MAGNETOSTRICTION IN IRON-CARBON ALLOYS.

By Herbert G. Dorsey.

SINCE Joule's discovery in 1842 that iron changes its dimensions when magnetized, much experimental work has been done on this interesting phenomenon. But of the numerous articles published I have found none which gives a complete chemical analysis of an iron specimen.

The excellent series of steel rods of known analysis used by Dr. C. W. Waggoner[1] being available it seemed desirable to make tests on their changes in length in a magnetic field.

After studying the various magnifying devices used by different workers and as a result of a few preliminary trials the method used by Guthe and Austin[2] in their work on magnetostriction in Heusler alloys was employed with but few changes.

Fig. 1 gives a general view of the entire apparatus with details of the magnifier. The steel rod shown in black was soldered into the ends of brass rods B and B' and the latter was clamped rigidly at one end of the frame of brass tubing. At the other end of this frame was attached the wooden base of the magnifier. This consisted of a glass plate freely movable on roller bearings of needles. Two glass plates were fixed at right angles in a block of wood and the third larger plate was held to the vertical needles by a rubber band R. The two proximate vertical surfaces were ground together with fine emery and this seemed essential to prevent slipping in the greater changes of length. The rubber band not only holds the plate in place but presses it lightly against the rod B so that any motion of B is communicated to the glass plate. To one of the vertical needles was fastened a glass pointer P, which was made by drawing a glass tube very small, bending at right angles near the larger end and fastening the point of the needle into the tube with

[1] Phys. Rev., Vol. 28, p. 393, 1909.
[2] Bul. Bur. Stds., Vol. 2, p. 297, 1906.

sealing wax. Opposite this needle in a hole in the wooden block was placed a glass tube bent at right angles, the other end being drawn down to a point. This glass arm being thus pivoted near the rotating needle gives a method of quick rough adjustment without changing much the sensibility. To the ends of the glass arm and pointer were fastened two silk fibers which supported the

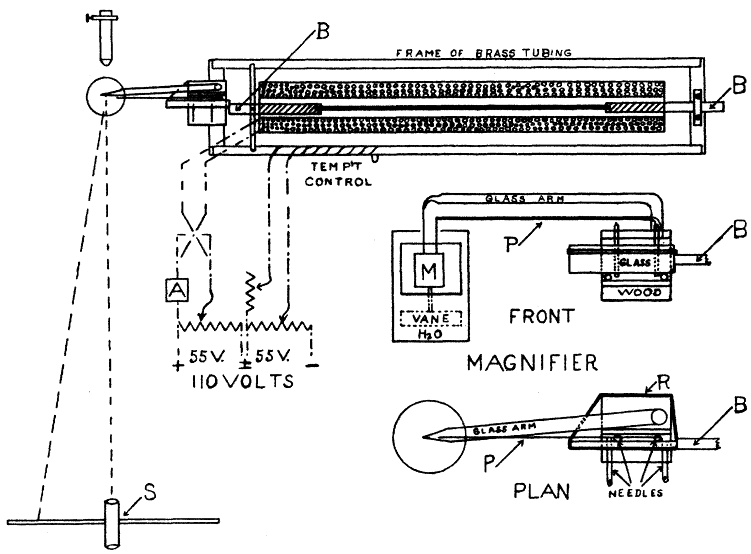

Fig. 1.

tiny mirror M. The mirror carried a small glass vane under water which rendered it nearly although not quite aperiodic. Mirror, vane and water were in a brass tube with glass window. The deflections of the mirror were observed by the telescope and scale.

If a = diameter of the needle = 0.0652 cm.
b = distance between threads = from 0.08 to 0.2 cm.
c = scale deflection.
d = distance scale to mirror, usually about 120 cm.
e = length of glass pointer = 6.90 cm.
l = length of rod tested,

then

$$\frac{\Delta l}{l} = \frac{abc}{2del}.$$

Scale deflections could be easily read to 0.1 cm. so that the smallest change in length which could be easily detected was

$$\frac{abc}{2de} = \frac{.0652 \times .08 \times .01}{2 \times 120 \times 6.90} = 3.15 \times 10^{-7} \text{ cm.}$$

At first a number twelve needle of 0.035 cm. diameter was used and a pointer 15 cm. long, but the magnifier was so much more sensitive than was necessary for such long rods that the larger needle and shorter pointer were chosen. With all of its high magnification, the parts were all so light that there seemed to be no lost motion of any sort and there was no appreciable trouble from vibrations. The entire apparatus was tested with a brass rod in the place of the iron rod and absolutely no effect was observable when the strongest magnetizing current was applied.

Temperature Compensation.

With a magnification of 300,000 a change in temperature of one degree for a 40 cm. rod would produce a deflection of more than a hundred scale divisions so that efforts to maintain a constant temperature were soon abandoned and compensation was employed. On the side tubes of the brass frame a few turns of fine insulated advance wire were wound, through which was sent a small current which could be closely adjusted by means of the shunt and rheostat shown in Fig. 1. This current warmed the brass frame and by its expansion the magnifier was moved forward an amount equal to the increase in length of the steel rod due to temperature changes. The changes in length due to magnetism are so nearly instantaneous and the magnifier responds so quickly that no trouble was experienced in getting readings before temperature produced any effect. In any case the temperature changes produce slow drifts while the magnetism changes are sudden, so that there is no difficulty in distinguishing between the two. With the harder steels the warming due to hysteresis of a single reversal of a current of one ampere could be detected.

