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AN ABSOLUTE DETERMINATION OF THE MINIMUM IONIZING ENERGY OF AN ELECTRON, AND THE APPLICATION OF THE THEORY OF IONIZATION BY COLLISION TO MIXTURES OF GASES.

BY EDWIN S. BISHOP.

THE theory of "ionization by collision" has been very beautifully worked out by J. S. Townsend and verified by him and others[1] for various gases at low pressures. The orignial purpose of these experiments was to submit the theory to a more rigid test by extending the observations to wide ranges of pressure, and by predicting, if possible, the electrical properties of a mixture of gases, the properties of the constituent pure gases being known.

Professor Townsend has shown that an ion in a sufficiently strong electric field will acquire sufficient kinetic energy between some of the collisions with the molecules to produce new ions from the neutral molecules. He has further shown that if the field strength is not too high, it is only the negative ion which is effective in producing new ones. Under these conditions, if we have n_0 ions set free by the action of the ultra-violet light at the negative plate of the two parallel plates between which the electric field is established, then the number of ions n reaching the positive plate is given by the formula

$$n = n_0 e^{\alpha d} \tag{1}$$

where e is Napier's base, α is the number of ions (either positive or negative) produced by the collisions of a negative ion in passing through one centimeter of the gas, and d is the distance between the plates in

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[1] Townsend, Nature, Vol. LXII., Aug. 9, 1900; Phil. Mag., Feb., 1901, June, 1902, Nov., 1903, March, 1905; Townsend and Kirkby, Phil. Mag., June, 1901; Townsend and Hurst, Phil. Mag., Dec., 1904; Hurst, Phil. Mag., April, 1906; Gill and Pidduck, Phil. Mag., Aug., 1908.

centimeters. Townsend[1] has also shown that for large values of X/p (field strength in volts per cm. ÷ pressure), the positive ions also produce new ones by collision, so that if β is the number of ions (either positive or negative) produced by a positive ion in passing through one centimeter of the gas, then the number of ions n which reach the positive plate is given by the equation

$$n = \frac{n_0(\alpha - \beta)e^{(\alpha-\beta)d}}{\alpha - \beta e^{(\alpha-\beta)d}}.$$

Inasmuch as all the experiments herein described were carried out at pressures much higher than those used by Townsend, the values of X/p are comparatively small, and consequently the first formula $n = n_0 e^{\alpha d}$ was found to satisfy all of the experimental data.

Description of Apparatus.

The apparatus which was used for finding the conductivity between two parallel plates is shown in Fig. 1. The conductivity was started by means of ultra-violet light falling upon the aluminum plate B, from the spark S, which consisted of two zinc electrodes. The spark was maintained by the discharge of four Leyden jars across the secondary of a Scheidel induction coil. The Scheidel coil was run on an alternating current from a rotary generator, the current which supplied the coil being

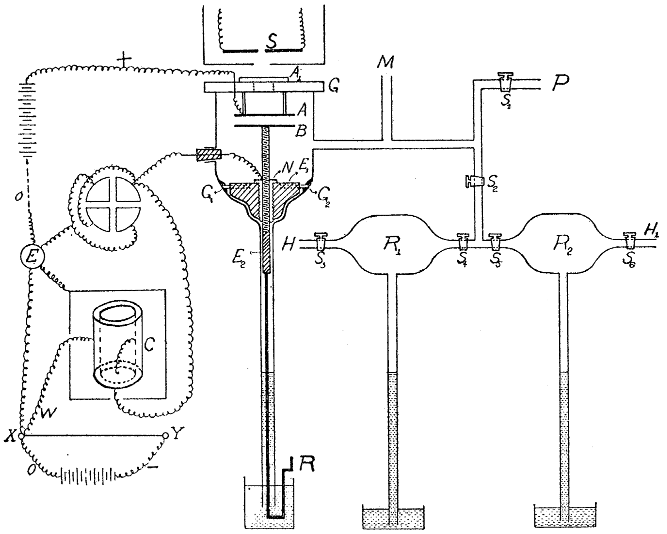

Fig. 1.

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[1] Townsend, Phil. Mag., Nov., 1903.

kept constant throughout a set of readings as well as the speed of the rotary generator. In this way the intensity of the light from the spark could be kept at any desired value, and a set of readings could be reproduced at any time with an error of less than three per cent. The ionization of the gas took place between the plates A and B, which were five centimeters in diameter. Plate G was a piece of heavy plate glass with a circular aperture one centimeter in diameter, over which was cemented the quartz plate A_1. Plate A was a silvered quartz plate with lines ruled every four tenths of a millimeter over an aperture one centimeter in diameter, to allow the passage of the light. Plate B was made of aluminum, mounted on a metallic screw which turned in the metal nut N, by means of the rod R. E_2 was a piece of ebonite about three centimeters long inserted between the screw and the rod to secure insulation. E_1 was an ebonite base which was held firmly in place by inserting into it glass rods, G_1, G_2 (G_3 not shown in diagram), which were fastened to the sides of the glass ionizing chamber with cæmentium. Plate B was made as accurately parallel with plate A as was possible before fastening plate G, which carried plate A by means of pillars, to the ionizing chamber. Plate G was sealed to the apparatus with De Khotinski cement. Connection was made between the plate B and the electrometer by a wire passing from the nut N through an amber plug in the side of the ionizing chamber. C was a cylindrical condenser the inside of which was connected to the plate B and the electrometer, and the outside of which was connected by the slide wire W to a high resistance bridge XY to which was applied about six volts for the measurement of the smaller currents. The positive potentials applied to the plate A were obtained from storage batteries, and varied from fifty to thirty-four hundred volts, depending on the pressure of the gas. These voltages were read on one of four Kelvin and White static voltmeters which ranged from 0 to 90 volts, 0 to 360 volts, 0 to 900 volts, 0 to 1,600 volts respectively. The voltmeters were first accurately calibrated by Mr. Kadesch and myself with a standard voltmeter, and were accurate to within one half per cent. A Gaede pump attached at P served to evacuate the whole system. A MacLeod gauge attached at M was used to measure pressures up to four centimeters. The higher pressures were read directly from a mirror scale placed back of the tube M. R_1 and R_2 were glass reservoirs of about 500 c.c. capacity which contained the gases to be studied. R_1 was kept nearly filled with hydrogen throughout the course of the experiments. The hydrogen used was produced by the electrolysis of phosphoric acid in distilled water. It was then bubbled through an alkaline solution of pyrogallic acid to remove the oxygen and passed into three drying tubes which contained caustic potash, calcium chloride, and phosphorous pentoxide respectively. All connections for this system were glass-sealed joints. After staying in the drying tubes for at least eight hours, the hydrogen was admitted to the reservoir R_1. The reservoir R_2 contained either air or else carbon dioxide, depending on the gas or mixture being studied. The carbon dioxide was carefully prepared and dried, as was also the air. The volumes of R_1 and R_2 were accurately determined before setting up the apparatus by weighing each empty and then filled with water at a known temperature. The barometer tubes leading down from R_1 and R_2 were each provided with a scale so that by knowing the change in pressure and volume of each reservoir, the quantity of each gas which was admitted to the ionizing chamber was known with an accuracy of one tenth per cent. The ionizing chamber for each set of observations was first exhausted to a pressure of one thousandth of a millimeter or less, with the Gaede pump, and then filled with a fresh supply of gas to a pressure which was always about ten centimeters greater than that for which the observations were to be taken. The ionizing chamber was then exhausted to the desired pressure. By this means, the traces of any gas left could never have been more than one hundred-thousandth part of the whole, and would therefore produce an entirely negligible effect. The apparatus was tested several times during the course of the experiments and when exhausted to one thousandth of a millimeter or less, the increase of pressure during three hours, the time required to make a set of observations, was never found to be more than two thousandth of a millimeter, so that any impurities in the gas arising from leakage or occluded gases in the apparatus must have produced an entirely negligible effect.

METHOD OF OBSERVING.

To make an observation the desired positive potential was applied to plate A; plate B, the pair of quadrants to which B was connected, and the inside cylinder of the capacity were earthed, and the slide wire W was at X (always zero potential). The spark was then started, plate B and its connecting system unearthed, and the time noted with a stop watch which was required for B to acquire a positive charge which could be just balanced by sliding the wire W of the inductive balance from X to Y. By this means the deflection of the electrometer could be kept within a fraction of a millimeter of its zero point, at a distance of 150 cm., during an observation. Three trials were usually made for an observation, the individual trials seldom varying by more than two per cent. from the mean.

To obtain a set of data the above process was repeated for several positive potentials applied to plate *A*, starting with potentials considerably less than was required to produce ionization by collision and continuing up to voltages which produced ionization currents that could just be balanced on the bridge.

If we let *V* represent the P.D. across the inductive balance, *c* the capacity of the condenser, electrometer, and plate *B*, and *t* the time in seconds required for plate *B* to acquire the potential *V*, then the currents through the gas will be proportional to cV/t. Since the capacity is constant the currents will be proportional to V/t, and these are the values for the current which are given in Figs. 2 to 10 inclusive.

