowed to come to rest before opening the key.

The reading of the electrometer is then taken. The process is repeated with the current in the coil reversed. If d denotes the double deflection of the electrometer, V the corresponding voltage, C the capacity of the system AB, and K the capacity of this system together with that of the electrometer and connections, we have

$$E = \frac{K}{C} \cdot V, \text{ or } V = \frac{C}{K} \cdot E = Ad,$$

where A is a constant. If d' denotes the double deflection corresponding to a known voltage V', we have also

$$V' = Ad'.$$

Hence, if E were equal to $n\phi$, the double deflection D to be expected would be

$$D = n\phi \cdot \frac{C}{K} \cdot \frac{d'}{V'};$$

and the immediate aim of the experiment is to determine the ratio d/D.

If the axial flux between the cylinders is not uniform, the mean value $\bar{\phi}$ throughout the length of the inner armature may be substituted for ϕ in the above formula without introducing much error, provided that ϕ does not depart greatly from uniformity and that the capacity of the system AB per unit length is nearly constant. From the dimensions and arrangement of the armatures it is evident that the second condition was satisfied in these experiments, and measurements on the magnetic field showed that the *minimum* and *maximum* values of ϕ did not differ by more than about 5 per cent. The axial flux was investigated with five test coils (set coaxially within the electric coil) ranging in diameter from a little less than the outer diameter of A to a little greater than the inner diameter of B, a properly tested Hibbert magnetic standard, and a ballistic galvanometer. Observations were made with each coil for every centimeter of the length of the condenser. The mean axial intensity thus found for the whole region occupied by the condenser was 119 gausses for a current of 1 ampere in the electric coil, the average departure from the mean given by the separate coils for the mean intensity within the cylinders described by them being about 1/6 per cent. This made $\bar{\phi}$ equal to 268×10 maxwells for a current of 1 ampere in the coil.

In order to eliminate errors arising from changes in the electrometer's sensibility and zero and from extraneous electromotive forces, the observations were made on a regular time schedule; the observations for d were repeated in inverse order; and the double deflections were obtained from the corresponding mean scale readings. If L denotes the mean scale reading when the current traversed the coil in one direction, and R the reading when the current was opposite, $d = L - R$. The four observations necessary to obtain one value of $L - R$ ordinarily constituted one *set*.

Nine sets of rotation experiments made in the early part of the work indicated that the average value of $(L - R)/D$, with due respect to sign, was zero; but the results were very irregular, the average numerical value of $(L - R)/D$ being about one fifth.

Much of the trouble was traced to the operation of the keys X and DS, which were therefore replaced by the keys D and L, Fig. 6, of the type

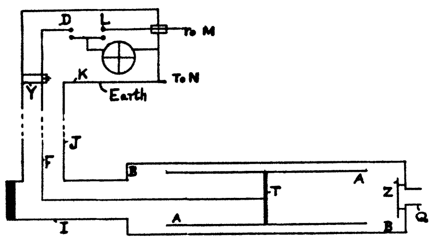

Fig. 6.

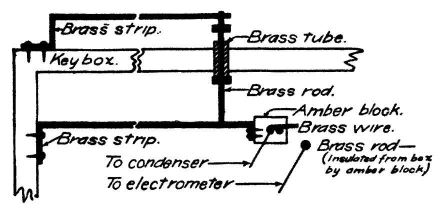

Fig. 7.

illustrated in Fig. 7, both placed in the key box. The difficulty with the old keys probably arose from the abrasion of fine particles of metal when the keys were operated, and from the presence of residual dust. Some of the discrepancies arose from the absence of perfect dryness, or constant conditions as to dryness, and some probably arose from the tremor given to the condenser and key DS by the rotation of the coil. The condenser, slightly altered, was therefore mounted directly on the wall; the amber insulators were all scraped, cleaned, and discharged in the hot gases from the flame of a bunsen burner; and the apparatus was more thoroughly washed out with filtered air. At the same time the speed was increased about 50 per cent. After these changes the discrepancies were greatly reduced, the average numerical value of $(L - R)/D$ in six sets being about 3 hundredths.

