ONE-WAVENESS IN WIRELESS TELEGRAPHY; PSEUDO-IMPACT EXCITATION.[1]

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I.

THE characteristics of impact excitation are the fulfillment of one-waveness, at the original wave-length of the secondary system and the absence of the swing of energy between the secondary and primary systems, so that oscillations continue alone in the secondary system, with its own low decrement. In pseudo-impact excitation, the latter characteristic is the result of the former, since at one-waveness there may not be any beat phenomena which may cause the swing of energy. The large value of the primary decrement is enough to account for rapid quenching of primary oscillations.[2]

Of different processes investigated for impact excitation and excitations taken as impact, we may enumerate the short quench sparks,[3] quench-tubes,[4] mercury vapor sparks,[5] hydrogen sparks,[6,7] ohmic resistance,[5,6] and partial sparks,[8,9] all of which are devices to increase the decrement of the primary system. Interest in this impact excitation and its investigation seems to center very legitimately upon the improvement of coupling, hitherto attained in most cases with a limitation in spark potential in the primary system.

The fundamental condition of impact excitation, namely the fulfillment of one-waveness, is also attained, employed and appreciated[10] in ordinary

[1] Excitation akin to the impact excitation.

[2] If δ_1 and ∂_1 are the damping factor and logarithmic decrement of the primary system, the time in which the initial amplitude falls to $1/e$, and the number of oscillations during the same time are, respectively,

$$\tau = \frac{1}{\delta_1}, \quad m = \frac{1}{\partial_1}.$$

In the history of wireless telegraphy, when we required a great number of m, m was called "Resonanzgrade," and when we require a short time for τ, τ is called "Abklingszeit."

[3] Wien, Jahrb., I, 469, 1908.

[4] Wien, Jahrb., IV., 135, 1910.

[5] Glatzel, Jahrb., II., 65, 1909.

[6] Espinosa de los Monteros, Jahrb., I., 480, 1908. Rau, Jahrb., IV., 52, 1910.

[7] Glatzel, Ann. Phys., 34, 711, 1911.

[8] Rohmann, Phys. Zt., 12, 649, 1911.

[9] Galletti, The Electr., Jan. 20, 1911.

[10] One-waveness is found to give the greatest range, though at a great reduction in coupling and efficiency.

excitation, with many kinds of dischargers,[1] though at lower primary decrement, and a looser coupling. The essentials of impact excitation are here almost realized.

Since the same name is sometimes given to different quantities, I write here the formula in full,[2]

$$K'^2 = K^2 - \left(\frac{\partial_1 - \partial_2}{2\pi}\right)^2,$$

where K' is the *degree of coupling*, ∂_1, ∂_2 are the original logarithmic decrements per complete period of primary and secondary systems, and K is the *coefficient of coupling*, defined by

$$K^2 = \frac{L_{12}^2}{L_1 L_2}.$$

The condition of one-waveness may be written in many ways, namely[3]

$$K'^2 = 0; \quad \text{or} \quad K^2 = \frac{1}{4\pi^2}(\partial_1 - \partial_2)^2; \quad \text{or} \quad L_{12} = \frac{1}{2\pi}\sqrt{L_1 L_2}\cdot(\partial_1 - \partial_2);$$

or

$$\partial_1 = \partial_2 + 2\pi K.$$

Here it is assumed from practical cases that the decrement in the primary with a spark gap is larger than the decrement in the secondary which consists of ohmic and radiation decrements. Of the above condition, the first comes from the theory of coupled systems; the second may be reached from practical trials.[4] Of the third form, we may say that the measurement of wave-length with a wavemeter employs this or less amount of mutual induction at perfect synchronism, and one wave is produced in the instrument, the wave-length of which is read and made the wave-length of exciting circuit. The fourth form is known from the experimental fact that the primary decrement must be much larger than the secondary decrement to effect a good impact excitation; the difference

[1] Fessenden's synchronous gap in U. S. N. Experiment; Austin, Bull. Bur. Stand., 7, No. 3, 1911. Marconi's rotating gap, The Electrician, July 14, 1911. Gaps in series, gaps with air blast have the same function.

[2] The expression of the degree of coupling and its connection with the two coupling waves seem to appear originally in Drude's paper, Ann. Phys., 13, 1904, p. 544.

[3] Pseudo-impact excitation occurs at the value of K'^2 very little less than zero. The value of K here expressed will be called the *critical coupling* and will be understood as the limiting value above which the pseudo-impact excitation is impossible.