Magnetization.

The magnetizing coil contained 2,240 turns of number 10 copper wire wound on a brass tube of 1.1 cm. external diameter and 70 cm. length. An alternate current of about 50 amperes was sent through it a few minutes while containing an iron core as a test for short circuits, for if there were any the short-circuited turns would get hotter than the rest of the coil. The heating was uniform, indicating no short circuits, but the coil length contracted permanently to 69.5 cm. The calculated value of the field strength is 40.5 gausses per ampere. This was checked by a small exploring coil, the same one described later, in two other long slim solenoids and the calculated values for all three differed by less than one per cent. The coil was then explored with the test coil and for a distance of over 40 cm. the field varied by not more than 1.3 per cent.

Current was taken from a motor generator set having nearly perfect automatic regulation of field excitation. To one side of the 55-55-110-volt circuit was shunted a tin resistance frame and from this 28 wires were connected to a dial switch so that currents through the magnetizing coil could be varied by steps from 0.05 to 42.5 amperes. Currents were measured by a Weston milli-voltmeter with two shunts, one for currents 0 to 5 amperes, the other 0 to 50 amperes.

All values of field strength H and magnetic intensity I, unless otherwise noted, are corrected for the demagnetizing effect of the ends of the rods. These corrections for H vary from 15 per cent. for $A3$ to more than 100 per cent. for PI. The values of m = length/diameter are given in Table X. Magnetization curves were taken by the ballistic method. The ballistic coil already mentioned as a test coil consisted of 185 turns of number 40 wire wound upon a brass tube of 0.80 cm. external diameter. The coil length was 3.1 cm. The galvanometer was of the D'Arsonval type of medium period and large damping. It was calibrated by means of the test coil used in two different long solenoids with several different currents measured by the same ammeter used in the rest of the work. A line was plotted from these different values and the galvanometer constant determined from the slope of the curve. It was also checked by a standard cell and condenser, proper corrections for logarithmic decrement being made for all measurements.

In taking data for the magnetization curves the specimen was first demagnetized by reversals and then the desired current reversed at least 25 times before the galvanometer throw was recorded and a second reading taken as a check. The current was then increased, 25 more reversals made and the throw recorded and this process repeated until the entire range was covered. With the harder steels 35 to 40 reversals were necessary. This approximately gives what C. W. Burrows[1] calls a normal magnetization curve.

In order to find the correction to be applied to the values of H for the length of ballistic coil used a magnetization curve for PI was made by the magnetometer method, and corrections applied for the demagnetizing effects of the rod ends according to the method of C. R. Mann.[2] The corrected curve was then plotted along with the curve made by the ballistic method and their differences found, from the average of which it appeared that if Mann's values of the correction factor N were divided by 1.66 the corrected values of I could be found up to $I = 800$. Above this value a sliding scale was used, and it is believed that the corrected values are not far from the truth. The values of N for the different specimens are given in Table X. This method of making corrections appears to be reliable for it is checked by an entirely different method. In Fig. 7, where the uncorrected susceptibility of $D1$ and $D2$ are plotted as a function of diameter divided by length these curves cross the Y axis at $K = 141$ and 115 respectively, which would represent the values in rings or rods without poles, and the values by correction are 139 and 113 respectively. The magnetic tests were made after the magnetostriction tests. To find values of I and H corresponding to values of current used in the magnetostriction tests large curves were plotted for each specimen and values taken from them.

The value of I, the intensity of magenetism, is given by the formula

$$I = \frac{\frac{\varphi}{n} - AH'}{4\pi\alpha - AN},$$

[1] Bul. Bur. Stds., Vol. 4, p. 205, 1908.
[2] PHYS. REV., Vol. 3, p. 367, 1895.

in which 2φ is the change in flux for a reversal of the current, n is the number of turns of the ballistic coil, = 185, A is the area of the ballistic coil, = 0.503 sq. cm., α = area of the specimen, N = the correction factor, and H' = uncorrected field strength = 40.5 × current in amperes.

$$2\varphi = q_0\delta(1 + \lambda/2)R \times 10^8,$$

in which q_0 = the quantity constant of the galvanometer = 100.7 × 10^{-9}, δ = the throw of the galvanometer, λ = the logarithmic decrement and R = the total resistance in the circuit. For any given specimen the above formula reduces to

$$I = a\delta(1 + \lambda/2)R - b \times \text{current},$$

where a and b are constants.

Specimens.

The rods were first tested in the following condition as received from Dr. Waggoner. They were about 40 cm. long and about .5 cm. in diameter, the ends being drilled longitudinally for about .5 cm. for lathe centers when they were turned down from the original rods. They were then annealed at 1000° for two hours and his magnetic tests made at room and liquid air temperatures. The chemical analysis is here repeated as given in his paper.

Chemical Analysis.

Mark.	C	P	Si	Mn	S
P.I.	.058	Trace	.008	.071	—
A1	.60	.013	.15	.14	.012
A2	.74	.012	.16	.14	013
A3	.89	.010	.19	.155	.013
A4	.98	.012	.16	.15	.013
A5	1.18	.012	.14	.14	.013
A55	1.26	.012	.16	.17	.014
A6	1.37	.011	.19	.16	.01?

Magnetostriction Tests.

