A STUDY OF THE MULTIPLE REFLECTIONS OF SHORT ELECTRIC WAVES BETWEEN TWO OR MORE REFLECTING SURFACES.

By L. E. Woodman and H. W. Webb.

I. Introduction.

THE phenomenon of "multiple reflections" between two parallel reflecting surfaces is well known, especially in its application to the optics of thin films, but its application to electric waves has been investigated only within the last few years. Several investigators[1] have worked with resonators on glass plates or with two or more screens of resonators placed parallel to each other and perpendicular to the direction of propagation and have evidently not considered the effect upon their results due to the multiple reflections between the various reflecting surfaces. On the other hand Blake and Fountain[2] while working with resonators pasted on glass plates encountered the peculiar phenomenon of a larger per cent. of incident energy transmitted and a smaller per cent. reflected when the resonators were of certain lengths than was transmitted or reflected by the bare glass. Blake and Fountain attributed the phenomenon to a change of refractive index due to change of responsiveness of the medium as required by the dispersion theory. Cartmel,[3] on the other hand, gave a different explanation based on the consideration of multiple reflections between the two surfaces of the glass plate. At bottom the two explanations are but different aspects of the same fundamental phenomenon, *i. e.*, phase change and change of velocity due to the varying response of the changed character of the medium. Blair,[4] following Stokes and Fresnel, derived the formulæ for the change of phase and also

[1] Garbasso and Aschkinass, Ann. der Physik, Vol. 53, p. 534, 1894. Aschkinass and Schaefer, Ann. der Physik, Vol. 5, p. 489, 1901.

[2] Blake and Fountain, Phys. Rev., Vol. 23, p. 257, 1906.

[3] W. B. Cartmel, Phys. Rev., Vol. 25, p. 64, 1907.

[4] W. R. Blair, Phys. Rev., Vol. 26, p. 76, 1908.

the intensity of the transmitted and reflected energy when electric waves are transmitted through thin plates. He considers the observed variations in intensity of both the transmitted and reflected energy in accord with the formulæ, but his study of the energy relations was very limited as he was more interested in the phase relations upon reflection.

About two years ago the writers began investigating the absorption of electric waves by several resonator screens in succession. Eleven screens were made by pasting tin-foil resonators on sheets of straw-board. These were placed one behind the other in a holder about half way between the vibrator and receiver. The screens were parallel to one another and inclined to the direction of propagation, making an incidence angle of 22.5°. The distance between them was approximately 5 cm. Measurements were taken of the energy transmitted by one screen, two screens, and so on, up to eleven. The energy fell off gradually for the first three screens but four screens were found to transmit about three per cent. more than three. For the next three screens, *i. e.*, the fifth, sixth and seventh, the energy again fell off gradually, but on placing the eighth screen in the holder no diminution in transmission was observed. On the contrary a slightly increased transmission was noted. When the distance between the screens was varied, these peculiar breaks in the orderly course of the absorption curve also varied, occurring for instance upon the addition of the fifth and tenth screens, or the third, sixth and ninth. Thus instead of building up a more or less homogeneous medium for electric waves, *i. e.*, one for which transmission varied regularly with the thickness, each screen of resonators acted like a new reflecting surface, and between each two screens "multiple reflections" took place, giving anomalous results which depended upon the combination of amplitudes and phase relations of the transmitted and reflected beams. The results were further complicated by the absorption of the medium. These phenomena, first observed in the attempt to solve a different problem, appeared sufficiently new and important to justify further study, so that the writers turned aside from their original purpose to seek a solution of these interesting interference phenomena complicated by resonance.

2. Apparatus and Methods of Measurement.

Vibrator.—The vibrator used was a special form of Righi vibrator,[1] chosen after several months' work with vibrators of the same general type, but of quite different construction. It was designed to meet the following needs: a vibrator which could be easily cleaned, easily and accurately reset at the focus of the collecting mirror after repolishing, and one which would have the smallest possible amount of metal and dielectric coming within the aperture of the

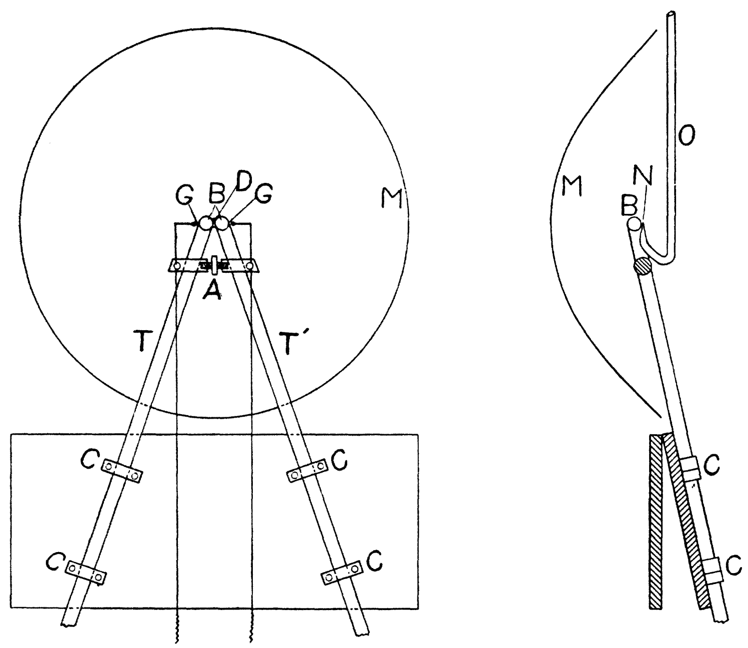


Fig. 1.

mirror. The essential details of construction are shown in Fig. 1. Two steel balls B, 0.95 cm. in diameter, rested in sockets accurately ground in the upper ends of two glass tubes TT', of one centimeter outside diameter. Below the aperture of the mirror M, these glass tubes were rigidly fastened to a plank by means of the hard rubber blocks C and about 10 cm. below the steel balls they were further stiffened by means of the hard rubber turn-buckle A. The screw

[1] Webb and Woodman, Phys. Rev., Vol. 29, p. 89, 1909.

of the turn-buckle was finely threaded and made a very satisfactory arrangement for adjusting the oil-gap. The turn-buckle also served as a convenient place for supporting the lead wires from the induction coil, which were bent at right angles just before coming to the air gaps *G* and terminated in small balls of copper made by fusing the ends of the wires in a Bunsen flame. The air gaps were about 3 mm. long and the spark took place along the axis of the vibrator. The source of energy was a 10-in. induction coil, with interrupter and condenser removed, connected to a 110-volt 60-cycle alternating current circuit. The current in the primary was usually kept at approximately seven amperes. The oil was fed through a glass tube *O*, ending in a capillary nozzle *N* directly in front of and slightly above the centers of the balls. By properly adjusting the flow of oil, a "film" *D* of any desired size would collect between the balls, the surplus of oil flowing down on the outside of the nozzle and dropping into a receptacle from the lowest point of the feed tube. The oil current served to dislodge any carbon bridges which might form due to the decomposing action of the spark as well as to keep a constant supply of fresh oil in the gap. As long as the film was kept constant and the length of gap small no trouble was experienced with the passing of the spark around the film instead of through it. Paraffin oil was used throughout these experiments.

The vibrator was set as nearly as possible at the focus of the parabolic mirror and then the final adjustments were made by setting up a bare receiver (one with no collecting mirror) 360 cm. from the vibrator and so adjusting the position of the vibrator that the distribution of energy in the reflected beam was symmetrical about the axis of the mirror in both the vertical and horizontal planes. This adjustment was tested from time to time, but it was not found necessary to reset the vibrator during a period of three months. In order to clean the vibrator it was only necessary to remove the balls, polish them and replace them in their sockets again, all of which could be done in less than five minutes without disturbing either the oil feed or the position of the vibrator with reference to the focus. This could be done at any time during a set of readings, as changing the vibrator usually made very little difference in the ratios of the galvanometer throws, especially when working with tuned receivers. Such a vibrator as this gives a much more symmetrical beam than does a vibrator having an enclosed oil receptacle. This is probably due to the small amount of dielectric in contact with the balls, as well as the small amount of metal and dielectric in the aperture of the mirror. The symmetry of the beam from a vibrator of this type will be more fully discussed under the section dealing with the parabolic mirrors.

