NOTE ON THE RELATION BETWEEN THE TEMPERATURE AND THE RESISTANCE OF NICKEL.

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THE splendid work of Callendar confirmed and extended by many others has placed in the hands of physicists that admirable and indispensable tool, the platinum resistance thermometer.

All this work demonstrates that the relation between the temperature and the resistance of platinum wires can be represented over a very wide range of temperature by a parabolic curve, such as given by an equation of this form:

$$R = R_0 + \frac{R_{100} - R_0}{100}\left\{T - \delta\left[\left(\frac{T}{100}\right)^2 - \left(\frac{T}{100}\right)\right]\right\} \qquad (1)$$

or in simpler form:

$$R = R_0 + aT - bT^2. \qquad (2)$$

This equation is due to Callendar who has pointed out that the constant δ depends only on the quality, or purity, of the particular sample of platinum employed. R_0 and R_{100} are the resistances of the thermometer coil at 0° and 100° C., respectively.

Probably no other metal can compete with platinum for use at high temperatures, but it is believed the striking advantages in nickel wire for the construction of resistance thermometers over a considerable range of moderate temperatures is not generally recognized.

Nickel exhibits an inversion point between 350° and 400° C., and sufficient data to determine its suitability for resistance thermometers at very low temperatures are not at present available. Its greater resistance, greater change of resistance with temperature, and its small cost as compared with platinum, all render nickel better than platinum for thermometric purposes over a considerable range of temperatures. While it may possibly be difficult to procure nickel wires of a uniformly high purity, yet the maximum purity seems unessential since what passes as commercially pure nickel seems to possess the advantages now in mind. Callendar has already shown that the temperature resistance curve of platinum is a parabola. At least three points must be located to define the parabolic curve.

The nickel resistance curve seems to be very accurately represented by a logarithmic curve of this form:

$$\text{Log } R = a + mT. \tag{3}$$

Table of Observed and Calculated Resistances of Nickel-Resistance Thermometers.

A				B				C			
Tem. °C.	Resistance.		Dif. Ob. — Cal. %.	Tem. °C.	Resistance.		Dif. Ob. — Cal. %.	Tem. °C.	Resistance.		Dif. Ob. — Cal. %.
	Obsd.	Calc.			Obsd.	Calc.			Obsd.	Calc.	
−25	11.030	11.036	−.05	−27.7	70.72	70.79	−.10				
−20	11.250	11.254	−.04	−27.6	70.74	70.82	−.11	0[1]	8.42	9.10	− 7.5
−15	11.475	11.477	−.02	−24.6	71.64	71.71	−.10	100	12.66	12.73	− 0.5
−10	11.700	11.703	−.02	−23.6	72.34	72.02	+.45	200	17.97	17.80	+ 0.9
− 5	11.935	11.935	±.00	−23.6	71.92	72.02	−.14	300	24.95	24.90	+ 0.2
0	12.173	12.173	±.00	−18.4	73.57	73.60	−.04	325	27.06	27.08	− 0.1
+ 5	12.420	12.413	+.06	−12.9	75.39	75.32	+.09	350	29.33	29.45	− 0.4
10	12.660	12.659	+.01	− 7.04	77.29	77.20	+.12	375	31.98	32.02	− 0.1
15	12.920	12.908	+.10	− 5.95	77.65	77.55	+.13	400[1]	33.67	34.84	− 3.5
20	13.173	13.163	+.08	+ 8.4	82.42	82.36	+.06	425[1]	34.70	37.87	− 8.4
25	13.435	13.423	+.09	8.8	82.54	82.50	+.05	450[1]	35.63	41.19	−15.6
30	13.700	13.689	+.08	9.0	82.61	82.57	+.05				
35	13.965	13.959	+.04	9.2	82.70	82.64	+.07				
40	14.240	14.235	+.04	11.38	83.44	83.40	+.05				
45	14.520	14.516	+.03	23.76	87.70	87.84	−.16				
50	14.800	14.802	−.01	24.05	87.82	87.95	−.15				
55	15.080	15.093	−.09								
60	15.385	15.393	−.05								
65	15.690	15.697	−.05								
70	16.000	16.007	−.04								
75	16.320	16.323	−.02								

[1] Not included in computation of equation.

A = Thermometer made by Leeds & Northrup for Weather Bureau for the measurement of solar radiation:

$$\text{Equation: Log } R = 1.08539 + .001699t. \tag{4}$$

B = Data supplied by Leeds & Northrup.

$$\text{Equation: Log } R = 1.90045 + .001818t. \tag{5}$$

C = Data supplied by Bureau of Standards on a single specimen of wire regarded as impure:

$$\text{Equation: Log } R = 0.96145 + .00145t. \tag{6}$$

Only two points are required to locate such a curve. The coefficient m depends upon the quality, that is, the purity, and possibly some other of the physical properties and conditions of the metal.

