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A SYSTEMATIC STUDY OF VIBRATORS AND RECEIVERS FOR SHORT ELECTRIC WAVES.

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I. INTRODUCTION.

THE work described in this paper was undertaken to find, if possible, such definite relations between the dimensions and properties of vibrators and receivers used in measurements with short electric waves, as would make possible a more systematic procedure in future investigations. In the past it has been customary to construct vibrators and receivers of any arbitrary form, and to separately determine the properties and the wave-length corresponding to each. This somewhat haphazard method has proved unnecessarily tedious and has led to many inconsistent results and in many cases it has not been possible for later workers adequately to check the wave-length determinations made by their predecessors. It has also been very difficult to select in advance apparatus of suitable dimensions for new experiments of a definite character. It seemed, then, of great importance to eliminate this chance procedure by determining how the wave-lengths depended upon the dimensions and other details of vibrators and receivers, and to find the absolute value for certain definite standard forms, which could be reproduced at any time from given data. In addition, the solution of several minor problems was attempted. The methods of measurement, such as the resonance and interference curves which have been extensively used in the past, also formed the object of considerable study, as it is probable that they have caused much of the inconsistency which disfigures this branch of physical investigation. In this manner it was hoped to bring the investigation of the short electric waves to such a condition, that future results could be interpreted without incorrect assumptions, based largely on the very misleading analogies with other types of radiation.

2. Method of Measurement.

The greater part of this investigation was based on wave-length determinations made by the Boltzmann mirror interference method, previously used by many investigators.[1] The essential feature of this method is the use of two plane parallel mirrors, which split the beam from the exciter, or "vibrator" as we shall call it, into two equal parts with a difference in path depending upon their relative displacements. The two reflected beams then fall on a "collecting" mirror, which brings them to a focus on the receiver. The method used in this work differed fundamentally from that employed by most of the earlier workers, in that a non-selective or untuned receiver was used in place of the selective or tuned receiver.

The "tuned receiver" method of measuring the principal wave-length emitted by a vibrator consists of two steps. First, the "tuning" of a receiver to the vibrator, that is, the determination of the form and dimensions of a receiver which gives the maximum energy; and second, the using of this tuned receiver to measure the wave-length by an interference method. Now it has always been found[2] that the wave-length measured in this way depends mainly on the receiver and only slightly on the emission of the vibrator, which indicates that the result depends mainly upon the accuracy of the tuning, and that any error here will introduce an equal error into the wave-length measurement. In the course of the present study, and as a result of over one hundred resonance curves taken under a variety of conditions, it was found that the tuning process, unchecked, oftener than not, gave untrustworthy results and was seldom free from errors that made accurate measurements impracticable. This is one probable cause of many of the discordant

[1] Klemencic and Czermak, Wied. Ann., 50, 177, 1893.

[2] G. F. Hull, PHYS. REV., V., 241, 1897; Willard and Woodman, PHYS. REV., XVIII., 18 1904; A. D. Cole, PHYS. REV., XX., 271, 1905.

results found in earlier work. A more complete discussion of this point will be taken up in the latter half of this paper.

This difficulty is avoided with the non-selective receiver and the wave-length determined is a function of the vibrator only. The disturbing factors and the difficulties of measurement are greatly increased by its use, but with proper care satisfactory and consistent results were obtained, which was not the case when tuned receivers were used. The non-selective receiver in wave-length measurements is not wholly new as the coherer may come under this head.[1] G. F. Hull employed a nail coherer with his interferometer method, although the question of its non-selectiveness was somewhat in doubt. For this study, however, the coherer was not suitable, as its usefulness in quantitative determinations is doubtful.

In seeking for a suitable non-selective receiver the effect of a resistance on the selective qualities of a resonator was investigated. If to the thermo-junction at the center of a Klemencic receiver[2] a large resistance be added, or if we employ a bolometer wire[3] of high resistance to measure the energy received, the resulting damping might be expected to prevent resonance.[4] This was tested by introducing near the thermo-couple short resistance wires of platinum, .045 mm. to .000 7 mm. in diameter, and "tuning," that is, measuring the energy for different receiver lengths. Although this resistance was varied from 0 to 500 ohms, this method did not give a non-selective receiver, since all the curves indicated a marked resonance of some kind. With a resistance of 500 ohms the energy received increased from 65 per cent. to 100 per cent. as the length of the receiver was shortened from 17 to 7 centimeters, decreased to 75 per cent. at 5.5 centimeters, and rose again to 100 per cent. at 3.5 centimeters. Again, with 150 ohms the energy dropped from 100 per cent. at 7 centimeters to 45 per cent. at 45 centimeters, and similar results were obtained with resistances of 25 and 70 ohms. Since the change for a receiver with no resistance added to the thermo-junction is not more than 75 per cent. between the lengths 14 centi-

[1] J. G. Bose, Phil. Mag., 43, 60, 1897; G. F. Hull, *l. c.*, p. 238.

[2] I. Klemencic, Wied. Ann., 45, 78, 1892.

[3] Rubens and Ritter, Wied. Ann., 40, 58, 1890.

[4] W. P. White, Phys. Rev., XXV., 138, 1907.

meters and 5 centimeters, we see that the introduction of large resistances does not greatly increase the non-selectiveness.

To explain this result we need only consider the independent resonance of the two antennæ or wings, between which lay the constriction consisting of the thermo-couple and the added resistance. When these wings were of proper length for individual resonance, the current passing between their adjacent ends through the constriction was a maximum, the action being very similar to that of the thermo-electric receiver employed by Lebedew in his work with short waves.[1] Thus the effect of increasing the resistance at the center was simply to split the single resonator into two, each of which possessed all the qualities of resonance.

The form which was finally adopted consisted of a Klemencic receiver, the length of which was ten or more times that of the average "tuning," or resonance length corresponding to the wave-lengths measured. The non-selective action of this arrangement is due to two causes. First, the absence of standing waves because of the rapid damping of the exciting wave, which has entirely ceased to affect the receiver by the time the first impulse has returned to the thermo-junction after reflection from the free ends. And, second, the overlapping of the broad resonance maxima of neighboring wave-lengths, which differ but little. For example, a receiver 70 centimeters long corresponds approximately to multiple tones of the wave-lengths, . . . , 6.5, 7.0, 7.6, 8.5, 9.4, 10.8, etc., centimeters, the lack of sharpness in the resonance, which is also due in part to the large damping in the incident wave, preventing any selective action. For the same reason a receiver whose length is very short compared with the resonance length should be non-selective, but in this case we should lose the advantage gained by the absence of standing waves, which is probably the chief factor in the success of the long receiver.

To prove the non-selectiveness of this new form by showing the absence of resonance maxima, a "tuning curve" was taken, varying the length of receiver from 110.0 centimeters to 3.0 centimeters. The result is shown in Fig. 1. No appreciable resonance could be observed until it was shortened to 27 cm., five times the funda-

[1] P. Lebedew, Wied. Ann., 56, 3, 1895.

mental resonance length, when a small maximum appeared (C). At three times the resonance length a sharper maximum was found (B), and at the resonance length (A) the resonance was very marked, the energy being six times that received with the greater

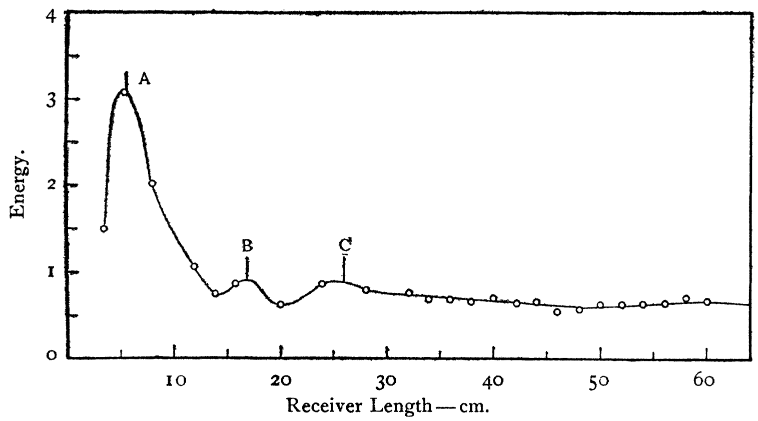


Fig. 1.

lengths. Thus beyond the first three maxima we have practically no resonance and still retain about 20 per cent. of the energy.

A further test was made by attaching capacities to the ends of a receiver, 58 centimeters long. As these capacities were diminished the energy received steadily increased without maxima or minima.

3. Details of Apparatus.

Receiver. — The arrangement of the non-selective receiver is shown in Fig. 2. The two collinear antennæ, or wings AB, were strips of copper foil, 0.15 mm. thick, 3 mm. wide, and 35 cm. long, making the total length of the receiver 70 cm. A wooden support L held the wings in position, their inner ends, which were 1 mm. apart, being attached to a wooden block A, glued to the support, while their outer ends were drawn tightly over similar blocks B. Care was taken to avoid bringing any dielectric into contact with the copper strips between the center and the ends, and to make them uniform. The effect at any point of a discontinuity of the

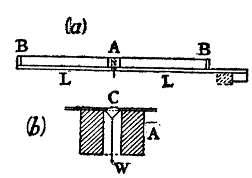


Fig. 2.

dielectric or of the metal would be a reflection of the current, producing there a loop of potential and giving the system a resonance period. For this reason the block *A* was made as small as possible, 1 cm.

The thermo-couple joining the inner ends of the wings was made of commercial iron and constantan wires, .04 mm. and .025 mm. in diameter, respectively, or, when a greater sensitiveness was required, of platinum and constantan wires of smaller diameter. The arrangement of the lead wires originally used by Klemencic[1] was found more satisfactory than that employed by recent writers.[2] Instead of attaching the leads from the galvanometers to the free ends of the thermo-couple wires, they were connected directly to the lower side of the wings, *W* (Fig. 2, *b*). The thermo-couple wires *C* were then simply short pieces of wire soldered to the wings, and bent around one another, so that they remained in contact. The junction between them was made permanent by soldering, or by an electric weld.[3] This method of connecting the leads makes the construction much simpler, and renders the junction less liable to breakage when jarred, than if the relatively heavy lead-wires were attached directly to the fine wires. Furthermore, it makes possible the use of very thin wire in the thermo-element. For very sensitive receivers a platinum-constantan couple of wire about .0045 mm. in diameter was used. The platinum was obtained from platinum-cored silver wire. For the constantan a piece of the commercial wire, .025 mm. in diameter, was dissolved by repeatedly dipping into warm concentrated nitric acid, until a finely pointed end was obtained. With the aid of a binocular microscope, this was soldered to one of the copper strips with the pointed end extending about 0.5 mm. The platinum wire, soldered to the second strip with about 1 mm. free, was then bent so as to make a spring contact with the end of the constantan, and the two metals were welded together by passing a very small electric spark to the copper wings. The sensitiveness of such a junction was about fifteen times greater than one made with the larger iron and constantan wires with a welded junction.