It will be noted from the figures that the so-called saturation current slowly increases with the potential applied to plate *A* until the field strength (potential ÷ distance between plates) is sufficient to produce new ions by collision, at which point the increase in current becomes much greater. Before that field strength is reached, equal small increments in current are produced by equal large increments in the potentials applied to plate *A*, giving a straight line when plotted up to the point where ionization by collision begins. This gradual increase in the so-called saturation current is probably due to two causes. First, as the field strength is increased, fewer of the electrons released by the action of the light are discharged back to the plate, and second, the energy of emission of the electron from the plate added to the energy it received

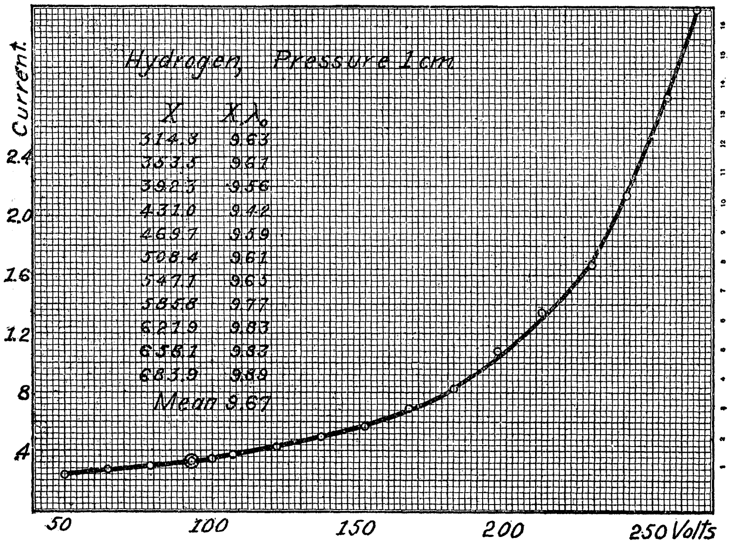
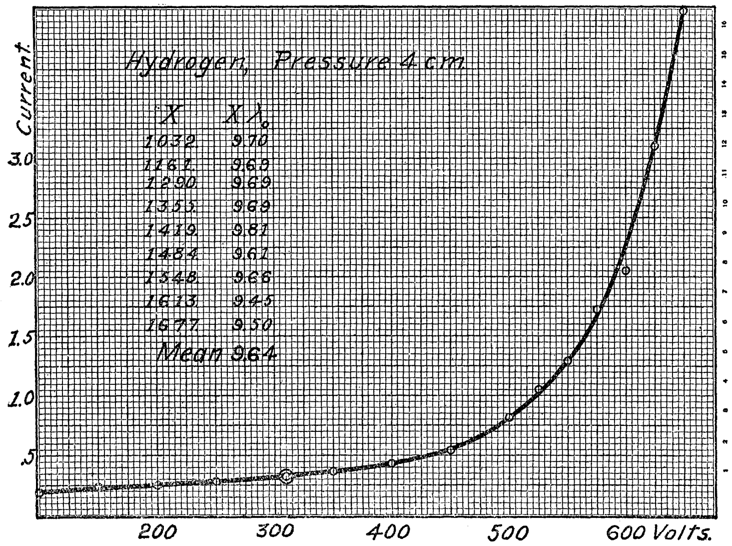
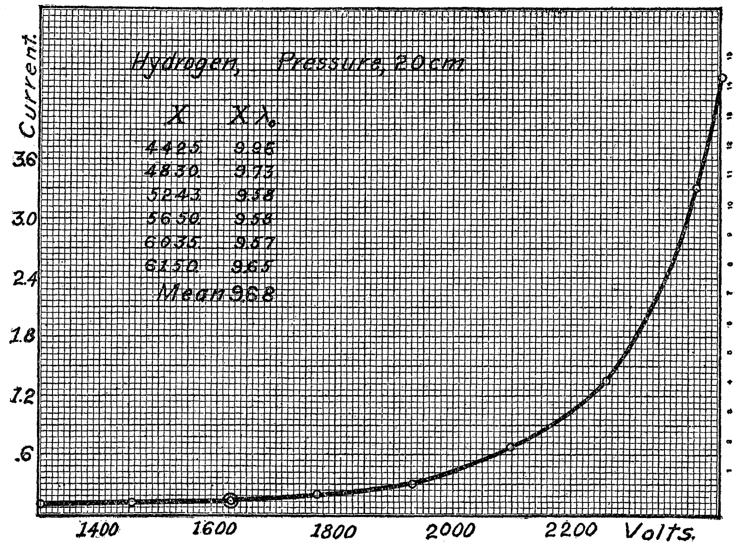


Fig. 2.



X	$X\lambda_0$
1032.	9.70
1161.	9.63
1290.	9.69
1355.	9.69
1419.	9.81
1484.	9.61
1548.	9.66
1613.	9.45
1677.	9.50
Mean	9.64

Fig. 3.



X	$X\lambda_0$
4425.	9.96
4830.	9.73
5243.	9.58
5650.	9.58
6035.	9.57
6150.	9.65
Mean	9.68

Fig. 4.

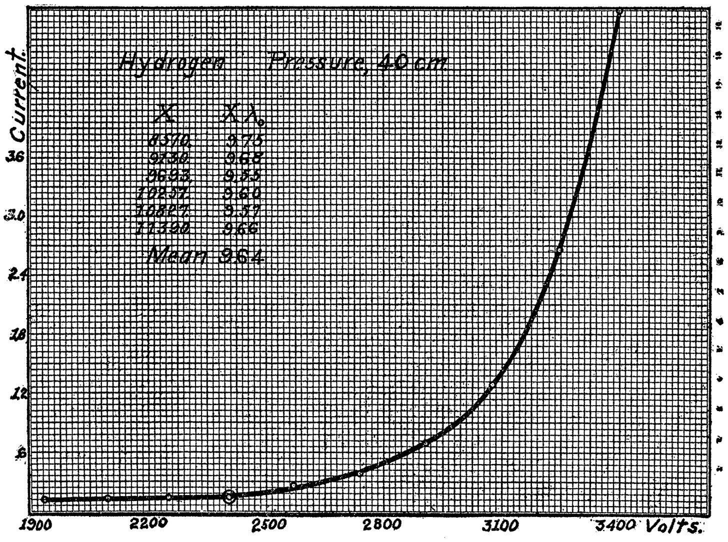

Current.
Hydrogen Pressure, 40 cm.
X
Xλ₀
8570. 9.75
9130. 9.68
9693. 9.55
10257. 9.60
10827. 9.57
11390. 9.66
Mean 9.64
36
3.0
2.4
1.8
1.2
.6
1900
2200
2500
2800
3100
3400 Volts.

Fig. 5.

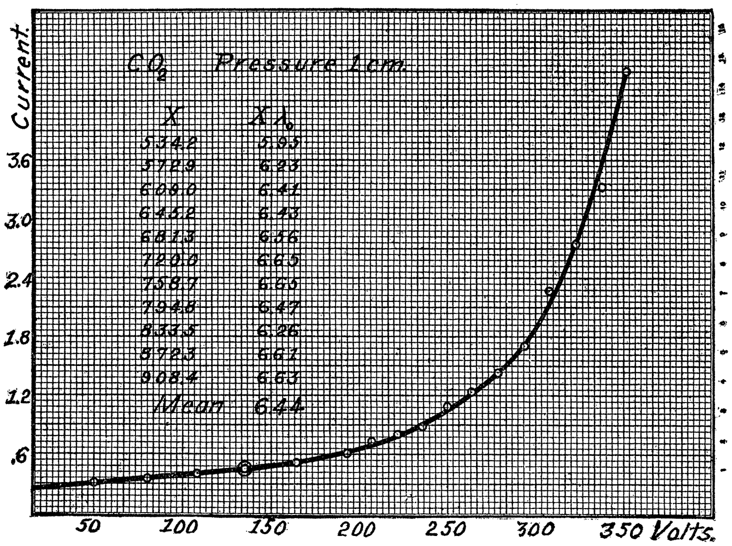
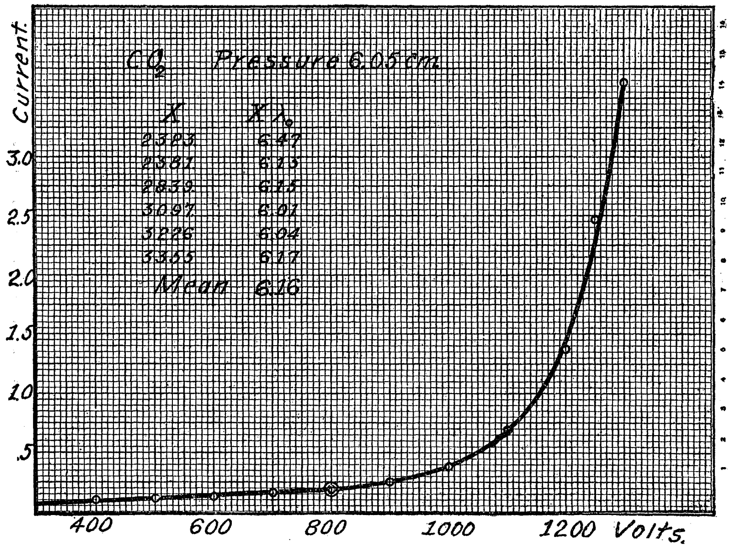
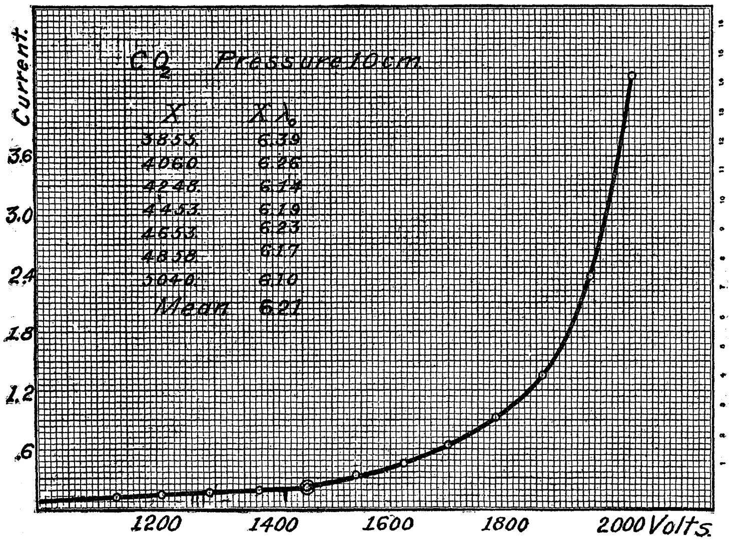

