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CONVECTION AND CONDUCTION OF HEAT IN GASES.

By Irving Langmuir.

Part I. Historical.

THE loss of heat by convection from a heated body has apparently always been looked upon as a phenomenon essentially so complicated that a true knowledge of its laws seemed nearly impossible. A. Oberbeck[1] gives the general differential equations for this problem but finds it impossible to solve them for actual cases. L. Lorenz[2] for the case of vertically placed plane surfaces is able to obtain some approximate solutions which agreed fairly well with some of the older experimental work. But as an illustration of the view taken even recently by one familiar with practically all the literature I might quote from a paper by A. Russell:[3] "The phenomenon of the convection of heat at the surface of a body immersed in a cooling fluid is one which does not lend itself readily to mathematical calculation. If the fluid be a gas the variations of the pressure, density, and velocity at different points of the gas so complicate the problem that little progress towards a complete solution has yet been made."

In his own paper Russell then feels compelled to make the following simplifying assumptions in dealing with this problem. "The liquid is supposed to be opaque to heat rays. It is also supposed to have *no viscosity* (italics mine). The liquid therefore slips past the surface of the solid. In addition it is supposed to be incompressible. Hence we should only expect the solutions to give roughly approximate values when applied to the problems of spheres and cylinders being cooled by currents of air."

[1] Ann. Phys., *7*, 271 (1879).
[2] Ann. Phys., *13*, 582 (1881).
[3] Phil. Mag., *20*, 591 (1910).

Kennelly[1] who made very elaborate measurements of the "Convection of Heat from Small Copper Wires" also finds the theory involved very complicated and is satisfied to derive empirical laws to express his results. He says "The lateral conduction through the air is negligible because the air does not remain at rest but expands and flows convectively. Consequently we may safely ignore conductive thermal loss." "Convection loss from the wire is a hydrodynamic phenomenon, involving the flow of air past the surface of the wire, and the amount of heat which this moving stream can carry off. Very little seems to be known quantitatively about convection."

Such views as these are not conducive to the finding of simple laws if such exist.

The writer has long felt that the assumption that the effect of the viscosity of the gas is negligible is unwarranted. In the case of convection from small wires it has seemed rather that it is one of the most essential of the factors involved. The writer's views on this were given in his thesis on some reactions around glowing Nernst filaments,[2] from which the following extracts are taken.

"In the case of electrically heated glowing filaments the rate of loss of energy is equal to the watts input. If, as is often the case, the radiation loss may be calculated, this may be subtracted and one thus obtains the energy lost by convection and heat conduction through the gas. Now according to the kinetic theory the viscosity of a gas increases with the square root of the absolute temperature; the driving force of the convection being proportional to the difference of density between the hot and cold gas, increases only very slowly with increasing temperature. Therefore in the immediate neighborhood of the filament the flow of gas is small and the heat must be carried away practically only by conduction."

"It would seem, however, as though heat conduction alone would come into account up to a distance of about 0.2 mm. from the center of the wire. It is highly probable that at very high temperatures, for example 2200°, the motion of the gas in the immediate neighborhood of the wire would not perceptibly increase but probably decrease, while at the same time the heat conductivity of the gas would increase very greatly. (For example the heat conductivity, **k**, at 2300° K. is 27×10^{-5} while at 273° K. it is only 4.7×10^{-5}.) Thus even at a distance from the wire where the motion of the gas is considerable, the conduction will be more important than the convection."

[1] Trans. Amer. Inst. E. E., *28*, 363 (1909).

[2] Über partielle Wiedervereinigung dissocierten Gase im Verlauf einer Abkuhlung. Inaugural Dissertation, Göttingen, 1906.

"Therefore up to a distance of a few tenths of a millimeter from the glowing body one may consider the heat to be carried only by radiation and *conduction*."

"If $\mathbf{W}$ is the rate of energy loss per cm. of length of the wire and $\mathbf{k}$ the coefficient of heat conductivity, a the radius of the wire and T_0 its temp., then for the temp. T at a distance r from the axis of the wire the relation holds:"

$$(1) \qquad T_0 - T = \frac{\mathbf{W}}{2\pi\mathbf{k}} \ln \frac{r}{a}.$$

Some later experiments by the author in this research laboratory[1] led to some very interesting results as to the heat losses in hydrogen from tungsten wires at very high temperatures. In connection with this work it was highly desirable to know the laws of heat "convection" more definitely, so the work described in the present paper was undertaken to test out the theory advanced in the above mentioned dissertation and, if the results should warrant it, to develop the theory further and give it more definite form. Several considerations had made it seem probable that the above theory would be fairly close to the truth. For example it had been noticed that the watts loss from a wire was very nearly independent of the position of the wire, that is, whether it were placed vertically or hoıizontally. Now the lines of flow of the heated air around the wire would be totally dissimilar in these two cases. Yet it was found that the energy necessary to maintain a piece of pure platinum wire at any given temperature (resistance kept constant) never differed by more than 6–8 per cent. for the vertical and the horizontal wire and at a bright red heat or above the difference became negligibly small. This was strong indication that the heat loss was dependent practically only on heat conduction very close to the filament and that the convection currents had practically no effect except to carry the heat away after it passed out through the film of adhering gas.

The thickness of the film of gas through which the conduction takes place can be calculated from equation (1) if the temp. of the wire, its diameter and the heat conductivity are known. This last quantity however varies considerably with the temp. and there is little data available on the heat conductivities of gases at very high temperatures.

Part II. Theoretical. Heat Conductivity of Gases at High Temperatures.

The literature on the heat conductivity of gases is relatively meager compared with the wealth of material on the viscosity of gas. For-

[1] Trans. Amer. Electrochem. Soc., *20*, p. 335 (1911).

tunately the kinetic theory furnishes us a means of calculating the heat conductivities from the viscosities.

