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A METHOD FOR MEASURING ELLIPTICITY AND THE DETERMINATION OF OPTICAL CONSTANTS OF METALS.

By A. Q. Tool.

THE object of this investigation was twofold, namely: to develop an accurate and convenient half-shade system for measuring the constants of elliptically polarized light and to apply it in determining the optical constants of metals by reflection. Since in every half-shade compensator method it is necessary to determine the orders of the analyzing plates of the system, and also since the orders of these plates vary greatly with the method used, any instrument designed for determining them must have a wide range of applicability. Such an instrument is also valuable since it can be used for measuring ellipticities in general. The instrument to be described was devised with this purpose in view.

I. Description and Use of Instrument.

The various methods employed for obtaining the constants of elliptically polarized light have differed greatly according to the conditions to be met. Babinet's compensator has been used chiefly for large ellipticities, while for small the Bravais bi-plate method with many modifications, earlier, and the Brace elliptic half-shade, more recently, have been employed. A number of years ago Stokes[1] devised a method by which a compensator of unknown order (approximately a quarter-wave plate) is used to restore the light to plane polarization. This possesses the advantage of allowing a

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[1] Stokes, G. G., Phil. Mag. (4), Vol. 2, p. 420, 1851.

compensator with considerable differential dispersion to be used throughout the spectrum and at the same time permits a wide range in the magnitude of the ellipticity to be measured. The chief difficulty with this instrument, however, is in setting the compensator and analyzing nicol with sufficient accuracy. The system used in the present investigation was devised to increase the sensibility of Stokes's method. In principle it consists in compensating the light down to a definite small ellipticity as tested by the Brace half-shade, instead of to plane polarized light.

The apparatus was designed after that of Stokes, the compensator being mounted on a circle movable with respect to a fixed vernier, and the analyzing nicol on a second vernier movable with respect to the compensator circle. (The circle and verniers were graduated to read to .01°.) The following modifications, however, were introduced. The analyzer consisted of a Jellett split nicol combined with an elliptic half-shade fixed relatively to the nicol and just before it (*i. e.*, toward the light source). This half-shade is best chosen with its dividing line making an angle of approximately 45° with its principal axis, so that when mounted with its dividing line perpendicular to that of the nicol, its optic axis will be at 45° to the axis of the nicol. A simultaneous "match" of the four parts of the field thus obtained constitutes a "setting". The ocular should therefore focus on both dividing lines at the same time.

The compensator was in most cases a plate of mica of such thickness that its order was approximately 90° for 560 $\mu\mu$. The average order of the two parts of the elliptic half-shade (also of mica) was varied over a wide range (from 0.5° to 30°) but the difference in the order of the two parts was never greater than 6°, nor less than 1°. This difference in order and also the angle between the principal planes of the split nicol (2° in the greater part of this work) were varied according to the intensity of the light tested.

This analyzing system was mounted on one arm of a spectrometer and just before the telescope, which was interchangeable with a short focus ocular — the telescope serving for all spectrometer adjustments and the ocular for matching the analyzing system. The polarizing nicol was mounted immediately after the collimator lens. Monochromatic light (as obtained from a spectral instrument) was focused on the collimator slit of the spectrometer, the range being within 4 $\mu\mu$ throughout the spectrum.

With monochromatic light and uniform illumination a simultaneous match of three parts of the field in this apparatus gives theoretically a match of all four. With thin half-shades and nearly monochromatic light this condition is realized experimentally, but under working conditions it is sometimes necessary to be satisfied with a match of three fields, which is sufficient provided the same three are always used.

The procedure for obtaining a match is to adjust first the compensator and combined half-shade system to approximately minimum intensity of field, then rotate them together for a match of the elliptic half-shade. With compensator fixed, the half-shade system is then rotated to a match of the nicol, which destroys to a slight extent that of the elliptic half-shade. A simultaneous match can then be obtained by repeating the last two adjustments. Having obtained a match, the positions of the compensator and nicol (R and r respectively) are observed and likewise for the complementary match (R' and r').[1] As in Stokes's method the direction of the major axis of the ellipse is given by $[(R + R')/2 - \pi/4]$. Tuckerman[2] has given the theory of this instrument in his paper on Elliptic Analyzing Systems, in which he gives two methods for calibrating the half-shade and compensator. First, when *the initial light* falling on the analyzing system *is plane polarized* letting

and
$$\left.\begin{aligned} R' - R &= c \\ r' - r &= n \end{aligned}\right\} \qquad (1)$$

he derives the following equations:

$$\cos 2\pi N_1 = \frac{\operatorname{tg} c}{\operatorname{tg} n} \quad \text{or} \quad \operatorname{tg} \pi N_1 = \sqrt{\frac{\sin (n - c)}{\sin (n + c)}}, \qquad (2)$$

and

$$\operatorname{tg} 2\eta = \frac{\sin n \sin 2\pi N_1}{\operatorname{tg} c} = \sqrt{\frac{\sin (n + c) \sin (n - c)}{\sin c}}, \qquad (3)$$

where $2\pi N_1$ is the order of the compensator (in degrees) and

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[1] It is well to eliminate circle defects by making the supplementary matches also.

[2] Tuckerman, L. B., University of Nebraska Studies, Vol. IX., No. 2, 1909.

$$\operatorname{tg} 2\eta = -\sin\alpha \operatorname{tg} 2\pi N,$$

2η being designated as the *effective order* of the half-shade,[1] $2\pi N$ the average order of the two parts, and α the angle between its axis and that of the nicol. Hence having measured c, n and α the orders of half-shade and compensator may be calculated. Usually the *effective order* only of the half-shade is necessary, therefore in the rest of this paper it will be referred to as the order of the half-shade.

If either N_1 or η be known, a convenient form is

$$\left.\begin{aligned} \operatorname{tg} 2\eta &= \frac{\sin(n+c)}{\sin c}\operatorname{tg}\pi N_1, \\ \text{or}\qquad &= \frac{\sin(n-c)}{\sin c \operatorname{tg}\pi N_1}. \end{aligned}\right\} \tag{4}$$

When the initial light is plane polarized and the order of the compensator large, that of the half-shade (2η) must also be large in order that the compensator and nicol rotations be of a magnitude such that $2\pi N_1$ may be determined with sufficient accuracy, maximum accuracy being secured when

$$\frac{\sin^2 n - \sin^2 c}{\sin^2 n} = \tfrac{1}{2}\sin^2 2\pi N. \tag{5}$$

The second method (to be used when 2η is small) provides sufficient accuracy when *the initial light* is *elliptically polarized*—the maximum accuracy in this case being obtained when

$$\frac{\sin^2 c - \sin^2 n}{\sin^2 c} = \tfrac{1}{2}\sin^2 2\pi N. \tag{6}$$

In this case ($e_0 \neq 0$) Tuckerman finds that

$$\operatorname{tg}\pi N_1 = \sqrt{\frac{\sin(c-n)}{\sin(c+n)}}\left(1 - a\operatorname{tg} 2\eta + \tfrac{1}{2}a^2\operatorname{tg}^2 2\eta - \cdots\right), \tag{7}$$

and

$$\operatorname{tg} 2\psi = \sqrt{\frac{\sin(c-n)\sin(c+n)}{\sin n}}\left(1 + \tfrac{1}{2}a^2\operatorname{tg}^2 2\eta - \cdots\right), \tag{8}$$