[1] I. Klemencic, Wied. Ann., 42, 416, 1891.

[2] I. Klemencic, Wied. Ann., 45, 78, 1892.

[3] Fountain and Blake, Phys. Rev., XXV., 257, 1907.

The two methods of arranging the lead wires were compared, but no difference was found either in the energy received or in the resonance length of the receiver.

The use of the non-selective receiver greatly increased the difficulty of measurement. In all this work a control or "check" receiver,[1] placed near the vibrator, was used to eliminate the errors due to the variations in the emitted energy. The result of any reading was then the ratio of the galvanometer throw, due to the non-selective or "main" receiver, to that obtained simultaneously from the check receiver. However, due to the differences in the character of the energy sent out by the vibrator, this ratio was not constant for any given arrangement of the apparatus,[2] the variation being very much larger with a non-selective main and a tuned check receiver, than with two similar receivers in resonance with the emitted wave-length. A non-selective check similar to the main receiver was tried, but it proved more convenient and satisfactory to have the auxiliary receiver tuned. To eliminate the errors due to the "change of ratio," the method of alternation was used, in which every second reading was taken with the apparatus in a standard condition. For example, when the position of a maximum or minimum in an interference curve was to be determined, alternate readings were taken on each point and some point of reference, chosen preferably near the maximum or minimum in question. The height of the different parts of the curve was thus given in terms of the height of the standard point, free from the error due to the gradual change of ratio. Two or three alternations were usually sufficient to accurately determine any point.

Interference Apparatus. — The arrangement of the interference apparatus is shown diagrammatically in Fig. 3 (*a*, plan; *b*, elevation). The "collecting" mirrors *M* and *N*, at the foci of which were, respectively, the vibrator *V* and the receiver *R*, were cylindrical-parabolic with horizontal axes, apertures 70 by 72 cm., and focal-length 36.0 cm. Mirrors of other types and dimensions described later were also employed. The two movable plane interference mirrors *B* were of plate glass, 76.0 cm. wide and 38 cm.

[1] Klemencic and Czermak, *l. c.*

[2] Blake and Fountain, PHYS. REV., XXIII., 261, 1906.

high, covered with tin-foil. Each was fastened with three adjusting screws to a vertical carriage *P*, with horizontal arms *A*, sliding on a rigid frame *F*. Millimeter scales on the arms and indexes on the frame measured the position of the mirrors. The adjustment of the planes of the mirrors was carefully tested with a plumb-bob and a

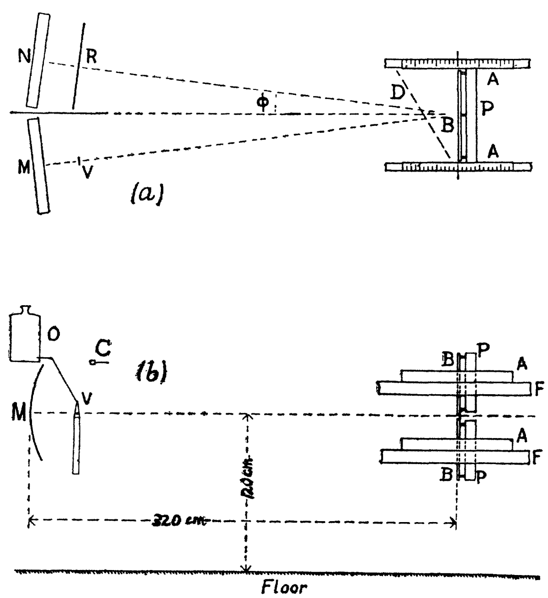


Fig. 3.

steel straight edge, as it was found that accurate alignment was very essential for correct results. The angle between the incident and reflected beams was $15° \, 30' = 2 \cos^{-1} .992$, so that the actual difference of path was about one per cent. greater than that indicated by the scale.

The main receiver (non-selective) *R* was mounted on a stand so as to permit readily altering its height by a measured amount. The check receiver *C* was fixed above the vibrator on a level with the top of the collecting mirror, as it was found that 9 per cent. of the energy in the direct beam was absorbed, when it was placed in front of the source. To facilitate changes in its length, it was made with detachable wings.

The galvanometers and the method of using them were similar to those described by Blake and Fountain.[1] The energy received by the check was measured on a du Bois-Rubens instrument, with a sensitiveness 4×10^{-9} volts, and for the main receiver a galvanometer was used of the type designed by Nichols and Williams,[2] with a sensitiveness 2×10^{-9} volts. The half-period of each galvanometer was 2.8 seconds.

Vibrators. — Two different types of vibrator were used in this investigation, the cylindrical or "rod" vibrator,[3] consisting of two metal cylinders placed end to end and separated by a very thin film of oil, and a modified form of Righi vibrator.[4] The former was selected as more suitable for an extended study, as it lent itself more readily to exact measurement, the conditions affecting its action being easily determined and controlled, as is not the case with the Righi or other similar forms. A convenient shape was arbitrarily chosen, which we shall call the "standard rod vibrator," and to which all the measurements of wave-length will be referred. Eight different sizes were used in which all the essential *relative* dimensions, including those of the supporting dielectric, were maintained constant. (See Table I., columns 1–5. The numbers 1–8 in column 1 will be used to refer to the individual vibrators.) The proportions between the dimensions are given in the following scheme (cf. Fig. 4):

Total length of vibrator $(l) = 8 \times$ diameter (d).

Length of dielectric (s), parallel to the axis of the cylinder, and consisting of 1 part oil, 2 parts wood-fibre $= \frac{3}{2} \times$ diameter.

Width of dielectric (w), perpendicular to the axis $= 3 \times$ diameter.

The details of the construction of the rod vibrator are given in Fig. 4, *a*, *b*, *c*.

The circular brass rods B (Fig. 4, *a* and *c*) were cut off plane at their free ends, while into the inner ends of each a steel sphere of the same diameter was set in such a way that only one hemisphere was exposed. The spheres were soldered into their sockets, being removed and again soldered into place, whenever fresh spark-

[1] Blake and Fountain, *l. c.*, p. 259.

[2] Nichols and Williams, PHYS. REV., XXVII., 250, 1908.

[3] Cf. A. D. Cole, PHYS. REV., IV., 55, 1896; K. F. Lindman, Ann. d. Phys., 7, 826, 1902.

[4] Cf. Willard and Woodman, *l. c.*

ing surfaces were required. The rods were supported by fitting them into holes drilled in wood-fibre plates *F*, which were attached to the upper ends of two flat wooden strips *W*, 35 cm. long, the lower ends of which were fastened rigidly to a wooden block *M*. The wood-fibre screw *S* served for the rough adjustment of the length of the "oil-gap," as we shall call the distance between the hemispherical ends of the rods. For the final adjustment the brass screw and nut *K* were employed. The effect of these was to spring together the two strips of wood and force apart the rods, the screw *S* acting as the fulcrum of a long lever. As the length of the oil-gap used was very small (.03–.07 mm.), such a delicate

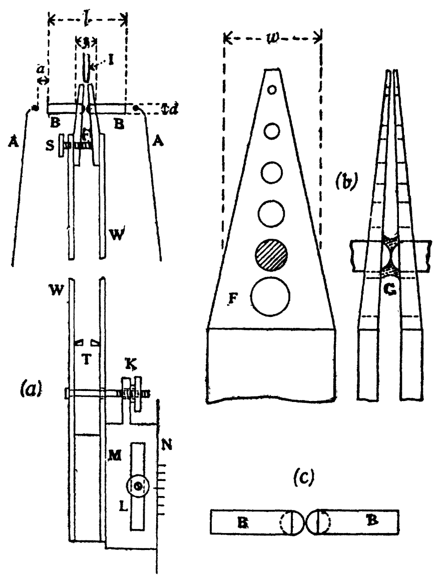


Fig. 4.

adjustment was necessary. To facilitate changing the size of the rods used in the vibrator, the wood-fibre plates *F* were made to accommodate six sizes, ranging from 0.24 to 0.79 cm. in diameter, being carefully tapered to keep the dielectric in its proper proportions (*b*, Fig. 4). To bring any desired set of holes into the focus of the collecting mirror, the block *M* was made to slide against a vertical stop *N* on the fixed supporting frame, a set screw *L* clamping it in the proper position, indicated by a scale.

The copper wires A, terminating in small knobs, were connected to the terminals of an induction coil which supplied the current to the vibrator. Except for a few millimeters, these wires were nearly vertical, so that they were approximately perpendicular to the electric force. The length of the "air gaps", a, was 7 mm. for all the rod vibrators, this dimension, as will be shown later, having no effect on the wave-length emitted. The axial position was selected for them, to avoid the fouling due to the decomposition of the oil straying from the center. When they were placed near the dielectric, they were soon choked by a carbon deposit, and the action of the vibrator became very irregular. The induction coil was operated on alternating current, 110 volts, 5–7 amperes, and was capable of a 3 cm. spark.[1]

The oil portion G of the dielectric was unenclosed, except on two sides by the wood-fibre plates. This was made possible by using a continuous flow from the nozzle I, connected by a glass tube to a reservoir O (Fig. 3, b). The escaping oil was caught by wooden troughs T and collected in a receiving jar. This arrangement was of great advantage, since it enabled the observer to examine or clean the sparking surfaces without dismounting the vibrator. Furthermore, it was necessary that the gap be in view at all times, so that its length could be regulated and kept nearly constant. During the earlier part of the work the oil-gap was measured by a microscope with a micrometer eye-piece, but it was later found possible to adjust it with sufficient accuracy from the appearance of the spark in the oil.

It should be emphasized that all the measurements described in this paper refer only to vibrators with *oil*-gaps *less than 0.1 mm.* in length. This is important as an air-gap at the center in place of the oil-gap, or an oil-gap several millimeters in length[2] introduces new conditions, and such vibrators should therefore be considered in another class.

