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THE TERMINAL VELOCITY OF FALL OF SMALL SPHERES IN AIR AT REDUCED PRESSURES.

BY L. W. MCKEEHAN.

§ I. INTRODUCTION.

THE terminal velocity of a freely falling sphere, when all other resistances are negligible compared to that due to the viscosity of the fluid through which the sphere falls, and when no slipping is supposed to occur at the surface of separation, is, as derived by Stokes,[1]

$$V = \frac{\frac{2}{9}ga^2(\sigma - \rho)}{\mu}.$$

In this expression μ is the coefficient of viscosity of the fluid, a the radius of the sphere, V the velocity of fall, g the acceleration of gravity, σ the density of the sphere, and ρ that of the fluid. The exclusion of all but viscous resistance restricts the applicability of the formula to very minute spheres, the condition to be fulfilled being that the radius of the sphere must be small compared to $\mu/V\rho$.

Experimental work at atmospheric pressure by Professor John Zeleny and the writer[2] has shown this formula to hold for spheres of wax ranging in radius from .002 cm. to .00035 cm., spheres of paraffin from .002 cm. to .0005 cm., and spheres of mercury from .001 cm. to .00016 cm., although the measurements on the last named are less accurate than for the other materials, owing to the high reflecting power of their surfaces.

In earlier experiments[3] using natural spores as approximating small spheres, large deviations from the formula given by Stokes were observed, all the spores going too slowly to agree with the formula. It was sug-

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[1] G. G. Stokes, Mathematical and Physical Papers, Vol. III., p. 59.

[2] PHYS. REV., Vol. XXX., p. 535; Phys. Zeitschr., Bd. 11, s. 78.

[3] Loc. cit.

gested by Sir Joseph Larmor at the Winnipeg Meeting of the British Association for the Advancement of Science, August, 1909, that experiments at lower pressures would serve to decide whether or not these deviations were due to the formation of eddies in the air near the not perfectly spherical spores. The force required to maintain these eddies would decrease with the density of the gas, whereas the purely viscous resistance should remain constant, since μ is independent of pressure, at least throughout a very great range.[1] If the large deviations observed with spores were due to this cause, the terminal velocity of fall at a lower pressure should be larger than at atmospheric pressure, and should tend to agree more closely with that given by the formula of Stokes.

Some experiments at low pressures, using lycopodium spores, were performed. These showed, indeed, an increase in terminal velocity, but one of far too great an amount to be due to the suspected cause, since the velocity became at low pressures several times as great as that calculated by the formula. Similar experiments were then performed with perfect spheres of wax, which satisfy Stokes's formula at atmospheric pressure. These showed likewise a large increase in terminal velocity with reduction of pressure, the data so obtained forming the experimental part of the paper here presented. This failure of Stokes's formula at low pressures is traceable to the omission of slip in his derivation. Such slipping of the gas along the surface of the sphere is easily shown to be inseparable from motion in a discontinuous medium, and becomes more important with reduction of pressure as the mean distance between the molecules increases.

The necessity of slip can be seen from the following considerations. The surface layer of molecules in a gas which is moving along a solid surface may be divided into two groups. The first group consists of those which have not yet struck the surface. Their mean velocity will have the component parallel to the surface of the more distant layer of gas from which they come. The second group consists of those molecules which have already struck. Their mean velocity will have a component parallel to the surface smaller than that of the first group, but greater than zero unless the impacts are entirely inelastic. In any case the mean velocity of both groups taken together, which is that of the gas layer as a whole, will have a tangential component, or, in other words, there will be slip between the gas and the solid surface.

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[1] Weinstein, Thermodynamik und Kinetik der Körper, Bd. I., s. 325.

§ II. THEORY.

A theoretical discussion of the case was given by Professor E. Cunningham[1] in a paper "On the Velocity of Steady Fall of Spherical Particles through Fluid Medium," which was communicated to the Royal Society of London in January, 1910, by Sir Joseph Larmor. He obtains as the general expression for the terminal velocity of fall of a small sphere in a gas

$$V = V_s\left(1 + A\frac{l}{a}\right),$$

in which V_s is the velocity given by Stokes's formula, l is the mean free path of the gas molecule, a the radius of the sphere, and A a constant whose numerical value depends on the assumption which is made regarding the impacts of the gas molecules on the surface of the sphere. An examination of the results there obtained has revealed some inaccuracies. When these are corrected different values of A than those he obtained are derived.

Suppose in the first instance that the collisions are of the nature of impacts of smooth elastic spheres; the value of A is then 1.5.[2] Suppose next that each impinging molecule enters the surface layer of the particle (whether the solid material or a layer of condensed gas is for the present purpose immaterial), and emerges again from the same area on which it impinged, but with a velocity independent in direction of the relative velocity of the sphere and the molecule before the collision, although of the same mean squared value; the value of A is then 1.2. Suppose that a fraction f of the impacts are of the former type and the rest of the latter; the value of A is then $1.5\frac{4}{5-f}$. These cover the cases treated by Cunningham, who obtained in the combined case $A = 1.63\frac{1}{2-f}$.

As an additional and limiting case suppose that after striking the surface of a particle a molecule of the gas is only able to leave it again when moving in the direction of the normal to the surface; the value of A is then 1.05.

Since the mean free path l of the gas molecule is inversely proportional to the pressure p of the gas, an alternative formula

$$V = V_s(1 + B/ap)$$

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[1] Proc. Roy. Soc., Vol. 83 A, p. 357, February, 1910.