The following procedure was taken in all measurements on magnetostriction. The specimen being placed in the coil and adjustments made was demagnetized by reversals. The desired current applied and deflection and current noted. Specimen again demagnetized and the same current applied in the opposite direction and deflection noted. This was then repeated, thus giving four readings for each step of the current. The deflections given in the following tables are the average of four measurements, each taken with the current in a direction opposite to that used in the preceding measurement. The last three scale deflections are exceptions and are the average of only two measurements on account of excessive heating of the higher currents.

TABLE I.

PI, 0.058 *Per Cent. Carbon.* $b=0.2892$ *cm.*, $d=120.5$ *cm.*, $l=39.5$ *cm.*

H	I	$\Delta l/l$	H	I	$\Delta l/l$
1.96	270	$.17\times10^{-6}$	63.9	1,308	3.24×10^{-6}
2.93	486	.37	73.0	1,322	3.10
4.33	712	.83	93.7	1,350	2.76
5.00	800	1.26	142.5	1,414	1.81
6.49	924	1.64	214.	1,486	.32
7.80	1,022	2.04	242.	1,511	− .26
11.00	1,115	2.50	300.	1,545	−1.55
14.0	1,163	2.84	352.	1,572	−2.57
18.4	1,202	3.13	493.	1,611	−4.88
24.0	1,230	3.30	580.	1,624	−5.57
29.2	1,244	3.39	757.	1,643	−6.58
36.8	1,260	3.45	1,090.	1,660	−7.41
44.3	1,278	3.45	1,580.	1,675	−8.05
52.0	1,292	3.39			

TABLE II.

A1, 0.60 *Per Cent. Carbon.* $b=0.1568$ *cm.*, $d=120.5$ *cm.*, $l=39.5$ *cm.*

H	I	$\Delta l/l$	H	I	$\Delta l/l$
4.7	263	$.14\times10^{-6}$	77.6	1,184	1.78×10^{-6}
6.8	413	.28	110.	1,240	1.41
8.1	490	.43	157.	1,298	.74
9.6	572	.56	194.	1,334	.16
11.4	653	.71	214.	1,344	− .11
14.0	760	.89	239.	1,370	− .50
17.7	870	1.07	272.	1,395	−1.03
24.0	972	1.45	353.	1,434	−2.30
30.0	1,022	1.54	428.	1,472	−3.17
36.4	1,058	1.67	583.	1,520	−4.76
43.0	1,090	1.68	770.	1,550	−5.80
49.6	1,120	1.70	1,190.	1,595	−6.63
59.5	1,150	1.73	1,685.	1,640	−7.35

TABLE III.

A2, 0.74 Per Cent. Carbon. $b=0.1205$ *cm.*, $d=121$ *cm.*, $l=39.6$ *cm.*

H	I	$\Delta l/l$	H	I	$\Delta l/l$
5.0	180	$.02\times10^{-6}$	61.5	1,162	1.40×10^{-6}
6.4	280	.07	70.1	1,180	1.39
8.0	386	.15	90.1	1,215	1.28
9.0	450	.20	138.	1,280	.76
10.5	526	.27	205.	1,346	− .12
13.4	650	.40	230.	1,365	− .44
15.7	732	.56	290.	1,404	−1.24
20.5	840	.77	342.	1,430	−1.90
25.8	922	.95	490.	1,482	−3.40
30.1	974	1.06	575.	1,503	−3.94
37.5	1,044	1.19	760.	1,532	−4.85
43.5	1,076	1.31	1,110.	1,571	−5.55
50.9	1,127	1.38	1,660.	1,618	−6.11

TABLE IV.

A3, 0.89 Per Cent. Carbon. $b=0.1385$ *cm.*, $d=120$ *cm.*, $l=39.4$ *cm.*

H	I	l/l	H	I	$\Delta l/l$
9.0	228	$.01\times10^{-6}$	73.5	1,130	$.68\times10^{-6}$
10.5	290	.03	90.0	1,175	.62
12.5	375	.04	138.	1,268	.29
16.6	545	.07	207.	1,333	.37
17.3	552	.14	230.	1,350	− .68
22.5	700	.24	290.	1,385	−1.37
28.0	790	.32	347.	1,410	−1.86
32.5	850	.40	482.	1,456	−3.07
39.5	928	.49	568.	1,474	−3.64
46.5	988	.56	760.	1,510	−4.52
54.0	1,036	.62	1,100.	1,550	−5.45
65.0	1,095	.67	1,635.	1,615	−6.11

In Fig. 2 the percentage change in length is plotted as a function of H for the first five rods of the series. The others are not plotted as they are so close to $A2$ that plotting them in this figure would only tend to produce confusion.

Fig. 3 shows the relation between magnetostriction and the intensity of magnetization.

Data for residual magnetostriction curves are plotted for specimens $A4$ and $A6$ in Fig. 4. To get these a reading of the telescope

