DEMAGNETIZATION FACTORS FOR CYLINDRICAL RODS.

By C. Riborg Mann.

THE effect of the form of a substance on the intensity of magnetization which it assumes when brought into a magnetic field has been a much-discussed question. F. Neumann[1] attacked the problem from the analytical side, and showed that the intensity of magnetization can be calculated when the magnetization is uniform. He further demonstrated that the magnetization induced in any substance, when brought into a uniform field, will itself be uniform only when the substance is bounded by a surface of the second degree.[2] As the ellipsoid is the only surface of second degree whose dimensions are finite, it is the only form which is of practical interest to us here.

For the special case of a prolate spheroid brought into a uniform magnetic field whose lines of force run parallel to the axis of revolution of the spheroid, Neumann gives the following formula for calculating the intensity of magnetization:[3]

$$C = \kappa Z \frac{1}{1 + N\kappa};$$

Or in the modern notation,

$$\mathfrak{J} = \kappa \mathfrak{H} \frac{1}{1 + N\kappa}, \qquad (1)$$

in which $\mathfrak{J}$ = intensity of magnetization,

$\mathfrak{H}$ = strength of field,

κ = susceptibility,

N = a constant depending on the ratio of the axes of the ellipsoid.

[1] Neumann, Crelle's Journal, Vol. XXXVII., p. 44.

[2] Maxwell, Electricity and Magnetism, § 437.

[3] Maxwell, Electricity and Magnetism, § 438.

For our special case, the one of greatest practical use, this factor N is given by the following formula:

$$N = +4\pi\left(\frac{1}{\epsilon^2} - 1\right)\left(\frac{1}{2\epsilon}\log\frac{1+\epsilon}{1-\epsilon} - 1\right), \tag{2}$$

in which $\epsilon = \sqrt{1 - \frac{b^2}{a^2}}$, when b stands for the smaller, a for the greater semi-axis of the ellipsoid.

We note that N depends only on a and b, *i.e.* on the form of the ellipsoid in question.

If we clear the equation (1) of fractions, we get

$$\mathfrak{J} = \kappa\,(\mathfrak{H} - N\mathfrak{J}). \tag{3}$$

When κ is very small, as in all substances except nickel, cobalt, and iron, the coefficient of N, *i.e.* $\kappa\mathfrak{J}$ is very small in comparison to $\kappa\mathfrak{H}$, and is, therefore, generally neglected, thus making the determination of κ practically independent of the form of the material investigated, *i.e.* independent of N. Hence in what follows, as we are to discuss N, we will speak only of the paramagnetic substances where κ is large.

When in formula (2) $a = \infty$, or $e = 1$, *i.e.* when the ellipsoid considered is endless, $N = 0$, and formula (3) becomes $\mathfrak{J} = \kappa\mathfrak{H}$, the fundamental equation for the intensity of magnetization. As a grows smaller, b remaining constant, N increases. Hence this N in formula (3) characterizes the effect of the free ends of the ellipsoid on its own magnetization, and is called, since the action of the free ends in paramagnetic substances is to diminish the effective magnetic field, the demagnetization factor.

If now N is known, $\mathfrak{J}$ and $\mathfrak{H}$ being measurable, formula (3) may be used to calculate κ, the magnetic susceptibility of any substance. Hence we may determine κ either on rings, where $N = 0$, as Rowland did, or on ellipsoids, where N can be calculated from formula (2). Both of these methods are open to the practical objection that it is difficult to obtain either a suitable ring or an ellipsoid made of the material whose susceptibility we wish to know. Could we but make our observations on cylinders, which are easily procured, the task of determining κ would in most cases be much facilitated. The following investigation was therefore undertaken to determine whether N is independent of $\mathfrak{H}$ and $\mathfrak{I}$ for cylinders as for ellipsoids, and whether or not its value is the same for a cylinder as for the corresponding ellipsoid, *i.e.* one whose greater axis is equal to the length, and whose smaller axis is equal to the diameter of the cylinder. The magnetization in the case of a cylinder is not uniform, as it is in ellipsoids; therefore, by the intensity of magnetization of a cylinder is understood a mean value obtained by dividing the total magnetic moment by the volume. Hence the magnetometric method was preferred for this work.