[2] One of the substitutions was made incorrectly in a note published in the PHYSICAL REVIEW, Vol. XXXII., p. 341, March, 1911, so that the numerical value of the constant multiplier of l/a is there given wrongly. This mistake was pointed out by Professor R. A. Millikan, who kindly read a part of the analysis leading to the results here given.

can be used, and is more convenient for experimental verification. Calculating the value of B from that of A, using $\mu = 1833 \times 10^{-7}$[1] at 20° C., we get when p is expressed in millimeters of mercury, for

$A = 1.0$	$B = .0075$
1.05	.0078
1.2	.0090
1.5	.0112

The terminal velocity of fall of small spheres should thus increase enormously at very low pressures, and should then become proportional to the radius of the sphere, and not to the square of the radius as at higher pressures. It should also be inversely proportional to the pressure. In Figs. 1 and 2 are plotted as abscissa the reciprocal of the computed velocity, that is, the time required to fall one centimeter at this velocity, and as ordinate the pressure in mm. of mercury. The temperature assumed is 20° C., and the value of the constant chosen is $A = 1.0$, since, as shown later, this is the value obtained by experiment. Six curves are drawn in Fig. 1 for different values of the radius, the value for each curve being given in cm. $\times 10^{-4}$ in the upper margin. Fig. 2 shows on a much larger scale the part near the origin omitted from Fig. 1. The close approach to a linear variation of time of fall with pressure should here be noted. It is found upon computation that the criterion for steady motion given above is fulfilled by all the sizes shown at all the pressures.

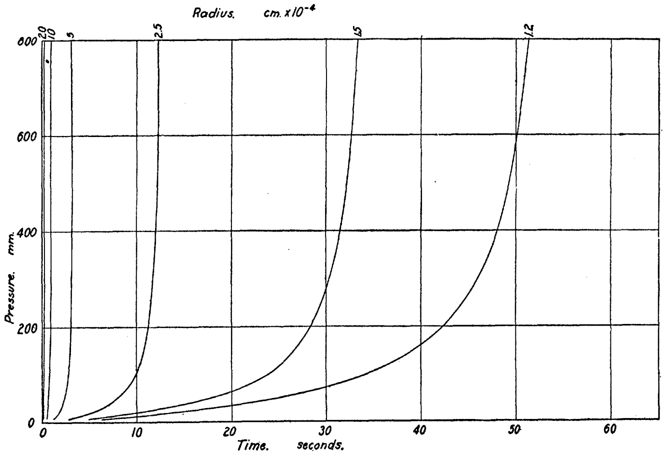

Fig. 1.

The assumption is made that μ has the same value at all pressures. This is required by the kinetic theory of gases and it seems likely that

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[1] R. A. Millikan, Phil. Mag., VI., Vol. 19, p. 215, February, 1910, $\mu = 1863 \times 10^{-7}$ at 26° C.

the values of μ obtained experimentally at low pressures are somewhat lower than at atmospheric pressure only because slip is neglected. For this reason the pressure at which μ shows a marked decrease depends on the method used in determining it, and upon the dimensions of the apparatus employed.

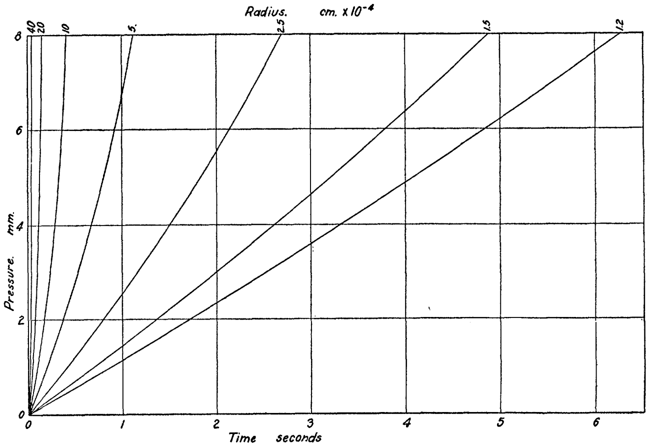

Fig. 2.

§ III. Experiments.

1. *Preparation of Material.*—Minute spheres of wax were made by an atomizer,[1] and were collected by allowing the cloud of spheres so formed to settle on sheets of paper in a closed space about 40 × 40 × 60 cm. The range of sizes obtained could be varied within certain limits by regulating the size and openings of the atomizer, and by varying the temperature of the wax.

2. *Measurements.*—In order to find the value of B in the formula

$$V = \frac{2}{9} \frac{ga^2(\sigma - \rho)}{\mu} \left(1 + \frac{B}{ap}\right),$$

the values of σ, the density of a sphere, a its radius, V its terminal velocity of fall, and p the pressure of the gas in millimeters of mercury, were determined for over six hundred spheres of wax.

3. *Density.*—The density σ of the wax was 1.058, as previously determined.[2]

4. *Terminal Velocity of Fall.*—The method of getting the terminal velocity of fall V was the same as that used in the previous experiments,

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[1] The same method was used in the earlier experiments cited. Microphotographs of these perfect spheres are there shown.

[2] Loc. cit.

and consisted in measuring the total time required to fall a known distance of about 31 cm.

