SPARK LENGTH, POTENTIAL AND FREQUENCY OF OSCILLATION: THE "LAG EFFECT" IN ELECTRIC DISCHARGE.

By J. C. Hubbard.

Introduction.

The spark potential for a given spark length is ordinarily defined as the greatest potential which can be applied to the spark gap for an indefinitely long time without causing the discharge to take place. The usual method of measuring spark potentials has accordingly been that of slowly increasing the potential, keeping the spark length constant, or of slowly diminishing the latter while the potential remains constant, until the discharge occurs. Extensive work has been done on the relation between spark length and potential for steady potentials, an admirable summary of which may be found in Sir J. J. Thomson's Conduction of Electricity through Gases (second edition). The study of short sparks has been especially prominent in recent years owing to the bearing of the phenomena upon the theory of ionization.[1]

It has long been known that a potential greater than the spark potential as defined above may be applied for a short time without producing discharge, especially if the gas be dry and free of dust. This effect has been called the "lag" of the spark, or the lag effect, and on it there has been much experimental work and discussion. Jaumann[2] pointed out some time ago that the time of lag is the interval necessary for some agency to convert the gas from an insulator into a conductor. It has been found that the presence of moisture, or the effect of such agents as ultra-violet light and Roentgen rays, in short, that the introduction by any agency of ions into the field between the electrodes, reduces the time of lag; though the ultimate discharge potential is not much affected.

[1] Conduction of Electricity through Gases, pp. 455–460. See also E. H. Williams, Phys. Rev., 31, pp. 216–240, 1910.

[2] Jaumann, Wied. Ann., 55, p. 656.

No very definite attempt seems so far to have been made to study quantitatively the lag effect. A promising method consists in applying electric oscillations of known amplitude and frequency to the spark gap. In a previous communication of mine to this journal[1] very positive indication was found that for sparks produced by an oscillating potential the spark length is a function not only of the maximum potential but also of the frequency of oscillation. The relation found was such that for very slow oscillations the spark potential approached the value for steady potentials, while with increasing frequency and constant spark gap the spark potentials became systematically higher. The work was not undertaken primarily with the object of studying the lag effect, and, as the spark gap in this case was at the break of an inductive circuit where complications due to the motions of parts might arise, in so far as it relates to the lag effect it is not to be considered of final value. There appeared almost simultaneously some interesting experiments by J. Algermissen[2] on the relation of spark length and potential for rapid oscillations of potential leading to similar results. The oscillations were produced in a system containing a spark gap by induction from a neighboring circuit. Sparks up to 2 cm. in length were studied, and complications arising from the length of spark being large in comparison to the radius of spark electrodes were introduced. The result was unmistakable, however, that as the frequency is increased the spark potential is increased for a given spark length.

The present study of alternating spark potentials makes use of a new method designed to give values of the potential and frequency under perfect control, and presents results of observation of a large number of cases together with some general conclusions.

The Source of Potential.

As a source of potential in the present work is used the oscillations set up in a circuit containing inductance and capacity when the initial current is interrupted. The conditions for breaking such a circuit without producing a spark at the break have been given in

[1] J. C. Hubbard, Phys. Rev., 22, pp. 129–158, March, 1906.
[2] J. Algermissen, Ann. der Phys. (4), 19, p. 1016, April, 1906.

my paper referred to above. In brief, if the speed of separation of the contacts exceed a certain critical value, called the velocity of break, no spark will be produced on breaking the circuit. This critical velocity was found to be proportional to the frequency and to increase almost linearly with the maximum potential, the effect of damping being negligible. Making use of a circuit breaker operated under the proper conditions it is possible, within the limits set by the attainable velocity of break, to secure oscillations of any desired frequency and initial amplitude.

Method and Apparatus.