It was formerly assumed that a cylinder could be used for its corresponding ellipsoid of revolution in observations on magnetization by induction.[1] Du Bois,[2] on the other hand, has shown from observations of Ewing[3] and Tanakadaté,[4] that this is not the case. From these observations he deduces a table giving N for various values of the ratio $\frac{\text{length}}{\text{diameter}}$ of the cylinder, this ratio being denoted m. These values for N may be used to reduce observations made on cylinders to those made on corresponding ellipsoids or on rings.

The observations on which this table is based, though the best that then existed, are not entirely satisfactory for computing these factors, and this for several reasons.[5] *First*, those of Ewing were made with too short a magnetizing coil, so that his magnetizing fields were not uniform throughout the whole space occupied by his core. *Secondly*, the two sets do not join, as Ewing's shortest cylinder had $m = 50$, while for Tanakadaté's longest cylinder m was only 39, thus necessitating an extrapolation over the interval $39 - 50$. *Thirdly*, the two sets of measurements were made in different ways, Ewing having used the ballistic, Tanakadaté the magnetometric method. In my opinion the results from these

[1] W. Weber, Electrodynamische Maasbestimmungen, III., p. 573, 1867; Kirchhoff, Ges. Abh., p. 221; Oberbeck, Poggendorff's Annalen, CXXXV., p. 84, 1868.

[2] Magnetische Kreise, Berlin, 1895.

[3] Philosophical Transactions, 176, II., p. 535, 1885.

[4] Philosophical Magazine, 5 Series, Vol. XXVI., p. 450, 1888.

[5] *loc. cit.* p. 535.

two different methods of measuring induction cannot be used together with certainty in a case like this.

Ascoli,[1] however, has recently published a table of these factors obtained from observations on cylindrical bundles of iron wire, which agreed very closely with that of Du Bois. Nevertheless it seemed doubtful to me if these numbers of Ascoli's could be used with certainty for the demagnetization factors of solid cylinders, because bundles of wire, as has been noted already by several physicists,[2] do not react towards magnetization as do solid cylinders of the same material, length, and cross-section, which we may term corresponding cylinders.

In order now to solve the problem satisfactorily, I proceeded as follows: I made a long series of magnetization curves, using cylinders of the same material but of different form, using also bundles of wire, being exceedingly careful that the results should be strictly comparable with each other, and from these deduced the conclusions given below. To give a detailed list of the observations were to take far too much space. The method used was briefly this:[3]

A long thin soft-iron wire was taken, for which $m=300$ (length 25.08 cm., diameter 0.0836 cm.). After determining the curve of magnetization, giving the valuation between $\mathfrak{J}$ and $\mathfrak{H}$, equal lengths were cut from each end, so that the wire assumed the form $m=200$. The magnetization curve was again determined and the wire again shortened to $m=150$, etc., until m became equal to 50. These observations were made after the usual magnetometric method, using a coil 38.5 cm. long of 1.5 cm. inner diameter, thus assuring me of a uniform field throughout the entire space occupied by the iron. The current was measured by a carefully calibrated ammeter and the strength of field calculated in the usual way by multiplying the number of amperes by the constant of the coil obtained by the well-known formula.

The effect of the magnetizing coil on the magnetometer was

[1] Rendiconti della royale Academia dei Lincei, 3, p. 190, 1894.

[2] v. Waltenhofen, Wiener Berichte, 61, II., p. 771, 1870; Warburg and Hönig, Wiedemann's Annalen, p. 828, 1883.

[3] The details of this work were published as a dissertation in Berlin, 1895, which may be obtained from the author.

balanced by another smaller coil through which the magnetizing current flowed in the opposite direction to that which it took in the main coil. This small coil was so placed that it just balanced the effect of the magnetizing coil on the magnetometer when the needle was at the zero point. Slight corrections had to be applied in many cases to the readings of the magnetometer when the needle was deflected, because this compensation was not perfect except at the zero point. Having adjusted this smaller coil so that the reading of the magnetometer remained the same when the current through both coils was flowing in either direction or

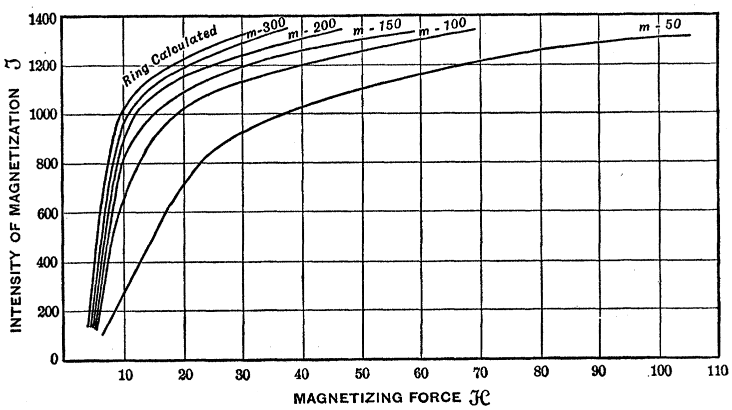

Fig. 1.