[4] For instance, Wien, Jahrb., I., 471. The theory neglects the presence of a spark in the primary system, and hence we are to expect some correction in practice, and this correction is very little. For example, in Wien's measurement (Wien, Ann. Phys., 29, 1909, p. 711, Tables 29, 30) values of K'^2 for $K = 0.013$, were -0.0003 and $+0.000098$, respectively, and in these cases maximum effects in the third circuit or resonator were observed.

between the two decrements being here $2\pi K$, depending on the amount of coupling. The theory of this one-waveness is very simple; we revert to the case of two systems at an extremely loose coupling,[1] and take the condition of isochronism[2] in theory, expecting a certain correction in case of resonance in practice.[3]

The resulting oscillation in the secondary system is the difference between two oscillations with the same frequency but different decrements and gives rise to an amplitude curve tapering at the beginning. The maximum current amplitude attainable in the secondary system depends on the fundamental quantities of the primary and secondary systems and is represented in practical units by

$$I_{\max}(\text{amp.}) = \frac{1}{2} \cdot \frac{\partial_1 - \partial_2}{\partial_2} \left(\frac{\partial_2}{\partial_1}\right)^{\frac{\partial_1}{\partial_1 - \partial_2}} \sqrt{\frac{C_1(\text{mf.})}{L_2(\text{mh.})}} \cdot V_f(\text{volt}),^4$$

where C_1, V_f are the capacity in microfarads and spark potential in volts in the primary system, and L_2 is the self-inductance in microhenries in the secondary system. Taking this formula for the pseudo-impact excitation and provisionally also for the impact excitation, let us try some calculations with some practical data.[5]

Example I.—One-waveness by large spark ordinary excitation. Given

$$\partial_2 = 0.004; \quad C_1 = 0.01 \text{ mf.}; \quad V_f = 6 \cdot 10^4 \text{ volts}; \quad L_2 = 100 \text{ mh.}$$

Pseudo-impact excitation is attained at $K = 4.5$ per cent.[6] for instance. Then we obtain

[1] Bjerknes, Wied. Ann., 44, 1891, 74; 55, 1895, 121; Zenneck, E. M. S., 584, 1905, L. D. T., 79, 1909. It is interesting to note that the theoretical and practical investigations were directed for some time to the reduction of primary decrement and to the production of maximum potential and current in the secondary system with two coupling-waves at close coupling, and now we have changed the course.

[2] In impact excitation, this condition must also be observed in order to obtain the most powerful oscillations in the secondary system.

[3] In practical trials with ordinary and impact excitations at the condition of one-waveness with maximum effect in the secondary or aerial system, isochronism and resonance (Zenneck, E. M. S., 556) often, though not always, become identical, within the reading-error of a wave meter. Perfect adjustment to one-waveness at maximum effect is not a very easy process, nor of a very definite delineation. Frequency and wave-length are to take account of the correction due to the larger values of decrements; the wave-meter indicates the corrected quantities.

[4] In case of $\partial_1 = \partial_2$ at isochronism, it requires a special treatment in theory, leading to quite different formulæ; as to the production of one-waveness, it is also a door case. This case is excluded in this paper.

[5] The theory of impact excitation will perhaps require the primary decrement ∂_1 to be treated as a function of time, practically to reach an infinitely large value at the end of first half beat time, and not as a constant as in case of ordinary excitation.

[6] Glatzel, Ann. Phys., 34, 711, 1911; Fleming and Dyke, The Electr., March 3, 1911; Wien, Jahrb., 4, 135, 1911.

$$\partial_1 = 0.2867;^1 \quad I_{\max} = 273 \text{ amp.}$$

Example II.—One-waveness by short spark impact excitation. Given

$$\partial_2 = 0.004; \quad C_1 = 0.01 \text{ mf.}; \quad V_f = 3\cdot 10^4 \text{ volts}; \quad L_2 = 100 \text{ mh.}$$

Impact excitation is attained at $K = 13$ per cent.[2] for instance. Then we obtain

$$\partial_1 = 0.821; \quad I_{\max} = 146 \text{ amp.}$$

Since the above formula, $\partial_1 - \partial_2$ is nearly equal to ∂_1, the factor consisting of decrements is almost unity, more so as ∂_1 is larger than ∂_2, and with no respect to coupling; therefore, under given constants of primary and secondary systems, the maximum current amplitude is almost proportional to the spark potential in the primary system. If ∂_2 is made larger, the same factor may differ more from unity. Now, with the condition of one-waveness, we may write,

$$D = \frac{\partial_1 - \partial_2}{\partial_2}\cdot\left(\frac{\partial_2}{\partial_1}\right)^{\frac{\partial_1}{\partial_1 - \partial_2}} = \frac{x^x}{(1+x)^{1+x}},^3$$

where

$$x = \frac{\partial_2}{2\pi K} = \frac{\partial_2}{\partial_1 - \partial_2}.$$

This factor is larger as x is smaller, or as ∂_2 is smaller at constant ∂_1, or as ∂_1 is larger at constant ∂_2. The value of ∂_2 can not be smaller than the radiation decrement; hence ∂_1 must be made larger and then the coupling is improved.[4]