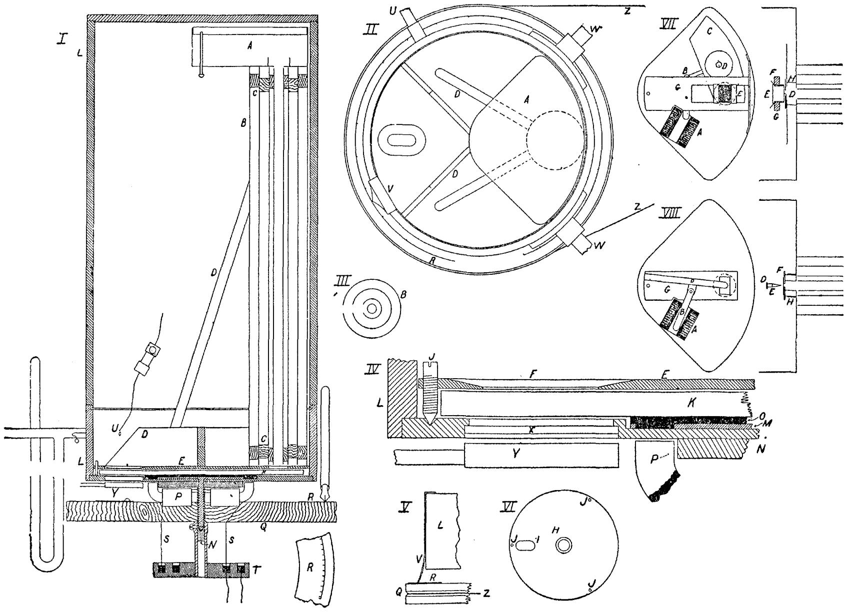

Fig. 3.

Referring to Fig. 3: drawing *I* is a vertical section through the apparatus; *II* is a plan of the same with the cover removed; *III* is a horizontal section through the tube in which the spheres fell; *IV* is an enlarged view of the lower left-hand corner of *I*; *V* and *VI* are details of parts not shown elsewhere; *VII* and *VIII* are plans and sections of two mechanisms used for releasing spheres. The lettering of corresponding parts is the same in the first six drawings.

The spheres released in the fall box *A* fell through the fall tube *B* to the revolving disc *K*. In releasing spheres by the method shown in drawing *VII*, a quantity of them several mm^3. in volume was placed in the cloth-bottomed trough *E*, which slipped tightly into a slot in the frame *G*. A solenoid *A* drew in the soft-iron armature *B*, and by it moved the sliding plate *C* across the short open tube *H* soldered to the bottom of the fall box just under *E*. The sliding plate carried a flexible pointed spring *D* projecting above a small hole in its center. The motion of *C* was stopped by a peg, not shown, when *D* reached the middle of the bottom of the trough *E*, against which it scratched lightly. The vibration produced by this scratch sifted some spheres through the cloth and they then fell through the small hole in *C*, through *H*, and into the fall tube *B*.

In the method shown in drawing *VIII* the spheres were left on the paper upon which they had been deposited after atomizing. A piece of this was put at *F*, with the sphere-coated side underneath, just over a small hole in the frame *G*, which fit tightly in the tube *H*. The solenoid *A*, by means of the armature *B* drew a brass block *C* from beneath a long metal strip *D*, which sprung downward, rapping the point *E* against the upper surface of the paper just over the hole in *G*. The blow was not sufficient to puncture the paper, but it set some of the spheres free and they then fell through *H* into the fall tube.

In the fall tube *B* the spheres, released by either of the methods described, fell through the innermost of four concentric brass tubes, designed to prevent air currents. The outer tubes were pierced at one point, as shown in drawing *III*, to facilitate changes of pressure in the spaces between the wooden rings placed near the ends. The three inner tubes were entirely supported by these rings and were not in contact with any other metal parts. The apparatus was used in a room with double walls and no windows, which provided an excellently steady temperature.

The outer tube was soldered at the top of the fall box *A*, and at the bottom to braces *D*, *D*, and to the brass plate *E*. This plate was supported on three leveling screws *J* (drawing *IV*), one of which rested in a socket, fixing the relative angular position of the fall tube and the outer case *L*. The plate was pierced by two holes, one a narrow radial slot just under the fall tube, and the other a window, 180° from the slot, for viewing the spheres after they had fallen. A plate *H* (drawing *VI*) similar to *E*, but without any fall tube, was substituted for it when measuring the spheres. This provided more room for the measuring microscope.

The surface for receiving the spheres was that of a plate glass disc *K* cemented to a circular brass plate *M*, and rotated under the plate *E* on a pivot in the bearing *N*. On a diameter of the plate *M* was imbedded a soft-iron armature *O* with thickened ends. The poles of this armature were directly over the poles of an electromagnet *P* carried on a wooden wheel *Q*, which was rotated about *N* by a chronograph motor through a belt *Z*. The current for *P* was supplied through wires *S*, *S*, dipping into circular troughs of mercury in the ebonite block *T*. By this device the bearing for the plate *K* was nowhere exposed to atmospheric pressure and required no packing.

At the instant of releasing the spheres the wheel *Q* was set in steady rotation by putting it in gear with the chronograph motor, and as the spheres arrived successively at the bottom of the fall tube they were received at different angular positions on the surface of the plate K. A chronograph pen connected in series with a seconds pendulum, rested on a paper strip R attached to Q, and marked each second as the wheel rotated. This time record is shown in plan just to the right of T, and also in drawing II for a different speed of rotation. After completing the experiment and stopping the rotation of Q, an index V, which fit over the edge of the case L was placed exactly 180° from the place the pen had ceased to mark. When this index pointed to any mark on the pen record as Q was rotated, the spheres which had fallen on K just as that mark was being made would be 180° from the bottom of the fall tube; that is they would be in a position to be seen through the window at F. By counting seconds from the beginning of the pen record to the index the elapsed time of fall was given directly.

This is a longer time than that which would be required if the sphere had started to fall with its terminal velocity, instead of starting from rest. The correction to be applied can easily be found. The following table gives its value for the apparatus used.

Observed Time. Seconds.	Correction. Seconds.
.50	−.06
1.00	−.03
2.00	−.02
3.00	−.01
6.00	−.01
7.00	0

These corrections have been applied to all times of fall less than seven seconds.