Meyer[1] gives the relation:

$$\mathbf{k} = 1.603\mathbf{h}\mathbf{c}_v,$$

$\mathbf{k}$ = heat conductivity, $\mathbf{h}$ = viscosity, $\mathbf{c}_v$ = specific heat (per gram) at constant volume.

Eucken in a recent paper[2] shows that the constant (which will be denoted by K) for the monatomic gases, helium and argon, is 2.50 instead of 1.603 and that for diatomic gases, H_2, O_2, N_2, and air it is 1.90. He also shows that K is independent of the temperature over a wide range.

In view of the serious difficulties involved in measurements of heat conductivities of gases and the ease and accuracy with which the viscosities can be measured it seems highly probable that the heat conductivities calculated in this way are much more reliable than those measured directly.

Viscosity of Gases.

The variation of the viscosity of gases with the temperature has been the subject of many careful researches in the last few years. In every case Sutherland's formula seems to agree within the experimental error with the results, at least in all cases of gases above their critical temperatures.

Sutherland's formula may be written

$$\mathbf{h} = \frac{KT^{\frac{1}{2}}}{1 + \frac{C}{T}}. \tag{2}$$

The values of K and C for H_2, air and Hg are:

Gas.	$10^6 K$	C	Observer.
H_2	6.6	77	Fisher, PHYS. REV., 24, 385 (1907).
Air	15.0	124	Fisher, PHYS. REV., 29, 106 (1909).
Hg	65.0	960	See below.

The viscosity of mercury vapor has been determined by Koch[3] and his results were confirmed by Noyes and Goodwin.[4] These results do not agree with Sutherland's formula, probably partly because of errors made at low temperatures and partly because the temperature of observation was much below the critical temp. of mercury. Koch gives for the

[1] Kinetic Theory of Gases.

[2] Physik. Zeitschr., *12*, 1101 (1911).

[3] Ann. Phys., *19*, 857 (1883).

[4] PHYS. REV., *4*, 207 (1896).

highest temperature at which he made measurements:

$$\text{at } 380^\circ \text{ C.} \quad 10^3\mathbf{h} = 0.654,$$

or extrapolating from his results:

$$\text{at } 420^\circ \text{ C.} \quad 10^3\mathbf{h} = 0.718.$$

This last result is probably far enough above the boiling point so that the viscosity is nearly normal and for temperatures above this, Sutherland's formula would probably give the correct values. A. O. Rankine[1] shows that

$$C = \frac{T_c}{1.15},$$

where T_c is the critical temp. As no data are available for the critical temp. of mercury it may be roughly calculated. The ratio of the boiling points (in ° K.) and critical temps. for most liquids is nearly constant, about 1 : 1.7. Thus the critical temp. of mercury would be about 1100° K. Hence $C = 1/1.15 \times 1100^\circ = 960$. From this and the viscosity, the value of K may be easily calculated. $K = 65.0 \times 10^{-6}$. This should be considered only a rough approximation.

Specific Heats.—The most reliable data seem to be those of M. Pier[2] who gives for the actual molecular specific heat (constant volume) at the temp. T (absolute):

$$\begin{aligned} \text{For } H_2 \quad \mathbf{c_v} &= 4.454 + .0009T, \\ \text{air} \quad \mathbf{c_v} &= 4.654 + .0009T, \\ \text{Hg} \quad \mathbf{c_v} &= 2.98. \end{aligned}$$

For the calculation of the heat conductivity we need the specific heats per gram. Assuming the "molecular weight" of air to be 28.8 we get

$$\begin{aligned} \text{For } H_2 \quad \mathbf{c_v} &= 2.21(1 + .0002T), \\ \text{air} \quad \mathbf{c_v} &= 0.1614(1 + .0002T), \\ \text{Hg} \quad \mathbf{c_v} &= 0.0149. \end{aligned}$$

Heat Conductivities.—For hydrogen and air $\mathbf{k} = 1.90\mathbf{hc_v}$.

For mercury vapor at high temperatures we may safely assume $\mathbf{k} = 2.5\mathbf{hc_v}$ although Schleiermacher[3] found experimentally a value of 3.15 for the constant at a temp. of 203° C.

[1] Proc. Roy. Soc. London, *A 84*, 181–92 (1910).

[2] Z. f. Electrochem., *15*, 536 (1909), and *16*, 899 (1910).

[3] Ann. phys., *36*, 346 (1889).

We thus obtain from the data for **h** and $\mathbf{c_v}$

$$\text{(3)} \qquad \text{For } H_2 \quad \mathbf{k} = 28 \times 10^{-6}\sqrt{T}\cdot\frac{1 + .0002T}{1+\frac{77}{T}},$$

$$\text{(4)} \qquad \text{air} \quad \mathbf{k} = 4.6 \times 10^{-6}\sqrt{T}\cdot\frac{1 + .0002T}{1+\frac{124}{T}},$$

$$\text{(5)} \qquad \text{Hg} \quad \mathbf{k} = 2.4 \times 10^{-6}\sqrt{T}\cdot\frac{1}{1+\frac{960}{T}}.$$

These equations should hold especially well at very high temperatures, when the gases are far above their critical temperatures.

But in the problem of the convection of heat from a hot wire, the difference in temperature between the wire and the atmosphere around it is often so great that we cannot consider the heat conductivity as being constant. We shall need to take into account the variation of the heat conductivity in the different layers of hot gas around the wire.

In any problem in heat conduction where steady conditions prevail we may write:

$$\text{(6)} \qquad \frac{dq}{ds} = \mathbf{k}\frac{dT}{dx},$$

where dq = heat flowing per second through the area ds, $\mathbf{k}$ = heat conductivity, T = temperature, x = distance measured perpendicular to the surface ds.