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[1] The effective order of the half-shade may be varied to a certain extent by changing its azimuth with respect to the analyzing nicol.

where

$$a = \frac{\sin c}{+\sqrt{\sin (c - n) \sin (c + n)}},$$

and

$$e_0 = \operatorname{tg} \psi = \frac{\text{minor axis}}{\text{major axis}}.$$

The series in (7) and (8) are not convergent for small values of e_0. For such cases the proper equations are given in the article mentioned, in which it is also shown that in the case of an uncalibrated compensator the maximum accuracy is obtained in the determination of e_0 when the compensator is a quarter wave plate.

Both of the above methods were used here in the calibration of compensators and half-shades.

CALIBRATION OF COMPENSATORS AND HALF-SHADES.

First Method. — In this method a 90° mica compensator (no. 1) and a 20° half-shade (no. 2) were used in the elliptic analyzer and calibrated simultaneously by observations taken on plane polarized light. In all calibration measurements the practice followed (unless otherwise stated) was to make but a single observation on R, R', r and r' for numerous points in the spectrum in preference to averaging several observations for relatively few points. The probable value for any wave-length was then obtained from the curve. The values of πN_1 and 2η were calculated from c and n according to equations (1), (2) and (3). Selected values are given in Table I. and all values plotted in Fig. 1 — the order of the half-shade being in all cases increased by 10°.

TABLE I.

Simultaneous Calibration of Compensator and Half-shade. Compensator No. 1. Half-shade No. 2. $e_0=0$.

Wave-length ($\mu\mu$).	c	n	Order of Half-shade (2η).	½ Order of Comp. (πN_1).
660	71.80°	85.70°	17.70°	38.24°
620	71.10	86.90	18.65	40.45
580	69.90	88.67	20.05	43.17
540	68.55	90.95	21.42	46.22
500	66.50	94.01	23.20	49.63
460	63.30	98.80	25.30	53.97

Table II. gives selected values also obtained by the same method for a selenite compensator (no. 2) and the same mica half-shade no. 2. Since the specimen of selenite did not give a good field, the probable error in the results is large. The plot of all values is given in Fig. 1.

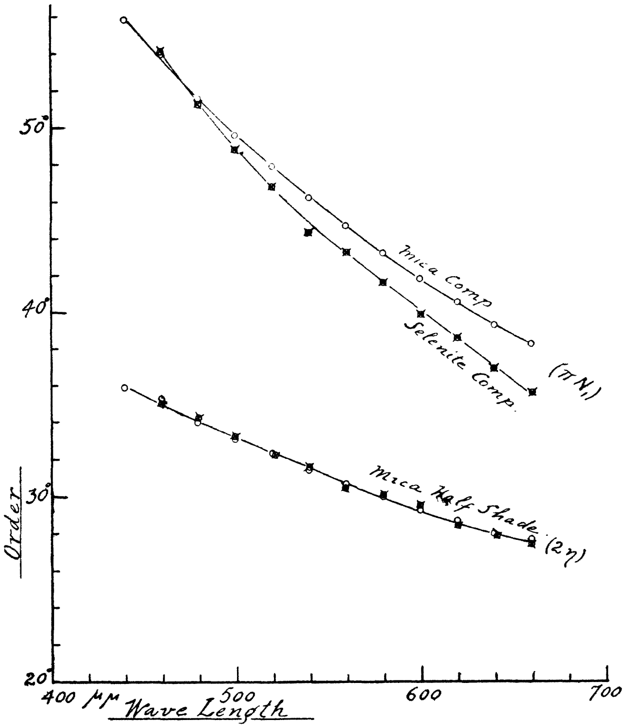

Fig. 1. Calibration of compensators Nos. 1 and 2 and halfshade No. 2.

TABLE II.

Simultaneous Calibration of Compensator and Half-shade. Compensator No. 2. Half-shade No. 2. $e_0=0$.

Wave-length ($\mu\mu$).	Order of Half-shade (2η).	½ Order of Compensator (πN_1).
660	17.4°	35.6°
640	18.0	36.9
600	19.5	39.9
560	20.5	43.2
520	22.1	46.8
480	24.1	51.3
460	25.0	54.1

This same method which serves very well for determining the order of thin half-shades by the use of a thick compensator was used to obtain the order of a thin half-shade (no. 1). Several observations made on one color indicated that the probable error in a single determination of R', R, etc., was not over .02°. Table III. gives the values obtained for half-shade no. 1 by compensator no. 1. The calculated values for 2η were obtained by taking the observed order for 590 $\mu\mu$ and assuming the dispersion in this to be the same as that obtained for compensator no. 1 (see Fig. 2).

TABLE III.

Calibration of Half-shade. Compensator No. 1. Half-shade No. 1.$e_0 = 0$.

Wave-length ($\mu\mu$).	Comp. Settings.	Nicol Settings.	c	n	Order of Half-shade (2η).	
					Obs.	Calc.
640	88.16° 359.06	93.03° 3.22	89.10°	89.81°	.88°	.87°
590	88.14 359.08	93.06 3.15	89.06	89.91	.94	—
540	88.12 359.11	93.13 3.11	89.01	90.02	.99	1.02
490	88.09 359.24	93.21 3.02	88.85	90.19	1.13	1.12

Table IV. gives the results obtained in the same way for another thin half-shade (no. 3). Greater accuracy was desired in these results in order to calibrate a thin compensator by comparison, hence they were obtained from the average of several observations — the probable error in 2η being not greater than .01°. Two compensators were used. No. 1 for 600 and 590 $\mu\mu$ and incidentally no. 2 for 580.

TABLE IV.

Calibration of Half-shade. Half-shade No. 3. $e_0 = 0$.

Comp.	Wave-length ($\mu\mu$).	c	n	Order of Half-shade (2η).
No. 1	600	88.24°	89.76°	1.745°
	590	88.22	89.80	1.770
No. 2	580	92.25	91.36	1.795

If the value of 2η be properly chosen and accurately known, a thin compensator such as used in the Brace instrument may be calibrated with sufficient accuracy by comparison. This method is identical with the above excepting that equations (4) are applicable in place of (2). Table V. gives the results obtained for compensator no. 3 by comparison with half-shade no. 3, and applying the *first* equation, since the factor $(c-n)$ which enters in the *second* is in this case too small to give the desired accuracy. The results are the average of several observations and the order accurate to .05°.

TABLE V.