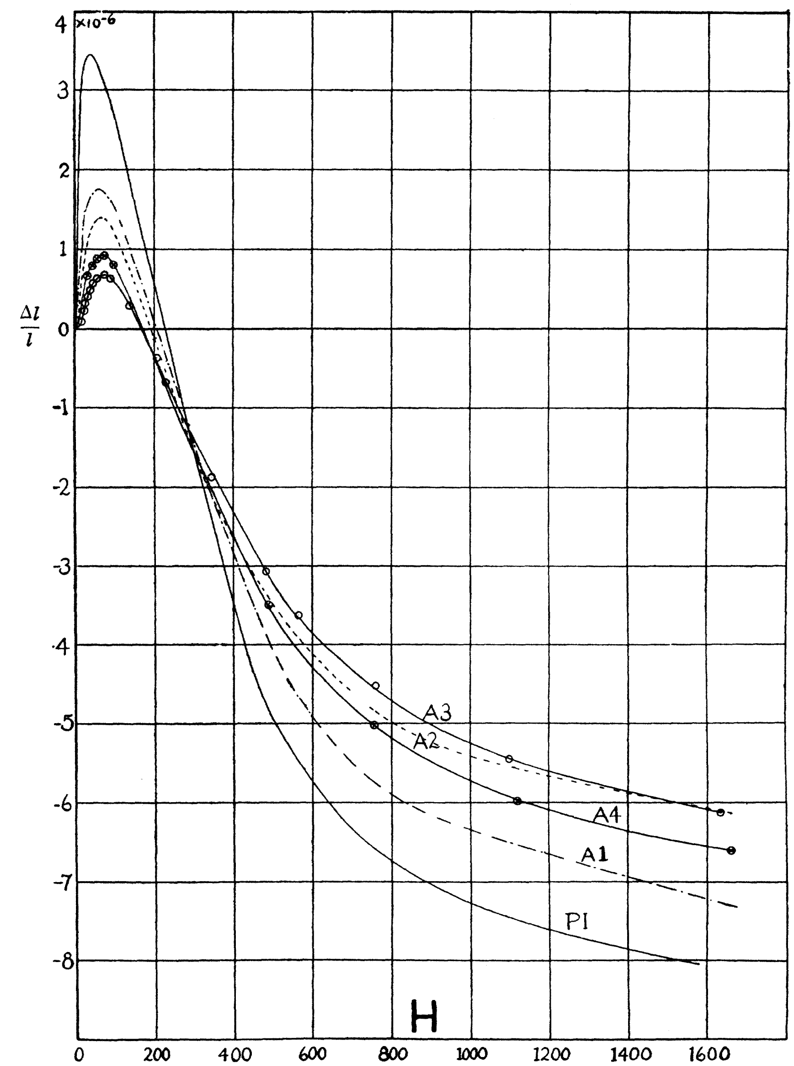

Fig. 2.

Magnetostriction as a function of field strength.

was made after the circuit was broken and as the deflection is then in the opposite direction as when the circuit is closed deflections for residual are recorded with an opposite sign. In the specimens magnetically soft scarcely any residual effect could be found but those which are harder remain longer after being magnetized unless the actual field strength reaches about 400 or more.

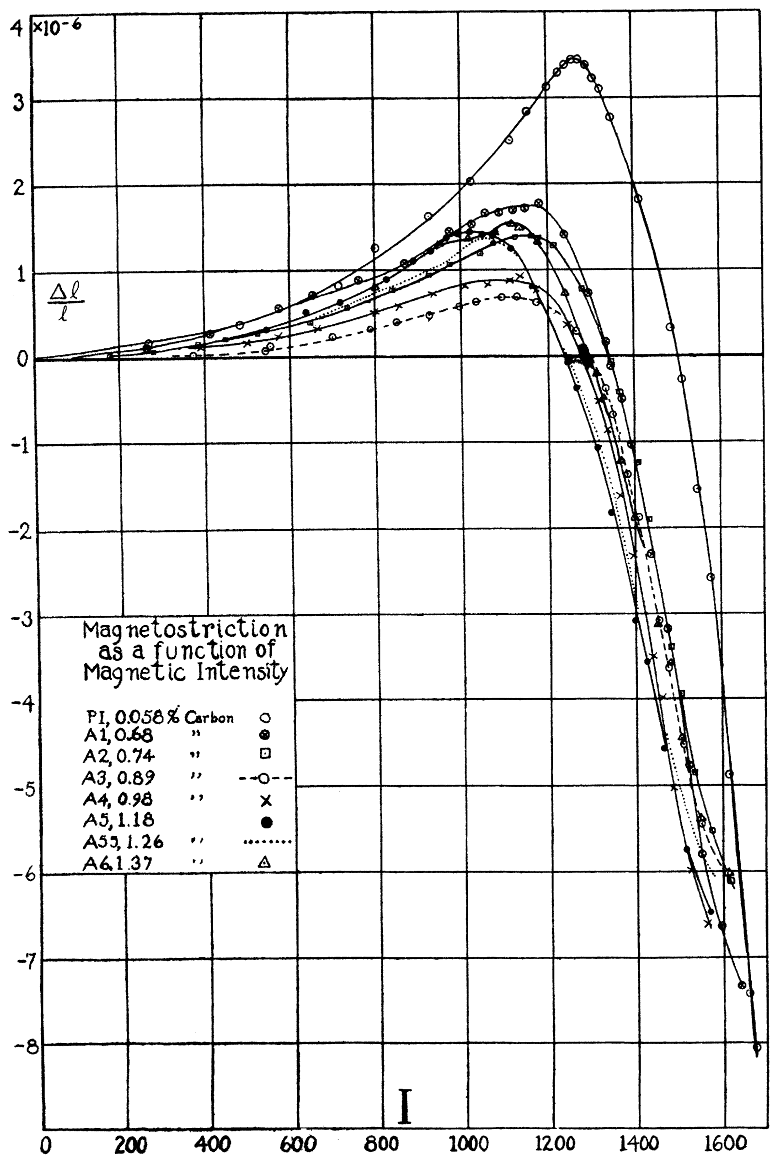

Fig. 3.

Table V.

