THE VARIATION WITH VELOCITY OF e/m FOR CATHODE RAYS.

By C. A. Proctor.

THE experimental investigation of the variation with velocity of electromagnetic mass has been carried out with great care by Kaufmann[1] and Bücherer[2] with the aid of the β rays from radium. The conclusions from these two researches are not in accord, and while the weight of evidence since the recent publication of Bücherer's results is certainly on the side of the Lorentz-Einstein rather than that of the Abraham theory, it is important that these results should be checked by different observers and different methods. There are two reasons at least why it appears the best check would be furnished by measurements on cathode rays in a very high vacuum. The range of velocities covered by the β rays is from about four to nine tenths of the velocity of light. Experiments with cathode rays would cover quite a different range — probably from about one to six tenths of the velocity of light. Furthermore a series of simultaneous measurements of discharge potentials, electric deflection and magnetic deflection furnishes a double check on the accuracy of the theoretical formulæ, since the longitudinal as well as the transverse inertia of the electrons here comes into play. This phase of the question has been discussed in some detail by Planck.[3] It is in this respect that the discharge tube has a distinct advantage over radium rays, since in observations upon the latter the only quantities measured are the two deflections and in consequence the transverse mass alone is involved.

Such a series of measurements has been carried out by H. Starke,[4] but the range of potentials was too small or the experimental error

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[1] Kaufmann, Ann. d. Phys., 19, p. 487, 1906.

[2] Bücherer, Ann. d. Phys., 29, p. 589, 1909.

[3] Planck, Verh. der D. Phys. Ges., 8, p. 418, 1906.

[4] Starke, Verh. der D. Phys. Ges., 8, p. 418, 1906.

too large for the results to be of value as evidence in favor of one theory or the other. The work of Classen[1] and Besthemeyer[2] also touches upon this question, but in neither case does the experimenter consider that any conclusive results have been reached.

It was in the hope of obtaining measurements over a sufficient range and of sufficient accuracy to be admissible as evidence on this matter that the work about to be described was undertaken. The results so far obtained are based on measurements of the two deflections alone, and cover a range of velocities only from twelve to forty-three hundredths of the velocity of light. Nevertheless they seem to be of sufficient interest to warrant their publication at this time especially as they are in much closer agreement with the Abraham than with the Lorentz theory.

That the discharge in a cathode tube may take place in the highest possible vacuum, whatever the discharge potential, a condition clearly desirable if not essential, it is necessary to excite it by external means. Two such means suggest themselves — ultra-violet light and the Wehnelt cathode. The former was tried by the writer. The discharge tube was provided with a short branch (*b*), Fig. 1, closed by a quartz window. Through this window the aluminum cathode was illuminated by a powerful oscillatory spark between zinc electrodes. While this spark was rich in ultra-violet light the discharge produced by it was insufficient to cause the screen in the tube to fluoresce even at high discharge potentials. Careful cleaning of the cathode and modifications of the spark circuit failed to change this condition. In view of the results obtained by Lenard this was most surprising, and the writer has been entirely at a loss to account for it. In subsequent work a Wehnelt cathode will be employed as experiment shows that there

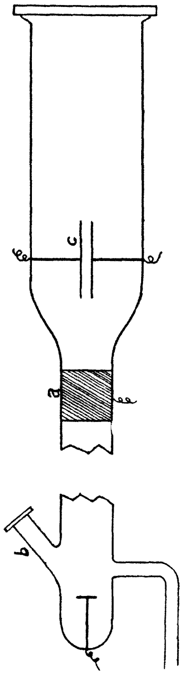

Fig. 1.

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[1] Classen, Verh. der D. Phys. Ges.

[2] Besthemeyer, Ann. d. Phys., 22, p. 429, 1907.

is no difficulty in obtaining a sufficient discharge from it. In the measurements so far made the potential was controlled largely by the degree of vacuum in the tube. The effect of this on the results will be considered later.

To obtain potential measurements of sufficient accuracy to be of any value, a voltometer which can be read quickly and whose readings can be relied on to a fraction of one per cent. is essential. None of the various arrangements of apparatus so far tried by the writer has met these requirements at high potentials, so that only readings of magnetic and electric deflections have as yet been obtained. A new voltometer of the general design of that used by Müller[1] is now under construction in the shop of Ryerson laboratory. Judging by Müller's results this should meet the needs of the experiment.