Receiver.—Both tuned and non-selective[1] receivers of the Klemencic type were used. In all receivers the wings were made of copper foil 0.15 mm. thick and 3 mm. wide and were fastened securely to a wooden base 12 mm. wide by means of wax, the space between them being 1 mm. The galvanometer lead wires were soldered to one side of the inner ends of the copper strips, while directly opposite to them on the upper side of these strips were soldered the fine wires of the thermo-junction. For the less sensitive receivers the thermo-couple was made of commercial iron and constantan wires 0.04 mm. and 0.025 mm. in diameter respectively, while for the more sensitive receivers platinum and constantan wires approximately 0.0045 mm. in diameter were used. The fine wires were soldered to the ends of the copper strips with a free end about a millimeter in length extending beyond the end of the strips. Under a binocular microscope these free ends were looped around each other with a spring contact and then either welded by means of an electric spark or soldered. The resistance of the thermo-junction was carefully determined before and after each set of observations and the sensitiveness was assumed to remain constant as long as there was no change in resistance.

In order to eliminate as far as possible the error due to the deterioration[2] of the vibrator and the consequent change in the energy emitted, a check receiver[3] was used, constructed as nearly like the main one as possible and the readings from both receivers were taken simultaneously. The ratio of the two galvanometer throws was always taken as proportional to the energy received by the main receiver.

[1] Webb and Woodman, *l. c.*

[2] Blake and Fountain, *l. c.*

[3] Klemencic and Czermak, Wied. Ann., Vol. 50, p. 175.

Several tuning curves were taken, some of which are plotted in Fig. 2. The total lengths of the main receivers are plotted as abscissae and the corresponding ratios of the galvanometer throws as ordinates. For convenience in plotting most of the ordinates have been reduced so as to make 100 the highest point on the curve. L_I shows a maximum at 23 mm., with an indication of a second maximum at 27 mm. In L_{II} the maximum occurs at 24 mm. while in L_{III} it comes at 25 mm. L_{IV} shows a decided maximum at 27 mm., L_V and L_{VI} have somewhat broader maxima, approximately at 24

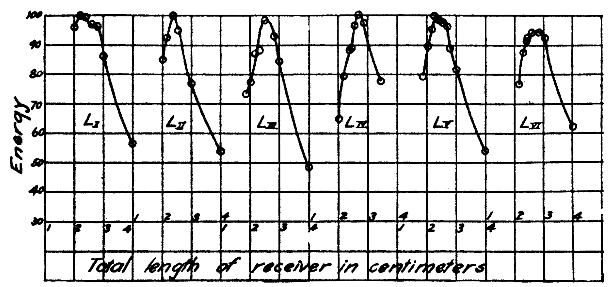


Fig. 2.

mm. in L_V and at 27 mm. in L_{VI}. From a study of ten or more such curves the conclusion is drawn that the vibrator emits a short spectrum, and that whether the maximum energy is obtained for a receiver length of 23 mm. or 27 mm. depends upon the characteristics of the receiver. This coincides well with the conclusion reached by Willard and Woodman,[1] that the form of the energy curves depends upon both receiver and vibrator. All of the receivers used were as nearly alike as possible, yet the distribution of the dielectric and the distance apart of the copper strips may have varied sufficiently to slightly change the resonance length of receivers of which the total length was only 25 mm. The fact that the vibrator cannot be considered a source of monochromatic radiation will be of considerable importance when we come to the discussion of the interference curves. Twenty-four mm. was chosen as the length of receiver to use in all work involving the use of a tuned receiver. The non-selective receivers were 310 mm. long, or approximately thirteen times the length of the tuned receiver.

[1] Willard and Woodman, PHYS. REV., Vol. 18, p. 3, 1904.

Parabolic Mirrors.[1] — The vibrator and receiver were each mounted at the focus of a copper parabolic mirror (paraboloid of revolution), silver-plated on the reflecting surface. Each mirror had an aperture of 56 cm., a depth along the axis of 15.2 cm. and a focal length of 12.2 cm. As most of the results in this paper are independent of any effect the mirrors may have had in determining the wave-length, no attempt was made to avoid "mirror action"[2] in the selection of these mirrors. Each mirror was mounted with its optic axis parallel to the floor and could be brought into proper alignment by rotation about either a vertical or a horizontal axis.

As the object in using these mirrors was to obtain a parallel beam of energy, it seemed worth while to test the beam for parallelism. This was attempted in three ways. The first consisted in comparing the actual distribution of energy in a plane perpendicular to the optic axis with that calculated on the assumption that the vibrator represented a point source placed at the focus of the mirror. The second consisted in comparing the distribution of energy perpendicular to the optic axis at a distance of 240 cm. from the mirror with the corresponding distribution at a distance of 360 cm. In a parallel beam the distribution at the two distances should be the same, provided the distances are sufficiently large to disregard the direct radiation. The third method consisted in taking readings with the main receiver placed at successive distances from the mirror along the optic axis. Unless the beam was diverging or the energy absorbed by the medium through which the waves passed, these readings ought to remain constant. In all three cases the distribution of energy was measured with a bare receiver (one with no collecting mirror), tuned to the incident energy and mounted in such a way that readings could be taken well outside the limits of the beam either horizontally or vertically, always starting from the center of the beam and taking the readings first on one side of the center and then on the other.

The theoretical distribution of energy from a point source placed at the focus of a paraboloid of revolution may be derived as follows:

[1] The mirrors were head-light mirrors, bought from the Adams and Westlake Company of Philadelphia.

[2] Webb and Woodman, *l. c.*

Let OP, Fig. 3, be a section of the paraboloid having its focus at F. About F as a center construct a sphere of any radius r.

Let E_s equal the energy per sq. cm. passing through the surface of the sphere. This will be a constant for a sphere of given radius.

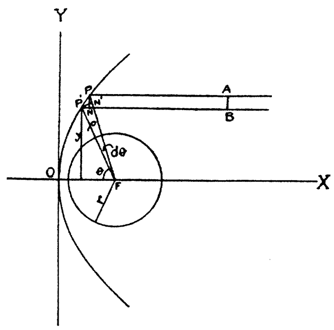


Fig. 3.

The total energy passing through the annulus on the sphere of width $d\theta$ and situated at an angle θ from the axis of the mirror is $2\pi r^2 \sin\theta d\theta E_s$. This same energy is contained in the hollow cylinder of thickness AB. The area of the cross-section of the hollow cylinder is $2\pi y dy = A_1$.

But $y = \rho \sin\theta$ where ρ is the radius vector of the paraboloid and $dy = AB = \rho d\theta$. Hence $A_1 = 2\pi y dy = 2\pi\rho^2 \sin\theta d\theta$.

Let E_p be the energy per sq. cm. passing through the plane AB. Then

$$E_p 2\pi\rho^2 \sin\theta d\theta = E_s 2\pi r^2 \sin\theta d\theta,$$

or

$$E_p = \frac{r^2}{\rho^2} E_s = \frac{r^2 E_s}{4p^2}(1 + \cos\theta)^2 = K(1 + \cos\theta)^2,$$

where p is the focal length of the parabola and K is a constant.