*A*4, 0.98 *Per Cent. Carbon.* $b=0.1568$ *cm.*, $d=120.5$ *cm.*, $l=39.5$ *cm.*

H	I	$\Delta l/l$	Residual.	
			Deflec.	$\Delta l/l$
8.6	244	$.02\times10^{-6}$		
9.8	300	.05		
11.7	390	.11		
14.4	500	.17	− .5	$.09\times10^{-6}$
16.4	575	.25	− .7	.14
19.5	665	.34	− .8	.22
26.2	800	.51	− 1.7	.25
30.0	854	.59	− 1.9	.30
36.3	932	.73	− 2.5	.34
44.5	1,008	.81	− 3.0	.34
52.3	1,060	.84	− 3.5	.30
63.4	1,110	.89	− 3.3	.34
71.7	1,137	.92	− 3.6	.34
92.0	1,173	.76	− 3.0	.30
138.	1,245	.36	− .8	.23
211.	1,315	− .53	+ 5.1	.26
235.	1,335	− .86	+ 6.8	.20
290.	1,367	−1.62	+11.8	.19
350.	1,395	−2.33	+16.0	.15
490.	1,440	−3.50	+23.4	.14
578.	1,460	−3.98		
758.	1,488	−5.01		
1,120.	1,528	−6.98		
1,660.	1,562	−6.60		

Table VI.

*A*5, 1.18 *Per Cent. Carbon.* $b=0.1036$ *cm.*, $d=120.5$ *cm.*, $l=39.6$ *cm.*

H	I	$\Delta l/l$	H	I	$\Delta l/l$
7.30	270	$.06\times10^{-6}$	74.5	1,073	1.40×10^{-6}
8.50	345	.13	98.	1,116	1.25
9.51	422	.20	143.	1,167	.77
11.5	542	.32	215.	1,248	− .07
13.5	638	.51	238.	1,267	− .37
16.0	718	.64	300.	1,314	−1.06
21.0	824	.90	352.	1,344	−1.82
26.2	890	1.11	495.	1,400	−3.08
31.6	930	1.23	580.	1,425	−3.56
38.0	964	1.36	780.	1,463	−4.56
45.8	996	1.43	1,147.	1,513	−5.75
53.7	1,020	1.45	1,680.	1,570	−6.48
65.6	1,050	1.43			

TABLE VII.

*A*55, 1.26 *Per Cent. Carbon.* $b=0.1568$ *cm.*, $d=120$ *cm.*, $l=39.5$ *cm.*

H	*I*	$\Delta l/l$	*H*	*I*	$\Delta l/l$
7.73	195	$.03\times10^{-6}$	78.0	1,107	1.26×10^{-6}
8.75	275	.06	100.	1,140	1.15
10.0	370	.11	148.	1,193	.59
11.8	465	.20	212.	1,265	− .23
14.7	595	.36	238.	1,285	− .50
17.0	678	.48	295.	1,322	−1.14
22.0	790	.72	353.	1,352	−1.73
27.2	867	.87	490.	1,400	−3.02
32.6	928	1.00	580.	1,425	−3.43
39.3	980	1.12	775.	1,460	−4.27
46.0	1,018	1.17	1,120.	1,508	−5.14
53.8	1,050	1.37	1,660.	1,575	−6.01
63.7	1,080	1.30			

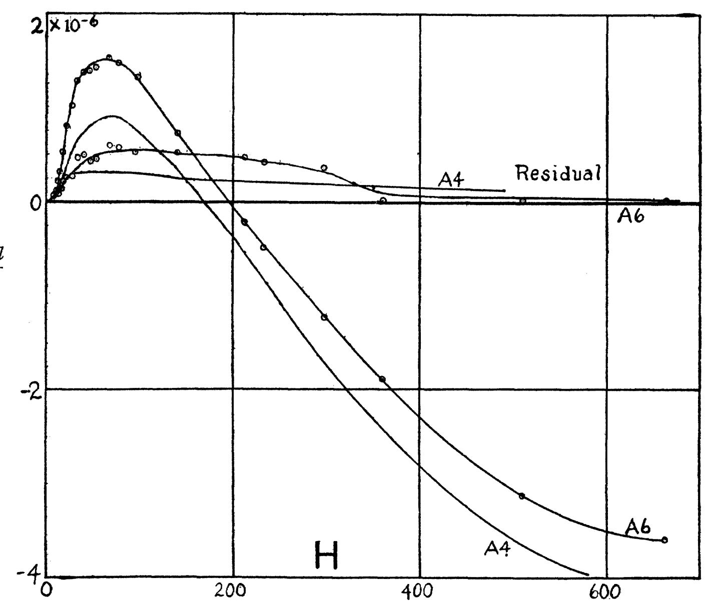

Fig. 4.

IN WHAT PORTION OF A ROD IS THE CHANGE IN LENGTH GREATEST?

Since iron gets longer for weak intensities of magnetization and shorter for stronger intensities in might be asked if all parts of a

TABLE VIII.