The data at present available to the writer in support of the logarithmic curve is conceded to be too scanty and inadequate to demonstrate anything like a general law, but the conformity is sufficient for a great many purposes, and is shown in the following table:

In the two thermometers from Leeds & Northrup the conformity to the logarithmic equation is really very close, although no great range of temperature is embraced. The residuals are of about the same order of magnitude as the probable accuracy of the resistance measurements, viz: one tenth of one per cent.

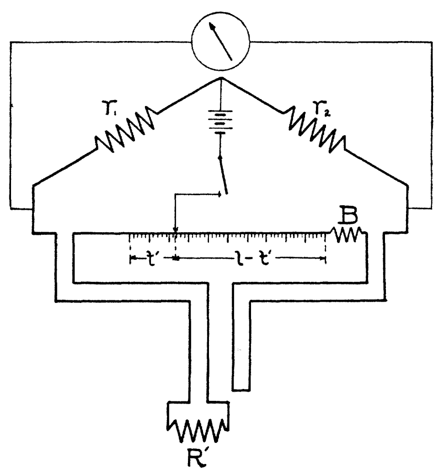

Fig. 1. Theoretical diagram of Wheatstone's bridge.

The data from the Bureau of Standards are not so well fitted by a logarithmic curve. The point at zero degrees, especially, is discordant, but from 100° to 375° the conformity is much closer.

Admitting that the nickel wire temperature resistance curve is approximately logarithmic, it remains to point out that resistance readings of a nickel thermometer on the usual form of slide-wire bridge, or equivalent, having a scale of equal parts, may also be made to give gas scale temperatures with a very considerable accuracy.

The two diagrams, Figs. 1 and 2, show two forms of bridge connections commonly employed in this kind of work. We consider only in each case a scale of equal parts. Any scale of unequal parts is an undesirable, if not an impracticable or impossible, thing.

The relation between resistance and scale reading in the Fig. 1 arrangement is strictly linear, and therefore the scale readings show temperatures on the gas scale only when the resistance of the thermometer varies according to a strictly linear law. This arrangement is used extensively with the platinum thermometer, and the scale readings are the so-called platinum temperatures.

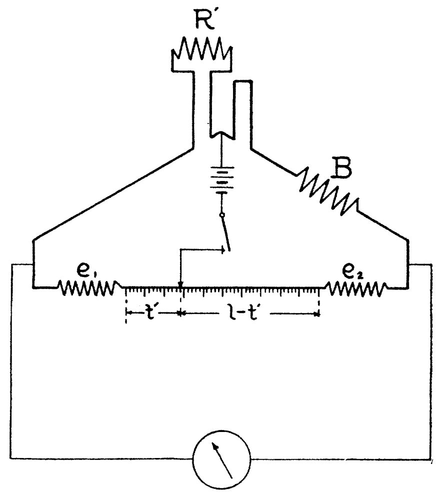

Fig. 2. Theoretical diagram of Wheatstone's bridge.

In the arrangement of Fig. 2 the relation between the scale readings t' and the resistance R' is given by the equation:

$$R' = B\frac{e + t'}{e + l - t'} \quad \text{when} \quad e_1 = e_2. \tag{7}$$

The curve represented by this equation between R' and t' is concave upward as is also the logarithmic curve representing nickel resistance, viz:

$$\text{Log } R = a + mt. \tag{3}$$

I now wish to show that a section of the $R't'$ curve, representing a temperature range of say 50° to 100° will conform with extreme closeness to the logarithmic curve over the same range; that is, the t' scale of the bridge is then accurately the gas temperature scale.

Suppose, for example, we want the extreme range of the bridge scale to embrace 100 degrees of temperature, say from t_1 to t_2. Now, the $R't'$ curve can be made to intersect the Rt logarithmic curve in two points determined by the two constants, B and e ($l =$ the resistance of the whole bridge wire, is determined by the range of temperature embraced, viz: 100 degrees in this case). The two curves will conform most closely over the whole range when the points of intersection lie at certain particular points between the two extremes t_1 and t_2 as at PQ, Fig. 3.

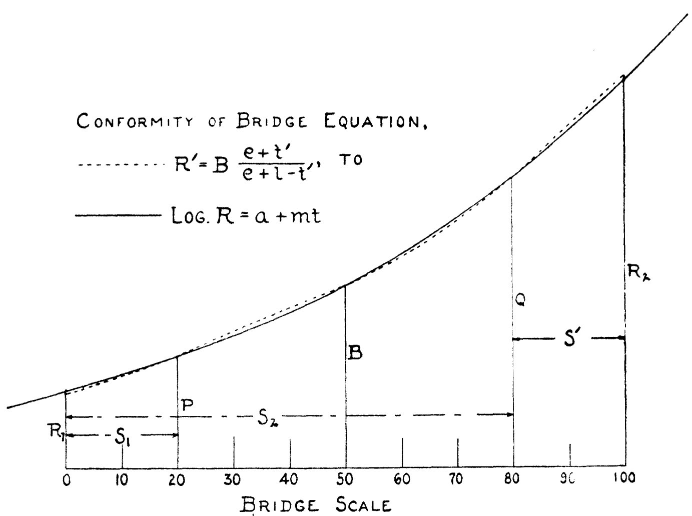

Fig. 3.