The theoretical curve can now be plotted by plotting as abscissæ the distances from the axis of the mirror perpendicular to the direction of propagation and for ordinates the intensities for the corresponding values of θ. Thus for abscissæ

$$\rho \sin\theta = \frac{2p \sin\theta}{1 + \cos\theta} = K' \frac{\sin\theta}{1 + \cos\theta}$$

and for ordinates

$$E_p = K(1 + \cos\theta)^2.$$

For convenience in plotting E_p is taken as proportional to

$$\frac{(1 + \cos\theta)^2}{4}.$$

The theoretical values are represented by the solid dots in Fig. 4, while the experimental values are represented by circles. Curves V_{I}, V_{II}, H_{I} and H_{II} represent the actual distribution of energy in the reflected beam. The abscissæ represent distances on either side of the center of the beam (the axis of the mirror), and the ordinates represent the energy expressed in per cent. of the energy at the center of the beam. It was found by trial that a metal screen having a circular opening 57 cm. in diameter placed 110 cm. from the vibrator cut off some of the direct radiation and hence gave a much more sharply defined *outside limit* to the beam. Most of the curves show that when the receiver had been moved 30 cm. from the center of the beam, which corresponds very closely with the half aperture of the mirror, the energy had decreased to approximately ten per cent. of its value at the center of the beam. For distances greater than 30 cm. the energy decreased very rapidly and at 40 cm. from the center of the beam barely five per cent. of the maximum energy could be detected.

V_{I} represents the vertical distribution at a distance of 360 cm. from the vibrator and H_{I} the horizontal distribution for the same distance. V_{II} and H_{II} are the corresponding distributions at a distance of 240 cm. from the vibrator. V_{I} and H_{II} correspond very closely with the theoretical distribution in a parallel beam. V_{II} and H_{I} do not agree so closely with the theoretical distribution, but even in these cases the agreement is remarkably close when one takes into consideration the fact that the vibrator is not a point source, as was assumed in working out the theoretical curve, and also the fact that the vibrator itself, together with its mountings, tends to disturb the symmetry of the beam. But perhaps the better test is to compare V_{I} with V_{II} and H_{I} with H_{II}. Such a comparison shows that the beam is slightly diverging, and also that the divergence is slightly greater in the horizontal plane than in the vertical. This difference in divergence in two planes at right angles to each other is probably due to the lack of symmetry of the vibrator, one distribution being measured along a line parallel to the direction of the electric force and the other along a line perpendicular to the electric force. The fact that the receiver is nearly ten times as long as it is wide would also tend to give different results as it is moved parallel to itself either horizontally or vertically across the beam.

Curve A, Fig. 4, shows the results obtained by moving the bare receiver along the axis of the mirror in the direction of propagation of the incident energy. Readings were taken at every ten centimeters between the two points used in determining curves V_{II}, V_{I}, etc., *i. e.*, between 240 cm. and 360 cm. from the vibrator. Distances from the vibrator are plotted as abscissæ and galvanometer

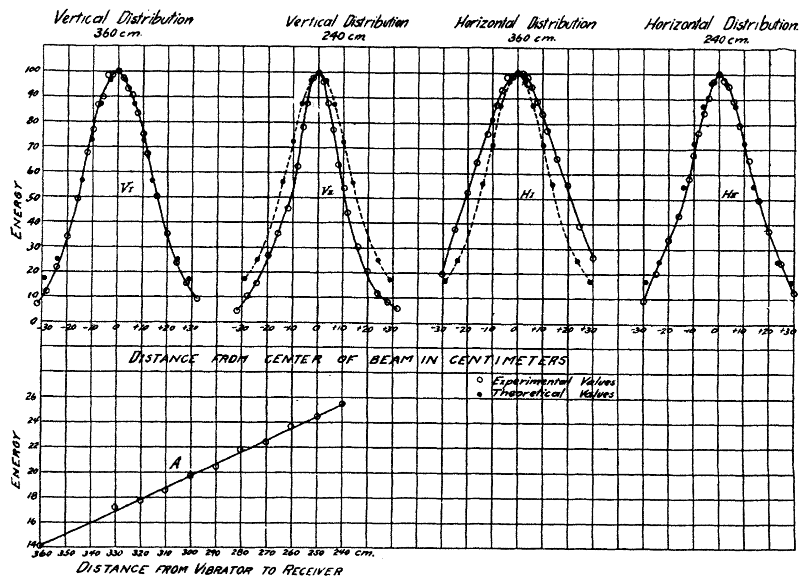


Fig. 4.

ratios, which are proportional to the energy, are plotted as ordinates. In moving the receiver a distance of 120 cm. towards the vibrator the energy nearly doubles, which would also indicate a diverging beam. The curve corresponds very closely to the distribution of energy in a beam diverging from a point source 110 cm. back of the vibrator, the intensity of the energy decreasing according to the inverse square law. If this inverse square relation were rigidly true, curve A could not be a straight line as plotted, but in any case such a limited portion of an inverse square curve taken at so great a distance from the origin would be very nearly a straight line. The angle representing the total divergence of the beam is approximately ten degrees, in which case the error would be very slight in considering a portion of the wave surface 60 cm. long taken at a distance of 350 cm. from the source as a plane wave front. An attempt was made to vary the divergence of the beam by moving the vibrator on either side of the focus along the optic axis of the mirror, but practically no change was found in the distribution of energy unless the vibrator was moved a centimeter or more from the focus, and as such a change seemed too large for continued use the vibrator was finally set back at the geometrical focus and kept there for the remainder of the work.

The three tests for parallelism showed that the beam was slightly diverging and yet nearly enough parallel to introduce no serious error in the experimental results which follow.

Galvanometers.—The galvanometer used for measuring the energy received by the check receiver was of the du Bois-Rubens two-coil type, triply iron-clad. Previous investigators[1] had found it necessary to increase the shielding from magnetic disturbances by adding additional armor. This reduced the throw due to outside disturbances to so small a quantity that it was disregarded. The sensitiveness of this galvanometer was 4×10^{-9} volts. For measuring the energy of the main receiver was used a galvanometer of the type designed by Nichols and Williams,[2] with a sensitiveness of 2×10^{-9} volts. The half-period of each galvanometer was 2.8 seconds.

3. Interference Curves with Tuned Receivers.

Airy[3] has shown that the intensity of the light transmitted and reflected by thin plates or films may be computed from the following formulæ:

$$I_T = \frac{a^2(1 - b^2)^2}{1 - 2b^2 \cos \delta + b^4}$$

for the transmitted energy, and

$$I_R = \frac{4a^2b^2 \sin^2 \frac{1}{2}\delta}{1 - 2b^2 \cos \delta + b^4}$$

[1] Blake and Fountain, *l. c.*

[2] Nichols and Williams, Phys. Rev., Vol. 27, p. 250, 1908.

[3] Sir G. Airy, Trans. Camb. Phil. Soc., Vol. 4, p. 419, 1830.

for the reflected energy, in which a is the amplitude of the incident ray, b the reflection coefficient when the reflection takes place in the rarer medium and δ is the phase retardation represented by the equation

$$\delta = \frac{2\pi}{\lambda} 2e \cos \omega,$$

λ being the wave-length in the material of the film, e the thickness of the film and ω the angle of refraction. These formulæ take no account of a possible phase change due to reflection or refraction and were therefore not directly applicable to the present problem. Professor R. C. Maclaurin assisted the writers in working out the following relations which more nearly express the conditions of the present investigation.

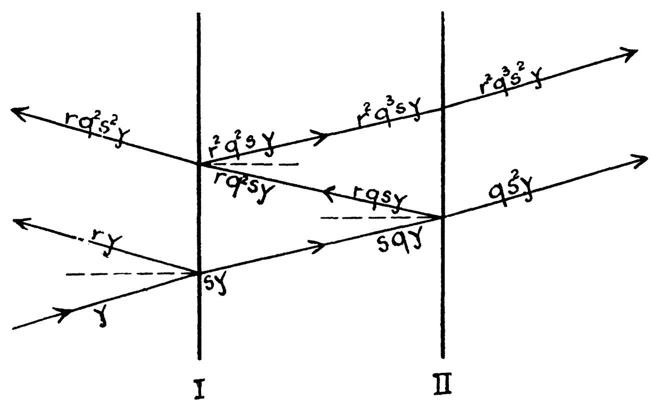


Fig. 5.