The arrangement of the discharge tube is shown in the accompanying sketch. The anode (Fig. 1) is a brass cylinder closed at one end except for a slit 0.5 mm. wide and 1.5 cm. long. The condenser (c) consists of two brass plates 4.2 × 2.4 cm. and 0.47 cm. apart. The end of the tube opposite the cathode is closed by a piece of plate glass coated on the inner side with calcium tungstate and cemented on.

The length of the condenser, distance from anode to screen, and distance from condenser to screen which enter into the computation of the electric field integral were measured with a steel scale graduated in hundredths of an inch. The distance between the condenser plates was measured by means of a cathetometer. From the dimensions so obtained the electric field integral

$$\int_a^s \int_a^x F dx \cdot dx$$

was computed with the aid of Maxwell's formula. The value obtained was held under suspicion, however, as the proximity of the walls of the tube, shielded by grounded tinfoil, to the end of the condenser next the anode rendered questionable the applicability of the formula. This suspicion was confirmed by the fact that the values of e/m at low velocities computed from this value of the field integral

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[1] Ann. d. Phys. (28), 3, p. 591, 1909.

were impossibly large. It was accordingly thought best to determine this quantity as follows. A series of forty readings were taken of the magnetic deflection and discharge potential at about 8,000 volts. At this value the potential, furnished by a twenty-four plate static machine, could be held very constant and measured with considerable accuracy with a Braun 10,000-volt electrometer. From these readings the values of e/m and the velocity were obtained in the usual way and the value of e/m for zero velocity computed with the aid of the Abraham formula. The value obtained was 1.859×10^7. This agrees within the limits of experimental error with that of Simon[1] but not with the more recently published results of Classen, Besthemeyer and Bücherer. It should be noted however that a very considerable change in this value would not appreciably affect the conclusions regarding relative values of e/m at different velocities. From this value of e/m and a series of readings of the electric and magnetic deflections also taken at discharge potentials of about 8,000 volts the value of the electric field integral was computed. The value thus obtained was about 10 per cent. less than that given by Maxwell's formula and was used in all subsequent computations. The justification for this method of determining the field integral is found in the work of Seitz,[2] who has shown that it is correct at the potentials here employed.

The magnetic field was obtained by means of a solenoid 24 cm. in diameter made in two sections each 48 cm. long and separated by a gap of 3 cm. to allow of the introduction of the discharge tube. This was mounted with its axis east and west and the discharge tube was so placed that the undeflected rays cut this axis at right angles. As the discharge tube was horizontal there was a considerable lateral deflection of the rays at low potentials due to the vertical component of the earth's field. This was compensated by a large horizontal coil immediately beneath the tube. The form of the magnetic field, proper position of the tube with reference to the solenoid, and correction for finite curvature of the magnetically deflected rays were determined by the methods given in detail by S. Simon.[3] The absolute value of the field on the axis of the solenoid

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[1] S. Simon, Ann. d. Phys., 69, p. 589, 1899.

[2] W. Seitz, Ann. d. Phys., 8, p. 233, 1902.

[3] S. Simon, *l. c.*

was determined as follows: A magnetometer with a heavy phosphor-bronze suspension was mounted so that the magnet was on the axis of the solenoid where the field is sensibly uniform, and adjusted until the magnet was perpendicular to this axis. This condition was judged to be fulfilled when the deflections on opposite sides of the zero were the same for any current within the possible range. The mean deflection for a known current was then determined from ten separate readings. This work was then repeated substituting for the solenoid employed in the experiment another solenoid whose field could be computed from its known dimensions. This reference solenoid was made, and has been repeatedly used, for the determination of the absolute value of the ohm. As in that work it gives results correct to within 0.1 per cent. there can be no doubt that its computed field is very little in error. Currents were measured with an "American" ammeter which was calibrated throughout its range by comparison with a Kelvin balance whose accuracy had been recently checked with the silver voltameter. Two determinations of the field made as above with different magnetometers and widely different currents gave values differing by 0.3 per cent. and their mean was taken as the value of the field.