Magnetization Curves for Cylinders for Values of *m* between 300 and 50.

not flowing at all, the iron to be investigated was brought into the center of the magnetizing coil and subjected to a small field. The readings of both ammeter and magnetometer were taken and the polarity of the field was reversed, its strength remaining as nearly constant as possible, and then a second reading was taken. The means of these two opposite readings were used to calculate, by the usual formula,[1] the corresponding values of $\mathfrak{H}$ and $\mathfrak{I}$. The field was then strengthened and the same operation repeated, — in short, the method of ascending reversals was used. From these cylinders made of iron wire, the magnetization curves in Fig. 1 for values of *m* between 300 and 50 were made.

[1] Wiedemann, Electricität, 3, § 428.

For shorter cylinders a different method was used. A short, thick rod for which m was only 5 (length, 11.850 cm.; diameter, 2.370 cm.), was gradually turned down, the length remaining constant, till m reached the value 50; *i.e.* till its form was the same as that of the shortest of the former set. Thus the two sets joined together, and in each the same iron was used throughout. This second set ($m=5$ to $m=50$) was made twice, using different qualities of iron and a different length.[1]

The first set was executed three times and the mean taken. Each curve of the other set was run at least three times and the

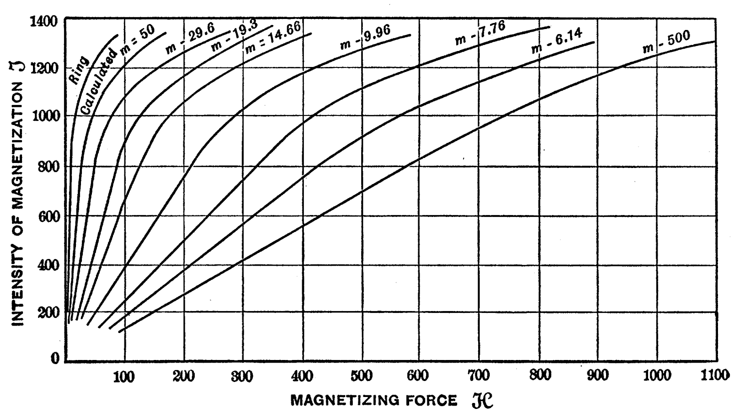

Fig. 2.

Magnetization Curves for the First Cylinder.

mean taken. The curves given in Fig. 2 are these mean curves. To get the demagnetization factors N from the curves, we proceed as follows: Let $\mathfrak{H}_1$ and $\mathfrak{H}_2$ be values of $\mathfrak{H}$ which belong to the same $\mathfrak{I}$ on any two curves, and N_1 and N_2 the corresponding values of N. Then from (3)

$$\mathfrak{I}=\kappa(\mathfrak{H}_1-N_1\mathfrak{I})=\kappa(\mathfrak{H}_2-N_2\mathfrak{I}),$$

or

$$N_2=N_1+\frac{\mathfrak{H}_2-\mathfrak{H}_1}{\mathfrak{I}}; \qquad (4)$$

i.e. we must measure the difference in $\mathfrak{H}$ of the two curves under consideration along the same $\mathfrak{I}$-line, and divide this difference

[1] For the second cylinder the length was 9.620 cm.

by the $\mathfrak{J}$ which belongs to that line, and add the quotient thus obtained to the demagnetization factor corresponding to the first curve.

This method presupposes the knowledge of N_1. There are several methods by which this factor can be determined. In my table below I assumed the curve for which $m=300$ as the curve 1. The value of the corresponding N, which I will designate N_{300}, I unfortunately did not have time to determine by an independent method before the work was of necessity broken off. I assumed $N_{300}=0.00075$, the value belonging to the corresponding ellipsoid, and for the following reason: Du Bois has shown theoretically that, for cylinders whose length is very much greater than their diameters, the quantity Nm^2 should be constant. From Ewing's observations he deduces the value of this constant as 45. If I assume this theoretical law of du Bois, I have the condition necessary to determine N_{300} from my observations as du Bois did.[1] The work is as follows, using the data from my observations:

m	N	x	Nm²
300	x		
200	0.00079 + x	0.00041	48.0 (200 and 150)
150	0.00172 + x	0.00041	48.0 (200 and 100)
100	0.00438 + x	0.00041	47.9 (150 and 100)
Mean		0.00041	48.0