The electrical system is shown diagrammatically in Fig. 1. The principal parts consist of the inductance L having the resistance R, a condenser K, battery E, interrupter U, an auxiliary resistance R_b for varying the initial current, and a micrometer spark gap S in parallel with the inductance and capacity. Let the current $i_0 = E/(R + R_b)$ be established, then provided the circuit at U is broken with sufficient suddenness to prevent a spark forming there, we shall have oscillations of potential at the electrodes of the micrometer spark gap, which, in view of the small values of K and R and the large values of L (see measurements below) may be accurately expressed by

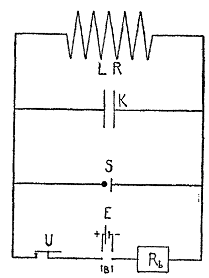

Fig. 1.

$$V = i_0\sqrt{\frac{L}{K}}\,e^{-\frac{R}{2L}t}\sin\left(\frac{t}{\sqrt{KL}}\right)$$

and which have the period $T = 2\pi\sqrt{KL}$.[1]

The method of observation consists in slowly lessening the width of the spark gap while interruptions of the current are being made until a spark takes place. The mean of several spark lengths so found is taken as the spark length corresponding to the initial current in the expression for V given above. The spark observations are made visually.

The condenser and inductance have been very fully described in previous communications from this laboratory and will be only briefly noticed here.

[1] Phys. Rev., 22, p. 131, 1906.

The *condenser*[1] consists of two thick steel plates of 25 cm. radius held apart by sets of three small cylinders cut from plane parallel plates of glass. The capacities were calculated from the formula of Kirchhoff. The range used in these experiments was between 200 and 600 cm.

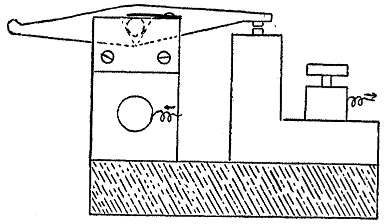

Fig. 2.

The *inductance*[2] consists of a coil of 3,600 turns of no. 18 (B. & S.) double cotton-covered copper wire wound in a large wooden spool. The channel filled with windings has a square cross-section 3 inches on a side, and its internal diameter is 8 in. The coil is divided into six independent sections of ten layers each; with all in series the inductance is 3.01 henries and the resistance 62 ohms. By using various combinations of sections a great range of inductance is possible.

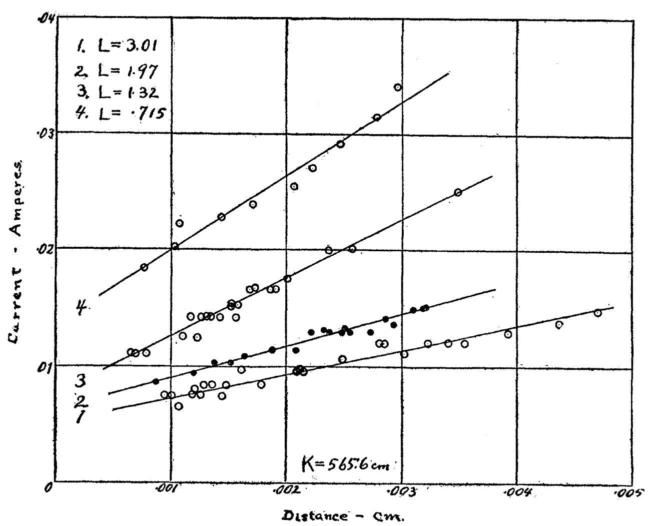

Fig. 3.

Besides this coil there were used two large coils from the Worcester Polytechnic Institute kindly loaned by Professor Duff. These had

[1] A. G. Webster, PHYS. REV., 6, p. 300, 1898.

[2] J. E. Ives, PHYS. REV., 14, p. 298, 1902.

an inductance when used in series of nearly a henry. It was found, however, that the insulation of these coils was not sufficient for work with any but the smallest sparks.

The *interrupter*, one of several designed by Professor Webster for use with the drop chronograph (*loc. cit.*), may be readily understood by a glance at Fig. 2. The circuit is broken by the falling of a weight on the end of a lever. The requirements are perfect rigidity and freedom from vibration when the lever arm is struck,

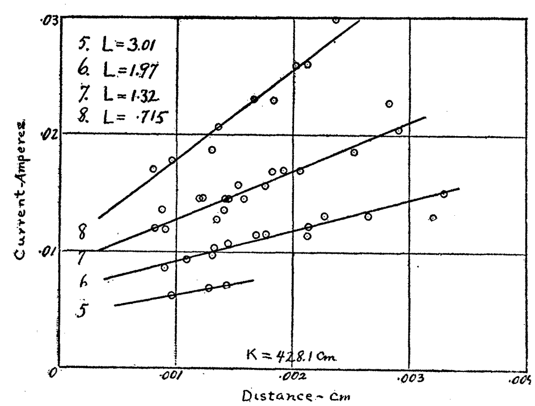

Fig. 4.