A6, 1.37 Per Cent. Carbon. $b=0.1399$ *cm.*, $d=119.5$ *cm.*, $l=39.4$ *cm.*

H	I	$\Delta l/l$	Residual.	
			Deflec.	$\Delta l/l$
8.8	270	$.06\times10^{-6}$	− .2	$.03\times10^{-6}$
10.4	368	.11	− .6	.03
12.3	476	.21	− .9	.08
15.0	600	.37	− 1.5	.15
16.9	674	.53	− 2.3	.21
21.5	800	.80	− 3.5	.31
27.0	898	1.01	− 4.9	.32
32.5	950	1.28	− 5.8	.46
40.0	1,012	1.37	− 6.2	.50
46.5	1,050	1.39	− 6.8	.44
54.1	1,080	1.42	− 6.9	.45
66.0	1,115	1.54	− 6.7	.60
75.6	1,136	1.49	− 6.4	.59
96.5	1,180	1.32	− 5.7	.52
140.	1,240	.72	− 1.4	.52
212.	1,313	− .20	+ 4.9	.49
233.	1,330	− .48	+ 6.4	.42
298.	1,368	−1.21	+11.2	.36
360.	1,400	−1.88	+13.5	.01
510.	1,450	−3.12	+22.0	.03
662.	1,480	−3.59	+25.7	.01
780.	1,502	−4.45		
1,114.	1,545	−5.40		
1,700.	1,610	−6.04		

rod are changing by the same amount when magnetized. To investigate this question several determinations were made by soldering slotted tubes on *PI* at equal distances from the center so as to leave the rod its original length and yet test the change in length for different portions of the rod. Only enough points were determined to find the maximum elongation although the maximum current was turned on each time to see that the retractions of the rod were about the same no difference what portion was tested.

After these tests were made a ballistic coil of a single layer was wrapped over one half of the rod and leads taken off at different portions so as to have the equivalent of several test coils of different lengths on the rod. It was assumed that the magnetism would be symmetrically distributed on each side of the rod center. A normal magnetization curve was then taken, the galvanometer being switched to the different test coils for each point. The maximum susceptibilities without any correction for end effects were thus determined for each of the different coils. These values are given in Table IX. and plotted in Fig. 5. It will be seen that the elonga-

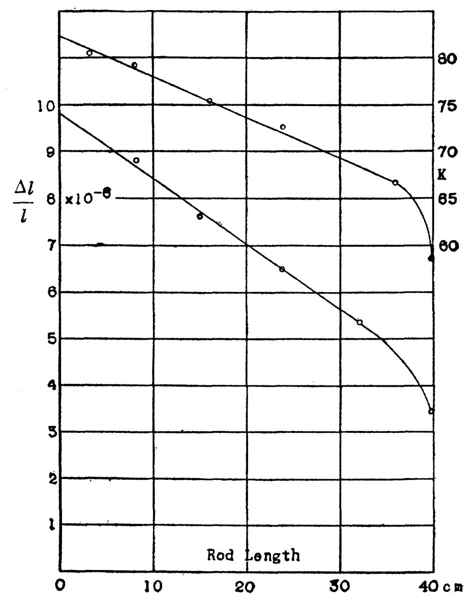

Fig. 5.

Maximum percentage elongation and maximum apparent susceptibility of *PI* as functions of the length of rod tested.

tion and apparent susceptibility increase regularly as the center of the rod is approached although there is a sudden decrease in both near the end of the rod. As would be expected, the different points of maximum elongation and points of maximum apparent susceptibility occurred at the same values of H as they did when the full rod length was tested, viz., $H = 44.3$ for magnetostriction and $H = 2.93$ for susceptibility.

Effect of Length of Rod.

In order to preserve the rods of the series another rod was taken for this test. It was of soft iron and cut to the same size as the others and annealed. It was called $D1$. After tests were made on the full length portions were cut from the end after each test so that eight different lengths were used. Values are given in Table IX. and in Fig. 6 maximum apparent susceptibilities and maximum

TABLE IX.

PI, Full Length, First Condition. $\Delta l/l$ and Uncorrected Susceptibility Tested in Different Portions of the Rod.

Length.	$\Delta l/l$ Max.	Coil Length.	k Max. (n.c.).
39.8 cm.	3.45×10^{-6}	39.8 cm.	58.6
32.0	5.35	35.9	66.6
23.9	6.50	23.8	72.6
15.0	7.62	16.0	75.5
8.1	8.80	8.0	79.2
7.9	9.15	3.1	80.5

D1, Soft Iron, No Analysis. Annealed and then Cut to Various Lengths. Diameter = 0.58 cm. Corrected k = 139.5.

Length.	Diameter / Length	$\Delta l/l$ Max.	k Max. (n.c.).
39.8 cm.	.0146	4.78×10^{-6}	75.1
35.0	.0166	4.08	65.5
30.0	.0193	3.93	54.3
25.0	.0232	2.23	42.8
22.0	.0263	1.87	38.6
20.0	.0290	3.19	30.8
12.0	.0474	2.33	
4.8	.121	1.90	
∞	0 from curve	5.80	141.

D2, Soft Steel, No Analysis. Eight Pieces Cut from the Same Rod. Ends Threaded and then Annealed in Nitrogen. Diameter = 0.638 cm. Corrected k = 113.

Length.	Diameter / Length	$\Delta l/l$ Max.	k Max. (n.c.).
40	.0160	3.10	62.9
35	.0182	3.32	54.7
30	.0213	2.90	43.9
25	.0255	2.85	34.4
20	.0319	2.78	24.4
15	.0425	2.70	16.1
10	.0638	2.40	8.7
5	.128	2.35	3.15
∞	0 from curve	3.80	115.

