A MECHANICAL EFFECT ACCOMPANYING MAGNETIZATION.

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ON the electron theory of matter the magnetic properties of bodies arise from the motion of the constituent electrons of their atoms. It is well known that all bodies may be divided according to their magnetic quality into three classes: (1) diamagnetic, (2) paramagnetic and (3) ferromagnetic. Bodies belong to the diamagnetic class if their atoms possess no resultant magnetic axis. They would also belong to this class even if they possessed a resultant magnetic axis provided this axis had no tendency to set in a definite direction in an external magnetic field. By the word atom in this discussion is meant a magnetic atom; that is, the smallest portion of the matter under consideration whose electronic properties are such that the magnetic properties of the whole can be built up by the piling together suitably of similar particles. The facts appear to indicate that in many cases the magnetic atoms are the same as the chemical atoms, in other cases possibly not.

Paramagnetic and ferromagnetic substances both appear to result when the constituent atoms both possess a resultant magnetic axis and also set themselves in a definite direction under the influence of an external magnetic field. The difference between the two classes appears to depend on the relative magnitude of the atomic magnetic moment and the forces called into play by the displacement of the atoms. When the restoring forces of non-magnetic type due to a small displacement give rise to a couple which is great compared with that exerted by the external field on the equivalent atomic magnet, the magnetic fields at our disposal will only produce small rotations of the atoms and these small rotations will be proportional to the external magnetic field. For small rotations the *resultant* intensity of magnetization will be proportional to the rotation. Thus the intensity of magnetization will always be proportional to the magnetizing field over the range of values of the latter which are practicable; the body will thus be paramagnetic.

Ferromagnetic substances are those for which the restoring couples are not great compared with those of magnetic type. In this case we are able to apply external fields great enough to bring all the atomic magnets into line with the applied field. The body will then possess its maximum or "saturation" value of intensity of magnetization. For a fuller discussion of this aspect of the subject the reader is referred to a paper by Langevin.[1]

On this view it will be seen that the distinction between paramagnetic and ferromagnetic substances is one of degree only. It depends only on the magnitude of the restoring forces of non-magnetic type compared with the displacing forces of magnetic type. It might be possible for a substance which was in general paramagnetic to become ferromagnetic by combining it with some substance which increased the freedom of motion of its atoms. This may be the explanation of the peculiar behavior of the alloys of manganese investigated by Heusler. It is obvious that the converse case is also to be expected and may be applied to explain the behavior of some of the alloys of iron and the other ferromagnetic metals.

On the view that we have taken the resultant magnetic fields which the atoms of magnetic (as opposed to diamagnetic) substances possess, arise from the motion of their constituent electrons in closed orbits. If e is the charge in electromagnetic units carried by an electron describing an orbit possessing approximately circular symmetry and lying approximately in one plane, then if a is the area of the orbit and t the time in which it is described it can be shown that the value of the magnetic force, at any point whose distance is considerable compared with the dimensions of the orbit, averaged over a revolution is the same as that arising from a magnet of moment ea/t in electromagnetic units. The equivalent magnet passes through the center of the orbit and is perpendicular to the plane of the latter. The component of the magnetic moment in any assigned direction is given by the formula ea_p/t where a_p is the resolved part of a perpendicular to the direction in question. If there are n of the above orbits per unit volume then the component, in the direction con-

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[1] Journal de Physique, Vol. IV, p. 678, 1905.

sidered, of the magnetic moment of the element of volume dv, supposed to contain a large number of them, is $nea_p/t \cdot dv$.

If the azimuths of the different orbits differ then the average value of a_p must be taken in the above formula. In the case of an unmagnetized bar of iron one direction is as probable as another for the azimuth so that the average value of a_p is equal to zero.

If we proceed to calculate the intensity of magnetization induced in the substance by a uniform external field in the direction of z, taking account of the possible presence of both positive and negative electrons in motion, we find

$$M_z = NE\left[\frac{\overline{A_p}}{T}\right] + ne\left[\frac{\overline{a_p}}{t}\right]. \tag{1}$$

where the bars denote average values. Here M_z is the magnetic movement per unit volume, and for the right hand side the capitals refer to the positive ions. In this formula the proper signs must be given to E and e and also to $\overline{A_p}$ and $\overline{a_p}$ according to the sign of the charge and the direction of rotation. If there are only rotating negative electrons present then M_z has the value given previously, namely, nea_p/t.