Calibration of Compensator by Comparison. Compensator No. 3. Half-shade No. 3. $e_0=0$.

Wave-length ($\mu\mu$).	c	n	$(c+n)$	(2η)	$(2\pi N_1)$
600	106.22°	106.14°	212.38°	1.745°	6.25°
580	106.20	106.12	212.32	1.770	6.44

Second Method.—As before stated, if the elliptic analyzer contain a thick compensator and thin half-shade, the calibration of the compensator can be accomplished if the initial light be transformed from plane to elliptically polarized light by some means, such as introducing a doubly refracting plate between the polarizer and analyzing system, the observations being taken as before. The initial ellipticity (e_0) and the order of the compensator $(2\pi N_1)$ are then calculated by applying equations (7) and (8).[1] Compensator no. 1 was recalibrated by this method, half-shade no. 1 being used. Selected results are given in Table VI. and all plotted in Fig. 2.

TABLE VI.

Simultaneous Calibration of Compensator and Measurement of Ellipticity. Compensator No. 1. Half-shade No. 1. $e_0 \neq 0$.

Wave-length ($\mu\mu$).	πN_1	$\frac{1}{2}$ **arc tg** e_0	**Wave-length** ($\mu\mu$).	πN_1	$\frac{1}{2}$ **arc tg** e_0
700	36.80°	29.33°	540	46.50°	30.30°
670	37.88	29.73	500	49.85	29.20
640	39.60	30.00	470	52.92	28.02
600	42.07	30.43	440	55.68	26.76
570	44.12	30.55	400	59.96	24.25

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[1] Since in these equations 2η enters only to produce small corrections, its values were obtained from Table III. and the uncorrected dispersion curve of the compensator as obtained from (7) assuming $\eta=0$.

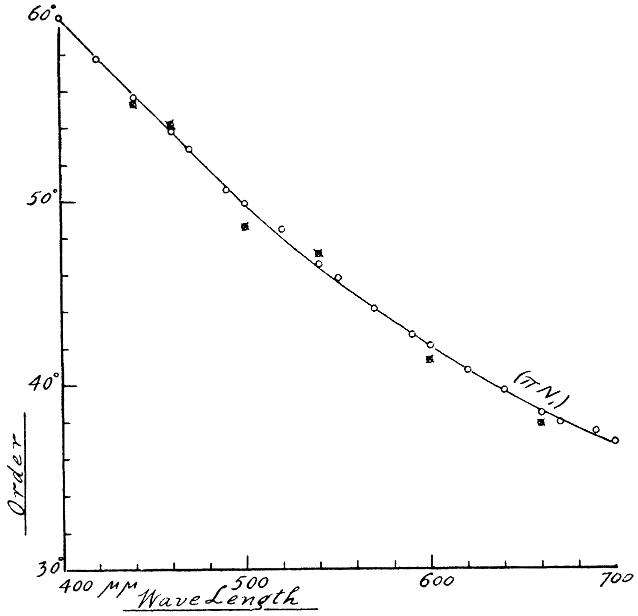

Fig. 2. Dispersion of mica compensator No. 1. (Crossed circles adapted from McComb's calibration of a 6° compensator.)

The accuracy obtained in the above calibrations shows that with the proper refinements these methods may be used to determine the order of half-shade and compensator of the modified Stokes analyzing system, and also for the analyzing parts of any other elliptic half-shade instrument now in use. The differential dispersion of any doubly refracting plate whatever may also be easily and accurately determined by calibrating it as a compensator of this modified system. These results for the differential dispersion of a mica compensator agree (as shown in Fig. 2) with McComb's which were obtained from a plate $\pi N_1 = 3^\circ$ for 600 $\mu\mu$, by an entirely different method.[1]

Since however the value of e_0 may be calculated without that of $2\pi N_1$ being known, the above calibrations for the compensators were used in the following work on metallic reflection only as a check on the results.

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[1] McComb, H. E., Phys. Rev., Vol. XXIX., p. 525, 1909.

II. Optical Constants of Metals.

The dispersion of metals has been studied by means of the two general methods, reflection and transmission. The latter comprises the prism and interferometer methods for obtaining the refractive indices and the use of the spectro-photometer for the absorption coefficients. Because of the difficulties involved in these, the reflection method has been used to a greater extent. This latter method of obtaining the metallic constants is represented by the work of many investigators, notably that of Drude.[1] He carried out a very careful investigation as to the effect of surface conditions, but unfortunately determined the constants for but two points in the spectrum from which no adequate idea of the dispersion throughout the spectrum is to be obtained. A number of other investigations have been made extending over a much wider range, notable among them being the work of Minor.[2] By the use of Voigt's[3] ultra-violet method he obtained the dispersion from 230 to 630 $\mu\mu$ for silver, steel, copper and cobalt. His curves for the refractive indices and absorption coefficients show strikingly the characteristic effect of transmission regions, especially with silver in the ultra-violet and copper in the yellow. With copper the color suggests that the effect is to be found in the visible spectrum. One would expect, as shown below, that gold also would possess similar peculiarities in this region, and perhaps other metals, provided the observations be properly distributed.

Methods and Equations.

The method and apparatus described in Part I. were used for measuring the ellipticity produced in plane polarized light by reflection from metallic mirrors and from these measurements the optical constants of the metals were calculated.

M'Connel's method[4] of an auxiliary nicol mounted upon the spectrometer table was used for adjusting the polarizer at 45° to the plane of incidence. The metal mirrors were mounted on the spec-

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[1] Drude, P., Wied. Ann., Vol. 39, p. 481, 1890.

[2] Minor, R. S., Ann. d. Physik, Vol. 10, p. 581, 1903.

[3] Voigt, W., Physik. Zeitschr., Vol. 2, p. 303, 1901.

[4] M'Connel, J. C., Phil. Mag. (5), Vol. 19, p. 317, 1885.

trometer table by means of a universal adjustment, the adjustments made and the angle of incidence (θ) determined in the customary way. Using compensator no. 1 and half-shade no. 3, the settings for the four different matches were made in every case.

With the *plane of vibration* (electrical) of the incident light as the *plane of reference*, the azimuth of the major axis of the reflected ellipse is given by the expression

$$2\gamma' = (R' + R) - (R_0' + R_0),$$

R' and R being the compensator settings on the reflected, while R_0' and R_0 are the corresponding settings on the incident light. This follows from the fact that the position of the major axis is always given by the expression $[(R' + R)/2 - \pi/4]$ when R' and R are complementary settings[1] with respect to the major axis as the axis of reference.

With the *plane of incidence* as the *plane of reference*, the azimuth of the major axis of the reflected ellipse is given by the corresponding expression

$$2\gamma = (R' + R) - (R_0' + R_0) - 2\varphi', \tag{9}$$

φ' being the azimuth of the incident vibration.