and a return to negligible resistance of contacts as soon as the circuit is closed. The lever should be of as small moment of inertia as is consistent with these requirements. The lever is of steel, shrunk on a steel axis which turns in V-shaped grooves in the tops of two upright projections of a brass frame. The latter is mounted on a block of ebonite which also carries the mounting for the other contact. A pair of stiff steel springs pressing the upper side of the axis keep it firmly in the grooves, and a stiff band of steel pressing against one end of the axis holds the other end against a metal stop mounted on the outside of the opposite post. Perfect and permanent adjustment is thus secured, the arrangement constituting a Kelvin so-called geometrical slide. The contacts are of silver and rubber bands (not shown) cause the contacts to press together as soon as the weight which operates the lever has done its work. This arrangement has given perfect satisfaction and stands the roughest usage. The resistance of the interrupter with the circuit closed was invariably found to be a small fraction of an ohm.

The *circuit breaker* first tried was the projectile of the drop chronograph. Several seconds were required to produce a single interruption and many minutes to find a single spark length. In its place was substituted a bicycle wheel operated by an electric motor. The wheel carried a two pound mass of iron on its rim for striking the lever arm of the break, suitable counterpoise being added to the inside of the rim on the opposite side. The wheel was usually rotated so as to produce about two breaks per second.

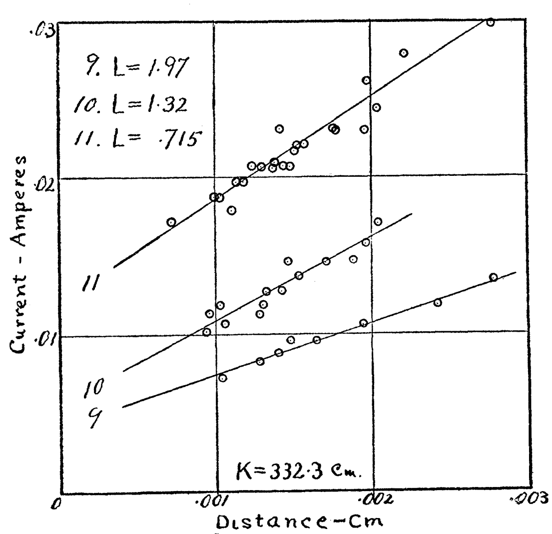

Fig. 5.

The *micrometer spark gap* is very similar in principle to many which have been described before.[1] The spark electrodes were of steel, one of them being a plane 10 × 12 mm., the other a ball 6.25 mm. in diameter mounted on the end of a brass cylinder of nearly the same diameter. These were ground with several pourings of flour of emery then given a high polish with diamantine on soft leather. The amount of labor involved in producing surfaces of high polish, free from scratches, on steel is extraordinary, and I am greatly indebted to Mr. H. F. Stimson, Scholar in Physics

[1] G. M. Hobbs, Phil. Mag. (6), 10, p. 620, December, 1905.

in Clark University, who undertook a careful study of the matter. Cleaning of the surfaces was effected by rubbing with soft, clean cloths, then with chamois skin and chalk or jeweller's rouge. Small particles of dust were removed by the use of an air jet. The plane was mounted in parallel ways and the ball with its axis of rotation at an acute angle with the plane. After the passage of a spark the plane was given a small translation and the ball a small rotaton so as to move the near surfaces in opposite directions, thus presenting fresh surfaces for the next spark. During the final measurements a jet of air (see below) was directed into the space between the electrodes. A separate circuit, the leads of which could be attached to the spark electrodes, containing a battery, resistance, and millivoltmeter was used to determine the position of contact of the electrodes. This zero was found immediately after the passage of each spark. It could be determined to one-tenth division of the drumhead of the micrometer screw, corresponding to .00002 cm. The results given here are mostly for sparks from .005 to .0005 cm. in length. The order of accuracy is therefore about one per cent. Difficulties of another sort discussed below render greater accuracy of reading useless for the present purpose.

Potentiometer and bridge circuits were arranged for frequently testing respectively the electromotive force E of the battery and the resistance of various parts of the system. For the battery E was used one or two storage cells.