Let I and II, Fig. 5, be two screens of resonators placed parallel to each other a known distance apart. An incident ray y will give rise to multiple reflections between the screens and the first two transmitted and reflected rays will be as shown. The total energy in either the transmitted or the reflected beam will be equal to the sum of all these rays that emerge after one or more reflections or refractions, taking into account the amplitude and the phase of each ray at the time when the addition takes place. For the sake of showing the separate rays the figure has been drawn with the incident ray making a small angle of incidence, but in the derivation of the expressions for the intensities as well as in the experimental work the incident ray was normal to the reflecting surface.

We can represent the incident wave by the expression

$$y = A \cos 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right) = A \cos \theta,$$

equals the real part of $Ae^{i\theta}$; or, $y =$ the real part of $e^{i\theta}$, since we can assume the amplitude of the incident wave as unity.

Let $r = Be^{i\psi}$ be a complex number such that when it is multiplied by the expression for the incident wave it will give the amplitude and also the phase of the reflected wave. In case there is no absorption B is the amplitude of the first reflected wave.

Let $s = Ce^{i\phi}$ be a similar complex multiplier for the transmitted wave. In case there is no absorption in the medium and no loss of energy at the reflecting surface, $B^2 + C^2 = 1$.

Let $q = e^{-Kx}\, e^{ia}$ be a complex quantity such that when multiplied by the expression for the transmitted beam at one surface, it gives the value of the transmitted beam just previous to the next reflection or refraction. K is the damping factor to take account of the loss of energy due to heating the resonators, scattering, etc., x is the distance between the screens, a is the phase change due to passing a distance x through the medium between the screens.

If y represents the incident wave,

ry will represent the first reflected wave,

sy will represent the first refracted wave at surface I,

qsy will represent the first refracted wave at surface II,

qs^2y will represent the first transmitted wave,

and so on.

The total transmitted wave will be represented by

$$y_t = qs^2y[1 + r^2q^2 + (r^2q^2)^2 + \cdots] = \frac{qs^2y}{1 - r^2q^2}.$$

Substituting the values given above for q, s, y and r, and writing $2\psi + 2a = \delta$ we find

$$\frac{qs^2y}{1 - r^2q^2} = \frac{e^{-Kx}\, C^2 e^{ia} e^{i(\theta + 2\phi)}}{1 - B^2 e^{-2Kx} e^{i\delta}} = Re^{i\beta},$$

where R is the amplitude of the transmitted wave and β is the phase angle.

The value of R is found to be

$$R = \frac{C^2 e^{-Kx}}{\sqrt{1 - 2B^2 e^{-2Kx} \cos \delta + B^4 e^{-4Kx}}}.$$

Therefore, since the intensity is proportional to the square of the amplitude,

$$I_t = K' R^2 = K' \frac{C^4 e^{-2Kx}}{1 - 2B^2 e^{-2Kx} \cos \delta + B^4 e^{-4Kx}}.$$

This reduces to the same form as Airy's formula if we put $K = 0$ and remember that $1 - B^2 = C^2$ on the assumption that the reflected energy plus the refracted energy is equal to the incident energy.

The expression for the transmitted energy will be nearly a maximum when

$$\delta = 2n\pi$$

and nearly a minimum when

$$\delta = (2n + 1)\pi,$$

In the expression

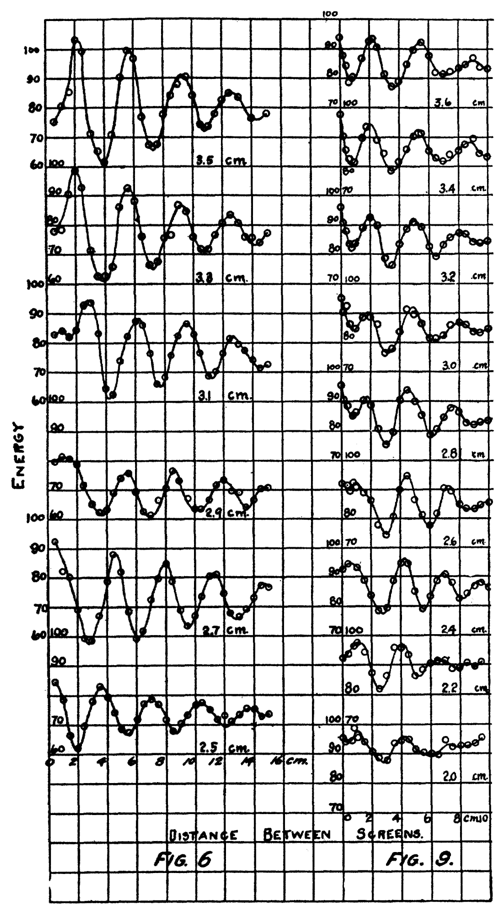
$$\delta = 2\psi + 2\alpha,$$

2ψ represents the phase change due to reflection and would remain constant for any one wave-length and any given distribution of resonators on the resonator reflectors. α, on the other hand, is the change of phase due to the passage of the wave through the distance between the screens. If now the distance between the screens is varied continuously through several wave-lengths, α and hence δ will take all possible values between zero and $2n\pi$, and the energy transmitted by the two screens will increase and decrease through a regular succession of maxima and minima.

This was tested in the following way. Half way between the vibrator and the main receiver there was placed a frame for holding the screens of resonators perpendicular to the direction of propagation. The first screen was fixed 180 cm. from the vibrator, and the second was so mounted that its distance from the first could be varied through a range of 20 cm. or more. The screens of resonators were made as nearly alike as possible. The resonators were pasted on straw-board nailed to wooden frames 86 cm. square, and consisted of strips of tin foil 2 mm. wide, and 3.5 cm. long, placed 1 cm. apart "end on" and 4 cm. apart "side on." On each screen were 18 columns and 21 rows.

The check receiver was 2.4 cm. long and was placed about 30 cm. directly in front of the vibrator. Owing to the lack of sensitiveness of the check and the lack of planeness of the reflecting screens, the effect of the reflected energy on the check was so small that it was disregarded. It was found necessary to change the position of the check in some later work with more sensitive receivers. The main receiver was also 2.4 cm. long and was mounted in the center of the beam 360 cm. from the vibrator. No collecting mirror was used. In order to further eliminate the errors due to change in ratios as the oil gap lengthened, alternate readings were taken on some standard condition and the condition under investigation. The average of the readings for any given distance apart of the screens was divided by the average of all the readings on the standard condition taken just before and just after the readings on the point in question. Inasmuch as the standard condition was a single screen, this quotient would represent the per cent. of the energy transmitted by the first screen alone that was transmitted by both screens.

The results with tuned receivers are plotted in Fig. 6. The distances between the screens are plotted as abscissæ. The ordinates marked 100 would mean that the same amount of energy was transmitted by two screens as by one, whereas the ordinate marked 60 would mean that two screens transmitted sixty per cent. of the energy transmitted by one alone. Six curves were taken, showing the results obtained with resonators of lengths 3.5 cm., 3.3 cm., etc., to 2.5 cm., the length for resonance being 2.9 cm. The curves all show very decided maxima and minima, which for the most part fall at regular intervals, the average distance between successive maxima or minima being 3.5 cm. This agrees fairly closely with the half wave-length of the incident energy, which is probably either 3.1 cm. or 3.2 cm. If the vibrator was emitting anything like monochromatic radiation we would expect the distance between the successive maxima or minima to equal the half wave-length in air, but when we remember that the wave-length determination is at best only an average of a very complex emission, and that the interference curves are taken under conditions where we get interference between certain groups of wave-lengths at a time depend-



Figs. 6 and 9.

ing on the distance between the reflecting surfaces, similar to the interference of white light when falling on a film of oil of varying thickness, it does not seem so remarkable that the half wave-length does not check more closely than ten per cent.