Sources of Error in Wave-length Measurements. — The measurement of wave-lengths by the system of mirrors described above is attended by several difficulties and sources of error. Among the

[1] Blake and Fountain, *l. c.*, p. 258.

[2] Cf. W. R. Blair, PHYS. REV., XXVI., 57, 1908.

most important of these is the disturbing action of the collecting mirrors, especially when a non-selective receiver is used. As this point is of considerable importance in all measurements of this nature, the results of an extended study of "mirror action" will be briefly described. The different collecting mirrors tested in the interference apparatus will for convenience be designated by the letters *A–E*, in the following way:

	Type of mirror.	Aperture.	Focal length (cm.).
A	Spherical.	50 cm. — circular.	19.1
B	"	50 cm. — "	18.8
C	Cylindrical-parabolic.	70 × 72 cm. — rectangular.	36.0
D	" "	70 × 72 cm. — "	50.0
E	" "	50 × 72 cm. — "	50.0

In the first stage of the investigation the spherical mirrors *A* and *B* were used with very unsatisfactory results. In nearly every

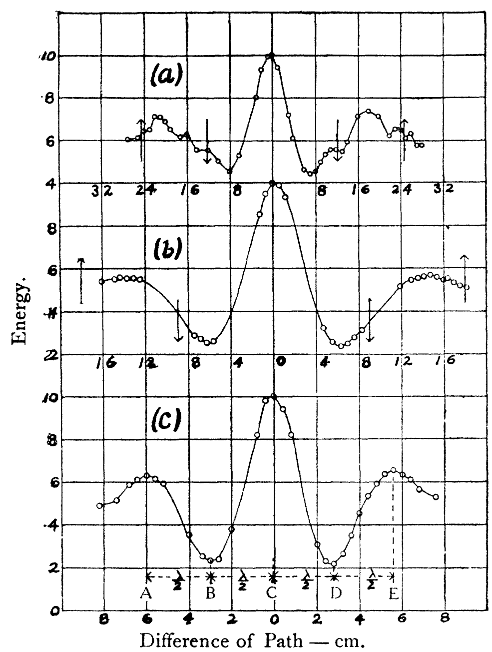


Fig. 5.

measurement of the wave-length the maxima and minima in the interference curves were displaced and distorted. This peculiar action of the mirrors can be illustrated best by a typical curve obtained under these conditions (see Fig. 5, *b*). Here the most noticeable features are the displaced maxima and minima, which indicate values of the wave-length, 14.4 cm. and 12.8 cm., respectively. The correct value, as determined by later experiment, was about 18.0 cm., the true position of the maxima and minima being indicated on the curve by arrows. It will also be observed that the maxima are considerably flattened, and that on the left hand side a double crest appears. With the cylindrical-parabolic mirror *C*, of double the focal length, and the spherical mirror *B* for vibrator and receiver, respectively, correct results were obtained for wave-length values less than 13.0 cm. For larger values the curves were extremely irregular with multiple maxima and minima. This is shown by the curve *a*, in Fig. 5. The true wave-length was about 24 cm., the arrows indicating as before the proper position for the maxima and minima. The distortion is due in part to other causes, such as the large difference in path between the two portions of the reflected beam, which tends to flatten the maxima. For purposes of comparison a normal wave-length curve free from mirror action is shown in Fig. 5, *c*.

With both mirrors of the cylindrical-parabolic type *C*, the wave-length curves were much more regular, and this combination was used for the greater part of the measurements on which this study is based. While it was evident that the disturbances were not wholly avoided, they were small and their effect could be largely eliminated by averaging the results over a great range of wave-lengths. As we shall see later the wave-lengths greater than 14 cm. were measured with errors of three or four per cent. For wave-lengths greater than 18 cm., the large difference in path made the maxima indeterminate, and in such cases only the minima could be used.

That the results for the shorter wave-lengths were not affected appreciably by the mirrors of the type *C*, was shown by comparing the results for a given vibrator with several different combinations of mirrors. With vibrator 5 the mirrors *A* and *B* gave a wave-length 11.6 cm., about five per cent. lower than the correct value, 12.2 cm. But with the combinations *C* and *C*, *D* and *D*, and *E* and E (see Table I., VII.–XI.) the values of the wave-length were approximately the same, showing that neither the change of focal length from 36 to 50 cm. (cf. C and D), nor the change in the height of the aperture from 70 to 50 cm. (cf. D and E) affected the results. With the mirrors C and B behind the vibrator and the receiver, respectively, the same value for the wave-length was obtained (VI.), proving that the use of the spherical mirror behind the receiver was permissible for wave-lengths less than 13 cm. Combinations C and B, and C and C were again compared with a vibrator emitting a wave-length of 10 cm., with the same result.

No complete explanation can be given for this peculiar action of the mirrors. It seems to depend principally upon their focal length, but it is also closely related to their form. The investigation of the phenomenon proved very complex, and in this study no attempt was made to push it further than was necessary to find mirrors which would give results free from such disturbances.

As will be seen later, the measurements made to determine the resonance lengths of receivers are also subject to mirror action, and in general this must be true of all measurements made with short waves. In a number of earlier investigations[1] the focal length of the mirrors used has been less than a single wave-length, and there the mirrors undoubtedly influenced the beam sent out from the combination of vibrator and mirror. However, the error produced was probably not of as great importance as would at first appear, since the effect of short focal lengths would be to give the emitted beam a definite "wave-length," which was determined by the focal length rather than by the nature of the vibrator, especially if the focal length differed but little from an odd multiple of the quarter wave-length of the vibrator. Nevertheless, the influence of the mirrors must be considered, whenever they are used, if satisfactory results are to be obtained.

The symmetry of the apparatus was found to be of very great importance in obtaining correct results. The two collecting and the two plane interference mirrors should be adjusted and aligned geometrically with great care. But it proved to be of much greater importance to so adjust the apparatus, that an equal amount of

[1] Cf. Willard and Woodman, *l. c.*; Blake and Fountain, *l. c.*

energy was reflected from each of the plane mirrors in its zero position. Failure to observe this precaution resulted in errors in measuring the wave-length, with tuned as well as with non-selective receivers, of five or more per cent. when this difference of energy was thirty to forty per cent. The test of this symmetry could be made by alternately removing one or the other of the mirrors and thus measuring the reflected energy from each, but it was more convenient and satisfactory to use an oblique deflecting screen *D* (Fig. 3) to intercept the upper or lower half of the beam. Usually, any dissymmetry could be eliminated by a slight vertical movement of the main receiver. In some cases the height of the vibrator was altered by a few millimeters, and in one or two measurements a metal screen was used to cut off some of the energy near the edge of the beam.

The "shadowing" of one mirror by the other[1] was avoided by having both the vibrator and receiver in the same horizontal plane with the dividing line between the interference mirrors. The sidewise shift of the reflected beams, when these mirrors were displaced from the zero position, resulted in some error, but this was small because of the small angle between the incident and reflected beams, and was still further reduced by the use of cylindrical mirrors focusing on the long non-selective receiver. When measuring large wave-lengths the center of gravity of the mirrors was kept nearly constant, to minimize the dissymmetry of the reflected beam, but even then wave-lengths greater than 20 cm. could be determined from the first minima only, as the maxima became indefinite.

In judging whether or not a wave-length curve was correct, the following criteria were employed: first, the maxima and minima should be well defined and single — in general the curve should be smooth and symmetrical; secondly, the wave-length determined from the maxima (*A* and *E*, Fig. 5, *c*) should agree with that found from the minima (*B* and *D*), within one or two per cent.; and thirdly, the central or principal maximum (*C*), corresponding to no difference of path, should not be displaced more than one or two millimeters. These criteria were not in themselves sufficient for correct results, but they were usually necessary, and hence served as guides.

[1] Willard and Woodman, *l. c.*, p. 12.

Definition of the Wave-length. — The quantity which we shall call the wave-length in these measurements was determined by averaging the two values found from the distances between the central maximum on the interference curve and the first maxima on either side, and the single value found from the distance between the first minima on either side. Thus, referring again to Fig. 5, *c*, we have $\lambda = \frac{1}{3}(CA + CE + BD)$. Greater weight is given the determinations from the maxima, as they are somewhat more accurrate than those based on the minima, since in the latter the errors are doubled. The distances AB and DE were not used, for averaging these values with the others would result in losing entirely the advantage gained by the use of the minima. Maxima and minima for differences in path greater than a single wave-length were not determined, as with a non-selective receiver they were very indefinite. Furthermore, they would depend upon the nature of the wave-train within the second wave-length, while with these highly damped radiations only the first wave-length is of importance. Hence, the wave-length is practically the distance between the first two crests in the wave-train emitted by the vibrator.

4. Wave-Lengths Emitted by Vibrators.

Rod Vibrator. — The results of the measurements of the wave-lengths emitted by the "standard" cylindrical or rod vibrators are tabulated in Table I. In the sixth column the Roman numerals are used to designate the individual determinations. In the seventh and eighth columns are given, respectively, the wave-lengths measured, and the ratios of the wave-lengths to the lengths of the vibrator $(\lambda \div l)$. In the last two columns the collecting mirrors used in the determination are indicated by the letters A–E, the meaning of which has already been given.

Fig. 6, *a*, gives these results in graphical form, the abscissas representing vibrator lengths and the ordinates wave-lengths. The curve is a straight line passing through the origin, showing that the wave-length is in exact proportion to the dimensions of the vibrator.