Control Experiments.

Frequency of Interruption of the Circuit.—On changing from the drop chronograph to the bicycle-wheel break no change of result was observed, showing that it was immaterial whether the train of oscillations was applied to the spark gap once in many seconds or twice per second.

Surface of the Spark Electrodes.—Different methods of polishing and cleaning gave approximately the same results except for the higher frequencies where the passage of the spark is extremely sensitive to the state of the surfaces. Especial care had to be taken to use only the freshest of materials in cleaning the electrodes.

State of the Gas.—First experiments gave results which varied by 50 per cent. among themselves. After much effort to get more uniform data it was found that a fine jet of air blown from a tapering glass tube into the discharge space reduced the necessary sparking potential to a very much lower value and enabled results to be repeated within a few per cent. of each other (curves 1–11, Figs.

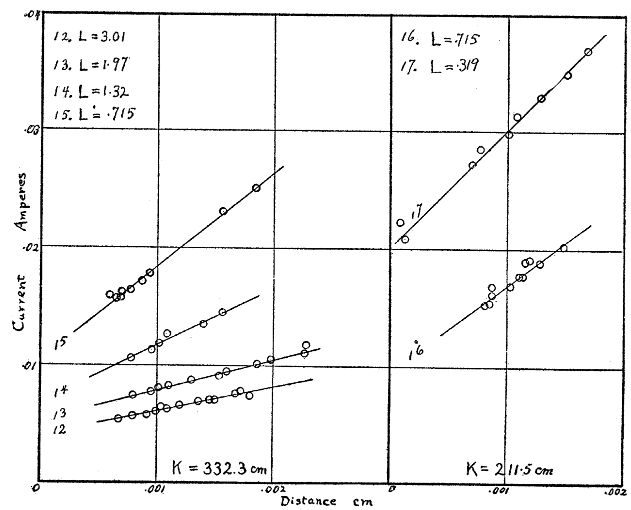

Fig. 6.

3–5).[1] Such discrepancies as then remained were found to be due to inefficient insulation in the windings of the coil, and before a last series of measurements was made were eliminated almost entirely by placing the coil in kerosene (curves 12–17, Fig. 6). Obviously, as the experiments go, there is no need to consider the effect of changes of atmospheric pressure. It may be remarked, however, that the most consistent results were obtained during rainy weather.

POTENTIAL AT THE TIME OF SPARKING.

While the damping in these experiments is small, it is of some

[1] The air coming from a filter-pump air-blast, the water in which was usually at about 6° C., passed through a long rubber tube and then through a plug of glass wool to filter out dust, before passing to the spark gap. The air was delivered at approximately the temperature of the room (20°) and was far from being saturated with moisture. The effect of the air was very striking, and seems to show that a part of the lag effect depends upon the number of ions initially present in the air. A tube containing a small quantity of radium placed under the spark gap produced similar results. A systematic study of the effect of various ionizing agents is to be made in later work.

interest to know whether the discharge begins during the first maximum of potential. It is conceivable that if the effect of a maximum were just unable to produce a spark it might succeed in forming ions which would enable the spark to pass at some following maximum. The first spark of condenser discharge has long

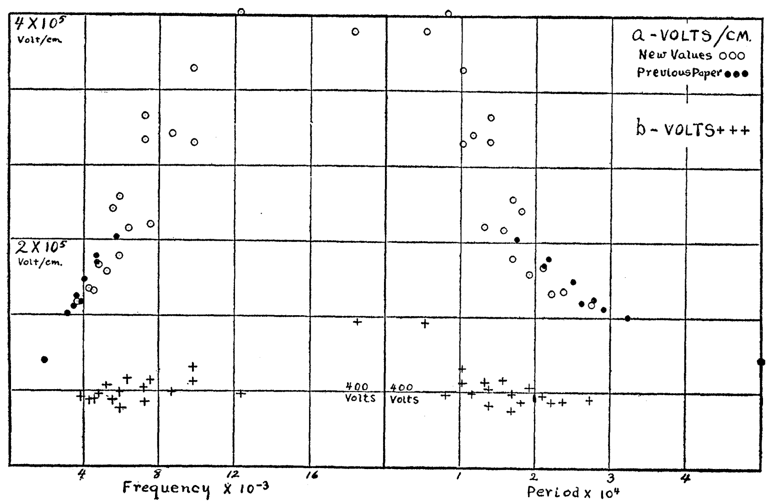

Fig. 7.