The potential difference in the plates of the condenser was obtained from a battery of small storage cells. The values used ranged from 70 to 800 volts. This potential was measured with a 320-volt Kelvin multicellular voltometer, calibrated by comparison with a 300-volt Weston direct current instrument. The Weston instrument itself was sent to the Bureau of Standards for calibration. A resistance of 10,000 ohms was placed in series with the battery and condenser. The potential was found to remain sensibly constant during any series of readings. Its value was therefore taken only at the beginning and end of each series and the mean used. These readings were commonly taken while the discharge was passing through the tube. Repeated tests showed, however, that the readings were the same when the discharge was not passing.

The general procedure in taking readings was as follows: The condenser potential and current in the solenoid were so adjusted that the deflection due to either was from one to two centimeters at the particular discharge potential employed. Readings of the electric and magnetic deflections were then taken alternately until a series usually consisting of five of each was obtained. The deflections were measured with a cathetometer which could be set on the fluorescent spot with an error not greater than 0.02 mm. The current in the solenoid was read with each of the magnetic deflections. The means of the currents and of each set of deflections were then taken. Readings were taken in this way to eliminate as far as possible the effect of the unsteadiness in the discharge potential which could not be held perfectly constant. The variations were commonly of the order of 1 per cent. The deflection per ampere in the solenoid and per 100 volts difference of potential on the condenser were then computed.

Forty-four pairs of magnetic and electric deflections obtained as above were for comparison with the theoretical formulæ divided into ten groups according to magnitude and the mean of each group taken. From the magnetic deflections the value of e/m for zero velocity, and the constants of the apparatus, the electric deflections to be expected on the Lorentz-Einstein and the Abraham theories were computed.

The reduction of observations was made by this method for the sake of comparing results with those of Kaufmann, to whom it is due. The method is in brief as follows:

If $z =$ magnetic deflection corrected for finite curvature,
$y =$ electric deflection,
$M =$ magnetic field integral,
$E =$ electric field integral,
$e =$ charge on electrons,
$m =$ mass of electrons,
$\beta =$ velocity of electron divided by velocity of light,
$c =$ velocity of light,

we have from the ordinary theory

$$z = \frac{eM}{m\beta c^2},$$

$$y = \frac{eE}{m\beta^2 c^2}.$$

If we set $m = m_0 \varphi(\beta)$ where m_0 is the mass of the electron at zero velocity and $\varphi(\beta)$ expresses the dependence of mass upon velocity we have

$$z = \frac{e}{m^0}\frac{M}{c}\frac{1}{\beta\varphi(\beta)},$$

$$y = \frac{e}{m_0}\frac{E}{c^2}\frac{1}{\beta^2\varphi(\beta)}.$$

If we know e/m_0, z and the form of the function $\varphi(\beta)$ we can compute y. For the details of this computation the reader is referred to Kaufmann's paper. For the Lorentz-Einstein theory we have

$$\varphi(\beta) = (1 - \beta^2)^{-\frac{1}{2}}.$$[1]

For the Abraham theory we have

$$\varphi(\beta) = \frac{3}{4}\frac{1}{\beta^2}\left\{\frac{1+\beta^2}{2\beta}\log\frac{1+\beta}{1-\beta} - 1\right\}.$$[1]

Table I. shows the results of computation compared with observed deflections. There is close agreement between the writer's results and those of Kaufmann, the variations of computed from observed values being in the same direction in both cases and about twice as great for the Lorentz formula as for the Abraham. On the other hand, both are quite different from those given by the recent experiments of Bücherer, who finds complete agreement between experiment and

Table I.