Current.
CO₂ Pressure 1 cm.
X
Xλ₀
5342 5.95
5729 6.23
6090 6.41
6452 6.43
6813 6.56
7200 6.65
7587 6.65
7948 6.47
8335 6.26
8723 6.61
9084 6.63
Mean 6.44
3.6
3.0
2.4
1.8
1.2
.6
50
100
150
200
250
300
350 Volts.

Fig. 6.



X	$X\lambda_0$
2323	6.47
2581	6.15
2839	6.15
3097	6.01
3226	6.04
3355	6.17
Mean	6.16

Fig. 7.



X	$X\lambda_0$
3855.	6.39
4060	6.26
4248.	6.14
4453.	6.19
4653.	6.23
4858.	6.17
5040.	6.10
Mean	6.21

Fig. 8.

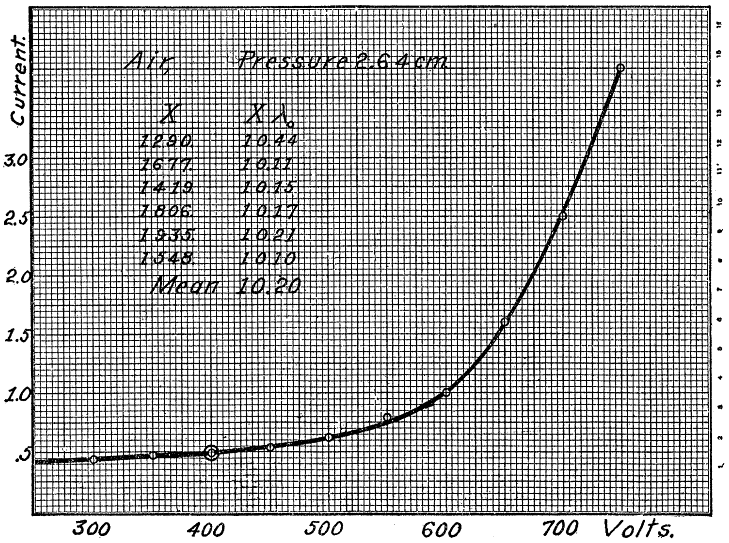

Air, Pressure 2.64 cm
X Xλ_0
1290 10.44
1677 10.11
1419 10.15
1806 10.17
1935 10.21
1548 10.10
Mean 10.20
Current.
3.0
2.5
2.0
1.5
1.0
.5
300
400
500
600
700
Volts.

Fig. 9.

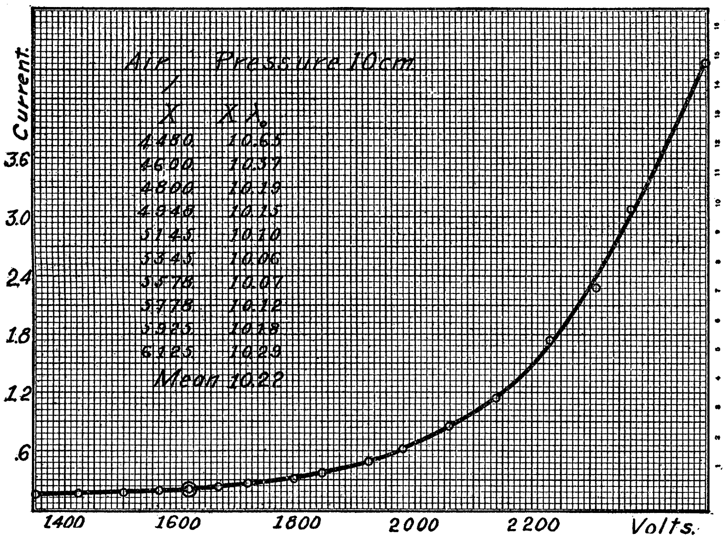

Air Pressure 10cm
X Xλ_0
4480 10.65
4600 10.37
4800 10.19
4948 10.15
5145 10.10
5345 10.06
5578 10.07
5778 10.12
5925 10.18
6125 10.29
Mean 10.22
Current.
3.6
3.0
2.4
1.8
1.2
.6
1400
1600
1800
2000
2200
Volts.

Fig. 10.

Table I.

Hydrogen.

Pressure 1 cm. $n_0 = .345$, $d = .3875$ cm.

X	n/n_0	α	X'
314.8	1.245	.544	312.0
353.5	1.435	.932	356.3
392.3	1.640	1.277	390.1
431.0	2.010	1.802	435.3
469.7	2.390	2.249	469.8
508.4	3.130	2.945	518.4
547.1	3.880	3.499	554.9
585.8	4.840	4.070	590.0
621.9	6.200	4.710	626.6
658.1	8.120	5.404	664.5
683.9	9.920	5.923	691.0

Pressure 1.9 cm. $n_0 = .318$, $d = .3875$ cm.

X	n/n_0	α	X'
451.6	1.11	.269	472.6
516.1	1.23	.534	521.5
580.6	1.45	.959	583.1
645.2	1.74	1.430	639.9
709.7	2.29	2.138	714.0
774.2	2.97	2.810	775.2
838.7	4.03	3.597	840.9
903.2	5.66	4.472	906.9
967.7	8.01	5,370	971.3
1,032.2	11.48	6.292	1,031.3
1,058.0	13.43	6.697	1,057.4

Pressure 4 cm. $n_0 = .323$, $d = 3875$ cm.

X	n/n_0	α	X'
1,032	1.32	.716	1,025
1,161	1.69	1.360	1,157
1,290	2.51	2.375	1,273
1,355	3.26	3.050	1,352
1,419	3.92	3.525	1,404
1,484	5.32	4.314	1,485
1,548	6.34	4.766	1,528
1,613	9.60	5.837	1,625
1,677	13.10	6.632	1,694

Pressure 5 cm. $n_0 = .481$, $d = .3875$ cm.

X	n/n_0	α	X'
1,161	1.16	.383	1,166
1,290	1.45	.962	1,294
1,419	1.83	1.564	1,398
1,548	2.61	2.476	1,529
1,677	4.17	3.685	1,676
1,806	6.65	4.889	1,805
1,871	9.16	5.716	1,887

Pressure 10 cm. $n_0 = .176$, $d = .3875$ cm.

X	n/n_0	α	X'
2,323	1.38	.831	2,350
2,581	1.94	1.710	2,547
2,839	3.31	3.089	2,789
3,097	6.40	4.790	3,035
3,226	9.37	5.773	3,161
3,355	14.27	6.852	3,293
3,484	23.50	8.13	3,436

Pressure 20 cm. $n_0 = .123$, $d = .4$ cm.

X	n/n_0	α	X'
4,425	1.56	1.112	4,546
4,830	2.43	2.220	4,838
5,243	5.32	4.179	5,239
5,650	10.97	5.981	5,548
6,035	26.90	8.220	5,884
6,150	35.90	8.942	5,982

Pressure 30 cm. $n_0 = .130$, $d = .3$ cm.

X	n/n_0	α	X'
6,543	1.67	1.709	6,696
6,997	2.43	2.960	6,977
7,503	3.84	4.485	7,264
8,020	6.96	6,467	7,586
8,580	13.65	8.702	7,906
9,100	33.98	11.740	8,293

Pressur 40 cm. $n_0 = .141$. $d = .3$ cm.