5. *Radius of Spheres.*—The radius a of each sphere was determined by measuring its diameter from two to eight times, the majority of the spheres being measured six times. An improved microscope plate micrometer[1] was used except in experiments II., 1, 2 and 3, and for fourteen spheres in other experiments which were too large to be measured by the plate micrometer used. These excepted spheres were measured by an ocular micrometer. The micrometer field was illuminated by diffused light reflected from the mirror Y (drawing IV, Fig. 3), through the window X and disc K. Test measurements using different magnifying powers gave concordant results for all the sizes used, and it appears that the measurements of radii, even for the smallest spheres, are not affected by constant errors due to diffraction, of an amount exceeding a small fraction of a wave-length of light. Some difficulty in the measurement of the smaller spheres was experienced on account of tremors of the

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[1] John Zeleny and L. W. McKeehan, PHYS. REV., Vol. XXXII., p. 530.

pier on which the apparatus stood. The trouble was eliminated by measuring diameters after midnight, when heavy traffic on the streets had ceased. The method of making exterior contact of the cross-hair and the circular image of a sphere was found the most convenient for rapid measurement. A correction for the width of the cross-hair had therefore to be applied to all diameters measured. The amount of this correction was determined by using other forms of contact for comparison. with other objects than spheres.

6. *Pressure.*—The pressure inside the case L, which was of brass about 5 mm. thick, was reduced by a water pump or by a Gaede mercury pump, depending upon the reduction desired. The pumps were attached at U (Fig. 3). The window X was of plate glass set in paraffin which made a permanently air-tight joint. The cover of the case L was ground with emery to fit against the lower part, and a thin coating of vacuum wax between the surfaces gave an excellent joint. A platinum wire for the electrical connection to the solenoid in the fall box was sealed through the glass manometer tube outside the case, the return circuit being through the metal parts of the apparatus and its supports W, W.

Pressure readings were taken by means of a cathetometer on a closed-arm mercury manometer, and when the pressure was low enough it was also measured by a McLeod gauge with a small factor. Glass stopcocks served to isolate the apparatus from the pumps. The apparatus was left to itself for at least three hours just preceding the releasing of the spheres, so that the pressure and temperature of all its parts was steady. A dish containing P_2O_5 was placed inside the apparatus before reducing the pressure. The drying agent was considered necessary, although experiments at atmospheric pressure showed the difference in viscosity due to a small amount of water vapor was not appreciable. At low pressures the same amount of vapor, forming a larger fraction of the whole gas, might change the viscosity considerably, since its viscosity is much less than that of air.

7. *Method of Finding and Measuring Spheres.*—In finding and measuring the spheres the microscope was placed in guides which permitted it to move only along a radius of the plate K, 180° from that passing under the fall tube. The distance from the center of the plate was controlled by a slow motion screw. With the microscope focused on the surface of K, the wheel Q and plate K were very slowly turned by hand through suitable gearing, and the spheres were measured as found. Each sphere was placed near the center of the field by moving the plate or the microscope, and the current for the magnet P was interrupted during the measurements in order to prevent creeping of the plate K. After sur-

veying the whole of one strip or zone the microscope was moved radially the width of the field, and the process was repeated until enough spheres had been measured to show the variation of time of fall with radius, or until no more could be found. The radii for an experiment were not computed from the plate micrometer readings until all had been measured. The total number of spheres that fell to the plate was not greater than three hundred in any one experiment, so that no effect on the velocity due to mutual action between the spheres occurred.[1]

§ IV. RESULTS OF EXPERIMENTS.

In the tables and curves *I* to *XVI*[2] are given the results of the experiments, arranged in order of decreasing pressure. The quantities tabulated and plotted are the radii in cm. $\times 10^{-4}$, and the reciprocals of the terminal velocities, that is the times in seconds required to fall one centimeter at this velocity. On the curves the radii are ordinates and the times abscissae. A small circle is drawn for each sphere measured.

The method of release shown in drawing *VIII*, Fig. 3, was found the most satisfactory of about seven methods, probably because the paper used closed the top of the fall tube throughout the experiment, and so no air currents set up by the movement of parts of the mechanism in the fall box could disturb the air in the tube. Large spheres did not, however, adhere to the paper and were not obtained by this method.

The results at atmospheric pressure using this method are given in Table and curve *I*, except that the spheres for which the radius exceeds .0013 cm. were obtained by the method shown in drawing *VII*, Fig. 3. The results at atmospheric pressure using other methods of release than the one shown in drawing *VIII* are contained in table and curve *II*. In neither of these tables are any spheres given which fell in the first twentieth of the period of a complete rotation of the plate *K*. The largest spheres are from experiments where the time of a rotation was 20 seconds, the smaller ones from experiments where the time was from 200 to 1200 seconds. The error in time measurements is thus kept small throughout the whole range.

The range of sizes used in all the experiments was from $a = .0025$ cm. to $a = .00012$ cm., and the range of pressures from atmospheric pressure (740 mm.) down to .32 mm. of mercury. The total time of fall for the various sizes and pressures varied from .74 sec. to 1,072 sec.

The value of the constant B in the formula was computed for each sphere in experiments VI., X., XIII. and XV., at pressures of 8.28, 2.11,

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[1] Cunningham, loc. cit.

[2] Some of these are omitted here for the sake of brevity.