Considering the other factor of $I_{\max}$, namely $\sqrt{C_1}\cdot V_f$, pseudo-impact excitation with larger coupling and smaller energy per spark and the same

[1] Measured with Marconi's decremeter, series gaps with 2 mm. spark each, gave $\partial_1 = 0.41$ almost constant, single-blasted gap gave $\partial_1 = 0.30$, larger for shorter spark, smaller for longer. Coupling must be adjusted every time for one-waveness when sparks or dischargers are changed.

[2] Glatzel, Ann. Phys., 34, 711, 1911; Fleming and Dyke, The Electr., March 3, 1911.

[3] A table of this quantity for values of x which occur in practice is given below.

[4] In similar manner, the time at which the secondary current reaches the maximum value $I_{\max}$ may be expressed in terms of x, namely,

$$t = \frac{2}{n}\cdot\frac{\log\partial_1 - \log\partial_2}{\partial_1 - \partial_2} = \frac{2}{n\partial_2}\, x\,[\log(1+x) - \log x]$$

$$= \frac{2}{n\partial_2}\log\frac{1}{(1+x)D}.$$

Hence this time is a function not only of ∂_1, ∂_2, or K and ∂_2, but it changes with the working wave-length. For example:

1. $\partial_1 = 0.4$; $\partial_2 = 0.08$; $n = 10^6$; $\lambda = 600m$; $t = 10\cdot 10^{-6}$;
2. $\partial_1 = 0.8$; $\partial_2 = 0.08$; $n = 10^6$; $\lambda = 600m$; $t = 6\cdot 10^{-6}$.

In the first case, the maximum amplitude occurs after 5 complete oscillations; in the second after 3.

excitation with smaller coupling and larger energy per spark are something like a balance between two above factors, and it can not be decided at once which factor is more important, best result being expected from the satisfaction of two factors, namely the closer coupling and larger energy per spark in the primary system.[1]

Example III.—Fleming and Dykes' Experiment.[2]

	Given Secondary Decrement.	One-waveness Observed. Per Cent.	Calculated Primary Decrement.	Number of Primary Oscillations till Amplitude Falls to 1/e.	$x=\partial_2/2\pi K$.	D	$I_{max}/\sqrt{\partial_2}$ Ratios.
Ordinary . .	$\partial_2=0.02$	$K=$ 4.5	$\partial_1=0.3027$	3.3	0.07074	0.7706	1
Excitation.	$\partial_2=0.18$	$K=$ 6.7	$\partial_1=0.6049$	1.6	0.4235	0.4204	0.2
Impact. . . .	$\partial_2=0.02$	$K=11$	$\partial_1=0.7111$	1.4	0.0289	0.8787	1
Excitation.	$\partial_2=0.18$	$K=13$	$\partial_1=0.9968$	1.0	0.2203	0.5620	0.2

In the next to last column, the factor of I_{max} depending only on the decrements is calculated. In the last column, these numbers are taken proportional to I_{max}. Anticipating the indication of a heat-working current measurement given below, currents in the secondary system are calculated in ratios, each for two excitations, and these ratios are about the same as those given in Fig. 4 in the authors' paper.

II.

The indication of a heat-working instrument in the secondary system takes a different form from that in case of damped trains of oscillations with one decrement and maximum initial amplitude. In the calculation I take the assumptions which are plausible in practical cases, namely,

$$(\partial_1 + \partial_2)^2 \ll 4\pi^2, \quad \partial_2 \ll \partial_1.$$

The initial value of current amplitude which is equal for the two synchronous oscillations in the secondary system, but which is not a measurable quantity, is

$$I_0 = \pi \frac{L_{12}}{L_2(\partial_1 - \partial_2)} I_{10} = \frac{1}{2}\sqrt{\frac{L_1}{L_2}} \cdot I_{10} = \frac{1}{2}\sqrt{\frac{C_1}{L_2}} \cdot V_f;$$

$$= \frac{\partial_2}{\partial_1 - \partial_2}\left(\frac{\partial_1}{\partial_2}\right)^{\frac{\partial_1}{\partial_1 - \partial_2}} \cdot I_{max};$$

[1] So far as theory goes, the advantage resulting from the coupling reaches a limit, when $x = 0$; $D = 1$, max. and limiting value. In practice, values 0.7 to 0.8 are already attained in case of impact excitation.