It is evident that for the incident light cot φ' gives the ratio of the component amplitudes in and perpendicular to the plane of incidence; likewise cot φ the corresponding ratio for the reflected light, φ being the azimuth of the restored vibration. If the difference in phase between these components of the reflected light be designated by δ the optical constants for the angle of incidence θ are given by the equations:

$$\nu_\theta = -\frac{\sin\theta \operatorname{tg}\theta(\cos 2\varphi + \cos 2\varphi')}{1 + \cos 2\varphi \cos 2\varphi' - \sin 2\varphi \sin 2\varphi' \cos\delta},$$

$$\nu_\theta \kappa_\theta = \frac{\sin\theta \operatorname{tg}\theta \sin 2\varphi \sin 2\varphi' \sin\delta}{1 + \cos 2\varphi \cos 2\varphi' - \sin 2\varphi \sin 2\varphi' \cos\delta},$$

$$\kappa_\theta = -\frac{\sin 2\varphi \sin 2\varphi'}{\cos 2\varphi + \cos 2\varphi'}, \sin\delta;$$

ν_θ being the refractive index, κ_θ the extinction coefficient per wave-

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[1] A complementary pair of settings satisfies the condition that $\cos c/\cos 2\pi N_1 > 0$. The condition stated by Tuckerman (loc. cit., p. 21) is incorrect.

length in the medium considered, and $\nu_\theta\kappa_\theta$ the same per wave-length *in vacuo* — all for the angle of incidence θ.

For $\varphi' = 45^\circ$ as customarily used, it follows, since from the well known equations $\sin 2\psi = \sin 2\varphi \sin \delta$ and $\cos 2\psi \sin 2\gamma = \sin 2\varphi \cos \delta$, that

$$\left.\begin{aligned}
\nu_\theta &= -\frac{\sin\theta \operatorname{tg}\theta \cos 2\psi \cos 2\gamma}{1 - \cos 2\psi \sin 2\gamma}, \\
\nu_\theta\kappa_\theta &= \frac{\sin\theta \operatorname{tg}\theta \sin 2\psi}{1 - \cos 2\psi \sin 2\gamma}, \\
\kappa_\theta &= -\frac{\operatorname{tg} 2\psi}{\cos 2\gamma}.
\end{aligned}\right\} \qquad (10)$$

The constants for perpendicular incidence ($\theta = 0$) were obtained from the following approximation formulæ which for all cases were accurate to within 0.2 per cent. Omitting terms of the second order they reduce to the equations as used by Drude.

$$\left.\begin{aligned}
\nu_0 &= \nu_\theta\left[1 + \frac{\sin^2\theta}{2\nu_\theta^2(1+\kappa_\theta^2)} + \frac{1}{2}\left(\frac{\nu_\theta^2\kappa_\theta^2}{\nu_\theta^2(1+\kappa_\theta^2)} - \frac{1}{4}\right)\frac{\sin^4\theta}{\nu_\theta^4(1+\kappa_\theta^2)^2} - \cdots\right],^1 \\
\nu_0\kappa_0 &= \nu_\theta\kappa_\theta\left[1 - \frac{\sin^2\theta}{2\nu_\theta^2(1+\kappa_\theta^2)} + \frac{1}{2}\left(\frac{\nu_\theta^2}{\nu_\theta^2(1+\kappa_\theta^2)} - \frac{1}{4}\right)\frac{\sin^4\theta}{\nu_\theta^4(1+\kappa_\theta^2)^2} - \cdots\right], \\
\kappa_0 &= \kappa_\theta\left[1 - \frac{\sin^2\theta}{\nu_\theta^2(1+\kappa_\theta^2)} + \frac{\nu_\theta^2}{\nu_\theta^2(1+\kappa_\theta^2)}\cdot\frac{\sin^4\theta}{\nu_\theta^4(1+\kappa_\theta^2)^2} - \cdots\right]
\end{aligned}\right\} \qquad (11)$$

The principle incidence ($\bar\theta$) and azimuth ($\bar\psi$) were also calculated by the following approximation formulæ which are easily obtained from the rigid formulæ.

$$\left.\begin{aligned}
\sin\bar\theta \operatorname{tg}\bar\theta &= \sqrt{\nu_0^2(1+\kappa_0^2)}\left[1 - \frac{1}{2}\frac{\nu_0^2(1-\kappa_0^2)}{\nu_0^2(1+\kappa_0^2)}\cdot\frac{\sin^2\bar\theta}{\nu_0^2(1+\kappa_0^2)} + \frac{1}{8}\left(2 - 3\left(\frac{1-\kappa_0^2}{1+\kappa_0^2}\right)^2\right)\frac{\sin^4\bar\theta}{\nu_0^4(1+\kappa_0^2)^2} - \cdots\right], \\
\operatorname{tg} 2\bar\psi &= \kappa_0\left[1 + \frac{\sin^2\bar\theta}{\nu_0^2(1+\kappa_0^2)} + \frac{\nu_0^2}{\nu_0^2(1+\kappa_0^2)}\cdot\frac{\sin^4\bar\theta}{\nu_0^4(1+\kappa_0^2)^2} + \cdots\right].
\end{aligned}\right\} \qquad (12)$$

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[1] The retention of ν_θ in both numerator and denominator in these equations renders the computation more convenient.

The reflection coefficient (J) was obtained from the equation

$$J = \frac{\nu_0^2(1 + \kappa_0^2) + 1 - 2\nu_0}{\nu_0^2(1 + \kappa_0^2) + 1 + 2\nu_0}. \qquad (13)$$

Effect of Surface Condition.

Surface films and impurities picked up during the grinding and polishing process are the chief sources of error in obtaining the optical constants by reflection. Since there is no evidence of their presence other than their effect on the magnitude of the constants, and since the different types of films produce different effects, and even the same type may have a different effect for every metal, it is necessary that a large mass of data be collected before it is possible to determine when a good surface has been obtained and even then there is doubt unless the general results of other investigators be taken as a criterion.

Because of the impossibility of producing a good reflecting surface without great risk of these disturbing effects, there has been considerable investigation concerning them. The results indicate that in general the films formed during the polishing process, whether arising from impurities or other causes, tend to increase the value of all the constants, especially the absorption and reflection coefficients. The type of film, however, which is formed when the surface is exposed to the air, usually has the opposite effect. Considerable study of this latter effect was made in the present investigation and it was observed that, with all metals, films were formed in a comparatively short time. Gold and nickel were the least affected of those investigated, while copper and zinc surfaces underwent an appreciable change with only an hour's exposure even in a very dry atmosphere. Unless care is taken, oil and moisture films form during the polishing process and also by exposure. The effect of these more or less transparent films, if they are very thin and have not attacked the surface, is comparatively small, as was found by testing a steel mirror before it was thoroughly cleansed.