If the heat flux is uniformly distributed over the whole surface s then we may separate the variables and integrate the equation as follows:

$$\text{(7)} \qquad \int \mathbf{k}dT = q\int\frac{dx}{s},$$

where $\mathbf{k}$ is a function of T only, and s is a function of x only.

If we measure the rate of loss of heat in watts (W) we have:

$$\text{(8)} \qquad W = \frac{4.19\int \mathbf{k}dT}{\int\frac{dx}{s}}.$$

Cylindrical Wires.—Let us consider a wire of diameter a surrounded by a cylindrical film of gas of a diameter b. Let T_2 be the temperature of the wire and T_1 the temperature of the gas at the outer surface of the film, *i. e.*, at a distance ½b from the center of the wire. Then, if l is the length of the wire,

$$\int \frac{dx}{s} = \frac{1}{2\pi l} ln \frac{b}{a}. \tag{9}$$

If $\mathbf{W}$ be the watts of heat energy conducted away from the wire *per unit* of length then

$$\mathbf{W} = \frac{4.19 \times 2\pi}{ln \frac{b}{a}} \int_{T_1}^{T_2} \mathbf{k} dT. \tag{10}$$

For convenience place

$$\varphi = 4.19 \int_0^T \mathbf{k} dT, \tag{11}$$

and place

$$\mathbf{s} = \frac{2\pi}{ln \frac{b}{a}}. \tag{11a}$$

Whence

$$\mathbf{W} = \mathbf{s}(\varphi_2 - \varphi_1). \tag{12}$$

Plane Surface.—Consider a plane surface of area s with an adhering film of gas of the thickness B. Equation (8) then becomes

$$W = 4.19 \frac{s}{B} \int_{T_1}^{T_2} \mathbf{k} dT = \frac{s}{B} (\varphi_2 - \varphi_1). \tag{13}$$

The function φ can be readily calculated and plotted as a function of T. For any gas the heat conductivity $\mathbf{k}$ can be put in the form

$$\mathbf{k} = A(1 + \alpha T) \frac{T^{\frac{1}{2}}}{1 + \frac{C}{T}}; \tag{14}$$

as a very close approximation we have

$$\varphi = 4.19 A(1 + 0.6\alpha T) \int_0^T \frac{T^{\frac{1}{2}} dT}{1 + \frac{C}{T}}, \tag{15}$$

and (accurately)

$$\int_0^T \frac{T^{\frac{1}{2}} dT}{1 + \frac{C}{T}} = \tfrac{2}{3} T^{\frac{3}{2}} - 2CT^{\frac{1}{2}} + 2C^{\frac{3}{2}} \tan^{-1} \sqrt{\frac{T}{C}}. \tag{16}$$

The values of φ for air, hydrogen and mercury vapor from 0° K. up to high tempreatures have been calculated (by slide rule) and are given in the following table:

TABLE I.

Table of φ, in Watts per Cm., as Function of Absolute Temp. (° K.).

T°K.	Hydrogen.	Air.	Mercury Vapor.
0°	0.0000	0.0000	
100°	.0329	.0041	
200°	.1294	.0168	
300°	.278	.0387	
400°	.470	.0669	
500°	.700	.1017	0.0165
700°	1.261	.189	.0356
900°	1.961	.297	.0621
1100°	2.787	.426	.0941
1300°	3.726	.576	.1333
1500°	4.787	.744	.1783
1700°	5.945	.931	.228
1900°	7.255	1.138	.284
2100°	8.655	1.363	.345
2300°	10.18	1.608	.411
2500°	11.82	1.871	.481
2700°	13.56		.556
2900°	15.54		.636
3100°	17.42		.719
3300°	19.50		.807
3500°	21.79		.898

Theory of Conducting Film.

Let us assume that the viscosity of the gas causes the heat to flow from a hot wire as though there were around the wire a stationary cylindrical film of gas (of diameter b) through which heat is carried only by conduction.

If we know the watts lost by a wire per cm. of length we are now in a position to calculate the diameter b of this film.

It is to be expected that b will vary with the diameter of the wire. Mr. E. Q. Adams of this laboratory has derived a relation between b and the diameter of the wire, a, which has been well verified by the experimental results. Mr. Adams' derivation is:

Derivation of $b \ln \frac{b}{a} = 2B$.

"The effective thickness of the film of air near a plate or wire is the distance the heat must travel before the heat flux due to temperature difference becomes negligible compared with that due to convection.

"At constant pressure the temperature and the temperature gradient at the outside surface of the film are assumed to be independent of the diameter of the wire.

"Consider now the analogy between the cases of heat conduction from a wire and from a plane under similar conditions of temperature, etc.

"Let r = the distance of any isotherm from the axis of the wire.

"And x = the distance of the corresponding isotherm from the plane.

"While r varies from $a/2$ to $b/2$, x varies from 0 to B.

"Since the temperature gradient at the surface of the film is assumed independent of the radius, at this point:

$$dr = dx.$$

"Elsewhere, since *within* the film convection is considered to be negligible, the total heat flux is constant, and since the conducting area is proportional to r and the heat conductivity *at the same temperature* independent of it, the temperature gradient is inversely proportional to r.

"Whence:

$$dr = \frac{r}{\frac{b}{2}}\,dx. \tag{18}$$

"Since the comparison is between points at the same temperature only, the temperature coefficient of heat conductivity does not enter at all.

"Integrating between limits:

$$ln\frac{b}{a} = \frac{2B}{b}.$$

"Multiplying by b:

$$b\,ln\frac{b}{a} = 2B. \tag{19}$$

Calculation of the Energy Loss from Cylindrical Wires.