0.69 and 0.34 mm. of mercury respectively. The average values of B for the four experiments were .00724, .00724, .00826 and .00729, giving a mean value

$$B = .0075 \pm 2,$$

corresponding to

$$A = 1.00 \pm 3.$$

The close agreement of the first, second, and fourth values is accidental, as seen by considering the probable errors in the experiments. The probable errors in the time of fall are large for the largest spheres, and the probable errors in the radius are several per cent. for the smallest spheres. The probable error in the average value of B for an experiment also depends upon that in the pressure, since this is not reduced in taking the average. The errors in pressure for these four experiments are different, since the manometer was used for the first and second, the McLeod gauge for the third and fourth. If the four values are weighted with reference to these probable errors, the weighted mean is found to be a little higher than the unweighted mean given above. These pressures were chosen for computing B because the change in the time of fall from that found for atmospheric pressure is large, and yet the total time of fall is not too small for accurate measurement, as it would be at still lower pressures.

Values of the time per centimeter were computed using the value of A corresponding to the assumption of normal emergence of the impinging molecules, $A = 1.05$, for each pressure used, and for the following values of the radius:

.004
.0035
.003
.0025
.002
.0017
.0014
.0012
.001
.0008
.0006
.0005
.0004
.0003
.00025
.0002
.00018
.00015
.00014
.00013
.00012

These times are plotted on curves *I*, and *III* to *XVI*, Figs. 5 to 9, as small crosses, and it will be seen that the circles representing observed spheres agree well with a line passing through these crosses.

An idea of the great amount of variation in time of fall will be obtained by referring to curve *I*, Fig. 5. The black dot near the left of the figure represents the time of falling one centimeter for a sphere of radius .00014 cm., at a pressure of 0.32 mm. of mercury. At atmospheric pressure this sphere would fall in a time shown by the last cross at the right of the figure. The value of the term B/ap in the formula varies from .003 for $a = .004$ cm., $p = 740$ mm., to 196 for $a = .00012$ cm., $p = .0.32$ mm. The agreement of the experiments with the formula throughout the entire range is excellent.

The differences in computed velocities produced by using one or another value of A can be seen by referring to Fig. 4. Three curves are drawn, showing the variation of the time required to fall one centimeter (abscissæ) with the variation of the radius in cm. × 10^{-4} (ordinates), at a pressure of 0.32 mm. of mercury, and at a temperature of 20° C. The curve at the left, and the one connected to it by horizontal lines are drawn for $A = 1.5$ and $A = 1.2$, respectively, and correspond to the limiting values possible under Cunningham's assumptions. The curve at the right is drawn for $A = 1.05$, the value corresponding to the assumption of normal emergence of the impinging molecules. Curve XVI, Fig. 9, which shows the experimental results at this pressure, agrees almost exactly with this last curve.

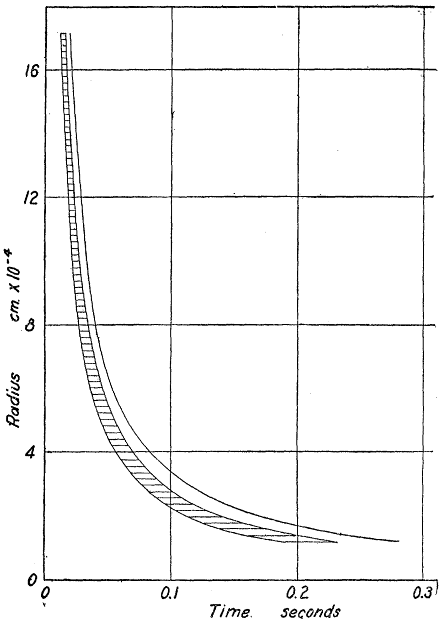

Fig. 4.

At atmospheric pressure the differences are not detectable by the method used, for the range of sizes obtained. This will be seen by comparing curves *I* and *II*, Figs. 5 and 6. The crosses in *II* represent times computed by assuming $A = 0$ (Stokes's formula), those in *I* times computed by assuming $A = 1.05$. The difference amounts to 7 per cent. in time for the smallest sphere shown, being less for all larger spheres, and the additional difference in case $A = 1.5$ would be about 3 per cent., again for the smallest sphere shown. An error of 1.5 per cent. in the measurement of the radius of such a sphere would entirely mask this additional difference in time of fall.

The only other experimental value of the constant in Cunningham's formula is given by Professor R. A. Millikan,[1] who found A to be 0.815. This value is determined from experiments on spheres of different sizes, but all at one pressure, namely atmospheric. The method is indirect in that the radii of the spheres were not measured but were computed from their time of fall on the assumption of uniform electrical charge for all the spheres. The material used was a liquid, and internal eddies in the drops may cause a consumption of energy, and consequently reduce the apparent value of A.

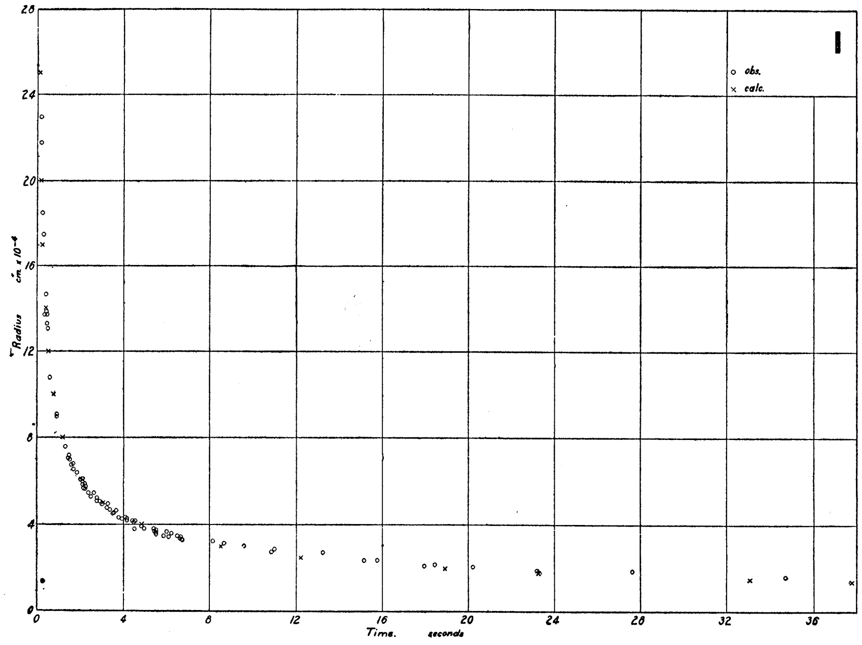

Fig. 5.