A matt surface or incomplete polish also reduces the value of the constants, the greatest change again being in κ_0 and J. That a granular or crystalline structure is productive of considerable error is revealed by changing the angle of incidence. This effect was most noticeable in the case of a cathode film of cobalt which when inspected by a low power microscope appeared to have a smooth glassy surface, but under a high power showed a very fine granular structure. The magnitude of the fluctuation in ν_0 and $\nu_0\kappa_0$ for a change of 30° in the angle of incidence was in this case about 5 per cent. though when carefully polished most fused metals do not show greater than 0.5 per cent. fluctuations.

Because of the large effect exercised by these surface conditions it is difficult to obtain results that will agree closely even when the changes in the method of polishing are slight. It must also be concluded from the large variation in the results produced by entirely different methods that it is impossible to burnish or polish a metal without mixing the surface with impurities which enter so deeply that no "purifying" process whatever will remove them. Furthermore, the oxide films for which such processes are chiefly intended, are in all probability formed after the surface has been fully prepared. One is inclined to question also whether burnishing and even polishing does not change the original structure of the metal sufficiently to cause a change in the constants.

The various investigators have used widely different methods in producing and treating their mirrors, and the results obtained have also differed widely. It appears, therefore, that the true value of the metallic constants cannot be determined to better than a probable accuracy of two to three per cent. It is important to note, though, that the *form of the dispersion curve* seems to depend much less upon the surface conditions and hence is more reliable.

In view of the above facts, several methods[1] of preparing mirrors were tried while endeavoring to produce a good reflecting surface with the least chance of contamination. The best results were obtained by first making the surface as smooth and plane as possible by the use of a fine clean file, then finally grinding with very fine emery dusted over soft dry paper laid upon a plane piece of glass. The mirror was then polished with rouge on kid or sometimes in the case of the softer metals on clean kid.

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[1] Several kinds of silver polish were tried and very brilliant mirrors obtained, but it was feared that the moisture or oils in them would produce disturbing surface films.

In this way, very good mirrors were obtained which were quite free from scratches. Difficulty was encountered in securing plane surfaces only when polished too long upon the leather. All mirrors were uniform over the whole surface though an area of 3 × 4 mm. only was needed.

EXPERIMENTAL RESULTS.

Nickel.[1] — The mirrors were made of pure rolled nickel (from Eimer & Amend) and were easily given a brilliant reflecting surface. Several series of observations were taken on different mirrors, these showing a large variation in the value of the constants. Those given in Table VII. for selected wave-lengths were obtained under

TABLE VII.

Nickel.[2] $\theta=65°$.

Wave-length ($\mu\mu$).	ν_0	κ_0	$\nu_0\kappa_0$	$\bar{\theta}$	$\bar{\psi}$	J
700	2.025	1.965	3.980	77.83°	32.05°	.676
660	1.945	1.975	3.840	77.42	32.15	.668
620	1.820	1.985	3.610	76.70	32.30	.653
580	1.725	1.975	3.405	76.00	32.30	.637
540	1.625	1.965	3.200	75.25	32.35	.621
500	1.540	1.930	2.975	74.33	32.25	.597
460	1.460	1.885	2.750	73.37	32.10	.572
420	1.415	1.790	2.530	72.33	31.70	.537

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[1] Kundt, A., Wied Ann., Vol. 34, p. 469, 1888. DuBois and Rubens, Ann. d. Physik, Vol. 8, p. 17, 1902.

[2] For the sake of brevity, the following example only is given to show the magnitude of the various observed quantities and the method of calculating the constants in this table:

Constants of the instrument:
$R_0'+R_0=267.60°$ $\phi'=45°$ $2\eta=1.93°$

Observations for $\lambda=540\,\mu\mu$, $\theta=65°$:
$R'=287.88°$ $r'=334.25°$ $R'+R=480.03°$
$R=192.15$ $r=292.99$ $c+n=137.08$
$c=95.73$ $n=41.35$ $c-n=54.38$

First calculations (from equations (7), (8), (9) and (10)):
$\pi N_1=46.25°$ $2\gamma=122.43°$ $2\psi=48.40°$
$\nu_\theta=1.575$ $\kappa_\theta=2.090$ $\nu_\theta\kappa_\theta=3.300$

Final results (from equations (11), (12) and (13)):
$\nu_0=1.625$ $\kappa_0=1.965$ $\nu_0\kappa_0=3.200$
$\bar{\theta}=75.25°$ $\bar{\psi}=32.35°$ $J=0.621$

what were considered the best surface conditions, observations having been taken within three hours after the mirror had been carefully polished. In general the other mirrors gave lower results for ν_0 and $\nu_0\kappa_0$, though κ_0 was usually larger. The lowest values for ν_0 were obtained from a highly polished mirror two weeks after it had been prepared. For this, ν_0 was found to be 1.63 (λ = 580) which is

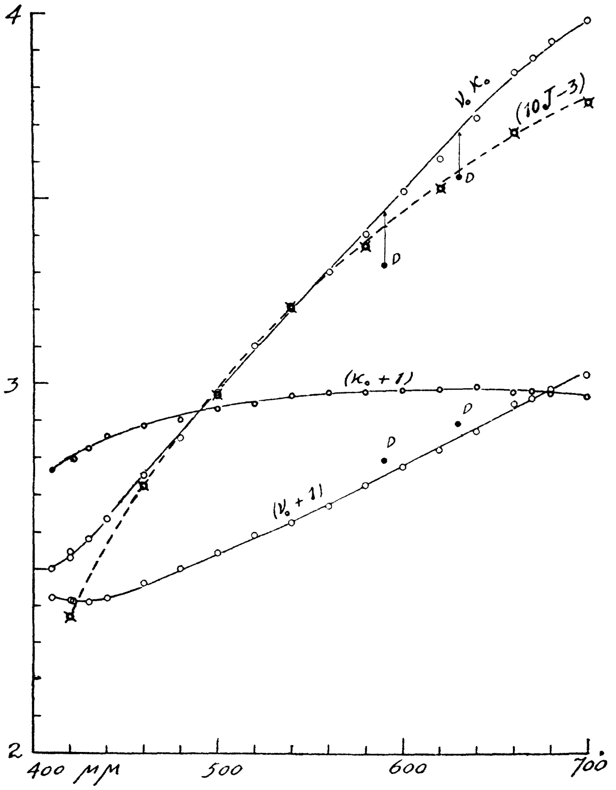

Fig. 3. *Nickel.* Drude's values for ν_0 and $\nu_0\kappa_0$ indicated by *D*.

about 6 per cent. lower than the value given in the table, while κ_0 was 2.05 or 3 per cent. higher. It is noteworthy that the slope and form of the curves through the spectrum were, however, the same in the various cases. Neither was there any great variation observed on changing the angle of incidence, there being in the case of one of the poorer mirrors only about 0.5 per cent. deviation between 55° and 75°.

On the curves (Fig. 3) are indicated all the values obtained from the chosen series of observations. To bring the various curves within the same plot their ordinates were changed as indicated.