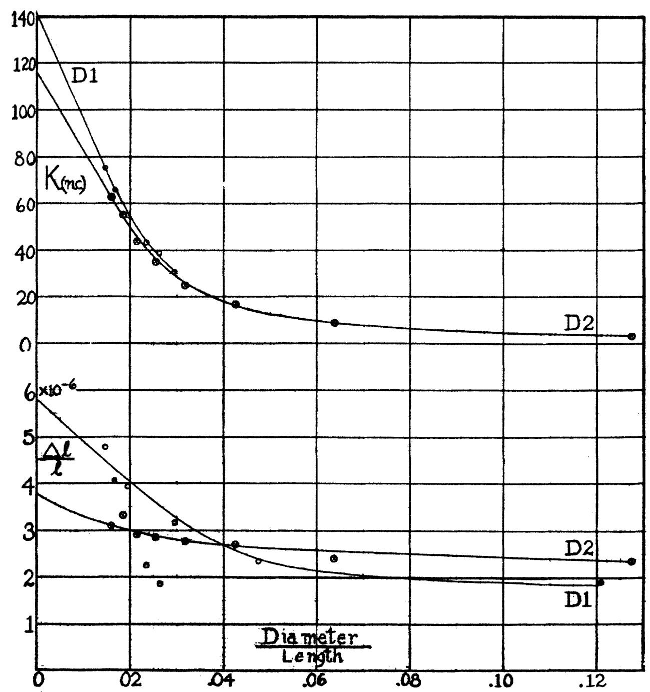

Fig. 6.

percentage elongations are plotted as functions of diameter divided by length. It will be noticed that two points for magnetostriction are far from the curve, but these are thought to be due to accidenta- conditions of the iron rather than to errors of measurements. All though carefully taken at the time they could not be checked afterwards as the rod had been made shorter. The rods were each time soldered to the brass end rods and the temperature reached in soldering might have been sufficient to have altered the condition. To avoid this another series of rods was made from a single long rod of commerical soft steel called *D2*. The ends were threaded so as to avoid soldering and then all were heated to 1000°C. in an atmosphere of nitrogen and furnace cooled. Each rod was then tested as before.

It should be noted that for the shorter rod lengths it required a much higher apparent field strength, uncorrected for end effects, to produce the maximum elongations. Thus for *D*1 full length maximum elongation occurred at apparent $H = 60$ while for the 4.8 cm. length it required apparent $H = 145$. For *D*2 the corresponding values are 54 and 243. These results quite confirm those of Sidney Lochner,[1] who showed that a short thick rod apparently expands for all fields only because the actual field strength is small due to the demagnetizing effects of the ends.

The values obtained for *D*1 and *D*2 are given in Table IX. and curves in Fig. 6. If these curves are projected back to cut the Y axis we have values for rods of infinite length as compared to their diameters, comparable to a ring without poles. As before mentioned the values of susceptibility check almost exactly with those obtained by making corrections for end effects. From similar reasoning then, a steel of the quality of *D*1 in the shape of a ring would have a maximum percentage elongation along the circumference of 5.8×10^{-6} and for steel of the quality of *D*2 it would be 3.8×10^{-6}.

Modulus of Elasticity.

Having found a close relationship between maximum elongation and maximum susceptibility it was thought desirable to see if there is any relation to the modulus of elasticity. This was tested by means of an Olsen testing machine and a mirror extensometer reading to 1/40,000 inch. The values are given in dynes per square cm. in Table X. and plotted as a function of the carbon content in Fig. 7. These values are in good general agreement with those found by Benedicks.[2]

It will be noticed that there is a general tendency for the curve to slope downwards for the higher per cent. carbon. Also that the retraction curve for a value of $H = 1{,}500$ has a general slope upwards so that roughly we can say that the amount of shortening in a strong field varies directly as the modulus of elasticity, a result exactly opposite to what I expected to find.

Effect of Heat Treatment.

To remove any possible hardening effect due to stretching during the modulus tests the specimens were annealed at about 800° and

[1] Phil. Mag., Vol. 36, p. 504, 1897.

[2] Reserches Physiques et physico chemiques sur l'acier au carbone. Carl Benedicks, Upsala, 1904.

furnace cooled. Curves were again taken which had the same general shape as those already given but there was a notable change in the maximum values for *A*55 and *A*6 both for magnetostriction and susceptibility. Both values had increased. *PI* had also increased very materially for magnetostriction with apparently no change in susceptibility, although only a rough test was made of it

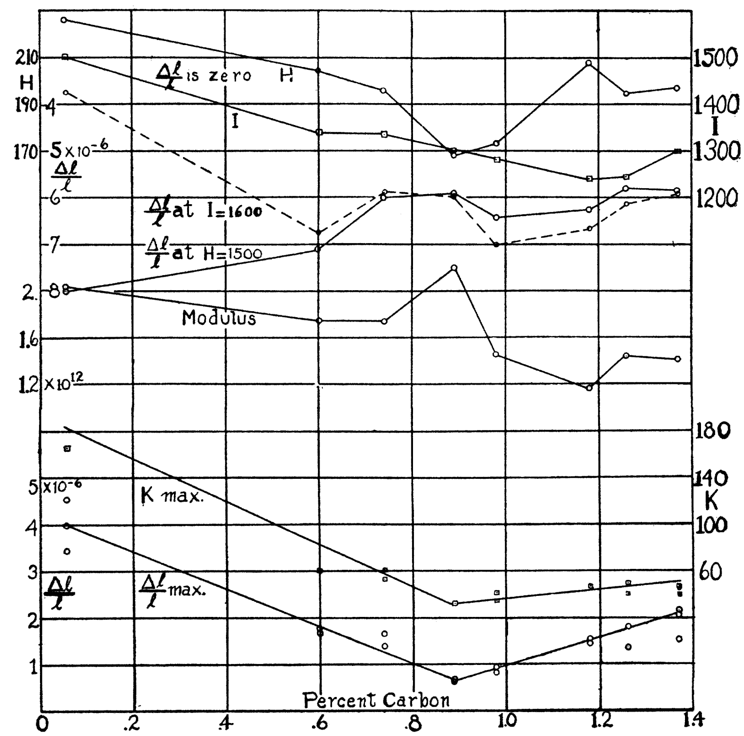

Fig. 7.