The mean value for the ratio $\lambda \div l$ for the eight vibrators is 2.40. In computing this, the determinations I. and II. were omitted, since the conditions under which they were taken were unfavorable. It will be noticed that the values for the determinations XII. and XIII. lie on either side of the mean value, the variation amounting to about three per cent. This was due mainly to the disturbing action of the mirrors, which as we have already seen becomes more

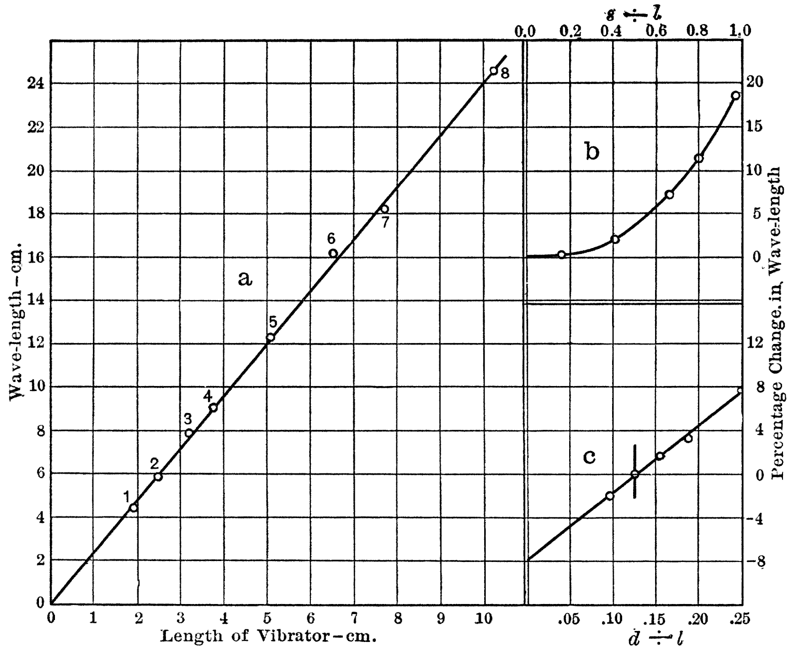


Fig. 6.

effective as the wave-length increases. However, the two errors very nearly balance one another, so that their mean value seems practically independent of the mirrors.

As mentioned before, the wave-length of vibrator 5 was determined with four different combinations of collecting mirrors, other than *A* and *B*, all of which give approximately the same values, the average of the six determinations VI.–XI. being 2.42. The wave-length of vibrator 8 (XIV.) was determined from the minima of the interference curve, since the maxima became indefinite for such large differences in path.

Conditions Affecting the Wave-length of the Rod Vibrator. — The results just described are of two-fold importance; first, because they prove a linear relation between the dimensions and the wave-length of a vibrator with constant *relative* dimensions ; and second, because of their statistical value. They enable us to obtain any desired wave-length by constructing a vibrator whose form and dimensions are given by the four relations (cf. Fig. 4);

TABLE I.

Vibrator.	Dimensions of Vibrators. Diameter of Rods $= d$ Inches.	cm.	Length of Rods $= l$ (cm.)	Length of Dielectric $= s$ (cm.)	Wave-length Determination.	Wave-length $= \lambda$ (cm.)	$\frac{\lambda}{l}$	Mirrors. Vibrator.	Receiver.
	2	3	4	5	6	7	8	9	10
1	$\frac{3}{32}$	0.24	1.90	0.4	I.	4.38	(2.31)	*A*	*B*
2	$\frac{1}{8}$	0.32	2.50	0.5	II.	5.78	(2.31)	*C*	*B*
3	$\frac{5}{32}$	0.40	3.20	0.6	III.	7.60	2.38	*C*	*B*
"	"	"	3.22	0.6	IV.	7.91	2.45	*C*	*C*
4	$\frac{3}{16}$	0.48	3.80	0.7	V.	8.96	2.36	*C*	*B*
5	$\frac{1}{4}$	0.64	5.08	1.0	VI.	12.15	2.40	*C*	*B*
"	"	"	5.10	"	VII.	12.30	2.41	*C*	*C*
"	"	"	5.10	"	VIII.	12.25	2.40	*C*	*C*
"	"	"	5.08	"	IX.	12.15	2.39	*C*	*C*
"	"	"	5.05	"	X.	12.27	2.43	*D*	*D*
"	"	"	5.05	"	XI.	12.40	2.46	*E*	*E*
6	$\frac{5}{16}$	0.80	6.55	1.2	XII.	16.15	2.46	*C*	*C*
7	$\frac{3}{8}$	0.96	7.78	1.5	XIII.	18.18	2.33	*C*	*C*
8	$\frac{1}{2}$	1.27	10.25	1.9	XIV.	24.60	2.40	*C*	*C*
						Mean	2.40		

$$l = 8d = \tfrac{16}{3}s = \tfrac{8}{3}w,$$
$$\lambda = 2.40l,$$
$$[a = 7 \text{ mm. Oil-gap} < 0.1 \text{ mm.}].$$

The relative dimensions of our standard vibrator are, however, entirely arbitrary, having been chosen for simplicity and convenience. For this reason a study of the rod vibrator was undertaken to determine approximately how the wave-length varies with varying relative dimensions and other conditions, thus enabling us to compute the wave-lengths for vibrators of other than the standard form. Vibrator 5 was used for this purpose, with the apparatus and non-selective receiver as before. To facilitate the measurements only the minima of the interference curves were taken to determine the wave-length, as it was found that this brought in no considerable error. The work was divided into three parts; determining the effect of varying,

1. The position and size of the air-gaps.
2. The length of the dielectric.
3. The ratio of the diameter to the length of the vibrator.

No change of wave-length was observed as the air-gaps, lying in the axis of the cylinders produced, were varied from 3 to 7 mm. With 3.5 mm. air-gaps, perpendicular to the axis of the cylinders, and 10 mm. from the oil-gap, unvarying results were obtained. The wave-length is therefore independent of the length and position of the air-gaps. Earlier work with tuning curves, however, indicated a more complex emission[1] when the air-gaps were very small. This will be shown to better advantage later, in the discussion of the Righi vibrator (see Fig. 8*b*).

As was to be expected, the length of the supporting dielectric at the center of the vibrator, measured along the axis, has a very appreciable effect, especially as it approaches the free ends. Without altering the metal rods of the vibrator, the dielectric was built up symmetrically from the center outwards, by slipping hard rubber collars, 2 cm. square, over their free ends. Fig. 6, *b*, shows the relation found between the length of the dielectric and the wave-length. For the sake of computation, the abscissas have been made to represent the fraction of the total length of vibrator covered by the dielectric, and the ordinates the corresponding percentage change in wave-length. The curves show that the dielectric is practically ineffective, if its length is less than one fourth the total length of the metal, but that the effect becomes more and more marked for larger values.

The relation between the wave-length and the diameter of the rods, for a given length of vibrator, is shown in Fig 6, *c*, in which the abscissas give the ratio of the diameter to the length ($d \div l$), and the ordinates the percentage change in wave-length, 100 per cent. corresponding to the standard condition $d \div l = 0.125$. The curve is a straight line and may be represented by the equation,

$$100\,\Delta\lambda \div \lambda = 60(d \div l - 0.125),$$

[1] Cf. K. F. Lindman, *l. c.*

where $\Delta\lambda$ is the actual change of wave-length. In this determination five sets of rods were used, with diameters ranging from 0.48 to 1.27 cm. The total lengths of the vibrator and of the dielectric were kept constant at 5.08 cm. and 0.8 cm., respectively.

If the straight line in Fig. 6, *c* be prolonged, it cuts the axis of zero diameter at $\Delta\lambda = -7.5$ per cent., or in the unreduced coördinates at $\lambda = 11.4$ cm. The length of the vibrator being 5.08 cm., we find the limiting value of the ratio of the wave-length to the vibrator length, as the diameter approaches zero, to be $11.4 \div 5.08 = 2.24$.

Within reasonable limits, we should now be able to calculate the wave-length of any rod vibrator from its dimensions alone, using the relation $\lambda = 2.4l$, found for the standard form, and the corrections for the dielectric and the diameter, just described. For example, the calculated wave-length for a vibrator with the dimensions $l = 4.2$ cm., $d = 0.40$ cm., $s = 0.55$ cm. is 9.9 cm., the correction for $d \div l$ being -2 per cent., and for $s \div l$, 0 per cent. The experimental value for this vibrator was 9.6 cm., but the interference curve was irregular, the indications being that this was somewhat low, so that the calculated value is probably more nearly correct.

The effect of varying the length of the oil-gap was not investigated with this type of vibrator, but the question will be discussed in the following section of this paper, when the results of a similar study with the Righi vibrator are described.

Wave-length of a Righi Vibrator.— For the purpose of obtaining further statistics of the relation of the wave-length to the dimensions of vibrators, a Righi vibrator[1] of special form and construction was next studied. The arrangement and dimensions of the eight sizes of vibrator used are given by Fig. 7, and Table II. The steel spheres *S* rested in shallow ground sockets in the ends of glass tubes *T*. The weight of the larger spheres was sufficient to keep them steady, but to hold the smaller sizes in place, it was necessary to use air suction, obtained by connecting an aspirator pump to the lower ends of the glass tubes. No oil vessel was used, the layer of oil *O* separating the spheres being maintained by a constant flow from a glass

[1] Cf. Willard and Woodman, *l. c.*, p. 2.

tube I, connected to a reservoir, while the wooden troughs L collected the escaping oil into a suitable receptacle. The adjustment of the length of the oil-gap was made by springing the glass tubes by means of the link P and the turn-screw K. A wooden clamp R, 3 cm. below the spheres, held the tubes rigidly apart at a fixed distance, and rendered the system stiff against vibration. Acting as a fulcrum, it further served to reduce the motion of the spheres

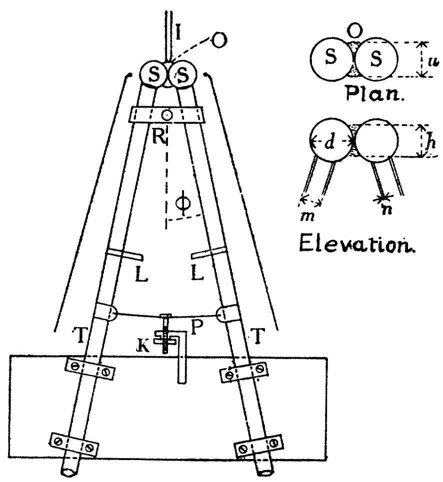


Fig. 7.

due to the screw K, thus making possible a very small change in the oil-gap. The air-gaps were placed in the axis of the vibrator, and were 6 mm. long for all sizes.

This peculiar form of the Righi vibrator was found to have very great advantage over that usually employed, in that it gave a very close approximation to a point source, and made the adjustment and renewal of the oil-gap very simple. Further details of its use will be given in a subsequent paper.

The dimensions for the eight sizes of the "standard Righi vibrator," as it will be called, are tabulated in Table II., columns 1–7, with the corresponding wave-lengths determined with the non-selective receiver in column 8, the letters used referring to Fig. 7. The constancy of the relative dimensions was not as carefully maintained as in the case of the rod vibrators. This was due partly to the necessity of choosing convenient sizes of glass tubes, and partly to the fact that the dimensions of the oil layer could not be kept proportional to the diameter of the spheres. With large spheres its size is limited by the surface tension, while with the smaller vibrators the vigorous flow, necessary to wash out the carbon deposit from the oil-gap, gave a relatively large layer. Since the variations in the relative dimensions of the glass mountings were small, and did not greatly alter the amount of dielectric in contact with the metal, they may be neglected.