[2] Fleming and Dyke, The Electrician, March 3, 1911. As I take the data from the abstract, calculation is only half way and regret not having access to their original paper.

where I_{10}, V_f are the initial and maximum current amplitude and the spark potential in the primary system and $I_{\max}$ is the maximum current amplitude attainable in the secondary system. The current in the secondary sytem takes the form

$$I = I_0(e^{-\delta_1 t} - e^{-\delta_2 t}) \sin \pi n t,$$

where δ_1, δ_2 are the original factors of damping in the two sytems, and n is twice the isochronous frequency. Therefore the heat produced in the instrument with a low resistance R in one second of time and at ζ sparks per second is represented by

$$Q = \zeta R \int_0^\infty I^2 dt$$

$$= \frac{\zeta R I_0^2}{2n}\left[\frac{1}{\partial_1\left[1+\left(\frac{\partial_1}{2\pi}\right)^2\right]} + \frac{1}{\partial_2\left[1+\left(\frac{\partial_2}{2\pi}\right)^2\right]} - \frac{2}{(\partial_1+\partial_2)\left[1+\left(\frac{\partial_1+\partial_2}{2\pi}\right)^2\right]}\right].$$

Whence we obtain by the above assumptions

$$Q = \frac{\zeta R}{8n}\cdot\frac{1}{\partial_2}\cdot\frac{L_1}{L_2}\cdot I_{10}^2$$

$$= \frac{\zeta R}{8n}\cdot\frac{1}{\partial_2}\cdot\frac{C_1}{L_2}V_f^2 = \frac{\pi^2}{4}\zeta R n\frac{C_2}{\partial_2}\cdot W$$

$$= \frac{\zeta R}{2n}\cdot\frac{1}{\partial_2}\cdot I_{\max}^2.$$

Reducing this to the indication of a hot wire ammeter, we have

$$I = \frac{\pi}{2}\sqrt{\zeta n}\sqrt{\frac{C_2}{\partial_2}w}$$

$$= \sqrt{\frac{\zeta}{2n\partial_2}}\cdot I_{\max}.$$

This is the usual expression for one group of damped trains, except that $I_{\max}$ takes the place of the initial amplitude.

III.

In order to study the results of coupling when K'^2 is positive, negative or zero, I start from the biquadratic Z-equation of Drude,[1] which is

$$Z^4 + Z^2[\tau_1^2 + \tau_2^2 - \tfrac{1}{2}(\vartheta_1 - \vartheta_2)^2] - Z(\tau_1^2 - \tau_2^2)(\vartheta_1 - \vartheta_2)$$

[1] Drude, Ann. Phys., 13, 1904, p. 534.

$$+\left[\left(\frac{\vartheta_1-\vartheta_2}{2}\right)^2+\tau_1^2\right]\left[\left(\frac{\vartheta_1-\vartheta_2}{2}\right)^2+\tau_2^2\right] = K^2(\tau_1^2+\vartheta_1^2)(\tau_2^2+\vartheta_2^2),$$

with

$$y = -\vartheta + i\tau = -\frac{\vartheta_1+\vartheta_2}{2} + Z.$$

Reducing to the notation previously used and at isochronism of the two systems and with the valid assumption, ∂_1^2 and $\partial_2^2 \lll 4\pi^2$,

$$\tau_1 = \tau_2 = \frac{1}{\pi n}; \quad \vartheta_1 = \frac{\partial_1}{2\pi^2 n}; \quad \vartheta_2 = \frac{\partial_2}{2\pi^2 n};$$

$$\tau_1^2 + \vartheta_1^2 = \tau_2^2 + \vartheta_2^2 = \frac{1}{\pi^2 n^2}.$$

The above equations then take the form

$$\left\{Z^2 + \frac{1}{\pi^2 n^2}\left[1 - \frac{1}{4}\left(\frac{\partial_1-\partial_2}{2\pi}\right)^2\right]\right\}^2 = \frac{1}{\pi^4 n^4}\left\{K^2 - \left(\frac{\partial_1-\partial_2}{2\pi}\right)^2\right\} = \frac{1}{\pi^4 n^4}K'^2,$$

with

$$y = -\frac{\partial_1+\partial_2}{4\pi^2 n} + z = -\frac{\Delta}{2\pi^2 N} + i\frac{1}{\pi N},$$

where Δ's and N's are the decrements and double frequencies after the coupling, to be found from the solution of above equation.