From (11*a*)

$$ln\frac{b}{a} = \frac{2\pi}{\mathrm{s}}.$$

Substituting in (19)

$$\frac{2\pi b}{\mathrm{s}} = 2B \tag{20}$$

or

$$\frac{b}{a} = \frac{\mathbf{s}}{\pi \frac{a}{B}}.$$

But from (11*a*)

$$\frac{b}{a} = \epsilon^{\frac{2\pi}{\mathbf{s}}}.$$

Whence

(21)
$$\frac{\mathbf{s}}{\pi \frac{a}{B}} = \epsilon^{\frac{2\pi}{\mathbf{s}}}$$

or

(22)
$$\frac{a}{B} = \frac{\mathbf{s}}{\pi} \epsilon^{-\frac{2\pi}{\mathbf{s}}}.$$

From this a curve may be drawn giving a/B as a function of **s**.

The following table gives values of **s** and a/B from which such a curve may be plotted (slide rule calculation).

TABLE II.

s	a/B	s	a/B	s	a/B	s	a/B
0.0	0.0	5.0	.453	10	1.696	30	7.738
0.5	0.735×10^{-6}	5.5	.558	12	2.263	32	8.370
1.0	0.594×10^{-3}	6.0	.671	14	2.844	34	8.995
1.5	0.725×10^{-2}	6.5	.788	16	3.438	36	9.622
2.0	2.752×10^{-2}	7.0	.908	18	4.040	38	10.25
2.5	.0644	7.5	1.032	20	4.645	40	10.87
3.0	.1176	8.0	1.160	22	5.263	42	11.50
3.5	.185	8.5	1.291	24	5.877	44	12.14
4.0	.265	9.0	1.424	26	6.505	46	12.77
4.5	.354	9.5	1.561	28	7.122	48	13.40
5.0	.453	10.0	1.696	30	7.738	50	14.03

If we know the value of B for any gas, that is, if we know the thickness of the film of gas in the case of a plane surface then we can very easily calculate the watts loss per unit of length from a wire of any diameter. The calculation is as follows:

Given.—B, the thickness of film for plane surface; a, the diameter of the wire.

Method.—1. Calculate a/B. 2. Look up the corresponding value of **s** from the above table (or on curve). 3. Look up the values of φ corresponding to the temperature of the wire and to the temperature of the gas some distance from the wire.

Result.—Then **W**, the watts lost per cm. of length of the wire, will be

$$\mathbf{W} = \mathbf{s}(\varphi_2 - \varphi_1).$$

Variation of B with the Pressure, Temperature and Nature of the Gas.

Although the effect of these factors on b, the thickness of the film around a cylindrical wire, would be complicated and difficult to foresee, yet it would seem probable that B, the thickness of the film for a plane surface, would vary in some simple way. The most natural assumption seems to be that B would be proportional to the viscosity of the gas and inversely proportional to its density. For it is the viscosity that causes the existence of the film and it is the difference of density between hot and cold gas (proportional to the density itself) that keeps the film from becoming indefinitely large.

Part III. Experimental.

Calibration of Platinum Wire.

Twenty feet of platinum wire, .020″ in diameter, was especially prepared for us by J. Bishop from the purest platinum.

We specified that it should have a temperature coefficient of electrical resistance of .0038 (from 0°–100° C.) but actually we found it to have only .00350. The purest platinum obtainable from Hereaus has a temperature coefficient of .0039. Nevertheless, we decided that this platinum would fill our needs.

Part of this wire was drawn down, through diamond dies, to the following sizes:

.010, .005, .0027 and .0016 inches.

The wires were annealed and the resistance of three of them was determined at the temperatures 0°, 100° and 445°, and the constants of Callendar's formula were calculated and found to be

$$\alpha = .00350,$$
$$\delta = 1.720.$$

Slight differences were observed between the different wires, but as no very great accuracy was sought it was assumed that the above constants would give the resistance of all of the wires.

In a previous paper[1] the author has shown that above a temperature of about 1100° C. the resistance of platinum no longer follows the parabola of Callendar, but is practically linear. The ratio between the hot resistance and the resistance at 0° C. was calculated from the parabolic

[1] J. Am. Chem. Soc., *28*, 1357 (1906).

formula up to 1300° K., and then continued as a straight line. A few points from the curve obtained are tabulated below.

Ratio of Resistance at T° to Resistance at 273° K.

T	273	473	673	873	1073	1273	1500	1700	1900
R/R_0	1.000	1.688	2.328	2.919	3.463	3.958	4.742	5.273	5.804

The accuracy of this calibration was such that the errors in the temperatures undoubtedly do not exceed 20° at 1300° K. and perhaps 50° at the melting point of platinum.

Table III.

Wires Used for Experiments.

The data for these wires are tabulated below.

Wire.	Diameter Inches.	Diameter Cm.	Resistance per Cm. at ° C. Ω/Cm.	Specific Resistance Cm. Cube. Microhms.	Temp. Coef. α, from 0° C.	δ, Callendar Equation.
I.	.00159	.00404	.882	11.26	.0035	1.72
II.	.00272	.00691	.2878	10.80	"	"
III.	.00497	.01262	.0878	11.00	"	"
IV.	.00987	.02508	.0218	10.77	"	"
V.	.02004	.0510	.00572	11.62	"	"

The diameters were found by weighing measured lengths of wire on a sensitive balance, and assuming the density of the platinum to be 21.48. These results agreed well with measurements with a micrometer.

Free Convection from Horizontal Platinum Wires in Air.

A piece of the wire about 40–50 cm. long was held horizontally between clamps in a wooden box open at the side. It was found that very steady readings could be obtained if the wire was merely protected from draughts by placing a few large screens around it. For convenience a box was used. It was about 1 meter long, 30 cm. high and 15 cm. deep. The wire was placed about 10 cm. from the top.