Atmospheric pressure.

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[1] Science, Vol. XXXII., p. 446.

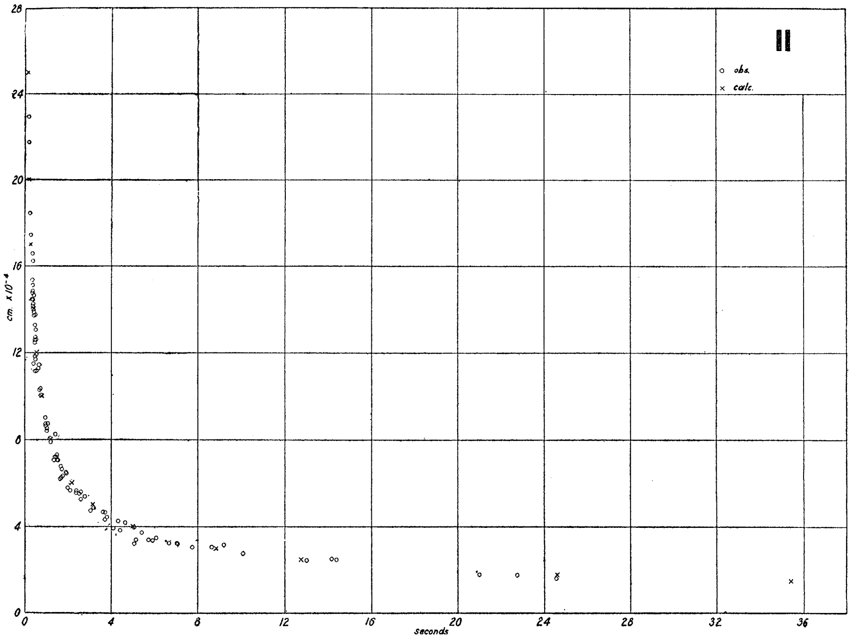

Fig. 6.

Atmospheric pressure.

§ V. Spores.

Lycopodium spores, which fell at atmospheric pressure only about half as fast as Stokes's formula required, were found at lower pressures to maintain the same ratio of observed velocities to velocities computed

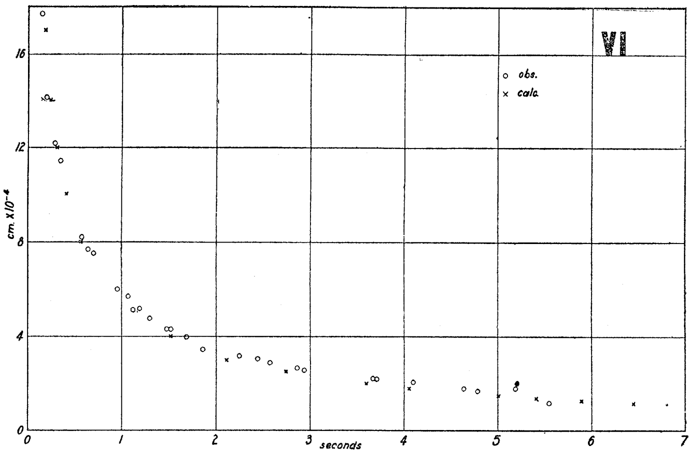

Fig. 7.

Pressure = 8.28 mm.

by the formula which held at all pressures for wax spheres. This means that the density, which is the only measured quantity appearing in the formula in the first degree, was very incorrect. The density determined by refined volumenometer measurements was 1.175. This may have been the density of a solid shell surrounding an air space or the density of a spongy mass containing many air spaces. The average density, meaning by this the quotient of the mass of a spore by the volume of a sphere having its mean radius, would have to be about 0.6 to make the spore behave as it does. In any event, turbulent motion of the air can no longer be held responsible for the variations observed at atmospheric pressure. Experiments with the other spores used before were not attempted, because the condition of the material appeared to have changed with time.

§ VI. Summary and Conclusion.

The formula

$$V = \frac{2}{9}\frac{ga^2(\sigma - \rho)}{\mu}\left(1 + A\frac{l}{a}\right),$$

in which A is a constant, expresses the terminal velocity of fall of small solid spheres in air throughout a wide range of radius and pressure. The constant A appearing in this formula is found to have a theoretical value depending on the assumption consistent with the kinetic theory of gases which is made concerning the mode of impact of the gas molecules on the surface of the sphere. Assuming elastic impacts the value of A is 1.5;

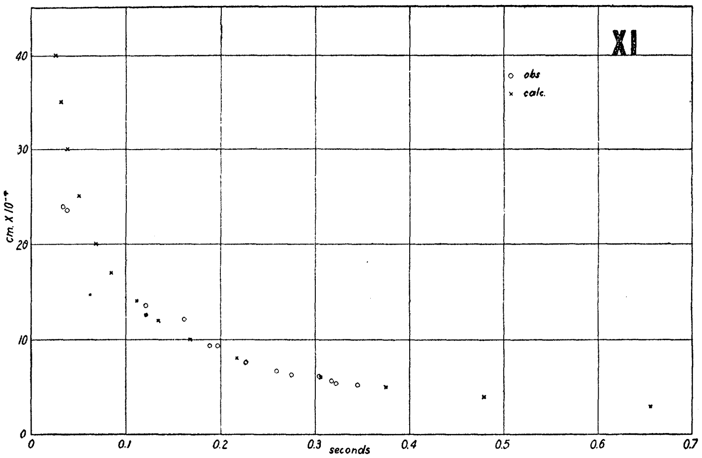

Fig. 8.