It is quite evident that my observations do not satisfy the theoretical conclusions of du Bois. A similar calculation for ellipsoids gives:

m	N	x	Nm²
300	x		
200	0.00085 + x	0.00044	51.6 (200 and 150)
150	0.00185 + x	0.00042	50.8 (200 and 100)
100	0.00465 + x	0.00040	50.0 (150 and 100)
Mean		0.00042	50.8

[1] Wiedemann's Annalen, XLVI., 1892.

These two tables are seen to be very similar. Therefore it is very probable that these long cylinders act very similar to their corresponding ellipsoids, as has always been assumed;[1] and hence I felt warranted in assuming the value I did for N_{300}.[2]

Having this for a starting point, the values of N for the other curves are easily calculated by formula 4.

The measurements on bundles of wire were conducted simultaneously with those on the second short cylinder. The wire used was 0.0801 cm. in thickness, and cut into lengths of 9.8 cm.

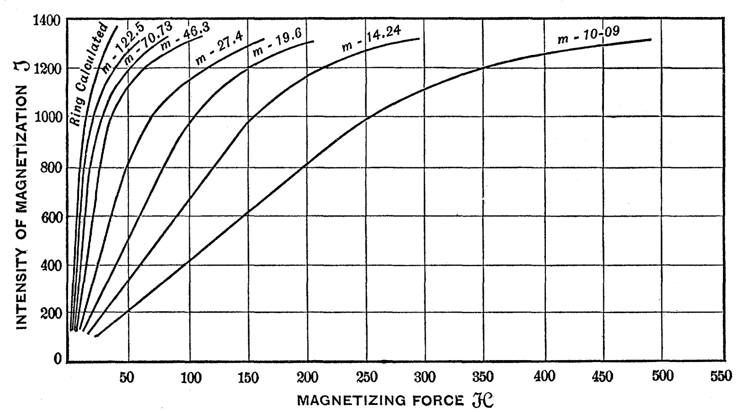

Fig. 3.
Magnetization Curves for Wire Bundles.

These lengths were bound into cylindrical bundles, and the magnetization curves (given in Fig. 3) determined as for solid cylinders.

The size of the bundles varied from a single wire, for which $m = 122.5$, to 171 wires, for which $m = 9.37$. The value of N_1 for the wire bundles was of course that for a cylinder of the form of a single wire; *i.e.* the value corresponding to $m = 122.5$. This factor was taken from the table of N for solid cylinders. In interpolating and comparing the observations, the factor N alone was not used, but rather the expression Nm^2, as this latter serves much better for this purpose. Figure 4 contains the curves

[1] Maxwell, Electricity and Magnetism, § 438.

[2] It is probable that it should be a trifle smaller, say 0.00070; but this difference of 0.00005 is not appreciable for the shorter cylinders.

$Nm^2 = f(m)$ for solid cylinders, ovoids, and wire bundles. The points represent the various observations.

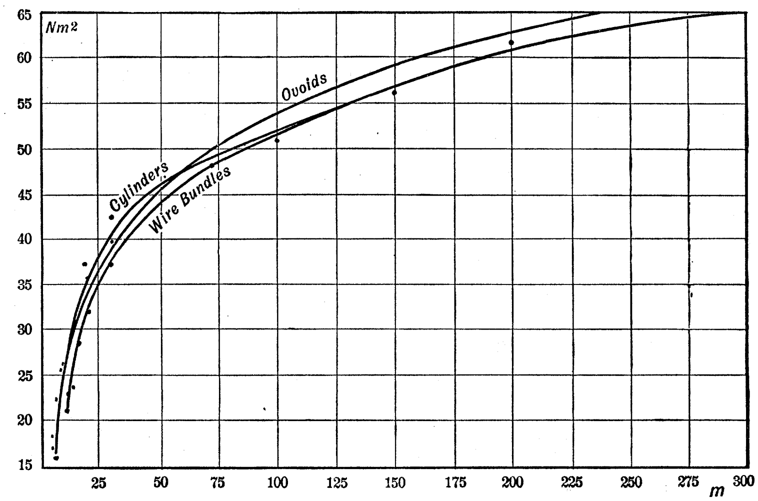

Fig. 4.