Direct current from a 125-volt line was passed through the wire. The current was measured by a calibrated ammeter. The voltage was measured with a voltmeter, connected to fine platinum wire leads which were welded to the hot wire at points far enough from its ends to avoid the cooling action of the latter.

From the volts and amperes the watts per cm. of length and the resistance were calculated. The cold resistance was measured by a Wheatstone bridge. The resistance at 0° C. was calculated, and the

TABLE IV.

Sample Series of Observations on Free Convection from Pt Wire in Air.

Wire No. II. Diam. .00691 cm. Total length 43.9 cm. Length between voltmeter lead 37.32 cm. Wire horizontal. Room temperature 300° K.

Volts.	Amperes.	Watts / Cm.	Resist. / R at 0° C.	Temp. ° K.
1.59	0.131	.00356	1.13	308
2.72	0.217	.0158	1.16	320
3.92	0.288	.0302	1.27	350
5.00	0.34	.0429	1.37	380
6.80	0.415	.0758	1.53	425
9.40	0.499	.1257	1.75	490
14.35	0.615	.2305	2.17	620
18.9	0.679	.344	2.59	760
25.8	0.771	.534	3.12	945
28.1	0.800	.603	3.28	1010
33.9	0.861	.788	3.66	1155
38.8	0.915	.952	3.95	1275
42.9	0.958	1.12	4.18	1370
45.1	0.98	1.183	4.30	1420
47.2	1.00	1.265	4.40	1460
69.6	1.22	2.275	5.31	1850
72.5	1.235	2.40	5.48	1920[1]

TABLE V.

Wire No.	Diam. Cm.	Length Cm.	Volts.	Amps.	Watts / Cm.	Resist. / R at ° C.	Temp. ° K.	Remarks.
I.	.00404	33.94	12.08	0.25	.089	1.62	450	
			47.9	0.45	.635	3.56	1112	
			70.0	0.53	1.082	4.41	1470	
			102.0	0.63	1.892	5.40	1890	Burnt out.
II.	.00691		—	—	given	above	—	
III.	.01262	36.5	5.52	0.95	.144	1.820	512	
			18.1	1.60	.794	3.54	1105	
			28.5	1.95	1.524	4.56	1530	
			47.5	2.62	3.41	5.65	1995	Burnt out.
IV.	.02508	37.45	3.60	2.30	.221	1.92	545	
			11.12	3.74	1.11	3.64	1145	
			14.5	4.24	1.64	4.19	1375	
			19.1	4.85	2.48	4.84	1650	Burnt out at thin
			23.7	5.42	3.44	5.35	1868	spot.
V.	.0510	44.5	2.19	4.75	.234	1.815	510	
			6.83	8.00	1.227	3.31	1020	
			11.99	10.60	2.85	4.45	1485	
			17.70	13.00	5.175	5.35	1868	Not burnt out.

[1] Burnt out.

ratio of the hot resistance to that at 0° C. was calculated and from this the temperature was determined from the calibration curve obtained as described above.

About the same number of observations were made with each of the other wires. A few only of these, taken at random, are given in the following table:

All the observed values of watts/cm. (87 observations) were plotted against temperature and *smooth* curves drawn as nearly as possible through the points. With two exceptions the maximum deviation of the observed watts/cm. from that of the curve was 3 per cent. and in most cases the deviation was less than 1 per cent. So it is evident that the convection currents were steady and that draughts of air were not influencing the results.

The following data were taken from the smoothed curves as obtained above.

TABLE VI.

Total Energy Losses from Horizontal Platinum Wires in Air (300° K.) in Watts per Cm.

Wire No.	Diam. Cm.	Temp. ° K.							
		500	700	900	1100	1300	1500	1700	1900
I.	.00404	0.11	0.24	0.41	0.61	0.84	1.14	1.54	2.13
II.	.00691	0.12	0.29	0.48	0.72	0.99	1.33	1.79	2.48
III.	.01262	0.13	0.31	0.53	0.79	1.11	1.46	1.95	2.71
IV.	.02508	0.17	0.39	0.68	1.02	1.45	2.00	2.68	3.55
V.	.0510	0.22	0.52	0.90	1.42	2.03	2.89	4.10	5.65

Radiation from Platinum.

The total radiation per sq. cm. of surface from a black body at temperature T is

$$5.32\left(\frac{T}{1000}\right)^4 \text{ watts.}$$

Or, the radiation from a wire of a cm. in diameter is

$$16.7a\left(\frac{T}{1000}\right)^4 \text{ watts per cm.}$$

Now platinum is far from being a black body. Lummer and Kurlbaum[1] have determined the ratio between the radiation from platinum and

[1] Verh. Phys. Ges., Berlin, *17*, 106 (1898).

that of a black body and found

Temp. ° K.	Ratio Pt : Black Body. Per Cent.
492	3.9
654	6.0
795	7.5
1108	11.2
1481	15.4
1761	18.0

From these data a curve was plotted and the radiation from the platinum wire used in these experiments was calculated as follows:

Table VII.

Energy Radiated from Platinum Wires in Watts/Cm.

Wire No.	Diam. Cm.	Temp. ° K.							
		500	700	900	1100	1300	1500	1700	1900
I.	.00404	.000	.001	.004	0.011	0.026	0.05	0.10	0.17
II.	.00691	.000	.002	.007	0.019	0.044	0.09	0.17	0.29
III.	.01262	.001	.003	.012	0.034	0.080	0.17	0.31	0.53
IV.	.02508	.001	.007	.024	0.067	0.159	0.33	0.62	1.06
V.	.0510	.002	.013	.049	0.137	0.323	0.67	1.25	2.15

Subtracting these corrections from the total watts lost in air we get

Table VIII.