Pressure = 2.01 mm.

assuming inelastic impacts and the same distribution of component velocities in the emerging as in the impinging molecules, the value of A is 1.2; assuming inelastic impacts and normal emergence of all impinging molecules, the value of A is 1.05; by experiment the value of A is 1.00 ± 3.

The close agreement of the experiments with the fomula derived for the assumption of normal emergence makes it probable that a gas molecule impinging on a solid is entangled in the surface layer of molecules, and emerges again after a number of collisions with these molecules, its direction of emergence being generally nearly normal to the surface. The experiments do not indicate whether this penetrable layer is composed of the same kind of molecules as the rest of the solid, or is a condensed layer of the gas, but only show that the distances between the molecules in it are small compared to the distances in a gas under normal conditions of temperature and pressure.

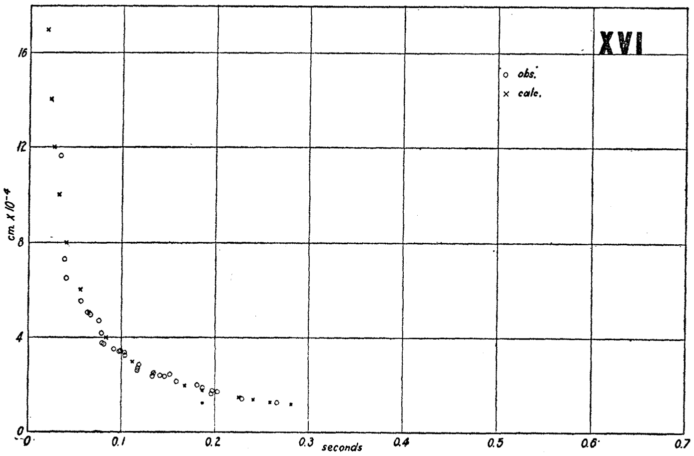

Fig. 9.

Pressure = 0.32 mm.

I take great pleasure in thanking Professor John Zeleny for his continuous interest in the progress of the work, and for his many valuable suggestions on the solution of experimental and theoretical difficulties.

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UNIVERSITY OF MINNESOTA,
May 17, 1911.

TABLE I.

Experiment 1. *p* 736 *mm. Temperature* 20°.6 *C. Tube length* 30.9 *cm. Maximum time of fall measurable* 20 *sec.*

a	$1/V$	a	$1/V$	a	$1/V$
.002293	.1929	.001746	.297	.001369	.481
.002170	.1995	.001461	.419	.001326	.474
.001846	.2404	.001382	.437	.001308	.518

Experiment 2. *p* 749 *mm. Temperature* 18°.3 *C. Tube length* 30.96 *cm. Maximum time of fall measurable* 226 *sec.*

a	$1/V$	a	$1/V$	a	$1/V$
.001369	.379	.000560	2.296	.000416	4.56
.001077	.600	.000541	2.45	.000407	4.51
.000906	.934	.000541	2.70	.000393	4.86
.000898	.934	.000522	2.56	.000385	4.97
.000757	1.365	.000505	2.86	.000385	5.40
.000719	1.524	.000492	3.36	.000382	4.52
.000696	1.566	.000466	3.47	.000373	5.45
.000679	1.718	.000461	3.75	.000368	5.49
.000674	1.653	.000453	3.65	.000355	5.52
.000653	1.748	.000449	3.59	.000350	5.88
.000637	1.916	.000433	4.10	.000346	6.10
.000608	2.169	.000432	3.86	.000346	6.67
.000604	2.088	.000427	4.18	.000339	6.63
.000585	2.270	.000426	3.96	.000335	6.72
.000580	2.189	.000417	4.18	.000334	6.79

Experiment 3. *p* 740 *mm. Temperature* 19°.7 *C. Tube length* 30.96 *cm. Maximum time of fall measurable* 1,080 *sec.*

a	$1/V$	a	$1/V$	a	$1/V$
.000704	1.492	.000370	6.03	.000238	15.18
.000571	2.335	.000361	6.26	.000238	15.79
.000562	2.205	.000351	6.52	.000218	18.49
.000518	2.854	.000327	8.17	.000213	17.96
.000502	2.984	.000317	8.69	.000210	20.24
.000490	3.081	.000303	9.63	.000192	23.22
.000473	3.31	.000287	11.03	.000191	27.64
.000413	4.44	.000274	10.86	.000162	34.76
.000378	5.52	.000273	13.26		

Table III.

Experiment 1. p 100.4 *mm. Temperature* 21°.0 *C. Tube length* 30.9 *cm. Maximum time of fall measurable* 212 *sec.*

a	$1/V$	a	$1/V$	a	$1/V$
.002660	.1462	.001176	.520	.000529	2.52
.002177	.2046	.001068	.695	.000497	2.95
.002069	.2795	.000992	.754	.000497	3.25
.001918	.296	.000833	1.170	.000440	3.49
.001854	.221	.000793	1.011	.000438	3.87
.001840	.260	.000712	1.436	.000320	6.76
.001531	.318	.000641	1.781	.000313	6.60
.001293	.482	.000543	2.64	.000302	6.88
.001207	.497	.000537	2.38		

Table IV.