It is interesting to note in Fig. 6 the change of curvature within the first two centimeters as the length of the resonators is changed. The first two curves start out with a very decided upward slope, while the last two have a downward slope in the same region, the change apparently occurring at or near the resonance length of the resonators. We may regard this merely as a shift of the maxima and minima to the right or the left. The position of the first maximum is undoubtedly determined in part by the phase change due to reflection. Although the curves do not admit of accurate numerical calculations of this phase change, it seems justifiable to conclude that the change in position of the first maximum indicates that the phase change due to reflection is not a constant for all lengths of resonators. On the contrary, it takes different values as the length of the resonators is varied, at least when the lengths are taken near the resonating length.

The change in the per cent. of energy transmitted by two screens becomes less and less as the distance between the reflecting surfaces increases. By the time the distance between the screens has become equal to two wave-lengths, the per cent. of energy transmitted by two screens has become nearly constant, being about seventy or eighty per cent. of the energy transmitted by the single screen. This apparent decrease in the energy is probably due partly to the complexity of the radiation and partly to the loss of energy by absorption in the resonators themselves as well as by scattering.

4. Interference Curves with Non-Selective Receivers.

(*a*) *By transmission.*—The arrangement of the apparatus for the interference curves obtained by transmission through two screens of resonators with non-selective receivers is given in Fig. 7. M_1 and M_2 are the parabolic mirrors, the vibrator, V, being at the focus of M_1 and the main receiver, MR, at the focus of M_2. The resonator screens were placed at S, halfway between the vibrator and the Boltzmann mirrors, BM. The energy transmitted by the resonator screens was reflected from the Boltzmann mirrors in the direction of M_2, the angle between the incident and reflected rays being 20°. S_2S_2 show the position of the screen used to cut off part of the direct radiation and so make the distribution of energy in the reflected beam correspond more closely to that in a parallel beam. At S_1 was placed a small zinc screen covering up the lower portion of the aperture of M_1. It was found that with sensitive receivers the energy of the check would increase nearly twenty per cent. due to the energy reflected back from the resonator screens. As the focus of the parabolic mirror was inside the plane of the aperture no position for the check could be found where it would not receive this reflected energy and still receive direct radiation from the vibrator. By placing the screen at S_1 a position could be found for

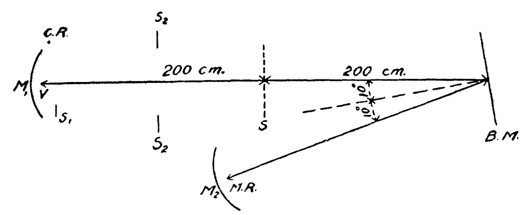


Fig. 7.

the check receiver, CR, where the readings of the check did not vary when readings were taken with and without the screen S in position.

The Boltzmann mirrors were placed in the position indicated, in order that wave-length determinations could be made using the energy which had been transmitted by the screens. For the interference curves the two mirrors were set in the same plane and used simply as a metal reflector. Each of the mirrors was made of plate glass, 76 cm. wide and 38 cm. high, covered with tin foil. They were mounted on vertical wooden frames with horizontal arms which in turn were made to slide on cross pieces on a rigid wooden frame. A more detailed description of the Boltzmann mirrors may be found in an earlier paper by the writers.[1]

The resonators were pasted on tracing cloth which had been moistened and stretched as tightly as possible over an artist's frame for stretching canvas. Two screens of resonators were pasted, each containing eighteen columns and twenty-nine rows. The resonators were 3.6 cm. long at the start, 3.0 cm. apart "side on" and 1.0

[1] Webb and Woodman, *l. c.*

cm. apart "end on." The resonators were shortened 0.2 cm. each time until the final length was 2.0 cm.

The non-selective receiver was chosen for this set of interference curves on account of the complex character of the energy given out by the vibrator. In fact the expression "tuned receiver" has very little meaning when used in connection with energy that has been transmitted through one or more screens of resonators. A receiver that is in tune with the incident energy without the screen of resonators is no longer in tune with the energy that has come through the screen. And in work of this kind where several different lengths of resonators are used a receiver of fixed length cannot possibly be in tune with the energy transmitted through the various lengths of resonators. This conclusion is based partly on a study of tuning curves taken with resonator screens placed in the path of the beam, but largely upon a study of the wave-length curves to be given later.

A comparison of the results obtained with the two types of receivers is given by the curves in Fig. 8. The abscissæ represent the lengths of resonators in centimeters and the ordinates represent the per cent. of the incident energy which is transmitted by the screens on which were various lengths of resonators. The curve marked *TR* gives the results obtained with a tuned receiver, while the curve marked *NSR* gives the results with the non-selective receiver. The most striking difference is the flatness of the curve taken with the non-selective receiver in comparison with the sharp minimum of the curve taken with the selective receiver. The latter very closely resembles the curves obtained by Blake and Fountain,[1] and would give 3.2 cm. as the length of resonator for a minimum of transmission and a maximum of reflection. While the curve taken with the non-selective receiver is too flat to admit of very great accuracy in locating the minimum of transmission, yet it seems to agree very closely with that obtained with the tuned receiver.

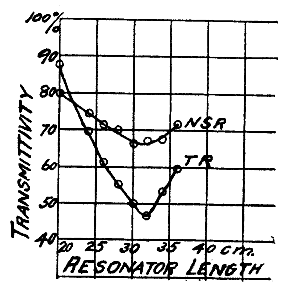


Fig. 8.

[1] Blake and Fountain, *l. c.*

Either receiver would measure the same per cent. of the incident energy when this energy has been transmitted through resonators 2.2 cm. long. Undoubtedly the curves would also have crossed for some length of resonator longer than the resonator length, but no attempt was made to find this length. From a study of such curves as these it seems to the writers that for work of this kind the non-selective receiver has very marked advantages over the tuned receiver, expecially when one is dealing with energy as complicated as that emitted by a Righi vibrator. The chief objection to the use of the non-selective receiver lies in the fact that the ratios of the energies measured by the two receivers decrease very rapidly as the oil gap becomes foul. This difficulty was overcome in a very satisfactory manner by adopting some standard condition with which readings could be taken as often as desired. For most of the work five readings were taken on the standard condition and four on the condition under investigation, always beginning and ending with the standard condition. The five readings on the standard condition were then averaged and also the four on the condition under investigation. Finally the ratio of the two averages was taken for the basis of comparison as well as for use in plotting the curves. These ratios usually agreed within two per cent., in which case only two sets of observations were taken for any given set of conditions, but in case the disagreement was more than two per cent. three and sometimes four sets of observations were taken.

The transmission interference curves taken with the non-selective receivers are given in Fig. 9. For the purpose of comparison they are plotted to the same scale as those obtained with the tuned receivers, the numbers just below each curve being the resonator length in each case. Most of the curves show well-defined maxima and minima, the average distance between two successive maxima or minima being 3.2 cm., which is a very close agreement with the half wave-length as determined with non-selective receivers. The total change in energy between the highest maximum and the lowest minimum is usually less than twenty per cent. of the energy transmitted through one screen, whereas with the tuned receiver it sometimes amounted to as much as forty per cent. The energy approaches a constant value much more rapidly than with the tuned receivers, so that it was useless to increase the distance between the screens beyond 10 cm. The last curve taken with a resonator length of 2.0 cm. is very flat and the maxima and minima disappear when the distance between the screens is only six centimeters. For the first seven curves the constant value towards which the energy approaches was approximately 85 per cent. of the energy transmitted by the single screen. However, when the resonators are 2.2 cm. long, this constant value is 90 per cent., and for resonators 2.0 cm. long it is approximately 92 per cent. This means in the last curve that the two screens placed six centimeters or more apart will transmit 92 per cent. of the energy that would be transmitted by a single screen, whereas, if the resonators were 2.6 cm. long, the same two screens placed 10 cm. or more apart would transmit 85 per cent. of this energy. A comparison of the positions of any particular maximum or minimum, like the second or third maximum for instance, in the curves taken in order down the page shows a gradual shifting of the position of the maximum towards the left. This shift is probably due to the fact that the change of phase due to the reflection gradually changes as the resonators are shortened. In some of the curves certain irregularities occur within the first centimeter or two. These are probably due partly to experimental errors and partly to other causes for which no adequate explanation can be given.