1. K'^2 positive, with any magnitude. In this case we have

$$Z = \pm i\frac{1}{\pi n}\sqrt{1 - \frac{1}{4}\left(\frac{\partial_1-\partial_2}{2\pi}\right)^2 \pm K'},$$

and the required quantities are

$$N = \frac{n}{\sqrt{1 - \frac{1}{4}\left(\frac{\partial_1-\partial_2}{2\pi}\right)^2 \pm K'}} = \frac{n}{\sqrt{1 \pm K'}},$$

$$\Delta = \frac{N}{n}\cdot\frac{\partial_1+\partial_2}{2}\cdot = \frac{1}{\sqrt{1\pm K'}}\cdot\tfrac{1}{2}(\partial_1+\partial_2),$$

the two coupling waves, each with different decrement.

2. K'^2 zero. Here we have

$$Z = \pm i\frac{1}{\pi n}\sqrt{1 - \frac{1}{4}\left(\frac{\partial_1-\partial_2}{2\pi}\right)^2}$$

and

$$N = \frac{n}{\sqrt{1 - \frac{1}{4}\left(\frac{\partial_1-\partial_2}{2\pi}\right)^2}} = n, \quad \Delta = \frac{N}{n}\cdot\frac{\partial_1+\partial_2}{2} = \tfrac{1}{2}(\partial_1+\partial_2).$$

Here only one wave is produced with nearly the original isochronous frequency, and the decrement is also unique but its value is nearly equal to the arithmetical mean of those of the two systems before coupling.

3. K'^2 negative, small in absolute value. In this case we use the expression of K^2 and we have

$$Z^2 = -\frac{1}{\pi^2 n^2}\left[1 - \frac{1}{4}\left(\frac{\partial_1 - \partial_2}{2\pi}\right)^2 \pm i\sqrt{\left(\frac{\partial_1 - \partial_2}{2\pi}\right)^2 - K^2}\right];$$

whence, for small absolute value of K'^2,

$$Z = \pm\frac{1}{2\pi n}\frac{\sqrt{\left(\frac{\partial_1 - \partial_2}{2\pi}\right)^2 - K^2}}{\sqrt{1 - \frac{1}{4}\left(\frac{\partial_1 - \partial_2}{2\pi}\right)^2}} \pm \frac{i}{\pi n}\sqrt{1 - \frac{1}{4}\left(\frac{\partial_1 - \partial_2}{2\pi}\right)^2}.$$

Therefore the required quantities are

$$N = \frac{n}{\sqrt{1 - \frac{1}{4}\left(\frac{\partial_1 - \partial_2}{2\pi}\right)^2}} = n,$$

$$\Delta = \frac{N}{n}\left\{\frac{\partial_1 + \partial_2}{2} \pm \pi\frac{\sqrt{\left(\frac{\partial_1 - \partial_2}{2\pi}\right)^2 - K^2}}{\sqrt{1 - \frac{1}{4}\left(\frac{\partial_1 - \partial_2}{2\pi}\right)^2}}\right\}$$

$$= \tfrac{1}{2}(\partial_1 + \partial_2) \pm \tfrac{1}{2}(\partial_1 - \partial_2)\sqrt{1 - \frac{4\pi^2 K^2}{(\partial_1 - \partial_2)^2}}.$$

Thus there are produced two coincident waves with the same frequency which is nearly the same with those original isochronous frequencies, but these two waves have different decrements. When we treat for small quantities, especially for K^2, these decrements become

$$\Delta_1 = \partial_1 - \frac{\pi^2 K^2}{\partial_1 - \partial_2}; \quad \Delta_2 = \partial_2 + \frac{\pi^2 K^2}{\partial_1 - \partial_2}.$$

Thus the decrement in the primary sytem is very little less than its original value ∂_1, and that in the secondary system very little larger than its original value ∂_2. Such is the pseudo-impact excitation, improved in the resulting decrements by making the primary decrement larger and the secondary decrement smaller.

4. The nature of *impact excitation* may be conceived in the following way as the result of the change in primary decrement. Starting with an inherently large value of primary decrement at a positive value of K'^2, hence with two coupling waves producing beats, the same decrement in-

creases at a rapid rate until it reaches a final large value at the end of one half beat time. Consequently K'^2 passes from its positive phase, through zero to a negative phase, and at the time when the primary decrements reach a final value, the two systems are at the condition of an extremely loose coupling with large ∂_1 and negative K'^2, with the resulting decrements and double frequency

$$\Delta_1 = \frac{N}{n}\partial_1; \quad \Delta_2 = \frac{N}{n}\partial_2; \quad N = \frac{n}{\sqrt{1 - \frac{1}{4}\left(\frac{\partial_1}{2\pi}\right)^2}};$$

where ∂_1 is the final value.