for this specimen. *PI*, *A*3 and *A*6 were then heated to about 1000° and quenched in water and tests made. *PI* and *A*6 were again annealed at 1000° and furnace cooled and *D*1 had been carried through the same process before being cut down. These values are all given in Table X. and plotted in Fig. 5 as a function of the per cent. carbon. The new values for *A*55 and *A*6 point to a probability that they were not entirely annealed in their first condition, and with the exception of *PI* and *D*1 it may be said that quenching or sudden cooling lowers the amount of maximum elongation while annealing raises it. This is in accordance with Joule's third law, that "the elongation is for the same intensity of magnetism proportional to the softness of the metal, greatest in iron, least in hard steel." *PI* and *D*1 which were soft iron both confirm and deny this general truth. For each in one so-called annealed condition gave values higher than in the quenched state. But, on the other hand, each in a quenched condition gave values higher than in one so-called annealed condition. I think this simply points to the fact that annealing in soft iron at least is not definitely stated until the exact temperature is stated, how long it is held at that temperature and how long a time is required in cooling. Some physical chemists claim that 5,000 hours at 1000° is necessary to anneal iron-carbon alloys.

This anomalous case for soft iron has also been reported by

Table X.

Collected Data for the Different Specimens.

	m	N	$E\times 10^{12}$	First Condition.		Annealed.		Quenched.		Annealed.	
				$\Delta l/l$ max.	k max.	$\Delta l/l$ max.	k max.	$\Delta l/l$ max.	k max.	$\Delta l/l$ max.	k max.
PI	66	.00663	2.02	3.45	166.	4.52		3.49	90.3	3.59	222.
*A*1	73	.00542	1.75	1.78	60.6	1.70					
*A*2	65	.00680	1.75	1.40	52.4	1.68	60.8				
*A*3	74	.00530	2.20	.68	32.8	.63		.26	10.9		
*A*4	64	.00693	1.45	.92	35.1	.81	42.3				
*A*5	66	.00663	1.16	1.45	47.3	1.54					
*A*55	70	.00595	1.44	1.37	40.5	1.80	49.8				
*A*6	68	.00608	1.41	1.54	40.0	2.08	46.4	1.21	20.9	2.15	46.9
*D*1	69	.00603				2.78	154.	3.55	91.8	4.78	139.5

Shelford Bidwell[1] who found one soft iron ring which in the "annealed" condition contracted for all values of field strength. He considered this a rare specimen, and was unwilling to quench it for fear he could not reproduce the unusual condition.

Accuracy.

It is believed that errors in all measurements of length were not greater than 0.3 per cent. and there were five such measurements

[1] Proc. Roy. Soc., Vol. 56, p. 94, 1894.

entering into the computations. Single readings of deflections may have been in error by from 0.5 per cent. to 3 per cent., depending upon the amount of deflection, but as these were mostly the average of four readings the error might be considered 1.5 per cent. Combining by square root of sum of squares gives 1.6 per cent. for measurements of magnetostriction. In the magnetic work currents may have been in error by 1 per cent., magnetic deflections on the average by 0.4 per cent., areas of rods and test coil by 0.6 per cent. each, resistances by 0.2 per cent., magnetic constants by 0.3 per cent. and field strength variation 1.3 per cent., giving a possible total error of 1.8 per cent. which is about the same as the discrepancy between the two different methods of arriving at the maximum susceptibilities of $D1$ and $D2$.

Conclusions.

From these experiments upon this series of iron-carbon alloys the following conclusions may be drawn:

1. The maximum elongation decreases with the carbon content to 0.9 per cent. carbon and then increases and may be represented by the equation

$$(\Delta + 4C - 4.2)(\Delta - 3.1C + 2.13) = 0,$$

in which $\Delta = \Delta l/(l \times 10^8)$ and C = per cent. carbon. Likewise the maximum susceptibility may be represented by the equation

$$(K + 185C - 194)(K - 40C + 4) = 0,$$

in which K = maximum susceptibility and C = per cent. carbon.

2. In a somewhat similar manner but not so definitely does the value of H at which the rods retract to their original length drop to a minimum value at about 0.9 per cent. carbon.

3. $\Delta l/l$ is practically the same, at a value of -1.4×10^{-6} for about $H = 300$.

4. Up to field strengths of about 1,600 the rods were still contracting at individual uniform rates. But for a field strength of 1,500 it may be said in a general way that the greater the carbon content the less the retraction. Also, the modulus of elasticity decreases with carbon. Or, the amount of shortening in strong fields varies directly as the modulus.

5. Steels slowly cooled have greater elongations and susceptibilities than when quenched. Accidental exceptions for soft iron. Magnetostriction depends upon the previous history of the specimen.

6. The percentage elongation in the middle of a 40-cm. rod of soft iron is nearly three times as much as for the entire rod.

7. Rods of different lengths of the same quality iron give different values, therefore

8. Absolute values of magnetostriction as determined by different workers cannot be comparable with each other unless experiments are made with rings or else some method of correction is devised.

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