One set of readings was taken using three screens of resonators. The two screens nearest the vibrator were kept at the constant distance apart of 4.5 cm. This distance was chosen because with it the energy transmitted by the two screens was neither a maximum nor a minimum, in fact it corresponded to a point on the axis of the interference curve. The resonators on each of the screens were 3.6 cm. long and were in all respects like the screens already described for the work with the non-selective receivers. The distance between the second and third screens was increased from zero to six centimeters, usually adding 0.5 cm. each time, and the energy transmitted by the three screens determined. The energy passed through a series of maxima and minima as for the two screens. Two successive maxima or minima were found to be 3.2 cm. apart, and the total change in energy amounted to about fifteen per cent. of the energy transmitted through the two screens, which was taken as the standard for comparison.

(*b*) *By Reflection.*—The arrangement of the apparatus for the work by reflection was very similar to that ordinarily used for wave-length determinations with the interferometer.[1] The separating surface[2] consisted of a wooden frame 101 cm. by 96.5 cm., on which were stretched strands of no. 34 copper wire spaced one centimeter apart. The frame was mounted with the wires horizontal, or parallel to the electric force. The energy reflected from this wire grid agreed with the transmitted energy within less than two per cent. The movable mirror was a piece of plate glass 76 cm. by 77 cm. covered with tin foil. It was mounted in the same position as that occupied by the resonator screen in the work on transmission, and could be replaced at any time by the resonator screen. The distance between the vibrator and the movable mirror was 200 cm. while the distance between the vibrator and the center of the wire grid was 95 cm. The fixed mirror was also of plate glass covered with tin foil, 76 cm. square, and was placed 105 cm. from the center of the wire grid. The distance between the main receiver and the center of the wire grid was 150 cm.

For determining the interference curves the fixed mirror was replaced by a large metal screen supported in such a way that the energy falling upon it was deflected to one side and produced no effect upon the receiver. The movable mirror was removed and the two tracing cloth screens already described were put in its place. The position of the check was not changed from that shown in Fig. 7. The curves marked *R* in Fig. 10 show the results obtained by reflection. The screen nearest the vibrator was kept in the same position throughout the observations and the energy reflected from it was used as a standard condition. Four readings were taken with two screens a known distance apart alternating with five readings on the standard cond tion, always beginning and ending with the standard condition. Then the readings of the energy reflected from one screen and those of the energy reflected from two screens were averaged separately and the latter average divided by the

[1] Willard and Woodman, *l. c.* Blake and Fountain, *l. c.*

[2] G. F. Hull, PHYS. REV., Vol. 5, p. 231, 1897.

former. These ratios are plotted as the ordinates in Fig. 10, and the distances between the screens are plotted as abscissæ. For the purpose of comparison, the corresponding curves by transmission are plotted just below the curves by reflection. The two curves, *i. e.*, either reflection curve with its corresponding transmission

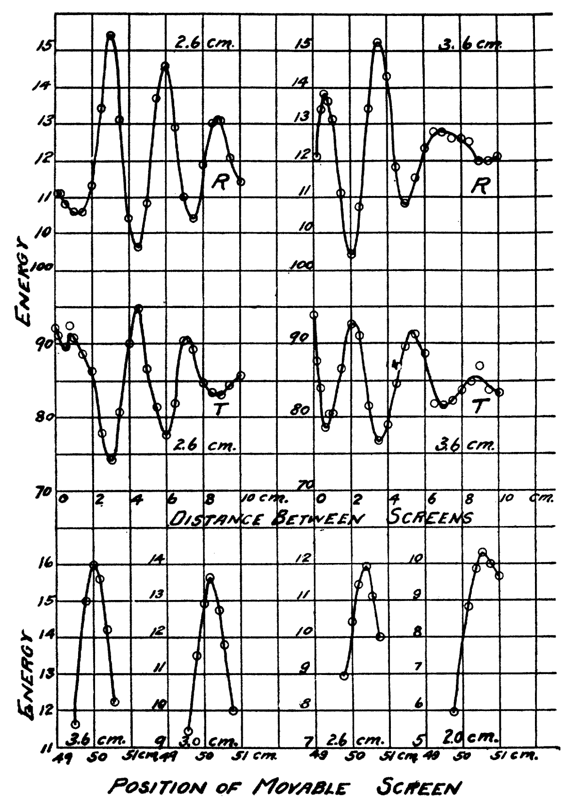


Fig. 10.

curve, are not plotted to the same scale on the Y-axis and no attempt was made to compare the absolute energy in the two beams. The same scale is used throughout for the abscissæ, so that the curves can be compared with reference to the position of the maxima and minima. Such a comparison shows that aside from a few irregularities within the first two centimeters, the corresponding curves agree very closely with each other, the maxima on the reflection curve falling at the same points as the minima on the transmission curve and vice versa. This also agrees well with the deductions which can be drawn from Airy's formulæ for the intensity of the reflected and transmitted energy. The resonators used in determining the two curves plotted were 2.6 cm. and 3.6 cm. long respectively.

The phase change due to reflection was studied as follows: The fixed mirror was replaced in its proper position and one of the screens of resonators was used in place of the movable mirror. Readings were taken around what would correspond to the center maximum on the ordinary wave-length curve as determined with the interferometer. If there is no phase change due to reflection the position of the center maximum should not change whether the readings are taken with the movable mirror in that arm of the interferometer, or with the screen of resonators substituted in place of it. Furthermore the position of the center maximum should not change for different lengths of resonators on the screen. The results of these tests are given in the four curves at the bottom of Fig. 10, from which it is seen that there is a gradual shifting of the center maximum as the resonators are shortened. By shortening the resonators from 3.6 cm. to 2.0 cm. the position of the center maximum has been shifted 6 mm., which would correspond to the phase change produced in the wave while going a distance of 1.2 cm. This corresponds very well with the variation of the phase change due to reflection as shown in the interference curves obtained by transmission. See Fig. 9. The position of the center maximum for resonators 2.0 cm. long was just the same as for the movable mirror, which would seem to indicate that the phase change due to reflection for resonators of this length was zero. The whole question of phase change upon reflection should be more carefully investigated than the writers have attempted to do in this work.

5. Wave-length Determinations.

Another factor that must be taken into consideration when sending the complex radiation from a Righi vibrator through the screens of resonators is the *selective* reflection which takes place, especially if the resonators are near the resonating length. The phenomena which take place as the length of the resonators is gradually shortened through the resonating length may be easily understood, at least qualitatively, from a consideration of Fig. 11. The continuous curve represents an assumed distribution of energy in the short spectrum of a Righi vibrator, the wave-lengths being plotted as abscissæ and the corresponding intensities as ordinates. The dotted curves represent the assumed transmission curves of a screen of resonators for five different lengths of resonators, the curve *C* corresponding to the resonating length. The same coordinates are used for these curves as for the continuous curve. When the resonators are of the length corresponding to curve *A* they will transmit the energy from the vibrator with practically no change of wave-length, by which is meant the wave-length as determined with the Boltzmann mirrors or the interferometer. But when the resonators are of the length corresponding to *B* they will transmit much more of the shorter wave-lengths than of the longer and the resultant wave-length as measured by the Boltzmann mirrors and a nonselective receiver will be shorter than that measured without the screen. When the resonators correspond to *C* they will transmit the shorter and the longer wave-lengths of the spectrum nearly equally but the intermediate wave-lengths with much less intensity. Now the wave-length determination will no longer give a smooth curve, especially around the second maxima. We would expect that such a combination of wave-lengths as this would give a very broad second maximum on either side of the center maximum. But when the resonators correspond to *D* they will transmit more of the longer wave-lengths than of the shorter and the resultant wave-length will accordingly be longer than that obtained without the screen. Resonators of the length corresponding to *E* transmit all of the wave-lengths represented in the characteristic curve of the vibrator with practically no change in relative intensity.