TABLE II.

Spheres.			Oil layer (cm.).		Supporting Rods (cm.).		λ (cm.).	Remarks.
Diameter $=d$		Metal.	Height $=h$	Width $=w$	Diam. $=m$	Thick. of walls $=n$		
Inches. 1	cm. 2	3	4	5	6	7	8	
$\frac{3}{8}$	0.95	Steel	0.6	0.6	0.60	0.10	6.7	From 1 max.
$\frac{1}{2}$	1.27	"	0.9	0.75	0.60	0.10	8.9	
$\frac{5}{8}$	1.59	"	1.0	0.7	0.60	0.10	10.4	
$\frac{3}{4}$	1.91	"	1.1	0.8	0.88	0.12	11.4	
$\frac{7}{8}$	2.22	Brass	1.3	0.85	0.88	0.12	12.8	
1	2.54	Steel	(1.35)	(1.0)	1.21	0.22	14.7	
$1\frac{1}{4}$	3.18	"	1.6	1.2	1.35	0.20	17.8	From 1 max. and 1 min.
$1\frac{1}{2}$	3.81	"	1.7	1.3	1.35	0.20	20.7	From min. alone.

As shown in Fig. 8, *a*, the relation between the wave-length and the diameter of the spheres, between the limits $\lambda = 7$ cm. and $\lambda = 20$ cm., can be represented very well by a straight line with the equation

$$\lambda = 4.95d + 2.2.$$

The intercept 2.2 cm. indicates a large change of curvature between $\lambda = 6.0$ cm. and $\lambda = 0$, for we should expect the curve to pass through the origin. This discontinuity in the relation between λ and d is probably due to the increasing size of the oil layer, which, as we have just seen, becomes larger and larger relatively to the decreasing diameter of the spheres. If the diminution in size were continued in the same way, the sphere would eventually become entirely immersed in the dielectric, and the relation between λ and d would then be given by a straight line with a much greater slope. We should therefore regard this equation simply as a convenient and close approximation, from which we can obtain, within the given limits, the wave-length for any vibrator having the standard form.

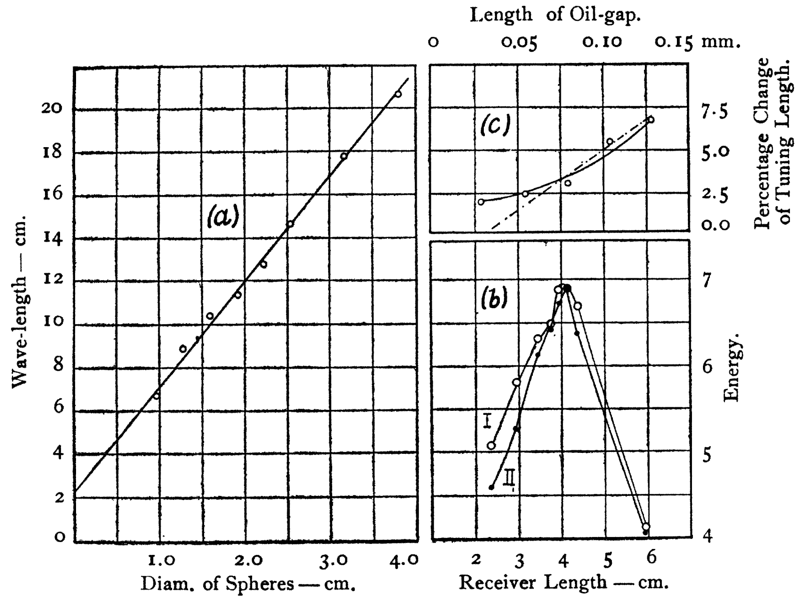


Fig. 8.

For convenience, the dimensions and wave-lengths of the Righi vibrators are given in the following compact form :

$$m = 0.5d$$
$$n = .07d$$
$$\varphi = \tan^{-1} \tfrac{1}{5}$$
$$h = .35d + 0.4$$
$$w = .25d + 0.4$$
$$\lambda = 4.95d + 2.2$$

The effect upon the wave-length of varying the conditions for this vibrator was investigated by means of tuning curves, the resonance length being taken as the measure of the wave-length. As already emphasized this method is open to objection, but in this case it was found sufficiently accurate. The following conditions were studied :

1. The length and position of the air-gap.

2. The dimensions and nature of the dielectric between the spheres.
3. The size of the oil-gap.
4. The nature of the metal.

No change in the wave-length was observed as the length and position of the air-gaps were altered. During this test the gaps were varied from 6 to 1.5 mm., and moved to positions both in front and above the vibrator. There was, however, as in the case of the rod vibrator, an indication of a greater complexity in the emission for the shorter air-gaps of 1.5 and 3 mm., with which the tuning curves were less smooth than those taken with 6 mm. gaps and showed a greater tendency to develop secondary maxima. This is shown very well by the Curves *I* and *II*, Fig. 8, *b*, which were taken simultaneously with air-gaps, 3 and 6 mm. long, respectively. The latter (*II*, *b*) is much sharper and has a well defined maximum, while the former (*I*, *b*) tends to give additional crests. Quantitative conclusions, however, cannot be drawn from these curves, since their form depends upon the collecting mirrors used, as well as upon the character of the emission of the vibrator.

The observations taken with these vibrators show that small variations in the dimensions of the dielectric are not sufficient to appreciably affect the wave-length. Complete data bearing on this point could not be obtained, as the nature of the vibrator prevented large variations in the size of the oil layer. Paraffin oil was used in the greater part of the work and proved very satisfactory. In the earlier measurements olive oil was tried, but because of its greater viscosity, the gap quickly clogged with the particles of carbon from the decomposed oil, causing the vibrator to break down after a very few discharges. The breaking potential of the olive oil was, however, much less than that of paraffin oil, with which it was impossible to use gaps larger than .08 mm., as then the spark between the spheres passed through the air instead of through the oil. Kerosene was also tried with this vibrator, but its action proved irregular and unsatisfactory. No difference in the wave-length emitted with the different oils was observed.

In a similar way the effect of the length of the oil-gap was studied by tuning curves. For measuring this length, a microscope with a micrometer eye piece was mounted on a long arm, so that it could be swung into the observing position in front of the vibrator, or out of the beam, at will. The measurement of the gap was always made at the beginning of a series of readings, before the oil was flowed on the spheres. The results of these observations are given by the curve *c* in Fig. 8, the ordinates representing percentage change of tuning length (*i. e.*, also of wave-length), and the abscissas the corresponding length of the oil-gap in millimeters. It is probable that the dotted line more nearly represents the relation than the heavy curve drawn through the observed points, since the pitting increases the length of the small gaps very rapidly, so that in determining the first point, the average length was about .04 mm., instead of .03 mm. as measured. We see, however, that within the limits, .03 to .08 mm., the effect of the gap length is small, and need not therefore be accurately determined, which conclusion applies also to the rod vibrators, since the effect of the gap is essentially the same in both types. Since the limit .03 to .08 mm. includes all sizes of gaps which can be used to advantage with either type of vibrator, this factor was neglected in all subsequent measurements. In practice it was found best to regulate the gap by the appearance of the spark, as the rapid pitting made micrometer measurements difficult and unreliable for long series of readings. This regulation was repeated at short intervals, as a large, bright, sputtering action at the oil-gap was usually accompanied by large variations in the ratios of the energy measured by the two receivers.

The tuning curves, taken with steel, brass, and aluminium spheres in the vibrator, coincided with one another almost exactly, showing that the emission was independent of the nature of the metal.

5. Energy and Damping.

Energy Emitted by the Vibrators. — A study of the energy emitted by different vibrators leads to some interesting conclusions. Much of the data available is not sufficiently accurate for the formulation of definite relations, because the measurement of the energy was complicated by the condition of the sparking surface, the presence of carbon particles in the oil, and the varying length of the spark gap, of which only the initial value could be determined.

We can, however, show approximately how the energy depends upon the various conditions and dimensions of the vibrator; viz., its size, the length of the oil-gap and of the air-gaps, and the nature of the metal and of the oil. These determinations were made on a non-selective receiver, which was taken as measuring the absolute energy, or when the wave-length was not changed by the variation, a tuned receiver was sometimes substituted. Unless otherwise stated the length of the oil-gap will be assumed constant in the following relations.

The energy emitted by a vibrator with constant relative dimensions was approximately proportional to the square of the wave-length. This was true for both types of vibrator, as shown by the following tables:

Rod Vibrator.

Wave-length, λ.	Energy.	Energy $\div \lambda^2$.
15.8	207	.83
12.2	114	.77
9.1	62	.75
7.7	51	.86

Righi Vibrator.

Wave-length, λ.	Energy.	Energy $\div \lambda^2$.
17.8	440	1.4
12.8	210	1.3
10.4	180	1.7
8.9	110	1.4

The energy corresponding to a given wave-length was very nearly the same for both types of vibrator.

The relation between the length of the oil-gap and the emitted energy was carefully studied with the standard Righi vibrator. As the wave-length and damping were practically constant, the energy was measured on a tuned receiver. The results varied considerably, but the mean of thirty measurements showed the energy to be inversely proportional to the length of the gap.

In both types of vibrator the energy emitted was found to be practically independent of the length of the air-gaps, within the limits 3–7 mm.

No difference in the energy was observed when spheres of brass or aluminium were substituted for the steel. With zinc spheres the energy was much smaller for the same initial length of oil-gap, but this was doubtless due to the extraordinarily rapid pitting of the soft metal, which caused a decrease of 75 per cent. in the emission during the first five discharges. The action of the steel, brass, and aluminium spheres was very similar. However, the life of the surfaces of the commercial steel spheres appeared to be somewhat longer, and furthermore their high polish and exact form and size made them much the best for continued work.

The energy varied with the oil used in the vibrator. With paraffin oil it was nearly 30 per cent. greater than with olive oil. This was due to the greater potential required to break through the layer of paraffin oil separating the sparking surfaces, for as we have seen the olive oil has a much lower breaking potential.

With a constant length of cylindrical vibrator, the energy was approximately proportional to the diameter of the rods.