We shall now proceed to obtain an expression for the moment of momentum of the revolving electrons about the direction of the applied magnetic force. Let the line about which the moment of momentum is calculated be the axis of z so that it is given by the equations $x = y = 0$. Consider any approximately circular orbit, the coördinates of whose center are $x_0 y_0 z_0$. Let the coördinates of the revolving electron referred to this center at any instant be ξ, η, ζ; thus its coördinates referred to the point on the axis about which the moment of momentum is to be taken are $x_0 + \xi$, $y_0 + \eta$, $z_0 + \zeta$. These are the displacements of the particle, and its velocity components are $d\xi/dt$, $d\eta/dt$ and $d\zeta/dt$. The moment of momentum about the z-axis is thus

$$m\left(\overline{x_0 + \xi}\,\frac{d\eta}{dt} - \overline{y_0 + \eta}\,\frac{d\xi}{dt}\right) = m\left\{x_0\frac{d\eta}{dt} - y_0\frac{d\xi}{dt} + \xi\frac{d\eta}{dt} - \eta\frac{d\xi}{dt}\right\}.$$

Averaging this over a complete revolution evidently the mean values are

$$\overline{x_0\frac{d\eta}{dt}} = 0, \quad \overline{y_0\frac{d\xi}{dt}} = 0$$

and

$$\overline{\xi\frac{d\eta}{dt}} = -\overline{\eta\frac{d\xi}{dt}} = \frac{\text{area of projected orbit}}{t} = a_p/t.$$

Hence the average moment of momentum about any axis is independent of the position of that axis, so long as its direction is the same and is equal to $2ma_p/t$. If there are revolving electrons with both positive and negative charges we find, with the same convention as to signs as that already used, that U_z the moment of momentum per unit volume arising from the motion of the electrons is given by

$$U_z = 2\left\{NM\left[\overline{\frac{A_p}{T}}\right] + nm\left[\overline{\frac{a_p}{t}}\right]\right\}. \tag{2}$$

We may regard the formulæ (1) and (2) as embracing all the electrons in the atom whether moving or not. In this case since the body as a whole is uncharged we have $NE = ne$. The quantities A_p/T and a_p/t will represent the resolved areal velocities of all the positive and negative electrons respectively; we may write then for brevity A and a respectively. With this understanding

$$M_z = nea\left[1 + \frac{A}{a}\right]$$

and

$$U_z = 2nma\left[1 + \frac{e}{E}\frac{M}{m}\frac{A}{a}\right] = 2\frac{m}{e}M_z\frac{1 + \frac{e}{E}\frac{M}{m}\frac{A}{a}}{1 + \frac{A}{a}}. \tag{3}$$

For small fields A and a will both be proportional to the applied field so that A/a will be a constant independent of the field strength and depending only on the constitution of the atom. So that in every case the moment of momentum per unit volume will be proportional to the intensity of magnetization, the factor of proportionality depending only on the charges and masses of the electrons and their configuration in the atom.

The most usual form of the electron theory of matter assumes that the negative electrons alone are in motion and most of the experimental facts seem to be in favor of this conclusion. In this case the above result acquires an important simplification. If the positive electrons are free from orbital motions $A = 0$ so that

$$U_z = 2\frac{m}{e}M_z. \tag{4}$$

Thus the ratio between the moment of momentum per unit volume and the intensity of magnetization is the same for all substances and is equal to twice the inverse of the specific charge (e/m) of the negative electrons.

Experiments are at present in progress in the physical laboratory of Princeton University with the object of detecting the existence of this moment of momentum experimentally. Consider a long thin cylindrical bar of iron suspended by a fiber passing through its axis of figure so that it is capable of vibrating about a vertical axis. When the bar is not magnetized its constituent electrons will not possess a resultant moment of momentum about any axis as on the average one azimuth is as probable as another for the orbits. Now consider the effect of suddenly applying a vertical magnetic field. The movable orbits will set so as to leave a balance in favor of the plane perpendicular to the direction of the field. There will thus be created a moment of momentum about the axis of suspension. But by the laws of dynamics the total moment of momentum of any self-contained system is invariable. The moment of momentum acquired by the revolving electrons must thus be balanced by an equal reaction elsewhere. It would appear that this reaction is to be sought for in one of two places. The most reasonable I think is to suppose that it is effective on the rest of the atom, the part which is not revolving in an orbit. In this case it would be made evident by a twisting of the suspended system as a whole. The twist would of course be purely temporary owing to the restoring effect of the torsional couple arising from the suspension. The other possibility is that there is a reaction of electromagnetic origin on the magnetizing system. Theoretically calculable reactions of this nature appear to be too small to account for the effect. In fact this explanation seems to be definitely excised by the fact that the moment of momentum is proportional to the mass of the revolving particles and independent of their charges (except in so far as their mass is electromagnetic).

Assuming that the magnetism of iron is due solely to the motion of negative electrons the torsional reaction should easily give a measurable effect. If any appreciable portion of the magnetization were to arise from the motion of heavier charges the effect would be proportionately greater. It will be seen that this method forms a valuable addition to our present equipment for investigating magnetic phenomena. So far experimental success has not been attained owing to the difficulty of eliminating disturbing effects.

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