Every such transition takes place within a very short time, and the law of change of primary decrement as a function of time is not yet known. As a consequence, the degree of coupling represented by

$$K'^2 = K^2 - \left(\frac{\partial_1 - \partial_2}{2\pi}\right)^2$$

and the primary double-frequency represented by

$$n_1 = \frac{1}{\pi}\frac{1}{\sqrt{C_1 L_1}}\sqrt{1 - \left(\frac{\partial_1}{2\pi}\right)^2}$$

are not constant, but simply decrease to the final values with a law not yet known, and also the time of half beat can not be written in the usual formula in terms of K', assumed constant. The value of ∂_1 calculated in part I. for impact excitation is only the *equivalent value*, reduced to the case of constant decrement and at zero phase of K'^2. Meanwhile, if we limit ourselves to the final phase at which the primary oscillation ceases, the condition of impact excitation is the same with that of pseudo-impact excitation and therefore we may take the expression I_{max} for the secondary maximum amplitude in part I. to be the same for the impact as well as for the pseudo-impact excitations, with ∂_1 as the equivalent value in the former and as a constant in the latter. Also the indication of a heat-working instrument, containing ∂_2 only in part II., may be taken, as it is, in case of impact excitation, and on the assumption $\partial_1 \gg \partial_2$ in case of pseudo-impact excitation. Distinction between these two excitations may be made only by observing the oscillation curves in the primary and secondary oscillations, but probably not in any other way hitherto known. These two excitations may also be a continuation with a difference only in degree, the phenomena not being fundamentally different, if we can prove the variation of the primary decrement existing also in the ordinary excitation.

TABLE I.

Values of $D = \frac{\partial_1 - \partial_2}{\partial_2}\left(\frac{\partial_2}{\partial_1}\right)^{\frac{\partial_1}{\partial_1 - \partial_2}} = \frac{x^x}{(1 + x)^{1+x}}$

for the Values of $x = \frac{\partial_2}{\partial_1 - \partial_2} = \frac{\partial_2}{2\pi K}$.

x	D	x	D
0.005	0.969	0.055	0.805
0.010	0.945	0.060	0.794
0.015	0.925	0.065	0.783
0.020	0.905	0.070	0.770
0.025	0.889	0.075	0.762
0.030	0.873	0.080	0.750
0.035	0.857	0.085	0.741
0.040	0.843	0.090	0.731
0.045	0.831	0.095	0.723
0.050	0.816	0.100	0.715

TABLE II.

Values of Critical Coupling, K in Per Cent.

∂_2	∂_1 0.1	∂_1 0.2	∂_1 0.3	∂_1 0.4	∂_1 0.5	∂_1 0.6	∂_1 0.7	∂_1 0.8	∂_1 0.9	∂_1 1.0
	Mean 0.76 Per Cent.	Mean 2.35 Per Cent.	Mean 3.94 Per Cent.	Mean 5.53 Per Cent.	Mean 7.12 Per Cent.	Mean 8.71 Per Cent.	Mean 10.31 Per Cent.	Mean 11.90 Per Cent.	Mean 13.49 Per Cent.	Mean 15.08 Per Cent.
0.005	1.51	3.10	4.70	6.29	7.88	9.47	11.06	12.65	14.24	15.84
0.010	1.43	3.02	4.62	6.21	7.80	9.37	10.98	12.57	14.17	15.76
0.015	1.35	2.94	4.54	6.13	7.72	9.31	10.90	12.49	14.09	15.68
0.020	1.27	2.86	4.46	6.05	7.64	9.23	10.82	12.41	14.01	15.60
0.025	1.19	2.79	4.38	5.97	7.56	9.15	10.74	12.33	13.93	15.52
0.030	1.11	2.71	4.30	5.89	7.48	9.07	10.66	12.26	13.85	15.44
0.035	1.03	2.63	4.22	5.81	7.40	8.99	10.58	12.18	13.77	15.36
0.040	0.95	2.55	4.14	5.73	7.30	8.91	10.50	12.10	13.69	15.28
0.045	0.88	2.47	4.06	5.65	7.24	8.83	10.42	12.02	13.61	15.20
0.050	0.80	2.39	3.98	5.57	7.16	8.75	10.35	11.93	13.53	15.12
0.055	0.72	2.31	3.90	5.49	7.08	8.67	10.27	11.86	13.45	15.04
0.060	0.64	2.23	3.82	5.41	7.00	8.59	10.19	11.78	13.37	14.96
0.065	0.56	2.15	3.74	5.33	6.92	8.52	10.11	11.70	13.29	14.88
0.070	0.48	2.07	3.66	5.25	6.84	8.44	10.02	11.62	13.21	14.80
0.075	0.40	1.99	3.58	5.17	6.76	8.36	9.95	11.54	13.13	14.72
0.080	0.32	1.91	3.50	5.09	6.68	8.28	9.87	11.46	13.05	14.64
0.085	0.24	1.83	3.42	5.01	6.61	8.20	9.79	11.38	12.97	14.56
0.090	0.16	1.75	3.34	4.93	6.51	8.12	9.71	11.30	12.89	14.48
0.095	0.08	1.67	3.26	4.85	6.45	8.04	9.63	11.22	12.81	14.40
0.100	—	1.59	3.18	4.77	6.37	7.96	9.55	11.14	12.73	14.32