been known to be of different character from the others and has been given the name pilot spark. I have found evidence to show that, given a spark length and the oscillating potential which will just produce a spark, but not greater, the spark will begin during the first maximum of induced potential. A micrometer microscope was focused upon the space between the contacts of the break, and the velocity of break, at first high enough to prevent sparking altogether, was reduced until a very faint spark occurred. The length of the spark together with the known velocity of break and the frequency of oscillation placed the time of occurrence of the spark very near the time of the first maximum of potential, *i. e.*, within one eighth of the period. A small further reduction of the velocity of break caused a complete change in the character of the spark. It was now thick, brilliant and longer and possessed a beaded appearance. This is because the rapidly moving electrode was in different positions at the times of successive oscillations. Measurement showed that the spark length for the first luminous condensation gave a time which invariably came in the second half period, again within one eighth period which was about the reading accuracy of the necessarily low power microscope. The pilot spark which must have preceded it was completely masked. In any event it would seem that the first spark takes place during the first maximum as was directly observed for the limiting case. For *constant* spark lengths at the micrometer spark gap no such changes were observed, which is easily understood as we do not there have the influence of the rapidly widening spark gap in preventing the second and successive discharges from passing. It seems probable, also, that unless the first maximum is great enough to produce a discharge, any ions that might be formed would be swept out of the field, making it less likely that the successive maxima would produce a discharge.

Further evidence on this question will be found in the discussion of results. Assuming that the spark takes place near the end of the first quarter period we have, in the most unfavorable case ($R = 27.1$ ohms, $L = .715$ henry, and $K = 6.28 \times 10^{-10}$ farads), as the value of the damping factor $e^{-\frac{R}{2L}\frac{\pi}{2}\sqrt{KL}} = .9987$, which is unity for our purpose. We therefore take

$$V = i_0 \sqrt{\frac{L}{K}} \sin\left(\frac{t}{\sqrt{KL}}\right)$$

and

$$V_m = i_0 \sqrt{\frac{L}{K}}. \tag{1}$$

This discussion ignores the fact that the resistance of the coil for rapid oscillations increases somewhat rapidly with the frequency.[1] Since we have a margin of safety of several hundred per cent., it is hardly likely that the effect of the damping need be considered.

Results.

Several series of observations were made to show the relationship between spark length and initial current. The results of these

[1] See, for example, A. Esau, Ann. der Phys. (4), 34, pp. 57 and 81, 1911.

measurements are shown graphically in Figs. 3–6. Abscissæ represent spark lengths in cm., and the ordinates represent the currents in amperes which were flowing at the various times of break and from which the potentials may be calculated from equation (1). Each point shown in the figures is a mean of several, usually five, readings.

The most important result of the measurements is at once seen from the figures; namely, that with given K and L and within the limits of the experiments the relation between initial current and spark length is a linear one. Lines best representing the results are drawn in the figures and are given numbers which correspond in general to the order in which the experiments were made. Series 1 and 2, however, contain observations weeks apart, some of the other series having been determined in the meantime. All the series obtained with the air jet in use are shown.

During most of the control experiments spark lengths were obtained up to .01 cm. Owing to the gradual deterioration of the insulation of the coils the available region of experiment became more restricted and by the time series 10 was reached it was almost impossible to get results at all, while such as were obtained scattered very badly. The coil was then immersed in kerosene for the remainder of the work, giving much more regular results (series 12–17). Immersion in kerosene did not, however, restore the available working region to its former value. Measurements taken with the two coils furnished by Professor Duff gave consistent readings for only the smallest initial currents which would produce visible sparks. These results indicate very clearly that for satisfactory work with these oscillations coils must be used of which there is no question as to high insulation. A coil is now being designed with which it is hoped to meet such objections in an extension of the work over a wider range of spark lengths and under varied conditions. It must be remarked, however, that no indication has been found of a change of slope of any of these curves due to the progressive deterioration of the insulation.