β	Magnetic Defn.	Elect. Defn. Observed.	Elect. Defn. Lorentz.	Per Cent.	Elect. Defn. Abraham.	Per Cent.
.432	2.678	0.1467	0.1511	3.0	0.1486	1.3
.408	2.866	0.1665	0.1712	2.8	0.1686	1.2
.387	3.022	0.1849	0.1887	1.9	0.1862	0.7
.341	3.485	0.2423	0.2465	1.7	0.2438	0.6
.285	4.205	0.3499	0.3525	0.7	0.3499	0.0
.229	5.353	0.5545	0.5657	2.0	0.5614	1.2
.187	6.535	0.8306	0.8313	0.1	0.8296	−0.1
.157	7.857	1.188	1.195	0.6	1.194	0.5
.140	8.749	1.483	1.478	−0.3	1.476	−0.5
.123	10.043	1.932	1.943	0.6	1.943	0.6

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[1] Abraham, Theorie d. Elect., Vol. II., pp. 191 and 203.

the Lorentz formula. A careful consideration of possible sources of error fails to reveal any that could account for the large discrepancy between the results and those of Bücherer, unless it be the residual gas in the dischage tube. As the discharge potential was controlled to a large extent by the degree of vacuum in the tube there was a considerable amount of gas present at the lower potentials. As has been pointed out by J. J. Thomson and others it is a matter of uncertainty how much this may affect the values of e/m obtained. In his discussion of Kaufmann's results Planck[1] has shown that some modification of the theory of the experiment is necessary, as the value of the velocity of the electrons computed from the apparatus constants and the smallest deflections is greater than the velocity of light, and that they conform more nearly to the Lorentz than to the Abraham formula if the assumption is made that the electric field integral is modified by the presence of residual gas. An attempt to apply this to the writer's results leads to difficulty, for the effect in question is a reduction of the electric field integral and we should expect this reduction to increase with the amount of gas present. In the present work then this effect should be greatest at the low discharge potentials and diminish progressively as the potential increased. If we took account of this in computing the results the apparent variation of e/m with velocity would become smaller. As it is already too small to fit either theory it is evident that this explanation in its present form does not suffice.

Another method of exhibiting the results is shown in Table II. From the observed deflections and constants of the apparatus β and e/m are computed. These appear in columns one and two of the

Table II.

β	Magnetic Defn.	Elect. Defn. Lorentz.	Elect. Defn. Abraham.	β	Magnetic Defn.	Elect. Defn. Lorentz.	Elect. Defn. Abraham.
.432	1.738	1.927	1.886	.229	1.838	1.888	1.877
.408	1.755	1.922	1.885	.187	1.828	1.861	1.854
.387	1.757	1.905	1.873	.157	1.848	1.871	1.866
.341	1.783	1.896	1.872	.140	1.835	1.853	1.849
.285	1.796	1.875	1.858	.123	1.856	1.871	1.867

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[1] Verh. d. El. Phys. Gesch., 9, p. 301, 1907.

table. From them are computed the value of e/m_0 by the Lorentz and by the Abraham formulæ. These are shown in columns three and four. If either formula exactly represented the observations e/m_0 should be constant. The values computed from the Abraham formula are perhaps constant to within the limits of observational errror, but those computed from the Lorentz formula certainly are not.

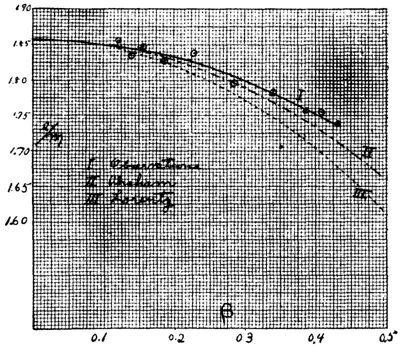

Fig. 2.

Fig. 2 shows the observed values of e/m as a function of β, and the values computed from the two formulæ in question.

In conclusion I wish to express my great indebtedness to Professor Millikan to whose suggestion this work is due, and who has during its progress been ever ready with friendly interest and helpful advice.

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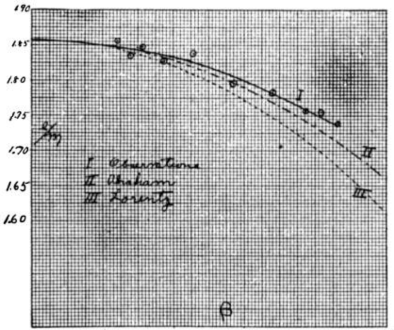
1.90
1.85
1.80
1.75
1.70
1.65
1.60
e/m
I
II
III
I Observations
II Abraham
III Lorentz
β
0.1
0.2
0.3
0.4
0.5

Fig. 2.