Experiment 2. p 34.85 *mm. Temperature* 21°.2 *C. Tube length* 30.9 *cm. Maximum time of fall measurable* 186 *sec.*

a	$1/V$	a	$1/V$	a	$1/V$
.003715	.0715	.001013	.676	.000634	1.501
.002989	.1040	.000975	.669	.000623	1.689
.002642	.1040	.000926	.767	.000532	2.053
.002591	.1234	.000909	.786	.000516	2.339
.001506	.351	.000908	.812	.000487	2.385
.001347	.429	.000808	1.027	.000460	2.781
.001199	.487	.000807	1.053	.000380	4.14
.001109	.604	.000737	1.170	.000331	5.10
.001095	.617	.000635	1.527	.000330	4.71

Table VI.

p 8.28 *mm. Temperature* 21°.3 *C. Tube length* 30.96 *cm. Maximum time of fall measurable* 172 *sec.*

a	$1/V$	a	$1/V$	a	$1/V$
.001769	.1427	.000517	1.118	.000263	2.86
.001411	.1978	.000477	1.287	.000257	2.94
.001216	.292	.000430	1.472	.000220	3.67
.001141	.353	.000429	1.517	.000219	3.71
.000819	.584	.000399	1.686	.000204	4.10
.000766	.652	.000343	1.861	.000175	4.64
.000752	.712	.000316	2.250	.000175	5.18
.000599	.960	.000303	2.445	.000169	4.79
.000569	1.067	.000287	2.57	.000119	5.54
.000518	1.183				

TABLE VII.

Experiment 2. *p* 7.70 *mm. Temperature* 20°.9 *C. Tube length* 30.9 *cm. Maximum time of fall measurable* 184 *sec.*

a	$1/V$	a	$1/V$	a	$1/V$
.002090	.1787	.000764	.650	.000466	1.495
.002038	.1787	.000704	.689	.000382	1.589
.001375	.276	.000701	.712	.000327	1.949
.001126	.370	.000688	.731	.000316	1.888
.001065	.491	.000621	.896	.000307	2.066
.001059	.445	.000555	.965	.000306	2.129
.001044	.406	.000525	1.104	.000274	2.556
.000922	.478	.000514	1.072	.000257	2.550
.000916	.500	.000510	1.153	.000210	3.17
.000873	.533	.000507	1.004	.000179	3.12
.000784	.630	.000494	1.134		

TABLE IX.

Experiment 1. *p* 3.14 *mm. Temperature* 20°.2 *C. Tube length* 30.9 *cm. Maximum time of fall measurable* 20 *sec.*

a	$1/V$	a	$1/V$	a	$1/V$
.002042	.0809	.001268	.1832	.000699	.390
.002027	.0923	.001240	.1914	.000687	.419
.002007	.0939	.001145	.1654	.000658	.401
.001966	.1134	.001106	.2012	.000609	.418
.001790	.1150	.001038	.2109	.000565	.460
.001775	.1118	.000930	.2275	.000538	.487
.001372	.1556	.000892	.2616	.000502	.496
.001365	.1540	.000890	.289	.000456	.565
.001327	.1703	.000709	.341	.000321	.625

TABLE XI.

Experiment 1. *p* 2.01 *mm. Temperature* 20°.6 *C. Tube length* 30.9 *cm. Maximum time of fall measurable* 20 *sec.*

a	$1/V$
.002396	.0341
.002358	.0387

Experiment 2. *p* 2.01 *mm. Temperature* 20°.5 *C. Tube length* 30.9 *cm. Maximum time of fall measurable* 19 *sec.*

a	$1/V$	a	$1/V$	a	$1/V$
.001357	.1208	.000768	.2278	.000624	.304
.001213	.1622	.000764	.2274	.000569	.317
.000940	.1888	.000670	.2596	.000515	.322
.000931	.1969	.000626	.275	.000510	.345

TABLE XII.

p 1.28 *mm.* *Temperature* 20°.7 *C.* *Tube length* 30.9 *cm.* *Maximum time of fall measurable* 18 *sec.*

a	$1/V$	a	$1/V$	a	$1/V$
.002222	.0426	.000808	.1270	.000391	.287
.001616	.0647	.000752	.1494	.000364	.311
.001533	.0672	.000623	.1595	.000340	.348
.001422	.0738	.000588	.1914	.000304	.340
.001323	.0838	.000554	.2034	.000294	.399
.001320	.0844	.000512	.259	.000293	.420
.001164	.0893	.000408	.280	.000281	.392
.001049	.1082	.000401	.324	.000245	.457
.001033	.0910	.000394	.321	.000233	.548
.000883	.1248				

TABLE XVI.

p 0.32 *mm.* *Temperature* 21°.3 *C.* *Tube length* 30.96 *cm.* *Maximum time of fall measurable* 17 *sec.*

a	$1/V$	a	$1/V$	a	$1/V$
.001163	.0341	.000349	.0992	.000239	.1472
.000730	.0379	.000347	.0973	.000216	.1598
.000647	.0405	.000340	.1028	.000202	.1816
.000548	.0564	.000327	.1031	.000190	.1871
.000501	.0638	.000289	.1177	.000178	.1968
.000494	.0668	.000273	.1167	.000172	.2024
.000468	.0752	.000263	.1161	.000163	.1961
.000415	.0788	.000253	.1346	.000142	.228
.000381	.0798	.000244	.1524	.000127	.266
.000379	.0814	.000242	.1417		
.000357	.0914	.000239	.1333		