In the hope of securing still better results several changes were made. The keys were slightly improved and every effort was made to operate them with extreme regularity, the outer air stream was abolished, attempts were made to free the condenser and key box more completely from dust and to keep the air in a more nearly uniform state of dryness, and the sensitiveness of the apparatus was increased by reducing the capacity of the electrometer and connections and increasing the capacity of the condenser, and by increasing the scale distance to 3.4 meters. The scale could still be read to tenths of divisions.

The length of the outer tube BB of the new condenser, Fig. 6, was 28 cm. and its internal diameter 6.64 cm. The length of the inner tube AA was 20.0 cm. and its external diameter 5.08 cm. The capacity C_1 of this condenser was thus approximately 37.3 es. units.

The connecting wire F, with diameter 0.023 cm., had an effective length of 4 cm. (from the end of AA to the end of BB) in the tube BB and a length of 114 cm. in the tube IJ, with internal diameter 3.8 cm. The capacity of this system was thus approximately 11.5 es. units. The capacity of the key D (to a point just below the amber piece Y) was found by experiment to be 2.0 es. units. The capacity C_2 of the key and connecting system was thus approximately 13.5 es. units.

The capacity C_3 of the electrometer and permanent connections was determined by comparison with $C_1 + C_2 = 50.8$ es. units.

From the magnetic observations already described and additional observations made necessary by the increased length of the new condenser the mean value of ϕ, the flux through the cylindrical space between its two armatures, was found to be 169×10 maxwells for a current of 1 ampere in the coil. The maximum and minimum values of ϕ differed by about 10 per cent. of the maximum.

The coil was driven at a speed of about 32 revolutions per second, and was traversed by a current of 20 amperes.

Ten sets of observations were obtained. During the eight hours (including four hours intermission) within which the rotations occurred C_3 decreased from 60.4 to 56.5 es. units, the voltage sensibility varied from 12,900 to 12,800 divisions per volt, and D increased from 47 to 48 divisions. The average value of $(L - R)/D$ without respect to sign for the ten sets was 1.4 hundredths, and the average value with due respect to sign was − 0.01 hundredth.

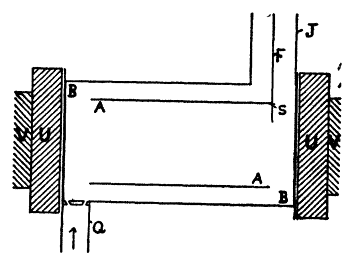

Fig. 8.

To investigate the matter when the magnetic field was produced by electromagnets the apparatus was suitably modified. Two large electromagnets UV, were substituted for the electric coil (Fig. 8). The cores were of steel shafting 7.5 cm. in diameter, and were capped with flat pole pieces UU, 12.3 cm. in diameter and 2.8 and 2.6 cm. thick, of wrought iron. The two electric coils surrounding the cores were similar, one of them being the coil C used in the experiments without iron. A was a brass cylinder 6.99 cm. in external diameter and 15.00 cm. long, B a brass cylinder 9.99 cm. in internal diameter and nearly 20 cm. long. The faces of the poles UU were 20.6 cm. apart, and the whole system of cylinders, rotating and fixed, was mounted symmetrically about the center of the magnetic field. The cylinder B was covered with brass caps 12.3 cm. in diameter, and provided with a tube Q by which a stream of filtered air could be sent through the condenser and electrometer systems. The electromagnets were driven by an electric motor at about 32 revolutions per second. In the earlier part of the work the condenser system was supported on the same base boards with the electromagnets, and the key system consisted of the same key X used in the experiments without iron together with another key constructed on the same principle as DS in those experiments and similarly located; but experience of the sort already referred to in the case of the experiments without iron led to changes quite similar to the changes made in those experiments. The calibrating apparatus and the method of observation were the same as in the experiments without iron.