Treatment of the Observations.—Since the relationship between the initial current and the spark length is a linear one, and the slopes of the lines are affected by change in the inductance or capacity only, it is at once evident that for oscillating potentials under such conditions the period alone and not the amplitude affects the spark constants. We may write

$$i_0 = \alpha x + \beta$$

as expressing the relation between the current i_0 and spark length x. The constants α and β depend only on K and L. Since

$$V_m = i_0\sqrt{L/K}$$

(see equation (1) and discussion above), we may write

$$V_m = \alpha\sqrt{L/K}\cdot x + \beta\sqrt{L/K} = ax + b \qquad (2)$$

where

$$a = \alpha\sqrt{L/K} = f_1(\sqrt{KL}), \text{ etc.}$$

It now remains to examine a and b as functions of the period. In Table I. are given the values of α and β from which the curves in Figs. 3–6 are drawn, together with the corresponding values of a and b, and the periods $T = 2\pi\sqrt{KL}$ and frequencies n. The values of a and b so found are plotted in Fig. 7 as functions both of frequency and of period. The figure at once suggests that, within the errors of experiment, the constant a is a linear function of the frequency. It must be remembered that a few observations more or less in Figs. 3–6 would change the slopes of the lines as well as the intercepts, and at higher frequencies especially the values of a and b are enormously sensitive to errors of original observations. The values of a obtained in my previous paper (*loc. cit.*, p. 154) by an indirect method are given in Table II. and are shown in the figure as filled circles. These agree in a striking manner with the results of the present measurements. This astonished me, inasmuch as the former work was done under entirely different conditions. In the first place, the air jet was not used, though the experiments were carried on in very damp spring weather. Again, in the former case the spark gap was widening at a rapid rate at the instant the spark took place, the rate being such that the electric force was practically uniform between the electrodes from the start. The agreement of the two sets of results raises questions which can only be settled by further experiments. It will be explained if the spark takes place at the time at which the potential gradient has reached its first maximum in both sets of experiments. This is further justification for our use of equation (1) in the present case and of the treatment of the results in the former.

TABLE I.[1]

No.	K, cm.	L, henries.	$T \times 10^4$	$n \times 10^{-3}$	α	β	$a \times 10^{-5}$ $\frac{\text{volts}}{\text{cm.}}$	b, Volts.
1	565.6	3.01	2.73	3.66	2.10	.00523	1.45	362
2	565.6	1.97	2.21	4.52	2.77	628	1.55	352
3	565.6	1.32	1.81	5.52	4.98	764	2.28	350
4	565.6	.715	1.33	7.52	6.34	1360	2.14	458
5	428.1	3.01	2.37	4.22	1.97	440	1.57	350
6	428.1	1.97	1.92	5.21	2.66	662	1.72	426
7	428.1	1.32	1.58	6.35	4.05	878	2.11	462
8	428.1	.715	1.16	8.63	7.63	1026	2.96	397
9	332.3	1.97	1.69	5.90	3.27	415	2.38	303
10	332.3	1.32	1.39	7.21	5.19	565	3.10	338
11	332.3	.715	1.02	9.80	6.53	1202	2.87	529
12	332.3	3.01	2.10	4.76	1.96	422	1.77	382
13	332.3	1.97	1.69	5.90	2.54	540	1.86	394
14	332.3	1.32	1.39	7.21	4.84	696	2.89	416
15	332.3	.715	1.02	9.80	8.02	1024	3.53	451
16	211.5	.715	.814	12.29	7.28	964	4.01	384
17	211.5	.319	.545	18.35	10.46	2092	3.86	771

TABLE II.

From this journal, Vol. 22, p. 154, 1906.

$T \times 10^4$	$n \times 10^{-3}$	$a \times 10^{-5}$ $\frac{\text{volts}}{\text{cm.}}$
1.75	5.72	2.03
2.13	4.69	1.80
2.16	4.63	1.86
2.50	4.00	1.65
2.61	3.83	1.46
2.78	3.60	1.50
2.91	3.44	1.41
3.23	3.10	1.34
5.05	1.98	.93

[1] The resistances of the coils L = 3.01, 1.97, 1.32, .715, and .319 henries are, respectively, R = 62.5, 51.3, 38.8, 27.1, and 17.6 ohms. No other appreciable resistance takes part in the oscillations.