The refractive indices (ν_0) are about 2 per cent lower than Drude's (indicated by D) while the absorption coefficients (κ_0) are 6 per cent. higher. The curves lie parallel to the two values he gives showing the same dispersion in that region of the spectrum. The values for the principal incidence (θ) and azimuth ($\bar{\psi}$) are higher than Drude's, but agree closely with those of Quincke.[1] Excepting those at the blue end, which are about 5 per cent. lower, the reflection coefficients (J) are about the same magnitude as those given by Hagen and Rubens.[2]

Some evidence of a transmission region in the extreme violet is shown by both the refractive index (ν_0) and the absorption coefficient (κ_0) curves.

Cobalt.[3] — The cobalt mirrors were made of the pure metal. They showed about the same effects of polish as did those of nickel. The results given in Table VIII. and Fig. 4 were obtained from three series of observations on the same mirror but at different angles of incidence. The refractive index is 3 per cent. higher and the absorption coefficient about 4 per cent. lower ($\lambda = 590$) than given by Drude. This divergence is less than Minor's original observations show but he corrected his curves to agree with the values given by Drude, the dispersion being the same.

Table VIII.

Cobalt.

Wave-length ($\mu\mu$).	ν_0	κ_0	$\nu_0\kappa_0$	$\bar{\theta}$	$\bar{\psi}$	J
680	2.500	1.775	4.440	79.20°	30.75°	.687
640	2.340	1.815	4.245	78.70	31.05	.679
600	2.210	1.825	4.030	78.15	31.15	.667
560	2.090	1.825	3.810	77.50	31.20	.653
520	1.975	1.830	3.615	76.90	31.35	.640
480	1.855	1.835	3.405	76.15	31.45	.623
440	1.705	1.870	3.190	75.30	31.75	.610
420	1.680	1.830	3.070	74.90	31.60	.596

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[1] Quincke, G., Pogg. Ann. Jubelbd., p. 345, 1874.

[2] Hagen & Rubens, Ann. d. Physik, Vol. 8, p. 17, 1902.

[3] Drude, P., Wied. Ann., Vol. 42, p. 189, 1891.

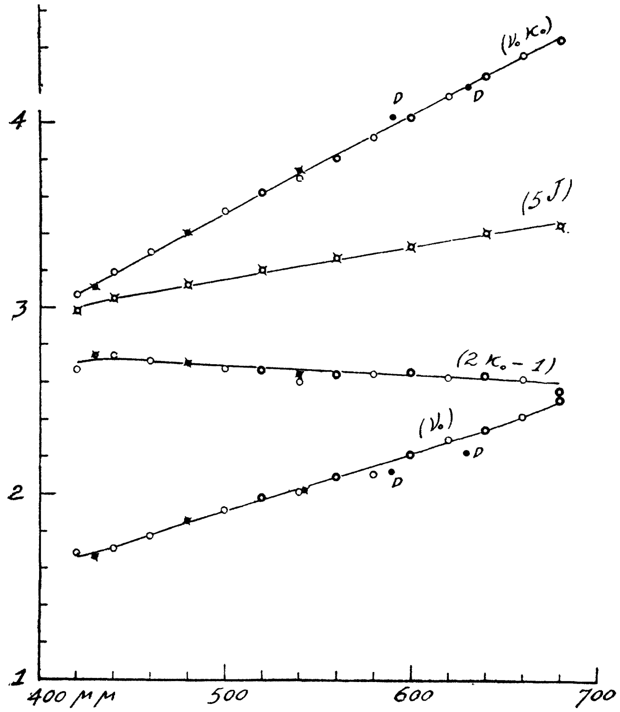

Fig. 4. *Cobalt.* Rings: $\theta = 75^\circ$. Circles: $\theta = 70^\circ$. Crossed circles: $\theta = 60^\circ$. Drude's values indicated by *D*.

Steel.—Two mirrors made from file steel which gave a good reflecting surface were tested. Different methods of polishing gave

Table IX.

Steel. $\theta = 70^\circ$.

Wave-length (μμ).	ν_0	κ_0	$\nu_0\kappa_0$	$\bar{\theta}$	$\bar{\psi}$	J
700	2.73	1.270	3.47	77.60	26.60	.580
690	2.72	1.270	3.46	77.55	26.60	.579
660	2.67	1.280	3.41	77.40	26.65	.575
620	2.58	1.280	3.30	77.05	26.70	.568
580	2.46	1.315	3.24	76.70	27.15	.564
560	2.40	1.340	3.21	76.65	27.40	.561
520	2.26	1.380	3.12	76.05	27.90	.556
480	2.07	1.455	3.01	75.40	28.65	.552
440	1.86	1.545	2.87	74.55	29.55	.547
420	1.79	1.575	2.82	74.15	29.80	.545

a considerable range of values for the constants. The series of observations chosen as representative were taken on a rouge polished surface. One with its surface prepared a year before gave for ν_0 the value 2.18 (580 $\mu\mu$) which is 10 per cent. lower than Drude's indicate; and for κ_0 the value 1.43 which is about 4 per cent. higher than his. Repolished this mirror gave the results in Table IX. and Fig. 5.

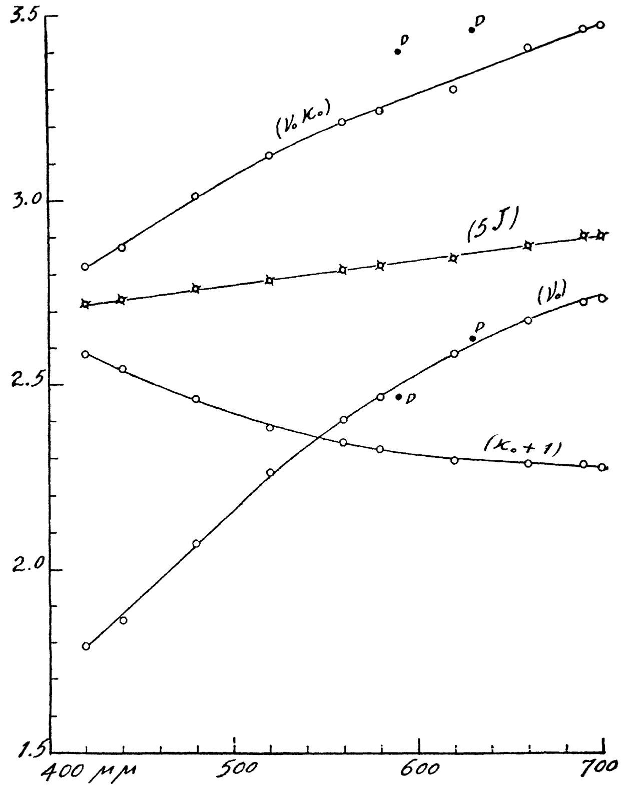

Fig. 5. *Steel.* Curves as obtained under "best" conditions.