Damping. — If we assume the wave-train emitted by a vibrator to be a simple damped sine wave with the equation $e^{-\gamma nt} \cos 2\pi nt$, where γ is the logarithmic decrement, n the frequency, and t the time, we find two expressions for γ,

$$\gamma = -\text{ nat. log.} \left(2\frac{E_3}{E_1} - 1 \right) \text{ for the first maximum,}$$

and

$$\gamma = -2 \text{ nat. log.} \left(1 - 2\frac{E_2}{E_1} \right) \text{ for the first minimum,}$$

E_1, E_2, and E_3 being the energy measured by the receiver in the interference apparatus for differences of path, zero, $\frac{1}{2}\lambda$, and λ, respectively. In this way Klemencic and Czermak[1] found $\gamma = .50$. In their determination the receiver used was tuned to the vibrator, and therefore this value of γ was much too low, as it depended chiefly upon the smaller decrement of the receiver. The non-selective receiver is not open to this objection since it measures the absolute value of the incident energy without regard to its phase.[2] But there is another source of error in determining γ, due to the wrong assumption of the form of the wave-train. For if we compare the values of γ found from the two expressions given above, we find that

[1] Klemencic and Czermak, *l. c.*, p. 181.
[2] Cf. G. F. Hull, *l. c.*

the minima give a value twenty to thirty per cent. lower than that computed from the maxima, showing that a damped sine wave does not nearly represent the emission. This arises from the fact that the damping is due almost entirely to radiation, and we cannot therefore express the differential equation for the vibrator in the form

$$\frac{d^2\varphi}{dt^2} + 2n\gamma\frac{d\varphi}{dt} + (\gamma^2 + 4\pi^2)n^2\varphi = 0,$$

as in the case of damping due entirely to the Joule effect. But γ has no real meaning unless it can be defined in this way. We must therefore arbitrarily define it as the average value of the two expressions given above and regard the sine wave $e^{-\gamma nt}\cos 2\pi nt$, which it determines in connection with the wave-length measurements, as a rough approximation to the true wave-form. This is useful as it gives an idea of the rate of disappearance of the train.

Most of the measurements with the non-selective receiver in the present investigation are not available for computing γ, since those in which the cylindrical mirrors were employed cannot be used because of the divergent beam, and many of those taken with the spherical mirrors are unreliable. However, among the latter were a few curves which appeared to be very nearly correct, and from these the following results were obtained:

From four curves taken with the rod vibrators the average value of γ was 1.28. With the Righi vibrator, steel spheres 1.9 cm. in diameter, the average of seven determinations gave $\gamma = 1.30$. With spheres 0.95 cm., 2.54 cm., and 3.18 cm. in diameter, γ was 1.48, 1.41, and 1.75, respectively, but these higher values were no doubt due to mirror disturbances, since the curves were all very much distorted. Moreover, we should expect the damping to be the same for all sizes of vibrator, especially in the case of the cylindrical type, in which all the relative dimensions were constant. With 1.9 cm. brass spheres γ was 1.33, with zinc 1.34, and with aluminium 1.28. The average for these three metals was 1.32, approximately equal to the values for steel, showing that the nature of the metal did not affect the damping appreciably.[1] No difference in γ was found for different lengths of oil-gap, up to .135 mm.[1]

[1] Cf. O. Bartenstein, Ann. d. Phys., 29, 245, 1909. These measurements were, however, made with a tuned receiver.

From the wave-length curves taken with a tuned main receiver, the computed values of γ were approximately equal to 0.50, agreeing with the results of Klemencic and Czermak. As already pointed out this value for γ has no real significance, since it depends on the damping of the receiver as well as on that of the vibrator.

6. Standard Receivers.

Description. — We have thus far considered the properties of what has been called a standard vibrator. We shall now take up the study of the receivers in resonance with given wave-lengths. For this purpose a "standard receiver" was constructed, with form and dimensions arbitrarily chosen, but based on previous experience. As in the standard rod vibrator, the essential *relative* dimensions of the standard receiver were maintained constant, as its size was altered.

The details of construction are given by Fig. 9. The copper wings A were flat strips, fastened with wax to a hard rubber base D. In the "constriction gap" C between their inner ends was the thermo-junction. The contact between the wires was electrically welded; or soldered, if the receiver was subject to jarring. The lead wires W were attached to the wings, as already described in connection with the non-selective receiver. The dimensions of the standard receiver are given by the following relations: cf. Fig. 9.

Length of dielectric, s } $= \frac{1}{4}$ length of corresponding standard rod
Height of dielectric } vibrator.
$= 0.104\,\lambda$ (approx. $\frac{1}{10}\,\lambda$) $=$ approx. $l/4$.
Depth of dielectric, $h = 3.0$ cm. for all sizes.
Thickness of wings, $t = 0.015$ cm. for all sizes.
Width of wings, $w = \frac{1}{4}s$.
Length of constriction, $c = \frac{1}{10}s$.

Diameter of thermo-couple wires in constriction: iron, 0.004 cm.; constantan, 0.0025 cm.

As an example, the standard receiver, corresponding to the standard rod vibrator 5 ($\lambda = 12.2$ cm.), had the dimensions $l = 5.3$ cm. (from experiment), $s = 1.27$ cm., $w = 0.32$ cm., and $c = 0.13$ cm., h and t having their usual values. This will be called "standard receiver 5," and this method of designating each receiver by the number connected with the corresponding rod vibrator will be observed in the rest of this paper.

Tuning Method. — The first step in determining the properties of a standard receiver was to find l, the resonance length corresponding to a given wave-length λ. This was first attempted by the usual method of tuning curves,[1] with unsatisfactory results. As mentioned earlier, the process of tuning by determining the length of receiver which takes up the most energy, is subject to numerous errors and accidental disturbances, which render the resulting curves uninterpretable. The question is one of very great importance in work with electric waves and therefore merits a somewhat detailed discussion.

The first great difficulty met with in these measurements was the distortion of the tuning curves by the vibrator and receiver mirrors, which were placed facing one another at a distance of 3 meters, and served to concentrate the energy from the source upon the receiver. This disturbance was especially marked in the earlier stages of the work, when spherical mirrors of focal length 19.0 cm. were used. In the majority of the curves two maxima appeared, one of which depended upon the focal length, and the true resonance maximum was displaced, as shown by subsequent tests. A typical instance of this is curve *a*, Fig. 11, in which the arrow indicates the correct position of the resonance maximum. With this mirror system not even approximate results were obtained for wave-lengths greater than 13 cm., as shown by the tuning curve reproduced in *b*, Fig. 11, taken with a wave-length of 18 cm. It will be seen that the resonance maximum, the position of which is marked by the arrow, is almost entirely covered up by the large maximum produced by the mirrors. Similar results were obtained with two paraboloidal mirrors, of focal length 12.0 cm., using a wave-length of 6.5 cm. The curves with this system show great variation, having sometimes two or three prominent maxima, as in Fig. 11, *c*. It should be remarked here that these variations may be due, to a small extent, to the form of the receiver used, as well as to the mirrors. This point will be subsequently discussed more fully.

[1] Willard and Woodman, *l. c.*

Smooth curves with single maxima were obtained with two sets of cylindrical-parabolic mirrors, of focal lengths 36 cm. and 50 cm., respectively, for wave-lengths less than 18 cm. However, the results obtained from them were inconsistent, the ratio of the wave-length λ (determined by the non-selective receiver) and the resonance length l varying from 2.14 to 2.38. With $\lambda = 18.7$ cm. two prominent maxima appeared at 7.0 and 8.5 cm. (Curve *I*, Fig.

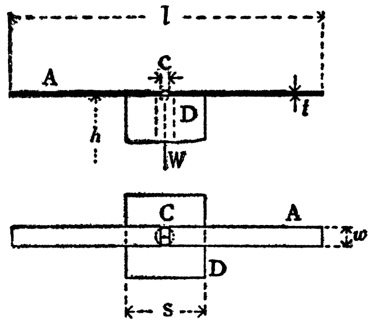


Fig. 9.

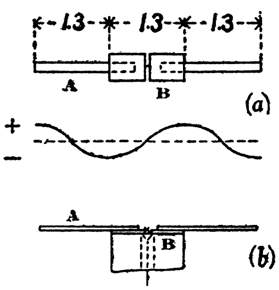


Fig. 10.

11, *d*). When the direct radiation and the portion of the reflected beam which passed close to the vibrator were eliminated by an oblique screen directly in front of the vibrator, a single maximum at 8.3 cm. was obtained (Curve *II*), indicating that the effect of the mirrors was to cause interference between the direct and the reflected beams. This would account for the simple displacement of the resonance maxima, as well as for the double crests.

To avoid this difficulty a series of tuning curves were taken without mirrors, with the vibrator and receiver 150 cm. apart and the check receiver above the vibrator as before. The results were still inconsistent, but agreed within 5 per cent. with the correct values subsequently found by another method. One possible cause of this variation is seen in the form of the resonance curves (Fig. 11, *e*, *f*.), which are not symmetrical, but very much flattened on the side of the longer receiver lengths. This was due to the greater energy taken up by the longer receivers, because of their greater length and capacity, which is superimposed upon the effect due to resonance. While it cannot be said that this flattening actually displaced the maxima, it can be readily seen that, since the variation of the energy measured by the receiver as its length is varied by several centimeters is only a few per cent., a slight disturbance would be sufficient to displace the maximum 5 or 10 per cent. Experience shows that such disturbances are present in these determinations and cannot be readily controlled. For example, in these measurements the method of gradually altering the length of a receiver, while determining its resonance length, was to start with the flat wings very long and cut small sections from the ends. This gave no opportunity of alternating on the two sides of the resonance length, to eliminate the error due to the change in the ratio between the energies received by the main and check receivers.[1] Now this is progressive with the age of the vibrator, and in the case which we are considering it was a continuous decrease. Hence the maxima of the tuning curves were displaced toward longer receiver lengths, the error being very large because of their flatness.