X	n/n_0	α	X'
8,570	1.87	2.086	8,877
9,130	2.68	3.286	9,164
9,693	4.93	5.318	9,568
10,257	9.15	7.379	9,919
10,827	18.78	9.765	10,284
11,390	36.09	11.940	10,586

from the field would be sufficient to ionize at the first impact, although the field strength were below the minimum ionizing field strength. When, however, the field strength attains such a value that the electron acquires sufficient energy between some of the impacts to ionize, then the current will increase rapidly for small increases in field strength. Thus the potential (or field strength) at which ionization by collision begins is obtainable either from the data or the curves and these points are indicated on the curves by two concentric circles. All the data for these experiments is obtainable from the curves, the abscissæ representing the number of volts applied to plate A and the ordinates representing the corresponding values of V/t (volts applied to inductive balance XY $\div$ time for plate B to acquire the potential V). The tables accompanying the curves show the values obtained for the ionizing energy for corresponding values of the field strength. The value of n_0 in all cases was taken as the value of the current at these doubly circled points, and n was the value of the current for any higher potential. The method of determining the minimum ionizing energy $X\lambda_0$ (free fall of the electron in volts) is given in the next general heading of this paper.

The values of α for various field strengths X were then calculated

Table II.

Carbon Dioxide.

X	n/n_0	α	X'
Pressure 1 cm. $n_0=.453$, $d=.3875$ cm.			
534.2	1.60	1.213	553.7
572.9	1.79	1.503	583.8
609.0	1.95	1.733	605.4
645.2	2.43	2.291	656.9
681.3	2.73	2.591	682.4
720.0	3.16	2.970	713.2
758.7	3.75	3.412	747.4
794.8	5.01	4.158	801.9
833.5	6.07	4.653	836.3
872.3	7.33	5.141	861.3
908.4	9.93	5.925	898.0
Pressure 10 cm. $n_0=.204$, $d=.4$ cm.			
3,855	1.60	1.175	3,983
4,060	2.16	1.975	4,146
4,248	3.19	2.900	4,329
4,453	4.51	3.766	4,474
4,653	6.62	4.725	4,621
4,858	11.60	6.120	4,819
5,040	21.66	7.680	5,023
Pressure 3.53 cm. $n_0=.197$, $d=.3875$ cm.			
1,548	1.59	1.197	1,558
1,806	3.22	3.017	1,810
1,935	5.00	4.152	2,009
2,064	7.97	5.357	2,066
2,194	12.48	6.508	2,177
2,258	18.73	7.554	2,272
Pressure 5 cm. $n_0=.240$, $d=.3875$ cm.			
2,052	1.60	1.213	2,122
2,263	2.53	2.395	2,316
2,472	4.17	3.683	2,491
2,681	7.04	5.038	2,655
2,888	14.80	6.947	2,862
2,942	18.75	7.556	2,923
Pressure 6.05 cm. $n_0=.189$, $d=.3875$ cm.			
2,323	1.31	.697	2,407
2,581	2.04	1.840	2,634
2,839	3.64	3.334	2,865
3,097	7.25	5.112	3,096
3,226	13.07	6.624	3,272
3,355	19.21	7.618	3,380

from the formula $n = n_0e^{ad}$, and are given in Tables I., II., and III. The computed values of the field strengths X' required to produce the same current, *i. e.*, the same value of n/n_0 are given in the last columns of the same tables. The method of obtaining these will be described in a later section.

Table III.

Air.

Pressure 2.64 cm. $n_0 = .498$, $d = .3875$ cm.			
X	n/n_0	α	X'
1,290	1.25	.576	1,287
1,419	1.59	1.197	1,419
1,548	2.00	1.789	1,522
1,677	3.21	3.009	1,699
1,806	5.02	4.163	1,843
1,935	7.53	5.210	1,959

Pressure 4.92 cm. $n_0 = .155$, $d = .3875$ cm.			
X	n/n_0	α	X'
2,323	1.43	.914	2,358
2,581	2.14	1.963	2,594
2,839	3.53	3.254	2,824
3,097	6.64	4.885	3,068
3,226	10.13	5.969	3,214
3,354	16.25	7.187	3,365
3,419	21.60	7.920	3,452

Pressure 10 cm. $n_0 = .197$, $d = .4$ cm.			
X	n/n_0	α	X'
4,480	1.56	1.112	4,576
4,600	1.86	1.551	4,709
4,800	2.46	2.250	4,892
4,948	3.13	2.852	5,033
5,145	4.28	3.635	5,200
5,345	5.78	4.386	5,348
5,578	8.83	5.445	5,542
5,778	11.57	6.114	5,656
5,925	15.58	6.857	5,778
6,125	23.20	7.851	5,933

Pressure 5.27 cm. $n_0 = .205$, $d = .3875$ cm.			
X	n/n_0	α	X'
2,323	1.20	.471	2,365
2,581	1.68	1.332	2,613
2,839	2.48	2.344	2,825
3,097	4.02	3.592	3,042
3,355	7.90	5.334	3,301
3,484	15.90	7.130	3,535

Determination of the Minimum Ionizing Energy.

Mr. Bergen Davis[1] has deduced a theory for the minimum kinetic energy which a negative ion must have in order to ionize a molecule by impact. He has shown that if α is the number of ionizing impacts which a negative ion makes in passing through one centimeter of a gas, l is the mean free path of the ion, and λ_0 is the minimum path which results in ionization, then the number of ionizing impacts is given by the following equation:

$$\alpha = \frac{1}{l}e^{-\frac{\lambda_0}{l}}\left[1 + \frac{\lambda_0}{l}e^{\frac{\lambda_0}{l}}Ei\left(-\frac{\lambda_0}{l}\right)\right],$$

which may be written

$$\alpha l = e^{-\frac{\lambda_0}{l}} + \frac{\lambda_0}{l}Ei\left(-\frac{\lambda_0}{l}\right). \tag{2}$$

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[1] Bergen Davis, Physical Review, Jan., 1907.

The values of the ponential integral $Ei(-\lambda_0/l)$ were obtained by him from tables by Dr. W. Laska (Sammlung von Formeln). The values of αl and the corresponding values of λ_0/l as computed by Davis are given in Table IV.

TABLE IV.

λ_0/l	αl	λ_0/l	αl	λ_0/l	αl
.04	.8536	.4	.3893	4	.0032
.06	.804	.6	.2762	5	.00099
.1	.7247	.8	.2008	6	.00032
.14	.656	1.	.1485	7	.000142
.2	.574	1.5	.0731	8	.000034
.25	.518	2.	.0375	9	.0000114
.3	.496	3.	.0106	10	.00000387

These values were plotted on three large curves about thirty centimeters each way, using the values of λ_0/l for the abscissæ and the corresponding values of αl for the ordinates. By referring to these curves the value of λ_0/l corresponding to any experimental value of αl could be interpolated within a fraction of one per cent.

The values of the mean free paths of hydrogen, carbon dioxide, and air at 76 cm. pressure and 23° C. in all these experiments were taken to be .0000185, .00000654, and .00000983 cm. respectively. At other pressures the mean free paths were taken inversely proportional to the pressure. In all cases the mean free path of the negative ion (electron) was taken to be 5.65 times the mean free path of the gas molecule at that pressure. If the electron is negligible in size, then its mean free path would be four times that of the gas under the same velocity conditions. Since, however, the electron when in fields sufficiently strong to cause it to ionize, moves with a velocity of order higher than that of the molecules,[1] its mean free path should be $\sqrt{2}$ times as great on that account.[2] Thus the mean free path of the electron in a gas should be $4\sqrt{2}$ times the mean free path of the molecule of the gas.[1]

Using this value of l and the experimentally determined values of α (see Tables I., II., III.), the values of λ_0/l corresponding to the values of αl for the various field strengths were found from the curves previously described. The minimum ionizing energy expressed in the number of volts through which the electron must freely fall would then be

$$\text{Energy (Volts)} = \frac{Xl\lambda_0}{l} = X\lambda_0.$$

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[1] From $e/m = 5.6 \times 10^{17}$ E.S.U. and equating $\frac{1}{2}mv^2$ to $Xe\lambda_0$, the minimum ionizing velocities of the electron in hydrogen, carbon dioxide, and air are 10^3, 4.3×10^3, and 4×10^3 times as great as the velocities of agitation of the molecules of these gases respectively.

[2] J. C. Maxwell, Phil. Mag., Ser. 4, Vol. 19, p. 29, 1860.

The data for the different gases is shown by the curves of Figs. 2 to 10 inclusive, and the results for $X\lambda_0$ corresponding to different values of X are shown in the tables accompanying those figures. A summary of all the values of $X\lambda_0$ for the different gases and pressures is given in Table V.

TABLE V.

Hydrogen.		Carbon Dioxide.		Air.	
Pressure in cm.	$X\lambda_0$	Pressure in cm.	$X\lambda_0$	Pressure in cm.	$X\lambda_0$
1	9.67	1	6.44	2.64	10.20
1.9	9.59	3.53	6.28	4.92	10.22
4	9.64	5	6.17	5.27	10.18
5	9.68	6.05	6.16	10	10.22
10	9.72	10	6.21		
20	9.68				
30	9.67				
40	9.64				

In finding the final mean of $X\lambda_0$ (Table VI.) for the different gases the above values of $X\lambda_0$ were given weights proportional to the pressures, since the accuracy of several observations increases with the pressure. The value of the minimum ionizing energy expressed in ergs would be

$$\text{Energy (Ergs)} = \frac{X\lambda_0}{300} \cdot e,$$

where e is the value of the elementary charge on the electron. The value of e used here was 4.89×10^{-10}, which was the value recently determined by Professor R. A. Millikan[1] by experiments on oil drops and which has been verified by E. Regener.[2]

TABLE VI.