Copper.[1] — The mirrors finally used were of ordinary commercially "pure" copper as the chemically pure obtained for the purpose was too full of flaws to give a satisfactory reflecting surface. On account of the rapid tarnishing it was found necessary to take the

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[1] Shea, D., Ann. d. Physik, Vol. 47, p. 202, 1892.

observations immediately after the mirror had been polished. The time effect reduces the value of all the constants much more rapidly at first than after several hours. This is probably the reason why Minor did not note an effect of this kind. Table X. shows the change observed in the value of the constants with time. In Fig. 6 are given curves which show the great variation arising from different treatment in polishing. Table XI. and Fig. 7 give values obtained under what was considered the best conditions, although curves *b* and *b′* in Fig. 6 represent values which agree much more closely with Drude's. Those chosen differ considerably from his, ν_0 being over 9 per cent. lower (590 $\mu\mu$), while κ_0 is about 25 per cent.

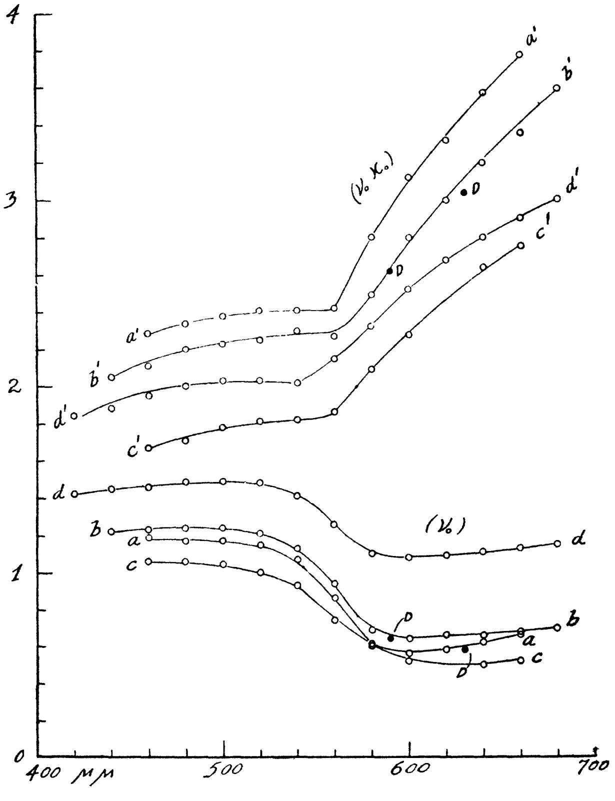

Fig. 6. *Copper.* Curves *a* and *a′* obtained under "best" conditions; *b* and *b′* from a mirror with incomplete polish; *c* and *c′* from one oxidized by heating; *d* and *d′* from one oxidized by exposure to moist air.

TABLE X.

Copper—Time Effect.

Wave-length ($\mu\mu$).	**Taken Immediately.**		**After Three Hours.**		**After Twenty Hours.**	
	ν_0	$\nu_0\kappa_0$	ν_0	$\nu_0\kappa_0$	ν_0	$\nu_0\kappa_0$
640	.615	3.580	.625	3.490	.605	3.395
600	.565	3.115	.555	3.040	.535	2.970
580	.605	2.720	.605	2.685	.570	2.620
540	1.070	2.410	1.050	2.395	1.025	2.345
500	1.170	2.380	1.170	2.370	1.170	2.350
460	1.180	2.290	1.175	2.250	1.155	2.210

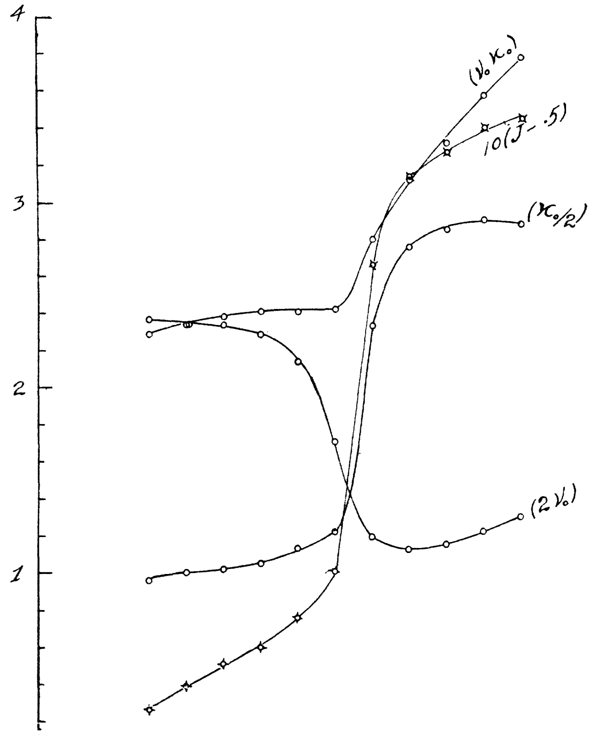

Fig. 7. *Copper.* Curves as obtained under "best" conditions.

higher. The form of the dispersion appears to be different also, his values suggesting a continuous slope toward the red, while the curves in Fig. 6 show in every case a definite rise from about 600 to 700 $\mu\mu$. The results of Minor in this region apparently agree with those of Drude though not necessarily since he calculated his value for 630 $\mu\mu$ from the dispersion given by Drude. All curves show the characteristics of a transmission band at about 560 $\mu\mu$.

Because of the large values of ν_0 and $\nu_0\kappa_0$ the calculated values for $\bar{\theta}$, $\bar{\psi}$ and J are also larger than usually given by other investigators.

TABLE XI.

Copper. $\theta = 60°$.

Wave-length ($\mu\mu$).	ν_0	κ_0	$\nu_0\kappa_0$	$\bar{\theta}$	$\bar{\psi}$	J
660	.655	5.770	3.780	76.20°	40.40°	.846
640	.615	5.820	3.580	75.55	40.45	.841
620	.580	5.720	3.320	74.60	40.40	.828
600	.565	5.510	3.115	73.80	40.30	.814
580	.600	4.660	2.795	72.35	39.55	.767
560	.855	2.830	2.420	70.75	36.35	.601
540	1.070	2.250	2.410	71.05	34.15	.576
520	1.145	2.105	2.410	71.15	33.60	.560
500	1.170	2.030	2.375	71.05	33.20	.551
480	1.170	1.995	2.335	70.80	33.10	.539
460	1.185	1.930	2.285	70.60	32.75	.526

Gold.[1]—Three series of observations were taken on a mirror of "pure" gold procured from Sargent & Co., Chicago. The values in the first series (Fig. 8, curves b and b') agree closely with those of Drude for 18-*carat* gold, but in the second (represented by the circles on curves a and a' in Fig. 8) taken after the mirror had been reground and polished, the results are about the magnitude of those obtained by other investigators for *pure* gold. It is remarkable to note that while the product $\nu_0\kappa_0$ is the same for both throughout the spectrum, the separate values of ν_0 and κ_0 are alike in the blue only and diverge widely in the red. The third series (represented by dots on curves a and a') was taken after the mirror had been again thoroughly repolished with rouge. The values in this are practically identical with those of the second series. Table XII. and Fig. 9 give the average value of the second and third series.