To avoid this difficulty by taking alternate readings on the two sides of the maximum, adjustable receivers have been used,[2] the length of which could be altered at will. This method was tried as a substitute for the "clipping" method just described, but a new source of error was found, which caused it to be abandoned. In the construction of most forms of replaceable wing receivers there occurs a discontinuity in the shape or cross-section of the metal wings. Now the effect of an abrupt change in the contour of a resonator is to produce at that point a loop of potential, introducing a new mode into the natural vibrations. Now if the total length is such that this superposed period bears a simple relation to the fundamental period, that is, if the loop of potential due to the discontinuity coincides with a loop in the normal vibration, the activity of the receiver will be a maximum, since the vibrations depend only slightly on the highly damped wave exciting them. Experiment showed that this action almost entirely covered up the effect of resonance with the vibrator, the maxima of these curves depending chiefly upon the position of the discontinuity in the resonator. The two adjustable receivers tested in this investigation are shown in Fig. 10, *a*, *b*. The first consisted of two brass blocks *B*, 3 mm. in cross-section, with holes

[1] Blake and Fountain, *l. c.*

[2] Cf. A. D. Cole, PHYS. REV., XX., 268, 1905.

in their ends, into which the variable wings *A* (rods 1 mm. in diameter) were inserted, perfect contact being secured by amalgamation. The total length of the two blocks plus the thermo-junction between them was 1.3 cm. With a vibrator correspondiug to a resonance length of about 2.5 cm., the maximum of the tuning curve

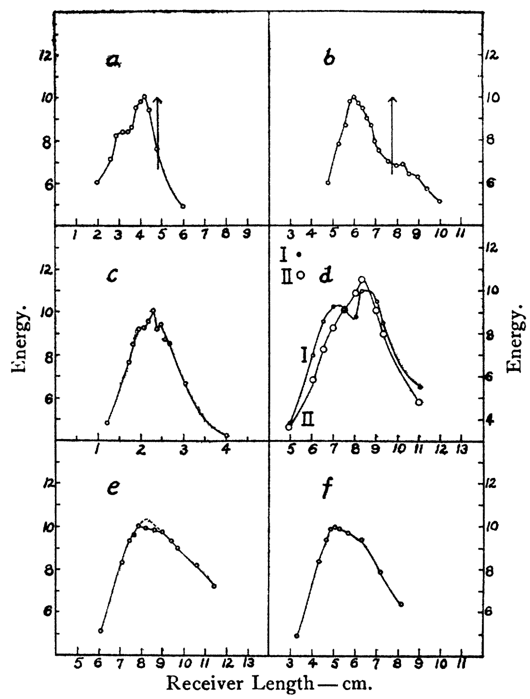


Fig. 11.

taken with this receiver came at 3.9 cm., three times the length of the enlarged part. With Fig. 10, *a*, is given a diagram of the distribution of potential in the vibrating system, showing the loop at the point of change of cross-section. Loops at the free ends and a node at the center, where the current is measured, are of course the normal conditions for maximum energy. In the second arrangement, shown in Fig. 10, *b*, the adjustable wings *A* were flat, and contact with the permanent part of the receiver was made by pressing them against similar strips *B*, fastened to a hard rubber base. The total distance between the extremities of the double thickness of metal was 1.6 cm. The tuning length found for a wave-length, corresponding to a 5.3 cm. resonance length, was 4.8 cm., again three times the length determined by the discontinuity. Similar results should be expected for any receiver with discontinuities in the dielectric in contact with the metal. These effects are, however, very much smaller, and hence could not be observed, but they were suspected in the case of the very flat resonance curves which have just been described.

Table III.

Standard. Rod Vibrator. 1	Standard. Receiver. 2	λ 3	Position of Maxima. 4	Resonance Length. By Tuning l 5	Resonance Length. By Wave-length Method. 6	$\lambda \div l$ 7
7	7	18.7	7.8	8.2		
			8.4 and 7.6	8.2		
			7.8	8.0		
				8.1 (mean)	(7.85)	2.32
6	6	15.8	7.2 and 6.6	7.1		
			7.35 " 6.7	(7.3)		
			6.5 " 7.5	(6.95)		
			7.2 " 6.8	7.0		
			7.2 " 6.6	7.0		
				7.0 (mean)	6.9	2.29
5	5	12.2	5.7 and 5.2	5.5		
			5.05	5.1		
				5.3 (mean)	5.3	2.30
					Mean	2.30

Returning to the tuning curves taken without mirrors by the clipping method, it was found that we could obtain the correct resonance lengths, if we used for their determination the maxima of the smooth curves drawn through the points lying on either side and at a little distance from the experimental maxima. (See dotted line in Fig. 11, *e*.) The results found in this way for three wave-lengths are given in Table III., column 5. The first three columns give the standard vibrators and the standard receivers used, with the corresponding wave-lengths. In the fourth column the individual maxima of the tuning curves are recorded, in the order of their importance, if there are more than one. In the sixth column are given the values of the resonance lengths found by the wave-length method, which we are next to consider. The last column gives the ratio of the wave-length λ to the resonance length l, the average for the three receivers being 2.30. The ratio corresponding to the wave-length 18.7 cm. was found from the value of l determined by the tuning method, as in this instance the result by the wave-length method was somewhat doubtful.

"*Tuned Receiver Wave-length*" *Method.* — As we have just seen, the method of tuning could not be used for the investigation of receivers, when great accuracy was required. It was therefore very necessary that a new method be sought, which would give consistent and definite results. This was finally found in the "tuned receiver wave-length" method, in which the special form of receiver under investigation was put in place of the non-selective receiver in the interference apparatus (cf. Fig. 3), and the "tuned receiver wave-length" L determined from the resulting wave-length curve. Without altering the vibrator, the receiver was then varied, and the effects of the change found in terms of L. In this way it was possible to study accurately and rapidly the relations of the receiver to the incident wave-length and the conditions affecting its action.

To apply this method to the determination of the resonance lengths, the tuned receiver wave-length (L) was measured for varying lengths (l) of the resonator, keeping constant the vibrator and all the other dimensions of the corresponding standard receiver. The results are plotted in Fig. 12, *I* and *II*. Curve *I* gives the relation between L and l for receiver 5, with vibrator 5 ($\lambda = 12.2$ cm.). It is a straight line with the equation $L = 1.54l + 4.0$, showing that l is not proportional to L. On the other hand, Curve *II*, taken with receiver 6, and vibrator 6 ($\lambda = 15.8$ cm.), deviates very much from a straight line, largely as a result of mirror disturbances. A curve similar to *II* was obtained for receiver 7, with vibrator 7 ($\lambda = 18.7$ cm.). From each of these curves the resonance length was then found by taking the value of l corresponding to $L = \lambda$, where λ is the true wave-length, found with a non-selective receiver. The results have already been given in column 6, Table III., and agree well with the averages obtained from a number of the modified tuning curves described above. It will be observed that this new definition of resonance length is exactly

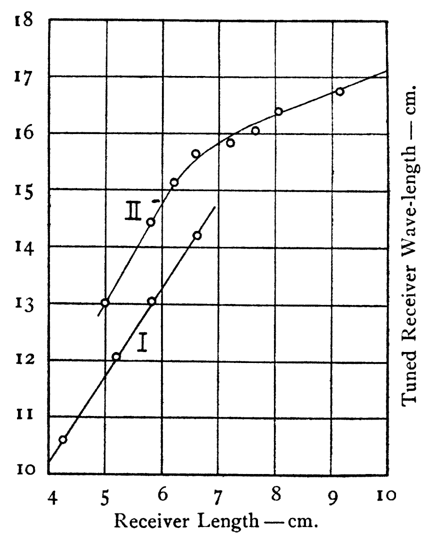


Fig. 12.

equivalent to the old, since there it was assumed that the true wave-length was measured by a receiver in resonance with the vibrator.

These three curves showing the relations between L and l appear to agree with the results of Willard and Woodman,[1] who found L to depend upon both the receiver and the vibrator. On the other hand A. D. Cole,[2] in his study of electric wave receivers found a direct proportionality between L and l and concluded that L was independent of the incident wave-length λ. If we consider the fact that the latter measurements were entirely free from mirror disturbances, it would seem that they were the most reliable, for in the present determinations, and probably in those of Willard and Woodman, the mirrors affected the results. But in none of these determinations are the data complete, or free from errors. We must therefore leave the quantitative determination of the extent to which

[1] Willard and Woodman, *l. c.*

[2] A. D. Cole, *l. c.*, p. 271.

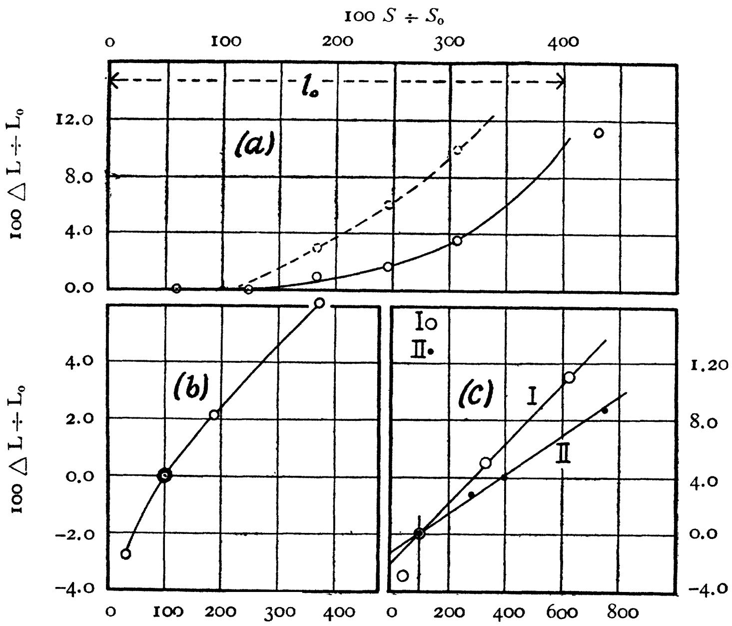
the vibrator and receiver influence these measurements to a more exact analytical study.

Effect of Varying the Relative Dimensions. — The curves giving the relation between L and l are of value in the further investigation of the properties of receivers. In using the method of tuned receiver wave-length curves the results are given in terms of L, and not of λ. As L depends upon the vibrator, the receiver, and the interference apparatus, it has no quantitative significance. However, from the data just obtained, we can express our results in terms of the equivalent length of a receiver, the other dimensions of which are those of one of the standard receivers. This somewhat extends the application of the method, but quantitatively its scope is still very limited, since L must not differ much from λ, if accurate results are to be obtained.

One of its most important applications was in the study of the effect of varying the relative dimensions of the standard receiver. This was undertaken partly to complete the statistical data which was the original object of this paper, but it was even more useful because of the light thrown on the question of receiver dimensions. The work was divided into four parts, the determination of the effect of —

1. The length of the dielectric along the receiver, s.
2. The width of the metal wings, w.
3. The thickness of the metal wings, t.
4. The length of the constriction, c.