Nm^2 as a function of m.

The following table gives corresponding values of m, N, and Nm^2 for the three cases:

	Cylinders.		Ovoids.		Wire Bundles.	
m	N	Nm^2	N	Nm^2	N	Nm^2
5	0.68000	17.0	0.7015	17.5	—	—
10	0.25500	25.5	0.2549	25.5	0.22750	22.8
15	0.14000	31.5	0.1350	30.4	0.12580	28.3
20	0.08975	35.9	0.0848	34.0	0.08225	52.5
25	0.06278	39.3	0.0579	36.2	0.05680	35.5
30	0.04604	41.4	0.0432	38.8	0.04213	37.9
40	0.02744	43.9	0.0266	42.5	0.02596	41.5
50	0.01825	45.6	0.0181	45.3	0.01760	44.0
60	0.01311	47.2	0.0132	47.5	0.01277	46.0
70	0.00988	48.4	0.0101	49.5	0.00951	47.8
80	0.00776	49.7	0.0080	51.2	0.00768	49.1
90	0.00628	50.8	0.0065	52.5	0.00623	50.5
100	0.00518	51.8	0.0054	54.0	0.00515	51.5
150	0.00251	56.5	0.0026	58.3	—	—
200	0.00152	60.8	0.0016	64.0	—	—
300	0.00075	67.5	0.00075	67.5	—	—

The column N for ovoids was calculated from formula (2). They are good for all values of $\mathfrak{J}$.

When I calculated the value of N for cylinders and wires from the observations by formula (4) under the supposition $N_{300} = 0.00075$, I found that N remains practically constant only for $\mathfrak{J} < 800$. Hence the numbers given in the table are obtained by calculating from the observations the value of N for every round hundred of $\mathfrak{J}$ from 300 to 800, and taking the mean. They are, therefore, called mean demagnetization factors, and are good only until $\mathfrak{J}$ reaches the value 800 c.g.s.

It will be seen from the curves that for $\mathfrak{J} > 800$ the magnetization curves fall off rapidly from the curve 1 for which $m = 300$, causing a correspondingly rapid increase in the values of N. This same effect has been noted by Lehmann[1] in experiments on radially cut rings. The wire bundles lie intermediate between cylinders and ellipsoids in this respect, the demagnetization factors remaining constant longer than those of their corresponding cylinders.

The following table for $\mathfrak{J}$, with the corresponding value of N, will illustrate the point in hand:

$\mathfrak{J}$	N		$\mathfrak{J}$	N	
	$m = 46.30$ wires.	$m = 29.60$ cylinders.		$m = 46.30$ wires.	$m = 29.60$ cylinders.
300	0.02167	0.04718	900	0.01950	0.05166
400	0.01907	0.04822	1000	0.02085	0.05877
500	0.02024	0.04873	1100	0.02480	0.07964
600	0.01984	0.04870	1200	0.03287	0.09768
700	0.01976	0.04892	1300	0.05181	0.12777
800	0.01962	0.04735			

The results may be summed up as follows.

The mean magnetization of a cylinder does not differ greatly in amount from the magnetization of the corresponding ellipsoid for values of $\mathfrak{J} < 800$ c.g.s. For $\mathfrak{J} > 800$ an ellipsoid assumes a much stronger magnetization for the same magnetizing force than its corresponding cylinder.

[1] Wiedemann's Annalen, XLVIII., p. 406, 1893.

Wire bundles assume, when $\mathfrak{J} < 800$, a much stronger magnetization for the same magnetizing force than either their corresponding ellipsoids or cylinders. The ellipsoid has, however, greater susceptibility for higher values of $\mathfrak{J}$.

Cylinders whose length is from 20 to 30 times their diameter differ most from the corresponding ellipsoids in their reaction towards induced magnetization.[1]

These values of N for cylinders, when used in formula (3), will give the correct value of κ, provided only that $\mathfrak{J} < 800$ c.g.s.

This result is practically of value, as it enables us to determine κ from observations made by the ordinary magnetometric method on cylinders.

The above investigation was carried on in the physical laboratory of the Berlin University, under the direction of the late Professor Kundt and Professor Warburg, whose kindness and assistance I wish here gratefully to acknowledge.

UNIVERSITY OF CHICAGO, January, 1896.

[1] Cf. Tanakadaté, Philosophical Magazine, 5 Series, Vol. XXVI., p. 453, 1888.