The secondary decrement, when the secondary system is an aerial circuit, consists of ohmic and radiation decrements and takes the form[1]

$$\partial_2 = \frac{\lambda(m)}{6\cdot 10^2 L_2(\text{mh.})}\cdot\left\{R(\text{ohm}) + K\frac{h^2}{\lambda^2}\right\},$$

where the quantity within the bracket is the total equivalent resistance of the same circuit. The radiation decrement increases as the wave-length is shorter for the same effective height h, while the ohmic resistance may be almost constant and may be kept low. In practical cases this radiation decrement alone may reach as much as 0.1. Since the value of ∂_2 changes with the working wave-length, coupling for one-waveness must be adjusted every time the working wave-length is changed. Table II. gives K in per cent. for one-waveness for various combinations of ∂_1 and ∂_2.[2] We see that for each value of ∂_1, K is less as ∂_2 is increased; but for the values of ∂_1 greater than 0.3, and $0 < \partial_2 < 0.1$, K is almost determined from the values of ∂_1 alone. For a constant secondary system, everything which increases ∂_1 increases the coupling for one-waveness and hence the efficiency is improved, and such may be the same with the improvement of "Löschwirkung"[3] in impact excitation.

IV.

One-waveness at the Receiver.

One-waveness is also important in the receiver as in the transmitter. In order to consider this question we will take one sample diagram shown in Fig. 1, by which couplings and connections will be made manifest and definite.

When the trains of waves arrive at the place of an isochronous receiving aerial circuit and there produce the field represented by $E_{ao}e^{-\delta t}\cos\pi nt$, there are set up oscillations in the aerial circuit, of which energy, one part is transferred to the inner circuit coupled to it, but the other part is partly lost as heat and partly radiated. This radiated energy has the same frequency as the field, and superpose another field on it with a sort of coupling, the result being likely to produce the coupled frequencies both in the aerial circuit and the surrounding field. This can

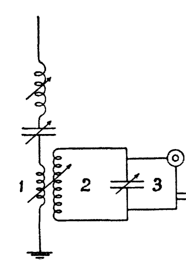

Fig.1.

[1] Rudenberg, Ann. Phys., 25, 1908, 451. The law $k\frac{h^2}{\lambda^2}$ appears to be confirmed experimentally by Austin, in case of k = constant; Bull. Bur. Stand., 7, No. 3, 1911.

[2] The relation (∂_1, K) with parameter ∂_2 are all parallel straight lines, easily extended for larger values of ∂_1 and ∂_2.

[3] Wien, Jahrb., 4, 135, 1911.

be avoided by making the aerial circuit less radiative by inserting an inductance without increasing the resistance in a proportionate amount. By this device the damping of the aerial circuit becomes much smaller than that of the arrived waves, and the resulting current oscillations in this circuit may be represented by

$$I_1 = I_{10}(e^{-\delta t} - e^{-\delta_1 t}) \sin \pi n t,$$

where δ_1 is the damping of the circuit and the maximum amplitude is represented by[1]

$$I_{10} = \frac{E_{ao}}{2\pi n L_1(\delta - \delta_1)}.$$

This is one-waveness and takes the maximum amplitude at the time

$$t = \frac{1}{\delta - \delta_1} \log n \frac{\delta}{\delta_1}$$

and

$$I_{1\max} = I_{10} \frac{\delta}{\delta_1} \left(e^{-\frac{\delta_1}{\delta - \delta_1}} - e^{-\frac{\delta}{\delta - \delta_1}} \right).$$

After this time we may write the equation for the aerial circuit in the form

$$I_1 = I_{1\max} e^{-\delta_1 t} \sin \pi n t.$$

In the next circuit 2, which is a closed condenser circuit made to have a very much smaller damping δ_2 than δ_1, and coupled to a one-waveness to the aerial circuit,[2] we may write the potential equation in the form

$$V_2 = V_{2\max} e^{-\delta_2 t} \sin \pi n t,$$

where

$$V_{2\max} = \frac{1}{4\pi n} \cdot \frac{\delta_1 - \delta_2}{\delta_2} \left(\frac{\delta_2}{\delta_1} \right)^{\frac{\delta_1}{\delta_1 - \delta_2}} \frac{1}{C_1} \sqrt{L_1 L_2} \cdot I_{1\max}.$$

And this oscillating potential works upon the next circuit. The circuit 3 consists of the detector and telephone in which the resistance and self-inductance are very large compared to the previous circuit. In this circuit we must make distinction betweeen wave frequency and group frequency. With respect to the wave frequency, circuit 3 is entirely out of tune or aperiodic and hence there is produced only the forced oscillations. The detector here works by rectification[3] and, if this rectification

[1] Bjerknes, Wied. Ann., 55, 1895, 121.