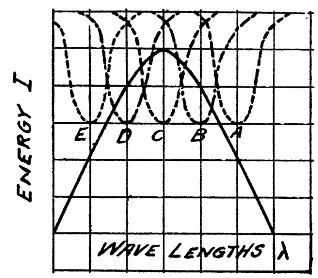


Fig. 11.

That this reasoning is correct, at least qualitatively, is clearly shown by the results plotted in Fig. 12. The arrangement of the apparatus was the same as that shown in Fig. 7, and used for the interference curves by transmission with non-selective receivers. For each length of resonators used in obtaining the interference curves a wave-length curve was taken by means of the Boltzmann mirrors. The non-selective receivers were also used for the wave-length determinations, thus doing away with the many errors[1] in-

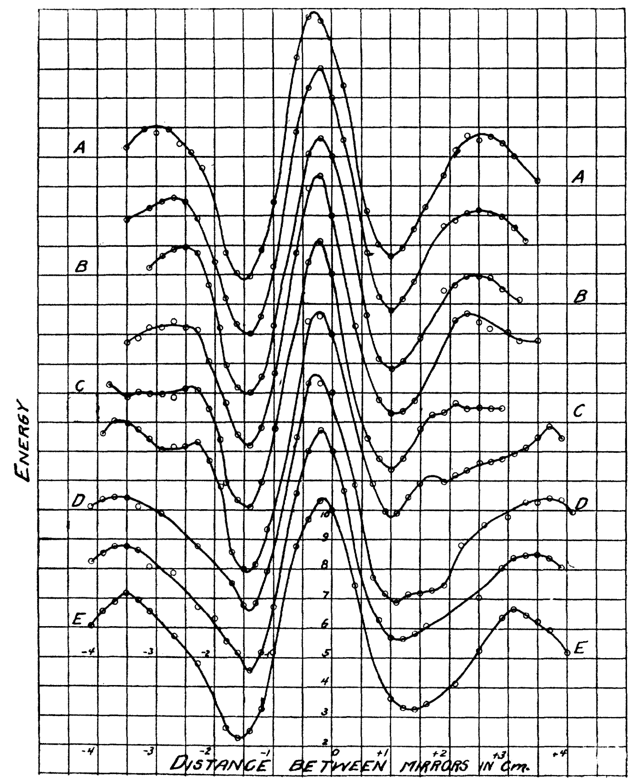


Fig. 12.

troduced by the use of selective receivers in wave-length work. For the curves given in Fig. 12 the energy was transmitted through one screen of resonators. The distances between the two Boltzmann mirrors are plotted as abscissæ and the corresponding intensities as ordinates. The readings were taken with reference to some position of the mirrors taken as a standard, a reading being taken for the standard position, then on the point in question, and finally

[1] Webb and Woodman, *l. c.*

on the standard again. The standard position was usually taken as near the center of a maximum or a minimum as possible, and then the ratio of the readings on the point in question to the average of the readings on the standard taken just before and after the point in question was computed. Then these ratios were multiplied by the proper factor to bring them into the proper relation to the center maximum, so that, as plotted, any ordinate represents the energy for the corresponding abscissa with reference to the energy received by the receivers for zero difference in path.

One great error in work of this kind arises from the fact that the characteristic curve of the emission of the vibrator changes as the spark gap deteriorates. This changes the selective action of resonators of a given length during a long set of observations and tends to make the wave-length curves irregular. For this reason the writers have made no attempt to use the curves in Fig. 12 for other than qualitative comparisons. The action of the receiver mirror as well as the comparatively large angle of incidence at the Boltzmann mirrors also tend to make the results unreliable for quantitative measurements, but do not make them any the less valid for the qualitative comparison intended by the writers. The curves in Fig. 12 are lettered to correspond at least roughly to the dotted curves in Fig. 11. The shortening of the wave-length as the resonators are shortened is very clearly marked in the first three curves. The next two show a flattening of the second maxima with the maximum energy on the side of the shorter wave-lengths, while the next two show the same flattening of the second maxima with the maximum energy on the side of the longer wave-lengths. In the last two the second maxima have become much sharper and indicate a wave-length longer than the true wave-length of the vibrator and gradually decreasing to the true wave-length.

Applying the same reasoning to the reflected energy one would expect that the wave-length of the reflected energy would be shorter than the true wave-length of the vibrator for the same length of resonators that gave a lengthening of the transmitted wave-length. This was tested by means of the interferometer arrangement already described in connection with the interference curves by reflection and found to be the case. The screen of resonators was used as the movable mirror in the interferometer and the wave-length curves taken in the usual way. For resonators 3.6 cm. long the wave-length by reflection was lengthened about ten per cent. of the true wave-length, while for resonators 2.0 cm. long it was shortened about the same amount. The curves are not given as they are very similar to those of Fig. 12.

6. Resonators on Glass.

Further data bearing upon multiple reflections was obtained by substituting a large piece of plate glass for the tracing cloth screens. The two reflecting surfaces in this case were the two surfaces of the glass plate. The glass surfaces themselves reflected a large per cent. of the incident energy, but the reflection coefficients of one or both surfaces could be varied by the addition of resonators of different lengths. The plate was 96 cm. by 122 cm. and was placed in the beam perpendicular to the direction of propagation. The vibrator and receiver were placed 360 cm. apart and the plate was arranged on a carriage so that readings could be taken with the plate in and out of the beam alternately. The readings without the plate in the beam were used as a standard, so that the ratios of the readings "in" to the readings "out" give us the transmittivity of the plate under the given condition. The readings were taken with both tuned and non-selective receivers.

The results are plotted in Fig. 13. Resonator lengths are plotted as abscissæ and the transmittivity as ordinates. Curves *I* and *II* were taken with a glass plate 0.8 cm. thick, curve *I* being taken with resonators pasted on only the surface of the glass towards the vibrator, and curve *II* with the resonators on both surfaces, those on one surface being directly opposite those on the other. In curve *I* the reflection coefficient for one surface of the glass was constant, while that for the other surface varied as the resonators were shortened. In curve *II* the reflection coefficients for both surfaces were changed whenever the resonators were changed. The resonators were 3.0 cm. long at the start, 0.5 cm. apart end on, and 3.0 cm. apart side on, and there were thirty rows and twenty-three columns. The curves marked *NSR* were taken with the non-selective receiver, those marked *TR* with the tuned receiver. The curves with the non-selective receiver tell us very little about the length of resonators for maximum resonance. Apparently the minima fall at the same places as those for the tuned receiver. With the tuned receivers the minima of transmission at the resonance length fall at 1.5 cm. for both curves *I* and *II*, showing that the addition of the resonators to the other surface of the glass did not change the length for resonance. It did slightly change the form of the curve, but more particularly the per cent. of the original energy that was transmitted. The transmittivity is in general less for the plate having the resonators on both sides. This may be due in part to

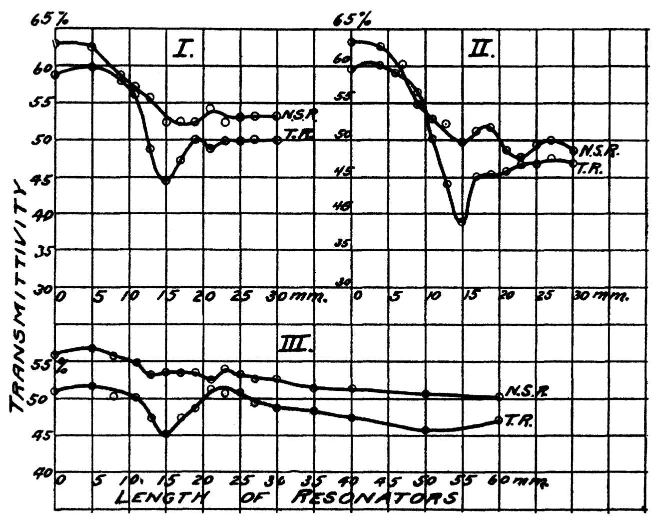


Fig. 13.

increased scattering, but is more probably due to the different combinations of reflection coefficients and phase changes due to reflection in the two cases. It is interesting to note that for resonators approximately 1.0 cm. long the transmittivity is the same for either type of receiver.