Energy Conducted from Platinum Wires by Air, or "Convection" Losses in Watts/Cm.

Wire No.	Diam. Cm.	Temp. ° K.							
		500	700	900	1100	1300	1500	1700	1900
I.	.00404	0.11	0.24	0.41	0.60	0.81	1.09	1.44	1.96
II.	.00691	0.12	0.29	0.47	0.70	0.95	1.24	1.62	2.19
III.	.01262	0.13	0.31	0.52	0.75	1.03	1.29	1.64	2.18
IV.	.02508	0.17	0.38	0.66	0.95	1.29	1.67	2.06	2.49
V.	.0510	0.22	0.51	0.85	1.28	1.71	2.22	2.85	3.50

From these data the thickness of the film of air, B, for a plane surface was calculated as follows:

$$\mathbf{s} = \frac{\mathbf{W}}{\varphi_2 - \varphi_1}$$

(φ_1 being taken at 300°).

Then from a curve giving the relation between $\mathbf{s}$ and a/B (equation 22) the corresponding value of a/B was found. From this B was calculated.

TABLE IX.

Thickness of Air Film for Plane Surface Calculated from Table VIII.

Wire No.	Diam. Cm.	Temp. ° K.								
		500	700	900	1100	1300	1500	1700	1900	Mean.
I.	.00404	.27	.43	.43	.47	.56	.47	.40	.25	.41
II.	.00691	.31	.32	.38	.40	.43	.47	.38	.26	.37
III.	.01262	.42	.42	.44	.49	.56	.69	.69	.47	.54
IV.	.02508	.30	.37	.37	.41	.45	.45	.51	.56	.43
V.	.0510	.28	.30	.33	.33	.36	.37	.35	.36	.34
Mean		.31	.37	.39	.42	.49	.49	.47	.38	.41

In drawing conclusions from the above table it should be borne in mind that a small error in the W/cm. will make a very large variation in B, as will be more clearly shown later.

Two facts stand out clearly from the above table:

1. The thickness of the film, B, calculated for a plane surface does not vary with the diameter of the wire. That is, within the experimental error, the expression

$$b \ln \frac{b}{a} = 2B$$

gives the relation between the thickness of the film and the diameter of the wire.

2. The film thickness B is surprisingly independent of the temperature. Considering the possible errors in the temperature measurements owing to the wires not being separately calibrated by resistance, it appears probable that B is independent of the temperature within the experimental error. To see if this is so and to judge the accuracy of the results the watts per cm. were calculated from equation (21) assuming the value of $B = 0.43$ cm. (a weighted mean of the above values of B).

TABLE X.

Calculated Energy Loss by Convection.

$B = 0.43$ cm.

Wire.	Diam.	500	700	900	1100	1300	1500	1700	1900
I.	.00404	0.10	0.24	0.41	0.62	0.85	1.12	1.42	1.74
II.	.00691	0.11	0.27	0.46	0.69	0.95	1.25	1.58	1.96
III.	.01262	0.13	0.31	0.53	0.79	1.09	1.44	1.81	2.24
IV.	.02508	0.15	0.36	0.64	0.94	1.30	1.72	2.17	2.67
V.	.0510	0.19	0.45	0.78	1.16	1.61	2.11	2.68	3.30

By a comparison of Table X. with Table VIII. it will be seen that the differences are relatively small, in fact probably within the experimental error. For example the greatest deviation is with wire III. at 1700°, the calculated power loss being 1.81 watts/cm. whereas 1.64 was found by experiment. This is an error of 10 per cent. but corresponds to an error in the resistance of the wire of 4.5 per cent. or an error in temperature of about 90° at 1700°. In nearly every other case the errors are much smaller than this. The general tendency for the calculated W/cm. for wire V. to be less than the observed may be connected in some way with the fact that the specific resistance of this wire is considerably higher (5 per cent.) than that of the others (see Table III.). An error in the temperature measurements of about 4 per cent. (*i. e.*, 40° at 1300°) from this cause would account for the discrepancy in the calculated W/cm.

Free Convections from Tungsten Wires in Hydrogen.

In a previous paper[1] the relation between power consumption and temperature for tungsten wires in hydrogen was studied. In those experiments the temperatures were determined for the most part from the change in the resistance of the wire and from its known temp. coefficient. In a few cases these results were checked by photometric measurements.

These experiments have now been repeated with much more care and in each case the temperature was determined both by resistance and by candle power except at such low temperatures that the candle power could not be measured. The two ways of measuring the temperature gave nearly identical results. The energy lost by radiation was calculated from the formula

$$\frac{W}{la} = 39.4\left(\frac{T}{1703}\right)^{4.74},$$

W, watts; l, length in cm.; a, diameter in cm.

This formula has been derived in the course of a careful study of the radiation from drawn tungsten wires in exhausted lamps.

The following table gives the W/cm. loss from tungsten wires in hydrogen corrected for radiation by the above formula. Most of the experiments were made with the wire vertically suspended in a tube about 5 cm. diam. Moderate variation in the size of the tube had little effect.

The results in column marked I. are those previously published.

Column II. gives results obtained by the same observer that obtained the results of column I., but in a different series of experiments.

[1] Trans. Amer. Electrochem. Soc., *20*, 225 (1911).

TABLE XI.

Wire: Drawn Tungsten .0045 Cm. Diam.