Gas.	$X\lambda_0$ (Volts).	$Xe\lambda_0$ (Ergs).
Hydrogen	9.66	1.58×10^{-11}
Carbon dioxide	6.21	1.01×10^{-11}
Air[3]	10.21	1.67×10^{-11}

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[1] R. A. Millikan, Science, Vol. XXXII., Sept. 30, 1910, and PHYSICAL REVIEW, April, 1911.

[2] E. Regener, Physikalische Zeitschrift, Feb. 15, 1911.

[3] The mean free paths used in determining the above values were taken from Meyer's Kinetic Theory. The mean free path of air at 23° C., computed from the most probable value of the coefficient of viscosity as given by R. A. Millikan in the PHYSICAL REVIEW, April, 1911, p. 386, is 5 per cent. less. This value of the mean free path of air gives a value for the ionizing energy which is 4 per cent. less than the above, or 9.8 volts. It will be noticed that O. V. Baeyer's value of 10 volts, which is the most direct determination which has been made, lies between these two values. This would indicate that Jean's correction for the persistence of velocities is not here needed.

Lenard,[1] by a very direct method obtained 11 volts for the ionizing energy of air, and O. v. Baeyer[2] by a similar direct method obtained 10 volts. For a review of other work on this subject the reader is referred to an article by Dr. Kleeman, published in the Proceedings of the Royal Society, June 9, 1910.

Relative Ionizing Energy for the Different Gases.

If we plot the field strengths corresponding to the doubly circled points of Figs. 2 to 18 inclusive as ordinates, and the corresponding pressures as abscissæ, we obtain the straight lines shown in Fig. 19.

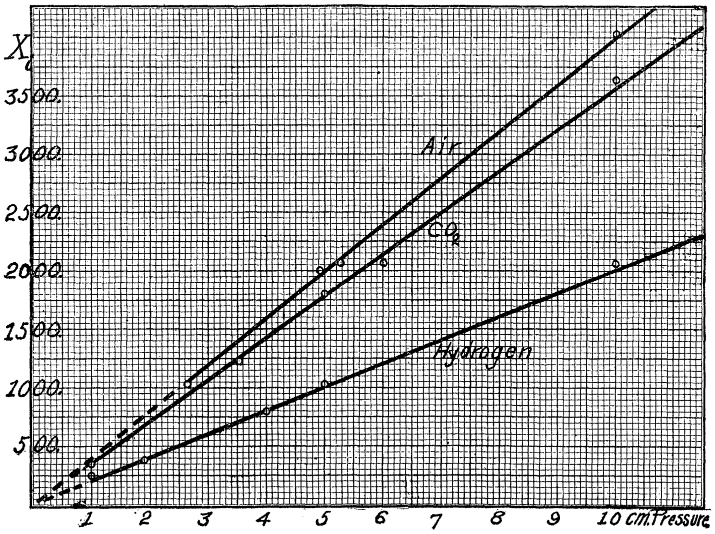

Fig. 11.

These are the minimum ionizing field strengths and are denoted by X_i. From these curves the very important law follows, that the minimum ionizing field strength for any gas is directly proportional to the pressure, or for any gas

$$\frac{X_i}{p} = \text{Cst.} \tag{3}$$

This equation satisfies the conditions at least for pressures above one centimeter. The straight lines are produced as dotted lines back to the origin merely to show that the law is a direct proportion. However, this law, as is well known, does not hold at low pressures and the experi-

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[1] Lenard, Drude's Ann. der Phys., Vol. 8, 1902.

[2] O. v. Baeyer, Verhandlungen der Deutschen Physikalischen Gesellschaft, Vol. 10, p. 100, 1908.

mental data would not follow the dotted portions of the curves, but would curve up as very low pressures were approached, the minimum ionizing field strength increasing very rapidly for high vacua. The straight line for hydrogen does not show the results for pressures of 20 and 40 centimeters. The results may be obtained from Figs. 4 and 5, X_i being equal to V/d (volts applied to plate A corresponding to doubly circled points ÷ distance d between the plates). These points fall on the straight line for hydrogen produced. Thus the law X_i/p = Cst has been verified up to 40 cm. pressure for hydrogen, and up to 10 cm. pressure for each of the gases, air and carbon dioxide. This constant was found to be 200 volts per cm. per cm. of pressure for hydrogen, 400 for air,[1] and 356 for carbon dioxide. In obtaining these values the mean of X_i/p for the different pressures was taken, each determination being given a weight proportional to the pressure of the gas.

First Method—Relative Energy: If the mean free path of the electron were the same in two gases, the relative ionizing energy of these gases would be

$$\frac{E_1}{E_2} = \frac{X_{i_1}}{X_{i_2}}, \tag{4}$$

where E_1 and E_2 are the minimum ionizing energies of the first and second gases respectively, and X_{i_1} and X_{i_2} are the corresponding minimum ionizing field strengths. But the ratio of the mean free paths of the electron in the two gases is the same as the ratio of the mean free paths of the molecules of the two gases, if the electron is negligible in size in comparison to the molecule. Since the energy acquired by the electron between impacts in the two gases is proportional to the mean free paths L_1 and L_2 of the gases, therefore the right hand member of Eq. (4) must be multiplied by this ratio. Hence we have for any two gases

$$\frac{E_1}{E_2} = \frac{X_{i_1}}{X_{i_2}} \cdot \frac{L_1}{L_2}. \tag{5}$$

For air and hydrogen,

$$\frac{E_A}{E_H} = \frac{400}{200} \cdot \frac{.00000983}{.0000185} = 1.063.$$

For air and carbon dioxide,

$$\frac{E_A}{E_{CO_2}} = \frac{400}{356} \cdot \frac{.00000983}{.00000654} = 1.689.$$

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[1] P. J. Kirkby, Phil. Mag., Feb., 1902, p. 222, states that the smallest value of X/p which gives an appreciable value of α is 40. This corresponds to X/p equal to 400 here since the pressure he used was given in millimeters.

For hydrogen and carbon dioxide,

$$\frac{E_{\text{H}}}{E_{\text{CO}_2}} = \frac{200}{356} \cdot \frac{.0000185}{.00000654} = 1.589.$$

Second Method—Relative Energy: The fact that the minimum ionizing energy remains constant at all pressures, taken in connection with the law $X_\iota/p = \text{Cst}$, shows that the mean free path of the electron in a gas must be inversely proportional to the pressure of the gas. Hence in order to make the mean free path l of the electron the same in two gases, all we have to do is to make the pressures of the two gases proportional to the mean free paths of the gases, *i. e.*, $p_1/p_2 = L_1/L_2$.

If we obtain the ionization curves for these pressures (see Figs. 12 and 13) in the following manner, we shall have the necessary data for determining the relative ionizing energy of the two gases. Accordingly the spark was adjusted so that the value of n_0 was the same in the two gases, *i. e.*, the currents were made the same for the minimum ionizing field strength in each gas. This was done in some cases by changing the intensity of the spark, and in others by adjusting the distance of the spark above the plate.

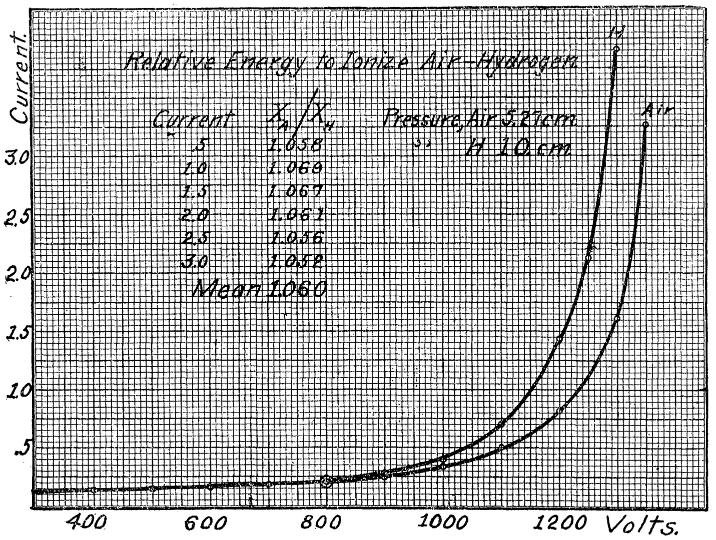
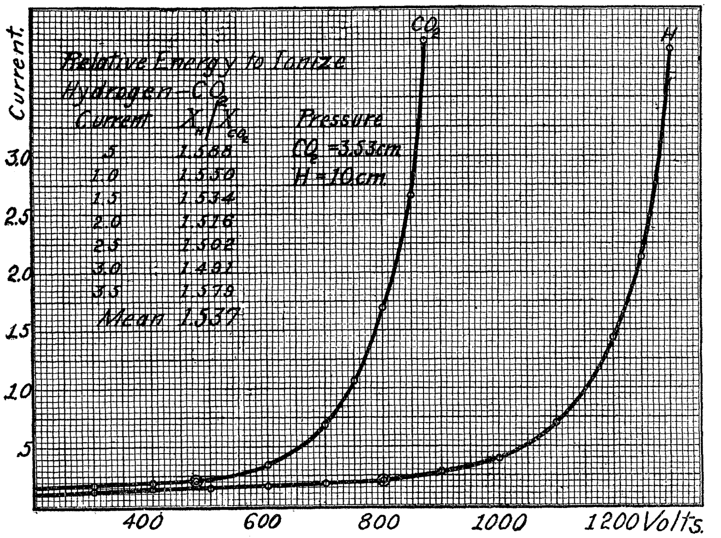


Fig. 12.