Curve *III* was taken with a plate of glass 0.6 cm. thick. The resonators were 6.0 cm. long at the start, 3.0 cm. apart side on and 1.0 cm. end on, and there were thirty rows and twelve columns. Very little need be said about the curve taken with the non-selective receiver, as the total change in the transmittivity was so slight that it made accurate work with such a receiver very difficult. The curve is published merely to show that the change in the total energy is very much less marked than the change for a particular group of wave-lengths in resonance with a tuned receiver.

If we compare curve *I* with curve *III*, we see that the transmittivity for bare glass depends upon the thickness of the glass. For the plate 0.6 cm. thick the transmittivity was 51 per cent. of the incident energy, while for the plate 0.8 cm. thick the transmittivity was 59 per cent., using the tuned receiver curves for the basis of comparison. This agrees well with what we might expect from the results of the work on the transmission of two screens of resonators on tracing cloth. If we should measure the transmittivity of a large number of glass plates of different thicknesses, we would undoubtedly find that the transmittivity would pass through a succession of maxima and minima, only of course in this case the results might be further complicated by the possible absorption of the glass. In all probability the thicker of the two plates used absorbed more than the thinner plate and yet it transmitted 8 per cent. more of the incident energy than the thinner plate.

There is no evidence of *extra-transmission* in either curves *I* or *II*, unless it be for resonators less than 1.0 cm. long. For resonators 0.5 cm. in length the transmittivity is apparently one or two per cent. higher than for bare glass. Of course it is difficult to measure to this degree of accuracy and the apparent increase in transmittivity may be due to experimental error, but the readings were repeated several times and the higher transmittivity always came when the resonators were on the glass. The fact that it occurs in both curves *I* and *II* makes it seem more probable that it is a case of *extra-transmission*. In curve *III* the transmittivity for the plate when the resonators were 2.3 cm. long was equal to that for the bare glass, which is only a special case of *extra-transmission*. As this curve was taken with the same plate and with the same distribution of resonators as that used by Blake and Fountain[1] it might be interesting to compare the curve with the tuned receiver with the corresponding curve (curve *T*, Fig. 9) in their paper. It must be borne in mind that their results were obtained with 10.0 cm. waves, while the wave-length used by the writers was only 6.0 cm. approximately. The length for resonance was 2.55 cm. for the 10 cm. waves and 1.5 for the 6.0 cm. waves. They found the greatest extra-

[1] Blake and Fountain, *l. c.*

transmission when the resonators were twice the resonating length, while the length for *equal transmission* in curve *III* is approximately 1.5 times the resonating length. This seems to strengthen the idea of Blake and Fountain that the *extra-transmission* does not always come for lengths twice the resonating length but may come for "any length greater than the fundamental length." The writers are of the opinion that it may also occur for lengths shorter than the resonating length. In fact in several of the curves published by Blake and Fountain the transmittivity is the same as that for bare glass when the resonators were still a centimeter or more in length, and it is an interesting conjecture as to what form the curves would have taken if the readings had been taken for lengths shorter than one centimeter. Comparing the two curves further we notice that the change in transmittivity is very much less for the 6.0 cm. waves than for the 10.0 cm. waves, varying from 45 per cent. at the minimum to 51 per cent. at the highest point for the 6.0 cm. waves and from 38 per cent. to 76 per cent. for the 10.0 cm. waves. Also the transmittivity of the bare glass was different for the different wave-lengths, being 51 per cent. for the 6.0 cm. waves and 60 per cent. for the 10.0 cm. waves. That this was not due entirely to a greater absorption of the shorter wave-lengths is well brought out in the following paragraph.

Using the adjustable system of rod vibrators described in another paper by the writers[1] and for which the wave-lengths were fairly accurately determined, the transmittivity of a plate of glass was measured for six different wave-lengths. For this purpose the plate of glass 0.8 cm. thick was used with no resonators on either side. It was mounted perpendicular to the direction of propagation of the incident energy and could be easily moved in or out of the beam. The energy without the plate in the beam was again used as a standard, the readings being taken alternately "out" and "in," always beginning and ending a set of readings with the standard condition. Readings were taken with both the tuned and non-selective receivers, the length of the check receiver as well as that of the main being changed each time to correspond to the wave-length used. The length of the non-selective receiver, was kept constant at 60.0 cm. The results are given in the accompanying table.

[1] Webb and Woodman, *l. c.*

Total Length of Vibrator.	Receiver Length (Tuned Receiver).	Approximate Wave-Length.	Per Cent. Transmitted.	
			Tuned Receiver.	Non-selective Receiver.
6.55 cm.	6.60 cm.	15.7 cm.	56.8 per cent.	68.2 per cent.
5.05	5.05	12.1	64.5	64.0
3.78	3.80	9.1	53.7	55.0
3.21	3.20	7.7	41.0	53.4
2.50	2.50	6.0	50.5	61.7
2.00	2.00	4.8	69.0	—

It will be seen that for the tuned receiver the transmittivity varies from 41 per cent. to 69 per cent. for different wave-lengths. If the change had always been either an increase or a decrease, it might have been ascribed to the varying absorption by the glass of different wave-lengths. But if it had been due to absorption we would expect to get the greater absorption and hence the smaller transmittivity for the shorter wave-lengths, but this is clearly not the case. The fact, too, that the thickness of the plate is, even for the shortest wave-length, only one sixth of a wave-length makes the explanation based upon absorption seem very improbable. As one would expect, the variation in the transmittivity for the non-selective receiver is much less than for the tuned receiver. Here, too, the transmittivity decreases for the first four wave-lengths tried and then increases again as the wave-lengths are made still shorter. These results all show that the transmittivity of a plate of glass of given thickness and dielectric constant passes through a series of maxima and minima for varying wave-lengths, so that in specifying the transmittivity of a given plate of glass it is always necessary to specify for what wave-length the transmittivity is found.

7. Conclusions.

As a result of the present study it appears:

1. That a Righi vibrator of small dimensions, mounted with no oil holder surrounding the balls and with as little dielectric as pos-posible in front of the aperture of the mirror and placed at the focus of a paraboloid of revolution acts very nearly like a point source and gives very nearly a parallel beam of energy.

2. That the tuning curves as well as the wave-length curves show that the radiation from a Righi vibrator consists of a short spectrum

of wave-lengths and that the maximum on the tuning curves is governed within the limits of this spectrum by the peculiarities of the receiver.

3. That the non-selective receiver is far superior for all work involving the determination of wave-lengths or the tuning of resonators pasted on screens and placed in the path of the incident energy.

4. That the intensity of the transmitted and reflected energy passes through a series of maxima and mimina as the distance between the reflecting surfaces is increased through a small number of wave-lengths. This agrees well with the theory as worked out for optics.

5. That a screen of resonators acts like a selectively reflecting surface for waves very nearly in resonance with the resonators. This was tested by determining the wave-length of the transmitted beam as well as of the reflected beam. These were found to check at least qualitatively with what one would expect from a consideration of an assumed emission curve for the vibrator combined with an assumed transmission curve for the screen of resonators.

6. That with a tuned receiver *extra-transmission* may be expected for plates of glass of the proper thickness and of the proper dielectric constant and for wave-lengths within proper limits.

7. That the transmittivity of a glass plate of a given thickness and given dielectric constant depends upon the wave-length of the incident energy.

8. That the change of phase due to reflection is not a constant but varies with the length of the resonators pasted on the reflecting surface.

In conclusion the writers wish to thank Mr. H. W. Farwell and Dr. L. B. Morse for kindly assistance in taking some of the observations. They also wish to acknowledge their indebtedness to Professor M. I. Pupin who gave the services of his assistant, Mr. W. E. Cushman, during an illness of one of the writers. They are indebted further to Professor E. F. Nichols who suggested the problem and many details of the method for solving it.

Phœnix Physical Laboratories,
Columbia University, February, 1909.