The variation of the length of the dielectric was studied with the standard rod vibrator 6 ($\lambda = 15.8$), with the corresponding receiver 6 ($l = 6.6$, $w = 0.4$, $c = 0.16$ cm., s variable). The hard rubber receiver base was made in 0.5 cm. sections held together with wax, so that its length could be varied from 6.9 to 0.9 cm., without disturbing the wings or thermo-junction. A thin layer of soft wax insured good contact between the metal and the hard rubber. The height of the base (1.6 cm.) was not altered, since this dimension is not of importance. The results are given in Fig. 13, *a*, the abscissas giving in percentages the ratio of the length of the dielectric to the standard length of dielectric, and the ordinates the percentage change in the tuned receiver wave-length. As in the case of the standard rod vibrators (Fig. 6, *b*) no appreciable effect due to the dielectric appears, until its length is more than one fourth of the receiver length. In the dotted curve the corrections given by curve



Standard dimensions : $l_0 = 4\ s_0 = 16\ w_0 = 40\ c_0\ L_0 = \lambda$.

Fig. 13.

II, Fig. 12, have been made, and the ordinates now represent the wave-length λ, on the approximate assumption that $\lambda = 2.3l$.

The width of the flat wings also proved to be an important factor. The effect of varying this dimension keeping the other conditions constant is shown in Fig. 13, *b*. Rod vibrator 5 ($\lambda = 12.2$) was used with the corresponding standard receiver 5 ($l = 5.22$, $s = 1.27$, $c = 0.13$ cm., w variable). The curve is not a straight line, the slope increasing as the width is diminished. But it may be represented within small limits of variation by the linear equation, $L = .055w + 94.5$, where L and w are both expressed in percentages of their values for $w = 0.32$, the normal width for this standard receiver. The same test was made with vibrator 6, showing the same variation of L with the width, but qualitatively only, because of the disturbing effect of the mirrors.

The width of the wings was previously investigated by Willard and Woodman,[1] who could detect no difference between flat strips 4.6 mm. and 3 mm. wide. Their failure to observe this was due to the small variation of width, which as the present results show gives only 1.5 per cent. change in L, too small to be detected by the tuning method used.

The effect of varying the thickness (t) of the wings was found to be small. The thicknesses, .06 cm. and .015 cm., were tried, the change being too small to necessitate intermediate determinations. With $\lambda = 12.2$ cm. and $l = 5.24$ cm., the corresponding values of L were 12.42 and 12.18, a variation of but two per cent. for three hundred per cent. variation in thickness. This may be represented by the equation $150\Delta L = \Delta t$, L and t being expressed in percentages in the usual way.

Similarly the variation in the length (c) of the constriction formed by the thermojunction was tested ($\lambda = 12.2$, $s = 1.27$, $w = 0.32$, $l = 5.23$ cm., c variable). The results are plotted in Curve *I*, Fig. 13, *c*. The curve gives the relation between c and L, expressed in percentage of the normal length of constriction and percentage change, respectively. This is represented by the linear equation, $L = .025c + 96$, or $\Delta L = .025\Delta c$. A similar linear relation was also obtained with rod vibrator 6 and the corresponding receiver, but the slope of the line was very much less than in the first case, as shown in Curve *II*. This difference may be due largely to error in determining the latter curve, since mirror disturbances were again operative.

The linear relation between ΔL and Δc can be explained by regarding the change in L as due to the increase of self-induction, which is approximately proportional to the change in length of the constriction. For if A is the self-induction of the standard form of receiver corresponding to the tuned receiver wave-length $L = \lambda$, x the increase of self-induction due to a change in c, and ΔL the resulting change in L, we have

$$\Delta L = K\sqrt{A + x} - K\sqrt{A} = \frac{K}{\sqrt{A}}\left(\frac{x}{2} - \frac{x^2}{4A} + \cdots\right),$$

where K is a constant depending upon the units and the capacity,

[1] Willard and Woodman, *l. c.*, p. 7.

which remains constant since the changes are all made near the center of the receiver. Now for values of c not greater than $l \div 5$, the error introduced by neglecting the term containing x^2 and subsequent terms is within the limits of the experimental error, so that we have practically a linear relation, $\Delta L = Kx \div 2\sqrt{A} = \text{const.} \times c$.

Summing up the results obtained with the standard receivers, we have,

$$L = \lambda = 2.30l,$$

for changes in

length of dielectric,	$\Delta L = 0$, when $s < l \div 4$,
width of wings,	$\Delta L = .055\Delta w$,
thickness of wings,	$\Delta L = .007\Delta t$,
length of constriction,	$\Delta L = .025\Delta c$,

the variations Δ being expressed in percentages of the corresponding dimensions of the standard receiver. In applying these data to receivers of other than the standard form, it must be remembered that the corrections which we have determined are only applicable in a very limited range. They should be regarded as a means of finding the safe limits of variation from the standard form, rather than as quantitative relations from which the properties of any receiver could be computed. This limitation is due partly to the fact that it is impossible to alter one condition without affecting the others to some extent. For example, the effect of the width of the resonator is closely related to the constriction of the stream lines at the center,[1] and on the other hand the lengthening of the constriction, by removing the point of convergence of the stream lines farther from the center, must affect the influence of the width. We cannot therefore make any great departure from our standard conditions and still use the data determined in this work.

Ratio of Length to Wave-length. — By both the tuning and wave-length method the value found for the ratio $\lambda \div l$ at the resonance length was 2.30, which differs from that found by other observers. The theoretical values deduced by Poincaré[2] and by Abraham[3] were approximately equal to 2.00. Macdonald's[4] theory gave 2.53, and

[1] Blake and Fountain, *l. c.*, p. 277.

[2] Poincaré, "Les Oscillations Electriques."

[3] M. Abraham, Wied. Ann., 66, 435, 1898.

[4] Macdonald, "Electric Waves," p. 111.

the work of a number of subsequent experimenters appeared to confirm this value. Willard and Woodman[4] found that for three of the four cases tested this ratio was approximately correct. A. D. Cole[1] obtained as an average of five determinations the value 2.52, and Blake and Fountain[4] the value 2.47. In all of these determinations the wave-length was measured by a tuned receiver, so that they were subject to the errors of tuning already considered. However, even if we assume that the results were free from such objection, they have no real bearing on the theoretical conclusions, since, as has been shown, the relative dimensions of the receivers used affected the resulting ratios, which should have a different value for every different form of receiver. The theory can only be verified with a receiver approximating the conditions assumed, that is, with a non-constricted receiver with a diameter or cross-section very small in comparison with the length.

The value of the ratio $\lambda \div l$ for such a receiver might be obtained by extrapolation on the curve (Fig. 13, *b*) giving the relation between L and w, but the indeterminate slope between 0 and 0.5 mm. makes this difficult. An infinitely thin receiver was, however, approximated by using wings .025 to .037 mm. in diameter, with no constricted portion. This was done by extending the iron and the constantan thermo-couple wires the full length of the receiver, a fine thread attached to the ends holding them in position. With $l = 5.22$ cm., $s = 1.27$ cm., and $\lambda = 12.2$ cm., the tuned receiver wave-length L was 11.45 cm., giving the ratio $L \div l = 2.20$. To avoid the error due to the difference between λ and L and to eliminate still further the effect of the dielectric, this measurement was repeated with $l = 5.50$ cm., $s = 0.47$ cm., and $\lambda = 12.2$ cm. L was then equal to 12.18 cm., giving for the limiting ratio of wave-length to receiver length, $L \div l = \lambda \div l = 2.22$. This agrees well with the limiting value, 2.24, already found for the ratio of the wave-length to the length of the rod vibrator, as the diameter approached zero. These results fail to confirm either of the theoretical values.[2]

For free linear resonators, consisting of strips of tin-foil 0.2 cm.

[1] *l. c.*
[2] Cf. O. Bartenstein, *l. c.*, p. 242.

wide and 4.4 cm. long, Blake and Fountain[1] found the ratio $\lambda \div l = 2.25$. This is in good agreement with the present results, even though the width may make a slight correction necessary, which, however, should be very small since the resonators of these observers had no constriction.

7. Summary.

Wave-length determinations for short electric waves should be made with a non-selective receiver. Measurements depending upon tuned receivers are liable to errors arising from the process of tuning, which in general is very unreliable.

A non-selective receiver may be obtained by using a Klemencic receiver of ten or more times the resonance length.

If the relative dimensions of a vibrator or of a receiver are maintained constant, the corresponding wave-length is directly proportional to the dimensions. Hence standard forms of vibrators and receivers may be selected, for which we may easily compute the dimensions for any desired wave-length, if we know from experiment the wave-length corresponding to any one size. Such a vibrator and a receiver were investigated, and their properties determined:

Vibrator; $l = 8d = \frac{16}{3} s = \frac{8}{3} w$; $\lambda = 2.4l$.
Receiver; $s = l \div 4 = 4w = 10c$, etc.; $\lambda = 2.3l$.

The wave-length emitted by a cylindrical vibrator of given length depends upon the extent and properties of the dielectric at the center, and the ratio of the diameter to the length, but not upon the nature of the metal, nor upon the length of the air-gaps. The length of the oil-gap, if small, was found to have little influence on the emitted wave-length.

The wave-length corresponding to a Klemencic receiver of given length depends upon the extent and nature of the dielectric at the center, the width and thickness of the antennæ, or wings, and the length of the constriction at the center.

A linear relation was obtained between the wave-length and the dimensions of a modified Righi vibrator.

[1] Blake and Fountain, *l. c.*

The limiting value of the ratio of the length of a rod vibrator or resonator to the corresponding wave-length, as its diameter approaches zero, was found to be 2.23.

The disturbing action of the "collecting" mirrors is very marked in wave-length and tuning curves, affecting both the form and position of the maxima and minima. This depends both upon the focal length and the type of the mirror. By the proper choice of mirrors the resulting error may be avoided in wave-length measurements, but it is a constant source of difficulty in tuning. Tuning without mirrors was tried, but in general the process was found unreliable, and interference curves taken with the receivers under investigation were substituted.

The damping for both types of vibrator studied was approximately 1.3.

In closing, we wish to thank Messrs. H. W. Farwell, J. L. Waldron, and C. R. Webb for their assistance in making measurements, and especially to express our indebtedness to Professor E. F. Nichols, who suggested the problem and gave many valuable suggestions for its solution.

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