Since we have n_0 the same in both gases, it is clear that if we take two points on the two curves where n is the same, *i. e.*, where the ionization currents are equal, then α, the number of ions produced by collision in each gas, must be the same. Having n_0, α, and l the same for both gases their relative energy must be the ratio of the field strengths of any two points on the curves for which n is the same, for since the distance d between the plates was the same for both gases, the relative ionizing energy must be simply the ratio of the positive potentials V, applied to the plate A, or



Current	X_H/X_{CO_2}
.5	1.588
1.0	1.550
1.5	1.534
2.0	1.516
2.5	1.502
3.0	1.491
3.5	1.579
Mean	1.537

Fig. 13.

$$\frac{E_1}{E_2} = \frac{V_1}{V_2}, \tag{6}$$

when

$$\frac{p_1}{p_2} = \frac{L_1}{L_2}$$

and n_0, α, l, and d are the same for both gases.

For air and hydrogen see Fig. 12 and table accompanying it. Here $E_A/E_H = 1.060$.

For hydrogen and carbon dioxide see Fig. 13 and accompanying table. Here $E_H/E_{CO_2} = 1.537$.

No data were taken by this method for comparing air and carbon dioxide.

Third Method—Relative Energy: A third method for obtaining the relative ionizing energy is by comparing the absolute values given in Table VI. While this is not a direct method, it serves somewhat as a check on the absolute values there given, since it gives the same relative ionizing energy as the first two methods.

For air and hydrogen $\frac{E_A}{E_H} = \frac{10.21}{9.66} = 1.057.$

For air and carbon dioxide, $\frac{E}{E_{CO_2}} = \frac{10.21}{6.21} = 1.644.$

For hydrogen and carbon dioxide, $\frac{E_H}{E_{CO_2}} = \frac{9.66}{6.21} = 1.556.$

For purposes of comparison the results of the three methods are given in Table VII.

TABLE VII.

Relative Ionizing Energies.

	E_A/E_H	E_A/E_{CO_2}	E_H/E_{CO_2}
First method	1.063	1.689	1.589
Second method	1.060	No data.	1.537
Third method	1.057	1.644	1.556

METHOD OF COMPUTING THE FIELD STRENGHTS REQUIRED TO PRODUCE A GIVEN CURRENT IN A PURE GAS.

Professor Townsend has shown that for small values of X/p

$$\frac{\alpha}{p} = F\left(\frac{X}{p}\right), \tag{7}$$

and that when X/p becomes large enough so that the positive as well as the negative ions produce new ones by collision

$$\frac{\beta}{p} = \phi\left(\frac{X}{p}\right), \tag{8}$$

where β is the number of ions (either positive or negative) produced by the collisions of a positive ion in passing through one centimeter of the gas.

In the present work the values of X/p are small compared to those used by Townsend, because of the fact that much higher pressures were used in this work than in his. According to his curves showing the relation between β/p and X/p, the values of β in the present experiments would be entirely negligible. Also since the values of α obtained in these experiments (see Tables I., II., III.) gives a constant value of $X\lambda_0$ for all field strengths and pressures, it follows that in these experiments the positive ions could not have produced new ions by collision.

In order to show that α/p is some function of X/p all of the values of α/p and corresponding values of X/p (taken from Tables I., II., III.), for all of the points observed (see Figs. 2 to 10) were plotted on a large scale, and the result is shown in Fig. 14, which is reduced to about one seventh of the original size. These points fall upon a smooth curve and consequently prove the existence of such a function. When they were plotted it was at once evident that these curves intercepted the X-axis at $X/p = X_i/p$, where X_i/p was the constant determined from Fig. 11

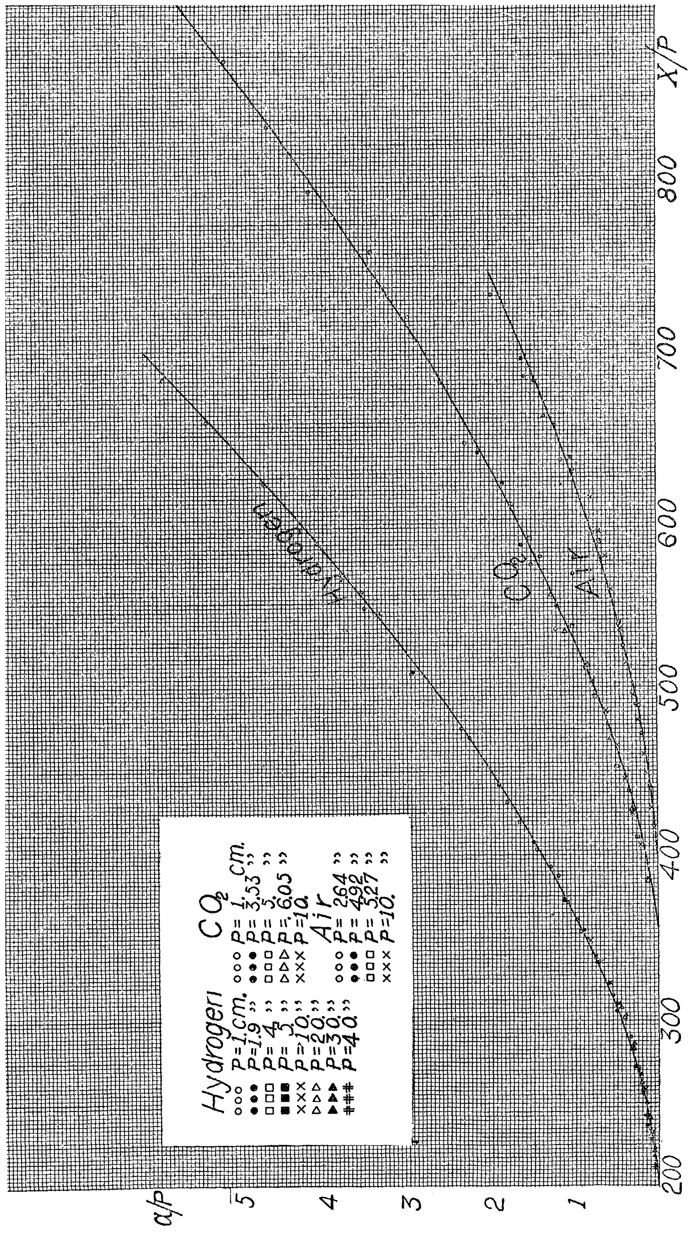


Fig. 14.

for the different gases. This would necessarily have to follow since α must be sensibly zero for field strengths which are very slightly less than the minimum ionizing field strength. Thus the function at once reduces to

$$\frac{\alpha}{p} = f\left(\frac{X}{p} - \frac{X_i}{p}\right), \tag{9}$$

where $X_i/p = 200$ for hydrogen, 356 for carbon dioxide, and 400 for air.

It was then found that all of the curves (Fig. 14) were satisfied by the general equation

$$\frac{\alpha}{p} = a\left(\frac{X}{p} - \frac{X_i}{p}\right)^b, \tag{10}$$

where a and b are constants depending on the gas and were determined from two points which fell exactly on the curve. This gave the following equations for the curves of Fig. 14 for hydrogen, carbon dioxide, and air respectively.

$$\begin{aligned} \frac{\alpha}{p} &= .0002667\left(\frac{X}{p} - 200\right)^{1.615}, \\ \frac{\alpha}{p} &= .0004029\left(\frac{X}{p} - 356\right)^{1.515}, \\ \frac{\alpha}{p} &= .0001594\left(\frac{X}{p} - 400\right)^{1.615}. \end{aligned} \tag{11}$$

From equation (10)

$$\alpha = ap\left(\frac{X}{p} - \frac{X_i}{p}\right)^b.$$

Substituting this value of α in the equation $n = n_0 e^{\alpha d}$ and taking the logarithm of both sides gives

$$\log_e \frac{n}{n_0} = apd\left(\frac{X}{p} - \frac{X_i}{p}\right)^b. \tag{12}$$

This equation with the values of the constants a, b, and X_i/p given in equation (11) was applied to all of the data on pure gases.