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[1] Shea. D., loc. cit.

Since the gold mirrors change very little by exposure the variation shown is probably to be ascribed to the polishing, especially since the observations in all three cases were taken within a few hours

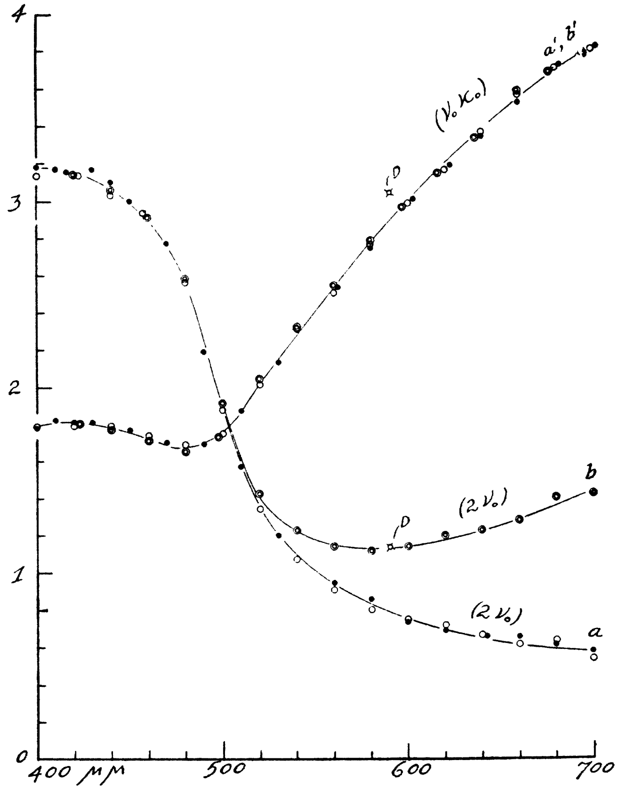

Fig. 8. *Gold.* Curves *b* and *b'* obtained from first series of observations; circles on *a* and *a'* from second series; dots on *a* and *a'* from third. Drude's values for *18 carat* gold as indicated.

after the mirror had been prepared. In view of these results it is probable that the comparatively low value obtained by Drude for the absorption coefficient of 18-carat gold was due to the condition of the surface, since it would be surprising to find the absorption of an alloy of gold, copper and silver less even than the average of gold and copper—the value for silver being greater than that for gold.

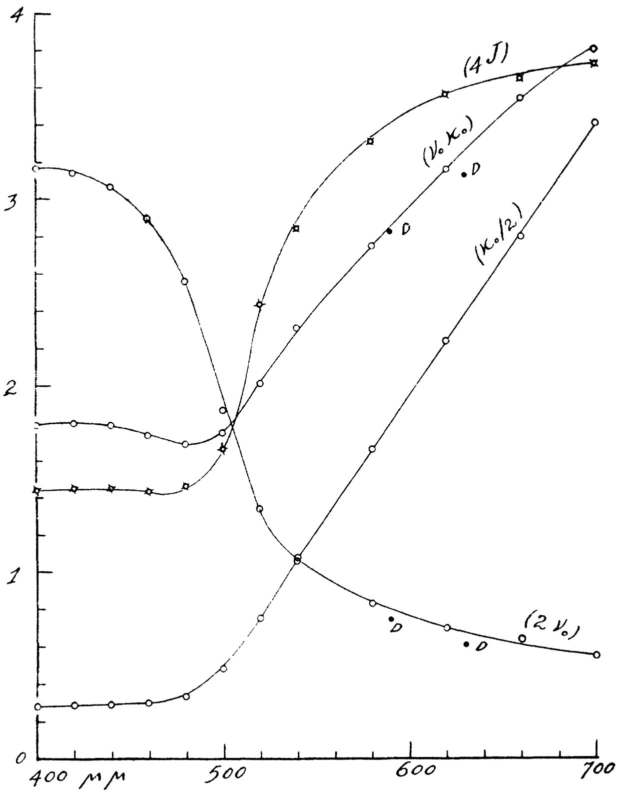

Fig. 9. *Gold.* Curves as obtained from the mean of the second and third series of observations. Drude's values for *pure* gold as indicated.

Table XII.

Gold. $\theta = 60°$.

Wave-length ($\mu\mu$).	ν_0	κ_0	$\nu_0\kappa_0$	$\bar{\theta}$	$\bar{\psi}$	J
700	0.280	13.60	3.800	76.10°	43.05°	.930
660	0.320	11.20	3.540	75.30	42.65	.910
620	0.350	8.95	3.160	73.90	42.10	.889
580	0.415	6.65	2.750	72.05	41.15	.827
540	0.535	4.32	2.305	69.70	39.35	.710
520	0.670	2.99	2.010	67.90	37.15	.608
500	0.935	1.870	1.750	66.55	33.15	.415
480	1.280	1.315	1.685	67.15	28.85	.364
460	1.450	1.200	1.740	68.20	27.25	.358
440	1.535	1.165	1.790	68.80	26.80	.362
420	1.570	1.145	1.800	68.95	26.55	.362
400	1.580	1.130	1.785	68.90	26.35	.360

As suggested by the color of the reflected light, the dispersion curves for gold show the effect of a transmission band, the center of which lies in the neighborhood of 490 $\mu\mu$. The form of the curves is much the same as those of copper, in which the transmission band is near 560 $\mu\mu$.

RÉSUMÉ.

An instrument which combines the Stokes and the half-shade principles for measuring ellipticity has been successfully used. It has been shown that this instrument will measure the order of thin doubly refracting plates, such as used in the Brace elliptic analyzer to 0.01°, which is an error of less than one per cent. for the thinnest half-shade so far advantageously employed. It will also calibrate a quarter-wave plate to at least 0.05°, an accuracy of about 0.05 per cent.; or, it will measure an ellipticity of 0.02 to at least one per cent., and an ellipticity of approximately unity to 0.05 per cent. The introduction of the elliptic half-shade enables it further to measure circularly polarized light—where Stokes's analyzer fails.

This method has been applied to the investigation of the optical constants of metals by reflection and determinations have been made throughout the visible spectrum. The maximum sensibility was not attempted here since the variation arising from surface conditions rendered such accuracy useless.

With all the metals studied observations were taken at sufficiently frequent intervals to determine the direction of the dispersion curve. This dispersion is found to agree in general with that of other investigators though the magnitude of the constants chosen as representative is often different.

Of especial interest are the results from copper and gold, showing the characteristics of a transmission band in the visible spectrum, the existence of which for copper, Minor has already shown.

In conclusion the writer wishes to thank Professors Skinner and Tuckerman for the aid given him.

THE BRACE LABORATORY OF PHYSICS,
THE UNIVERSITY OF NEBRASKA