Temp. ° K.	Watts per Cm. Observed.			Calculated W/Cm.	Ratio.
	I.	II.	III.	IV.	III./IV., V.
500		0.42		0.48	
700	——	1.10	——	1.11	——
900	——	2.00	——	1.90	——
1100	2.5	2.90	2.2	2.84	0.77
1300	3.5	3.70	2.9	3.9	0.75
1500	4.5	——	3.9	5.1	0.76
1700	5.9	——	5.0	6.4	0.77
1900	7.9	——	6.9	7.9	0.88
2100	10.1	——	8.9	9.5	0.94
2300	13.0	——	11.2	11.2	1.00
2500	17.4	——	16.0	13.0	1.23
2700	24.8	——	24.5	15.0	1.63
2900	36.4	——	39.0	17.3	2.26
3100	56.4	——	60.2	19.4	3.11
3300	96.2	——	88.2	21.8	4.06

In column III. are the results obtained in the recent experiments referred to. At low temperatures they are probably not any more reliable than those of columns I. and II., but at high temperatures (above 2300°) where the candle powers were used for the temperature estimation, the results of column III. are much more trustworthy than those of column I.

The calculated W/cm. given in column IV. were obtained as follows:

For air at atmosphere pressure and room temp. B is equal to 0.43 cm., the weighted mean obtained from Table IX. We assume, for different gases or the same gas under different conditions, that B would vary directly as the viscosity and inversely as the density at the outside surface of the film. At 300° K. the viscosity of hydrogen from equation (2) is 0.496 that of air. The density is .070 that of air. Hence B for hydrogen should be 0.496/0.070 or 7.1 that of air, that is, for hydrogen at 1 atmo.

$$B = 3.05 \text{ cm.}$$

For a wire .0045 cm.

$$\frac{a}{B} = .00145,$$

whence

$$s = 1.131.$$

The W/cm. (calculated) given in Table XI. are obtained by multiplying this value of s by the values of $\varphi_2 - \varphi_1$ for hydrogen (see Table I.)

calculating φ_1 for the temp. 300° K. From the value of **s** by (11*a*) it is found that *b*, the effective diameter of the film of stationary gas around the wire, is 1.14 cm.

Up to the temperature of 2000 the observed value of *W*/cm. in columns I. and III. are less than the calculated. This is to be expected from two causes: (1) The gas in the tube is at a temp. above 300° K. which would increase φ_1 and decrease $\varphi_2 - \varphi_1$, and with it, **W**. (2) The walls of the tube would prevent to some extent "free convection" and would tend to increase *b*, thus decrease **s** and therefore **W**.

On the whole at temperatures below 2300° the agreement is strikingly good in view of the fact that the results are calculated from experiments with platinum wires in air and that no arbitrary constants have been employed.

The increasingly large deviations above 2300° are due to dissociation of hydrogen into hydrogen atoms. This was suggested in the previous paper and has now been amply verified. The results on the measurement of the extent of the dissociation and the heat of the reaction were given in a paper presented before the Washington meeting of the American Chemical Society, December, 1911. These results will soon be published in the Journal of the Society.

Free Convection from Tungsten Wires in Mercury Vapor.

The apparatus used in this experiment is illustrated in Fig. 1. *A* is a glass tube 2.5 cm. in diameter, containing mercury at its lower end which serves to supply the mercury vapor and also to make electric contact with a platinum wire fastened to the lower end of the tungsten wire *W*. The wire *W* about 7 cm. long is welded at its upper end to a heavy platinum wire which is fastened to a steel rod *F* through which the current is supplied. The tube *A* is wound with resistance wire with leads *E–E*. Another coil with leads *GG* is placed around that part of the tube containing the mercury. Asbestos insulation *K* prevents too great heat loss from this lower end whereas a second glass tube *C* placed around *A* serves to prevent heat loss from the central part of the tube. The procedure of the experiments was as follows: The entire apparatus was exhausted to less than 1 mm. pressure. Pure hydrogen was admitted to about 2/3 of an atmosphere pressure and the mercury was raised to boiling by means of the coil *GG*. Then sufficient current was applied to the winding *EE* to prevent condensation of the mercury except above the level of the asbestos placed at *J*. Mica disks *B* served to prevent hydrogen gas from diffusing down into the mercury vapor. Liquid air was placed around the tube, *L*, to dry the hydrogen and to dry out the whole system while exhausted before admitting the hydrogen.

In the first few experiments no mica disks were used, for it was thought that the strong blast of mercury vapor would prevent any hydrogen from diffusing down into the heavy mercury vapor. But it was found that the energy consumption by the wire increased with about the 15th power of the absolute temperature and that the tube *A* which was of lead glass blackened opposite the wire *W* because of reduction of the lead. By using the mica disks the energy consumption became nearly linear and showed no tendency to increase abnormally at high temperatures. But experiments since that time showed that the correction for radiation which we then applied was too large. So the results obtained with our new values for radiation do show a distinct tendency to increase at very high temperatures. However, this is believed to be due entirely to minute traces of hydrogen which did make their way down through the mercury vapor. In fact in several trials, even with the mica disks, the tubes showed signs of blackening due to lead reduction.

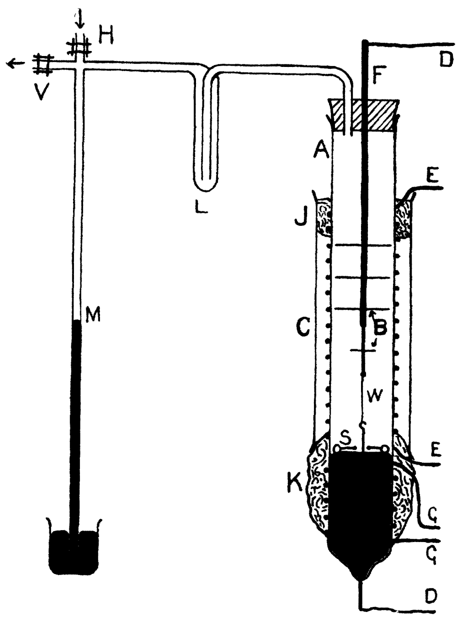

Fig. 1.