The observed values of n/n_0 (see Tables I., II., III.) were substituted in equation (12) and the values of the field strengths X' computed which ought to give that current according to the theory. These computed values of X are given in the fourth column of Tables I., II., III., under the heading X' to distinguish them from the applied field strengths given in the first columns of the same tables under the heading X. The observed and computed values are in nearly all cases within the experimental error, the mean per cent. of difference between them for any gas being less than 1.5 per cent.

APPLICATION TO MIXTURES.

In the following discussion, if we have m per cent. of one gas present in a mixture and $(100 - m)$ per cent. of another gas, we mean by this that the partial pressure of the first is m per cent. of the total pressure of the mixture and that therefore m per cent. of the molecules present are of the first gas and $(100 - m)$ per cent. are molecules of the second gas.

Let Q_1 be the cross section of the first kind of molecules.

Let Q_2 be the cross section of the second kind of molecules.

Let L_1 and L_2 be the mean free paths of the pure gases respectively.

Let the total cross section Q_1 of the first pure gas be represented by 100. Then if we take a mixture of gases containing m per cent. of the first kind and $(100 - m)$ per cent. of the second kind, the total cross section Q_m of the mixture for the same pressure will be

$$Q_m = mQ_1 + (100 - m)Q_2$$

or

$$Q_m = mQ_1 + (100 - m)Q_1\frac{L_1}{L_2}, \tag{13}$$

since from the kinetic theory $Q_1/Q_2 = L_2/L_1$.

Then if the electron is negligibly small and moves with a velocity of order higher than that of the molecules, we may consider the molecules at rest and take the mean free path of the electron in the mixture to be inversely proportional to the cross section Q_m of the mixture. Then if l_1 is the mean free path of the electron in the pure gas and l_m in the mixture we have

$$\frac{l_1}{l_m} = \frac{Q_m}{Q_1}. \tag{14}$$

For hydrogen and air $L_H/L_A = 1.88$. If we let $Q_H = 100$ represent the total cross section of pure hydrogen, Q_{25}, Q_{50}, and Q_{75} represent the total cross sections of mixtures of hydrogen and air containing 25, 50, and 75 per cent. of air respectively, and Q_A the total cross section of air, all for the same pressure, we have, applying equation (13)

$$\begin{aligned} Q_H &= 100, \\ Q_{25} &= 75 + (25 \times 1.88) = 122, \\ Q_{50} &= 50 + (50 \times 1.88) = 144, \\ Q_{75} &= 25 + (75 \times 1.88) = 166, \\ Q_A &= 188. \end{aligned} \tag{15}$$

From the above it is evident that in a mixture of 75 per cent., hydrogen and 25 per cent. air, for example, the mean free path of the electron would be 1.22 times as small in the mixture as in the pure hydrogen at the same pressure. Thus the mean free path of the electron in this mixture is the same as it would be in pure hydrogen at a pressure 1.22 times as great.

In general the electron will behave in ionizing the hydrogen in any mixture as though the pressures were Q_m/Q_H times p where Q_m is the cross section of all the molecules in any mixture and Q_H is the cross section that they would have were they all hydrogen molecules. This new value will be called the apparent pressure and will be denoted by P_1 or

$$P_1 = \frac{Q_m}{Q_H} p. \tag{16}$$

Similarly the electron will behave in ionizing the air in the mixture as though the pressure were

$$P_2 = \frac{Q_m}{Q_A} p. \tag{17}$$

Since there are m per cent. of the hydrogen molecules present at an apparent pressure P_1 and $(100 - m)$ per cent. of the air molecules at an apparent pressure P_2, then equation (12) for a pure gas will become for a mixture of two gases

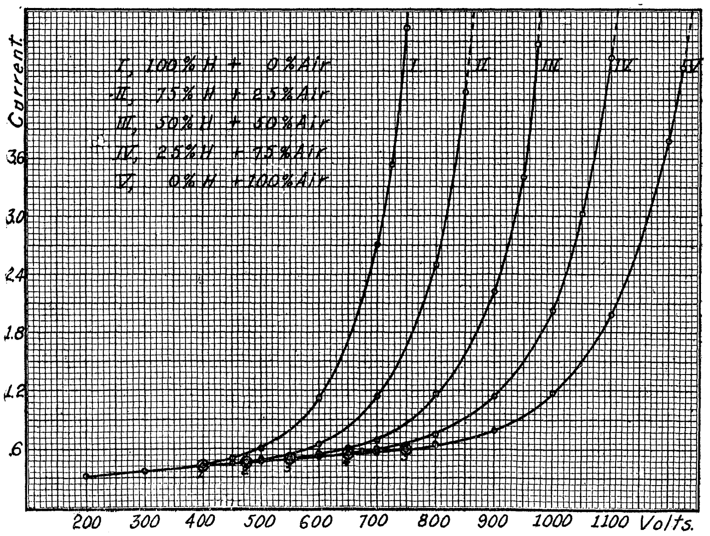


Fig. 15.

$$\log_e \frac{n}{n_0} = m a_1 P_1 d \left(\frac{X}{P_1} - \frac{X_{i_1}}{p}\right)^{b_1} + (100 - m) a_2 P_2 d \left(\frac{X}{P_2} - \frac{X_{i_2}}{p}\right)^{b_2}, \tag{18}$$

where the subscripts refer to the first and second gases respectively. In first term of the right-hand member of equation (18) represents the part of the current due to the ionization of the first gas and the second term that due to the second gas.

For the ionization curves for mixtures of air and hydrogen (See Fig. 15) equation (18) becomes for the different mixtures

$$\begin{aligned}
&\text{I.}\ \log_e \frac{n}{n_0} = .0002667 \times 5 \times .3875\left(\frac{X}{5} - 200\right)^{1.615}, \\
&\text{II.}\ \log_e \frac{n}{n_0} = .75 \times .0002667 \times 6.1 \times .3875\left(\frac{X}{6.1} - 200\right)^{1.615}, \\
&\qquad + .25 \times .0001594 \times 3.245 \times .3875\left(\frac{X}{3.245} - 400\right)^{1.615}, \\
&\text{III.}\ \log_e \frac{n}{n_0} = .50 \times .0002667 \times 7.2 \times .3875\left(\frac{X}{7.2} - 200\right)^{1.615}, \\
&\qquad + .50 \times .0001594 \times 3.83 \times .3875\left(\frac{X}{3.83} - 400\right)^{1.615}, \qquad (19) \\
&\text{IV.}\ \log_e \frac{n}{n_0} = .25 \times .0002667 \times 8.3 \times .3875\left(\frac{X}{8.3} - 200\right)^{1.615}, \\
&\qquad + .75 \times .0001594 \times 4.415 \times .3875\left(\frac{X}{4.415} - 400\right)^{1.615}, \\
&\text{V.}\ \log_e \frac{n}{n_0} = .0001594 \times 5 \times .3875\left(\frac{X}{5} - 400\right)^{1.615}.
\end{aligned}$$

To apply the formulæ of (19) the values of the field strengths X' given in the third column of Table VIII. were substituted in the equations and there resulted the values of n/n_0 given in the second column of that table. From the curves (Fig. 15) it was possible to determine what experimental value of X would be required to give the same value of n/n_0, *i. e.*, the same ionization current. These experimental values are given in the first column of Table VIII. headed X. The actual pressure in these mixtures was 5 cm. and the distance between the plates A and B in all cases was .3875 cm. The agreement between the experimental values X and the computed values X' of the field strengths required to produce a given current in the mixtures in all cases was within the experimental error.

It will be noticed that the curves on mixtures of air and hydrogen, II., III., and IV., Fig. 15, are spaced between the curves for the pure gases, I. and V., almost proportionately to the percentage of the mixture. This is what might have been expected since the ionizing energy for air is only six per cent. more than that for hydrogen, so that in mixing the two gases the main effect was to decrease the mean free path of the electron as we increase the content of the air in the mixture, thus increasing the field strength required to produce a given ionization current in approximately the same proportion that the mean free path of the electron has been decreased. The above proportion would be exactly true were the ionizing energies for air and hydrogen the same.

Table VIII.

Mixtures of Air and Hydrogen.

I. 100 % H, $n_0=.420$.			IV. 25 % H, 75 % Air, $n_0=.548$.		
X	n/n_0	X'	X	n/n_0	X'
1,128	1.15	1,161	1,735	1.03	1,807
1,290	1.44	1,290	2,034	1.34	2,064
1,564	2.77	1,548	2,334	2.13	2,323
1,817	6.68	1,807	2,600	3.92	2,581
1,858	8.18	1,871	2,716	5.57	2,710
1,923	11.10	1,935	2,830	8.16	2,839
II. 75 % H, 25 % Air, $n_0=.458$.			V. 100 % Air, $n_0=.600$.		
1,388	1.16	1,419	2,000	1.02	2,064
1,548	1.42	1,548	2,307	1.30	2,323
1,688	1.85	1,677	2,581	1.95	2,581
1,814	2.53	1,807	2,848	3.35	2,839
1,941	3.64	1,935	3,107	6.46	3,097
2,070	5.46	2,064	3,166	7.65	3,161
2,173	8.55	2,194			
III. 50 % H, 50 % Air, $n_0=.496$.					
1,475	1.03	1,548			
1,798	1.39	1,807			
2,059	2.32	2,064			
2,335	4.60	2,323			
2,452	6.83	2,452			

Consequently it was thought desirable to further test the theory by using mixtures of gases which varied largely in their ionizing energies, and for this purpose mixtures of hydrogen and carbon dioxide were chosen, the ionizing energies of these gases being 1.58×10^{-11} ergs and 1.01×10^{-11} ergs respectively.