It is interesting to notice the trend of a assuming it to be a linear function of the frequency n. The results in Tables I. and II. give, by least squares, for

$$a = pn + c$$

the values p, = 21.49, and c = 81,800 volts/cm., the constant c is the value for $n = 0$, that is, the value of a for steady potentials. This value is not far from the values of a which have been determined by others with spark lengths of the same order for constant potentials; the results of Earhart,[1] for instance, giving a as about 65,000 volts/cm. A still better agreement will be shown if the observations for the higher frequencies, which are very sensitive to the condition of the electrode surfaces, are left out of account.

It cannot be said whether the constant b depends on the frequency. The values of b are very sensitive to errors in a. The results for b show a trend to larger values with increasing values of n, and indicate a value of about 350 volts for zero frequency, again in agreement with steady potential work. Values of b from the previous paper are not shown for the reason that their absolute values are too largely affected by the residual errors of the method of approximation used in their calculation.

It is unfortunate that means were not available for the measurement of the inductance and resistance of the coils at the various frequencies used. The inductance diminishes slowly, while the resistance rises more rapidly with the frequency.[2] Both these effects, if taken into account, would diminish the calculated values of a and b by several per cent., but not nearly by enough to account for the changes observed in a. For example, for the moderate frequency of 4,000 vib. per sec., our value of a is about three times that for zero frequency. Since a involves the square root of L, the latter, in order to account for the whole change, would have to be diminished by eight ninths or 89 per cent. by the effect of the frequency, an amount which is of entirely different order of magnitude from that indicated by such experiments and theory as are available. These considerations will, however, be taken into account in the new experiments which are being planned.

[1] Earhart, Phil. Mag., VI., 1, p. 147, 1901.

[2] Esau, *loc. cit.*; see also J. G. Coffin, Proc. Amer. Acad., 41, p. 789, 1906, etc.

Summary and Conclusions.

This paper presents a study of the relation between spark length, potential, and frequency for oscillating potentials having frequencies from 4,000 to 18,000 vib. per sec., and for spark lengths ranging from .005 to .0005 cm. The potentials are produced by interrupting a known initial current in a circuit containing inductance and capacity. The interruption is made in such a manner that no spark is produced at the break, thus giving definite values of potential at the electrodes of an auxiliary spark gap.

The effect of different amplitudes and frequencies of potential upon the spark constants a and b in the equation $V = ax + b$, usually employed to relate steady potentials V and spark lengths x, is studied. The experiments relate to air at atmospheric pressure.

1. Evidence is presented showing that the discharge produced by a train of oscillations begins near the time of the first maximum of the potential (control experiments, etc.).

2. It is found that the spark lengths are linearly related to the strengths of initial cu rent (Figs. 3–6), the inductance and capacity in the circuit remaining the same; *i. e.*, the spark constants in the equation $V = ax + b$ are independent of the amplitude alone of the oscillating potential.

3. The spark constants vary with the frequency. The constant a in the equation is, within the limits of the experiments, a rapidly increasing linear function of the frequency. When the frequency is zero, that is, for steady potentials, the value of a reduces to its normal value which for these short sparks is about 6.5×10^4 volts/cm. At a frequency of 4,000 vib./sec., for instance, the value of a is nearly three times normal. This statement means that for an oscillating potential of this frequency the maximum potential of an oscillation necessary to produce a spark must be about three times as great as the sparking value of the steady potential, and so on. The value of b is practically the same as for steady potentials, *i. e.*, about 350 volts, except that there is a slight tendency to increase with the frequency.

4. It may be concluded finally that the lag effect of the electric spark depends upon at least two independent factors: (1) Upon the amount of ionization initially present in the spark gap. This

factor is of an accidental nature and is subject to regulation and control (see control experiments). (2) The lag effect depends also upon some process which takes place in the gas and which takes time for its completion such that the shorter the time the greater is the necessary sparking potential. The latter factor will probably be found to depend upon the nature and condition of the gas (pressure, etc.).

These facts accord in a general way with the theory of the spark discharge as developed by Sir J. J. Thomson.

Apparatus for a further study of this phenomenon is under construction, it being evident (Fig. 6) that it will be possible to obtain results of considerable precision. With such results it may be possible by a study of different gases at different pressures to obtain much greater insight into the problem of ionization by an electric field.

Clark University,
January, 1911.