It is quite sufficient to select the two points, PQ, arbitrarily for example, at say $\frac{1}{4}l$, and $\frac{3}{4}l$. We can even pass the $R't'$ curve through the points t_1 and t_2, or through t_1 and the middle point m, or through m and t_2. The deviation of the curves one from the other is smaller, as a rule, than the ordinary errors of observation, except in work of the highest precision. The general equations for computing the values of B and e giving two intersections intermediate between the extremities of the range comprised by a bridge scale are:

$$B^2 = PQ,\text{[1]} \tag{8}$$

$$e = \frac{P(l - s_1) - BS_1}{B - P}. \tag{9}$$

In expressing these equations it is assumed for simplification that S_1 and S_2 are symmetrically located on the bridge scale so that $S_1 + S_2 = l$ in all cases.

A single example will illustrate the magnitude of the deviations between the two curves, and, in order to bring this out clearly it has been necessary to carry out the computation to six figures, which, of course, represents a much higher order of precision than obtains in ordinary work.

TABLE II.

Table of Corrections to Reduce Bridge Scale Readings, t', to Gas Scale Temperatures, t, with a Nickel-resistance Thermometer:

Equation: Log $R = 1.0859391 + 0.00169881t$. $S_1 = 20°$, $S_2 = 80°$, $l = 100°$.

Bridge Scale t'	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°	100°
Corrections to gas scale t	+.10	+.04	−.00	−.01	−.01	.00	+.01	+.01	.00	−.04	−.10

Suppose we have fixed upon values of B and e in the equation

$$R' = B\frac{e + t'}{e + l - t'}. \tag{10}$$

Such that the $R't'$ curve intersects the logarithmic curve at two points, PQ. We may now imagine the $R't'$ curve to slide upward or downward along the logarithmic curve, and it will still conform closely to the logarithmic curve. This means that by increasing or diminishing the resistance B we can shift the whole thermometric scale to a higher or lower range of temperatures, while still retaining the value of $e_1 = e_2$ on the bridge, and without changing the deviations of the bridge scale from the temperature scale.

[1] It is a fundamental property of the logarithmic curve that the ratio of any two ordinates, $P \div R_1$ for example, is the same as the ratio, $R_2 \div Q$, of any two other ordinates which are separated by the same horizontal space; that is, when $S_1 = S'$. As a corollary of this proposition, the middle ordinate between two ordinates is the square-root of their product. Therefore, in equation (8), $B = \sqrt{PQ}$ is a point on the logarithmic curve. Consequently, the $R't'$ curve must always intersect the logarithmic curve in the middle point as well as at P and Q. The product PQ is a constant for every pair of ordinates of which B is the middle ordinate.

The data whose examination has called forth the results herein presented are admitted to be open to uncertainty, but it seems that easily available samples of nickel wire show a temperature resistance curve very nearly represented by the logarithmic equation, or, what is nearly the same thing, by the equation (10).

It is therefore worth while to bring out the useful mathematical relations existing between the logarithmic and the bridge scale equations. The matter is of the greatest practical utility, not only in scientific investigations, but in a large field of commercial work demanding a more or less accurate knowledge of moderate temperatures.

In view of the foregoing, we hope those in a position to do so may investigate more fully the variations of resistance of nickel with temperature, and especially the determination of the relation between the composition of the material employed and the character of the resistance curve.

The practical application of all this theory is very simple. The maker of bridges sets up the arrangement shown in Fig. 2. The resistance B must be accurately adjusted to equal the resistance the thermometer will have when its temperature is that represented by the middle point of the bridge scale. The resistances e_1 and e_2 must be accurately *equal*, but the exact value is not of much importance. If e is a little greater or smaller it simply shifts the P and Q points of intersection either nearer to or farther from the extremities of the bridge scale. Having approximated the e coils it remains only to "point" the bridge scale for graduation into equal parts. Put into the bridge any convenient known resistance equal to the thermometer resistance at some known temperature, preferably near the P and Q points of the range. The point at which the bridge balances corrected for the small errors in table II is the particular temperature point of the bridge scale, and nothing more is necessary than to run a scale of equal subdivisions through this point and the middle point of the bridge. The careful man will probably prefer to locate another point of the scale near the upper temperature limit. Any inequalities of resistance of the bridge wire introduce errors, as is well understood, but which can often be neglected or corrected for by calibration if necessary.