Convection from Tungsten Wire in Mercury Vapor.

Temp. of Wire ° K.	Watts per Cm.		Convection.	*B* Cm.	Watts/Cm. Calculated.	
	Total.	Radiated.			*B* = .0784.	*B* = 0.41.
1500	0.61	0.13	0.48	.083	0.487	0.15
1700	0.90	0.26	0.64	.072	0.624	0.19
1900	1.24	0.45	0.79	.074	0.777	0.24
2100	1.71	0.76	0.95	.077	0.943	0.29
2300	2.29	1.19	1.10	.084	1.125	0.34
2500	3.04	1.74	1.30	.082	1.315	0.40
2700	4.24	2.54	1.70	.055	1.52	0.46
2900	5.82	3.47	2.35	.034	1.74	0.53
3100	7.74	4.64	3.10	.022	1.97	0.60
3300	9.94	6.00	3.94	.016	2.21	0.67
3500	12.37	7.62	4.75	.013	2.46	0.75

Experiments will probably be undertaken with mercury vapor in the absence of foreign gases to verify this conclusion and obtain more accurate data on the energy loss in mercury vapor.

The following tables gives the results of the final experiment with mercury vapor at atmospheric pressure and a wire of .0069 cm. diameter.

The values of B are calculated as in the case of convection in air. The average value of B up to about 2700° is 0.0784 so this value is used as a basis for calculating the watts/cm. as in next to the last column. It is seen that the agreement between the observed and calculated values is excellent up to about 2700°, but that above this the energy loss is much greater. This is probably due to traces of hydrogen as mentioned above.

If we assume that for different gases the thickness of film, B, for a plane surface should be proportional to **h** and inversely proportional to the density then we calculate that B should be 0.41 cm. For at 600° K. the density of mercury vapor is 3.46 times that of air and its viscosity is 3.27 times that of air. The watts/cm. calculated on this basis are about 69 per cent. too low. It is possible however that the blast of mercury vapor may have caused a greater loss of energy from the wire than would have occurred with "free convection."

At any rate the order of magnitude of the results is right and up to 2700° K. there is no perceptible temperature coefficient to the value of B.

Summary and Conclusions.

It has been shown that:

1. The loss of heat from wires by free convection takes place exactly as if there were a film of stationary gas around the wire, through which the heat is carried entirely by conduction.

2. The thickness of the film is independent of the temperature of the wire, but probably increases with increasing temperature of the surrounding gas.

3. The loss of heat from very small platinum (also copper) wires by radiation is negligibly small up to temperatures of several hundred degrees.

4. The thickness of the film of gas varies in a simple way with the diameter of the wire, namely,

$$b \ln \frac{b}{a} = 2B,$$

B being a constant for any gas, b diameter of film of gas, a diameter of wire.

5. The rate of convection of heat from any wire is equal to the product of two factors, one the shape factor **s** involving only the diameter of the wire and the constant B (for any gas); and the other, a function φ of the heat conductivity of the gas.

Thus if **W** is the energy loss from wire in watts per cm., then

$$\mathbf{W} = \mathbf{s}(\varphi_2 - \varphi_1),$$

where **s** may be found from the equation

$$\frac{\mathbf{s}}{\pi}\epsilon^{\frac{-2\pi}{\mathbf{s}}} = \frac{a}{B}$$

and

$$\varphi = 4.19\int_0^T \mathbf{k}\,dT.$$

k is the heat conductivity of the gas at the temperature T in cal./cm. ° C. φ_2 is taken at the temperature (T_2) of the wire, and φ_1 is taken at the temperature (T_1) of the atmosphere.

6. Tables are given by which curves may be plotted showing the relation between φ and T and between a/B and **s**.

7. The fact that B is found to be independent of the temperature is a strong indication that Sutherland's formulæ may be applied to the heat conduction of gases up to extremely high temperatures.

In a subsequent paper the author will show that the formulæ here developed agree extremely well with Kennelly's results on the "Convection of Heat from Small Copper Wire," that for air the thickness of film B varies inversely as the 0.75 power of the air pressure and that for forced convection B varies inversely as the 0.75 power of the wind velocity. It will also be shown that a similar relation holds foı the value of B when the temperature of the atmosphere around the wire varies from 90° up to 800° K.

Several experiments will also be described in which the presence of such a "stationary" air film is demonstrated. The thickness of the film has been determined by direct measurement and the temperature distribution around the filament has been studied with care.

The author wishes to express his indebtedness to Mr. S. P. Sweetser, Mr. H. Huthsteiner and Mı. E. Q. Adams for most of the experimental work in connection with this investigation.

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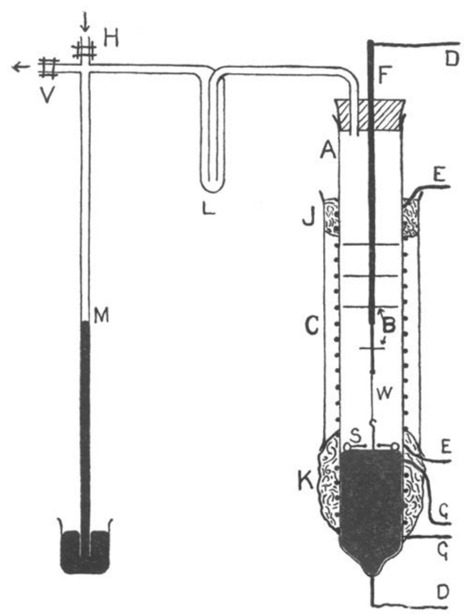

Fig. 1.