[2] At close coupling, presence of two coupling waves may be observed, see also L. W. Austin, Bull. Bur. St., Vol. 7, No. 2, 1911.

[3] This is one explanation, and in mineral detectors, some much more complicated action is suspected, though it takes no definite shape. So far as known, all forms of present detectors possess the property of polarity, whether electric rectification or magnetic hysteresis. The sense of rectification is constant while the potential working on the mineral detectors is

is perfect, current which passes through the telephone coil may be represented by

$$I_3 = I_{3\max}\left[\int_0^{1/2n} + \int_{3/2n}^{5/2n} + \int_{7/2n}^{9/2n} + \int_{11/2n}^{13/2n} + \cdots\right] e^{-\delta_2 t}\cos \pi n t dt,$$

where

$$\measuredangle (I_3, V_2) = \frac{\pi}{2},$$

$$I_{3\max} = \frac{V_{2\max}}{\sqrt{R_3^2 + (\pi n L_3)^2}} = \frac{1}{\pi n L_3}\cdot V_{2\max}.$$

This current is taken as a simple summation for this reason. The frequency of the oscillatory current is of the order 10^6 and that of the telephone diaphragm of the order 10^3; hence the most part of the oscillations in one train of currents passes through the telephone coil while the diaphragm is making one quarter vibration, from its normal position to the extreme amplitude. When the above integrations are made, the current through the telephone coil is represented by

$$\begin{aligned} I_3 &= \frac{1}{\pi n} I_{3\max}\left[\frac{1}{2\pi}\partial_2 + \frac{e^{\frac{1}{2}\partial_2}}{e^{\frac{1}{2}\partial_2} - 1}\right] \\ &= \frac{1}{\pi n} I_{3\max}\left(\frac{2}{\partial_2} + \frac{1}{2} + \frac{1}{2\pi}\partial_2\right) \\ &= \frac{2}{\pi n \partial_2}\cdot I_{3\max}, \end{aligned}$$

where ∂_2 is the decrement of the circuit 2. Thus there is apparently an accumulative action of current wave-trains in the receiver, but this is due to the mechanical slowness of the telephone diaphragm and does not extend its integrating action to the group frequency as in a heat-working instrument. It is the above current which is indicated in a moving-coil galvanometer placed in the circuit 3, and it is this current which produces the amplitude of vibration of the diaphragm, and which may be smaller and still keep up audibility, according as the group frequency approaches the natural frequency of the vibrations of the diaphragm. As regards the group frequency ζ, the intermittent current I_3 flows through the telephone coils, ζ times per second and gives rise to the note of ζ. For this frequency we have mechanical resonance and a certain kind of coupling. For the former, the damping of the diaphragm is very great; consequently it vibrates with forced group frequency. If the lowest natural frequency

large, but when it is small we observe reversals, the galvanometer deflecting in opposite senses irregularly, and the tone audible in the telephone while the galvanometer shows no deflection at all.

of the diaphragm is p, and the group frequency ζ, the intensity of tone produced will be proportional to

$$\frac{1}{(p^2 - \zeta^2)^2},$$

which expression of course excludes the idea of damping.[1] As to the kind of coupling, transfer of energy from the diaphragm back to the circuit 3 is in the form of an opposite current in the same double frequency ζ due to the vibration of an iron plate in front of the pair of permanent magnets, and there is need of something like one-waveness, by the adjustment of coupling. Such is already attained by Pickard's adjustable telephone, in which the magnet and coils are adjusted to a proper distance from the diaphragm, and the effect of this adjustment on the intensity of tone is something marvellous, while in some other telephone such adjustment is made at the manufacturing place. Thus, in the telephone for use in wireless telegraphy, mechanical resonance of the diaphragm to the group or spark frequency and the proper coupling or distance between it and the magnet, are important items of manufacture, while its resistance is immaterial within a wide limit, since it is negligible in comparison with its inductance.

[1] M. Wien, Ann. Phys., 18, 1049, 1905.