Application to Mixtures of Hydrogen and Carbon Dioxide.

For hydrogen and carbon dioxide $L_H/L_{CO_2} = 2.83$. If we let $Q_H = 100$ represent the cross section of the hydrogen, Q_{25}, Q_{50}, Q_{75}, represent the cross sections of mixtures of hydrogen and carbon dioxide containing 25, 50, and 75 per cent. of carbon dioxide respectively, and Q_{CO_2} the cross section of the carbon dioxide, then applying equation (13) we have

$$\begin{aligned} Q_H &= 100, \\ Q_{25} &= 75 + (25 \times 2.83) = 145.75, \\ Q_{50} &= 50 + (50 \times 2.83) = 191.5, \\ Q_{75} &= 25 + (75 \times 2.83) = 237.25, \\ Q_{CO_2} &= 283. \end{aligned} \tag{20}$$

If then we apply equations (16) and (17) to those in (20) we shall obtain the apparent pressures for each of the gases hydrogen and carbon dioxide in the mixture. Substituting these values of P_1 and P_2, the values of a_1, a_2, b_1, b_2, from equation (11), the values of m and $(100 - m)$ given in Fig. 16, the values of X_{i_1}/p and X_{i_2}/p for hydrogen and carbon dioxide respectively, and the value of d, all in the general equation (18) and we obtain the following five equations for the curves of Fig. 16.

$$\begin{aligned} \text{I.}\ \log_e \frac{n}{n_0} &= .0002667 \times 5 \times .4 \left(\frac{X}{5} - 200\right)^{1.615}, \\ \text{II.}\ \log_e \frac{n}{n_0} &= .75 \times .0002667 \times 7.288 \times .4 \left(\frac{X}{7.288} - 200\right)^{1.615}, \\ &\quad + .25 \times .0004029 \times 2.575 \times .4 \left(\frac{X}{2.575} - 356\right)^{1.515}, \\ \text{III.}\ \log_e \frac{n}{n_0} &= .50 \times .0002667 \times 9.575 \times .4 \left(\frac{X}{9.575} - 200\right)^{1.615}, \\ &\quad + .50 \times .0004029 \times 3.383 \times .4 \left(\frac{X}{3.383} - 356\right)^{1.515}, \\ \text{IV.}\ \log_e \frac{n}{n_0} &= .25 \times .0002667 \times 11.86 \times .4 \left(\frac{X}{11.86} - 200\right)^{1.615}, \\ &\quad + .75 \times .0004029 \times 4.192 \times .4 \left(\frac{X}{4.192} - 356\right)^{1.515}, \\ \text{V.}\ \log_e \frac{n}{n_0} &= .0004029 \times 5 \times .4 \left(\frac{X}{5} - 356\right)^{1.515}. \end{aligned} \tag{21}$$

The values of the field strengths given in the last column, Table IX., headed X' were then substituted in the equations of (21) and the corresponding values of n/n_0 computed. Then from the curves of Fig. 16 the experimental values of X which would give the same values of n/n_0 were determined and are given in the first column of Table IX., headed X.

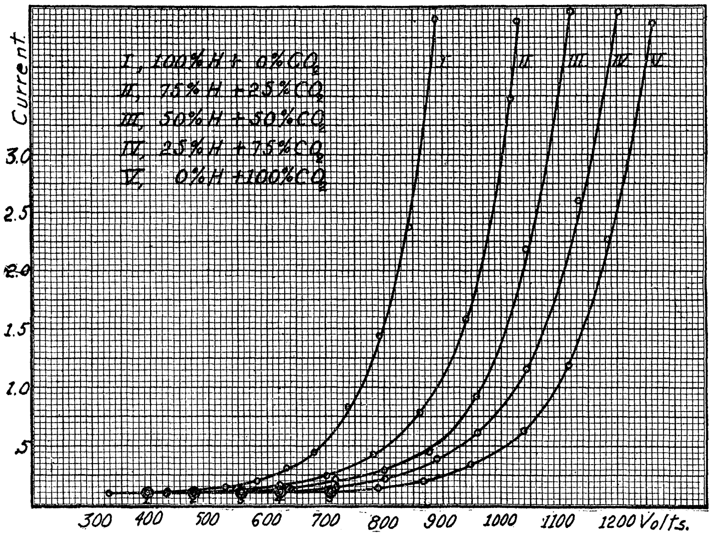


Fig. 16.

TABLE IX.

Mixtures of Hydrogen and Carbon Dioxide.

I. 100% H, $n_0 = .095$.

X	n/n_0	X'
1,512	2.47	1,500
1,730	5.72	1,750
1,875	9.36	1,875
1,995	16	2,000
2,133	28.6	2,125

II. 75% H, 25% CO_2, $n_0 = .097$.

X	n/n_0	X'
1,600	1.83	1,625
1,720	2.31	1,750
1,990	4.92	2,000
2,272	11.9	2,250
2,530	33.4	2,500

III. 50% H, 50% CO_2, $n_0 = .099$.

X	n/n_0	X'
1,737	1.83	1,750
2,000	2.96	2,000
2,265	5.89	2,250
2,490	13.5	2,500
2,765	37.1	2,750

IV. 25% H, 75% CO_2 $n_0 = .101$.

X	n/n_0	X'
1,990	2.07	2,000
2,225	3.79	2,250
2,482	7.74	2,500
2,763	19.2	2,750
3,012	42.7	2,950

V. 100% CO_2, $n_0 = .103$.

X	n/n_0	X'
2,225	2.20	2,250
2,487	4.49	2,500
2,763	10.5	2,750
3,043	28.0	3,000
3,140	38.6	3,075

The average per cent. of difference between the experimental values X and the computed values X' for the mixtures of hydrogen and carbon dioxide in Table IX. is .9 per cent.; for the mixtures of air and hydrogen in Table VIII. it is also .9 per cent., and for the pure gases, hydrogen, air, and carbon dioxide, in Tables VIII. and IX. it is .9 per cent., 1.5 per cent., and 1.1 per cent., respectively. From this it is seen that the theory has been checked with the same degree of accuracy in mixtures of gases as in pure gases, and that in practically all cases the computed and observed field strengths do not differ by more than the experimental error.

Summmary and Conclusions.

1. The theory of ionization by collision has been verified for pressures as high as 40 centimeters.

2. The minimum ionizing energy for any gas is constant, being independent of the pressure and field strength so long as the latter is sufficiently high to produce new ions by collision.

3. These ionizing energies were here found to be 9.66, 6.21, and 10.21 volts, for hydrogen, carbon dioxide, and air, respectively.

4. The minimum ionizing field strength is directly proportional to the pressure, or X_i/p = cst, for pressures at least above one centimeter.

5. The relative ionizing energy of any two gases may be determined by

$$\frac{E_1}{E_2} = \frac{X_{i_1}}{X_{i_2}} \cdot \frac{L_1}{L_2}, \tag{1}$$

where E, X, and L are the ionizing energy, minimum ionizing field strength, and mean free path of the two gases. (2) Or by the second equation

$$\frac{E_1}{E_2} = \frac{V_1}{V_2},$$

where V_1 and V_2 are the potentials applied to plate A, when the pressures of the two gases are in the ratio

$$\frac{p_1}{p_2} = \frac{L_1}{L_2},$$

and n_0, α, l, and d are the same for both gases.

6. The general function

$$\frac{\alpha}{p} = f\left(\frac{X}{p}\right)$$

has been reduced to the special one

$$\frac{\alpha}{p} = a\left(\frac{X}{p} - \frac{X_i}{p}\right)^b.$$

This, combined with the equation $n = n_0 e^{ad}$ gives

$$\log_e \frac{n}{n_0} = apd\left(\frac{X}{p} - \frac{X_i}{p}\right)^b .$$

The second of these equations enables us to determine α for any field strength and pressure, and the third one enables us to compute the field strength required to produce a given value of n/n_0 for any pressure, and distance between plates, within the range of these experiments.

7. By combining the latter expression for two pure gases the value of n/n_0 for any field strength, pressure, and mixture can be computed by a formula of the general form

$$\log_e \frac{n}{n_0} = ma_1P_1d\left(\frac{X}{P_1} - \frac{X_{i_1}}{p}\right)^{b_1} + (100 - m)a_2P_2d\left(\frac{X}{P_2} - \frac{X_{i_2}}{p}\right)^{b_2} .$$

In conclusion the writer wishes to thank Professor Michelson and the staff of the Physics Department for their continued interest in this work, Mr. Fred Pearson for his kind assistance in setting up the apparatus, and especially Professor Millikan for suggesting the problem and for his valuable help and encouragement.

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