The mean value of the axial magnetic flux in the space between A and B was determined by ballistic observations as in the former experiments, but with two sets of test coils in the shape of two uniformly wound solenoids with the same number of turns per cm., one with diameter slightly less than the external diameter of A, the other with diameter slightly greater than the internal diameter of B, and each somewhat longer than A. Each solenoid was provided with four pairs of leads—two leads with 20 turns between them, two with 40, two with 60, and two with 68, making four test coils for each solenoid, all with a common center. Each solenoid in turn was placed symmetrically in the magnetic field and the flux through each of its four coils determined when a current of 10.0 amperes traversed the electric coils of the magnets. By plotting the difference between the fluxes for the corresponding coils as a function of the number of turns, obtaining from the curve the flux corresponding to the number 64.9, the number of turns in 15 cm., the length of A, and multiplying by the ratio of the right cross-section of the space between A and B to the right cross-section of the space between the mean circumferences of the solenoids, the value of $\bar{\phi}$ for a current of 10 amperes in the electric coils was found to be about 289 × 100 maxwells. Table I.

TABLE I.

Number of Turns in Each Coil.	20	40	60	68
Magnetic flux in maxwells × 10,000	55	122	189	220

gives the relation between differential fluxes and turns for the solenoids and shows that, as in the previous experiments, the flux ϕ is not far from uniform. Magnetic tests for currents above 10 amperes were carried out, but the comparatively small increase in flux (20 per cent. in changing from 9 to 16 amperes) did not appear to justify the large additional amount of heat generated in the coils; so that the experiments on rotation were always carried out with currents in the neighborhood of 10 amperes. Except in the early experiments, the currents were always somewhat *greater* than 10 amperes.

The earliest experiments were still less precise than the corresponding experiments without iron, but gave, like them a null effect. With the condenser on the wall and the key system improved, but not in its final state, the average magnitude of $(L - R)/D$ in 14 sets was reduced to about 5 hundredths. A slight further reduction to about 4 hundredths in 23 sets was made after the readjustment of the electrometer.

Finally, after still further attempts to improve conditions, 12 sets were obtained in which the average magnitude of $(L - R)/D$ was reduced to 1.7 hundredths, the average value with due respect to sign being $+ 1.2$ hundredths. During nearly all of the time in which the last five sets were obtained the electrometer was in unusually stable condition; before these sets were begun the clamp of one of the keys had been improved; and throughout the five sets the keys were operated with even greater care than formerly. For these sets the average magnitude of $(L - R)/D$ was 0.8 hundredth, and its value with due respect to sign was $+ 0.6$ hundredth. About an hour after the last set was obtained rotations were again started; but conditions had become poor, giving large discrepancies, and observations were discontinued.

In the interval of four hours devoted to these 12 sets the voltage sensibility of the electrometer ranged from 12,700 to 12,600 divisions per volt; the capacity C_3 of the electrometer decreased from 54.4 to 50.3 es. units; and D changed from 28 divisions to 29 divisions.

The capacity of the condenser AB was approximately 21.0 es. units; the capacity of the wire (about 106 cm. in length, 0.023 cm. in diameter) and tube $SFY - J$ was about 10.4 es. units; and that of the key DY, about 2.0 es. units, (Figs. 6 and 8). C_2, the capacity of the key and connecting system, was thus approximately 12.4 es. units. C_3 was again obtained by comparison with $C_1 + C_2$.

The investigation leads conclusively to the result that the condenser system, when it remains at rest and the system producing the field rotates, receives not more than a minute fraction of the charge it would receive for the same relative motion if the system producing the field remained at rest. Within the limits of error of the experiments—about

1.4 per cent. in the experiments without iron, and about 1 per cent. in the experiments with iron—the fraction is zero.

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