THE EFFECT OF A TRANSVERSE MAGNETIC FIELD ON METALLIC RESISTANCE.

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THE effect of a transverse magnetic field in changing the resistance of a conductor has been investigated by many observers; Patterson,[1] and very recently Grunmach[2] have made very careful measurements of this effect in a number of metals. Considering the size of the effect, the agreement between the results of these two observers is, on the whole, very good. In all the metals which have been tried, with the exception of the ferromagnetic metals, iron, nickel, and cobalt, a transverse magnetic field is found to produce an increase in the resistance. Grunmach found that in the case of the ferromagnetic metals, weak magnetic fields produced an increase in the resistance. On increasing the magnetic force the effect became smaller, until for a certain value of the magnetic force it disappeared; for greater values the resistance was diminished. With one specimen of iron, and with nickel, the effect appeared to be a decrease of the resistance from the start.

A theory of the effect of a transverse magnetic field in increasing the resistance of conductors has been given by J. J. Thomson.[3] He assumes that the electric current is carried by free corpuscles, which, when no external forces act, are in motion in all directions, just as the molecules of a gas inside a closed vessel. The corpuscles are, in fact, assumed to behave just as the molecules of a gas, with this exception; the collisions which take place are not between corpuscles and corpuscles, but between corpuscles and atoms of the metal. When an electric force is applied, a drift of the corpuscles in one direction is produced, which is the electric current. In this explanation, in order to get an effect of the right sign, it is necessary to assume that the collisions between corpuscles and atoms are greatly influenced by the electric charges carried by the former.

[1] Philosophical Magazine, 3, p. 643, 1902.

[2] Annalen der Physik, 22, p. 141, 1907.

[3] Rapports presentes au Congres International de Physique, III., p. 138, 1900. Philosophical Magazine, 3, p. 353, 1902.

The assumption of collisions like those between hard elastic spheres cannot therefore be made. This theory can be made to apply to the ferromagnetic metals only by assuming that in them the collisions are of an altogether different character from the collisions in other metals. Lorentz,[1] on the other hand, expressly assuming collisions uninfluenced by the electric charges of the corpuscles, has successfully applied this theory to the thermoelectric properties of metals; it thus seems as if the assumption of collisions between corpuscles and atoms like those between hard elastic spheres ought also to explain the change in resistance of conductors in a magnetic field.

In the present paper it will be shown that, keeping to the assumption of collisions of this nature between corpuscles and atoms, and introducing the probable effect of the magnetic field in changing the molecular configuration of the metal, it is possible to explain the sign of the effect in the ordinary para- and diamagnetic metals, and in the ferromagnetic metals, as well. The theory of the effect as given by J. J. Thomson will first be developed, and afterwards this theory will be modified by introducing the effect of the magnetic field on the molecular configuration of the metal.

We assume that the corpuscles are all of one kind and move uniformly in all directions with the velocity U. In the paper of Lorentz, referred to above, the theory of electric and thermal conductivity has been given where the distribution of velocities among the corpuscles is assumed to be that given by Maxwell's law for the distribution of velocities among the molecules of a gas. But for the present purpose, since we can hope to get an idea of the order of magnitude only of the effect, this complication is unnecessary.

In a magnetic field the velocity of the corpuscles moving in a plane perpendicular to the magnetic force will be altered. Taking the xy plane as the plane of motion, the current being in the x-axis, and the magnetic force in the z-axis, the equations of motion of a single corpuscle are:

$$\left.\begin{aligned} m\frac{\partial^2 x}{\partial t^2} &= eX - He\frac{\partial y}{\partial t} \\ m\frac{\partial^2 y}{\partial t^2} &= He\frac{\partial x}{\partial t} \end{aligned}\right\} \qquad (1)$$

[1] Archives Neerlandaises des Sciences Exactes et Naturelles, 10, p. 336, 1905.

m being the mass, e the charge, and H the intensity of the uniform magnetic field.

From this we get:

$$m\frac{\partial y}{\partial t} = Hex$$

since when $x = 0$,

$$\frac{\partial y}{\partial t} = 0.$$

As the effect produced by the magnetic field is known to be sma l it will be permissible to take for x in this equation the value it would have if no magnetic field were applied; *i. e.*:

$$x = \frac{1}{2}\frac{e}{m}Xt^2 + Ut$$

$$\frac{\partial y}{\partial t} = \frac{He}{m}\left(\frac{1}{2}\frac{e}{m}Xt^2 + Ut\right).$$

Substituting this in the first of (1):

$$\frac{\partial^2 x}{\partial t^2} = \frac{e}{m}X - \frac{H^2e^2}{m^2}\left(\frac{1}{2}\frac{e}{m}Xt^2 + Ut\right)$$

and hence:

$$\frac{\partial x}{\partial t} = \frac{e}{m}Xt - \frac{H^2e^2}{m^2}\left(\frac{1}{6}\frac{e}{m}Xt^3 + \frac{1}{2}Ut^2\right) + U.$$

Calling $\bar{u}$ the mean velocity of the corpuscle due to the electric and magnetic field between two collisions, and T the time of free path, we get:

$$\bar{u} = \frac{1}{T}\int_0^T \frac{\partial x}{\partial t}\,dt = \frac{1}{2}\frac{e}{m}XT - \frac{H^2e^2}{m^2}\left[\frac{1}{24}\frac{e}{m}XT^3 + \frac{1}{6}UT^2\right] + U. \qquad (2)$$

If there are n corpuscles per unit volume, the current in the direction of the x-axis will be:

$$I = ne\bar{u} = \frac{1}{2}\frac{e^2}{m}nXT - \frac{1}{6}\frac{H^2e^3}{m^2}\left[\frac{1}{4}n\frac{e}{m}XT^3 + nUT^2\right]. \qquad (3)$$

The last term of (2) drops out because there will be as many corpuscles for which U is positive as negative. We cannot, however, say that the second term in the brackets in (2) drops out, since, owing to the factor T^2, this will be different for corpuscles whose initial velocities are in opposite directions. A corpuscle for which U is negative will have a greater value of T, as a result of the electric force, than one for which U is positive. This term can be easily evaluated if we consider the collisions between corpuscles and atoms to be like those between hard elastic spheres. For half of the corpuscles U will be negative; so this term can be written:

$$\frac{1}{2} nUT'^2 - \frac{1}{2} nUT''^2 = nUT\Delta T,$$

T being the time between collisions when no electric force is applied. If L is the free path of a corpuscle:

$$\Delta T = T' - T'' = L\left\{\frac{1}{U + \frac{1}{2}\frac{e}{m}XT} - \frac{1}{U - \frac{1}{2}\frac{e}{m}XT}\right\}$$

so that:

$$nUT^2 = -\frac{e}{m} nXT^3$$

and the electric current is given by the expression:

$$I = \frac{1}{2}\frac{e^2}{m} nXT + \frac{1}{8}\frac{H^2e^3}{m^2}\frac{e}{m} nXT^3.$$

The first term on the right is the current when no magnetic force is applied. The effect of the magnetic force is thus to increase the current, or decrease the resistance, which is contrary to observed facts.

J. J. Thomson, however, considers that the collisions between corpuscles and atoms are strongly influenced by the electric charges carried by the corpuscles. Without going into any calculation it is easy to see that if this influence is sufficienly pronounced, the difference, ΔT, between the time of the free path in the direction of the current and the opposite direction, will be very small; hence the second term in the brackets of (3) may be neglected in comparison with the first term, and we get:

$$I = \frac{1}{2}\frac{e^2}{m} nXT - \frac{1}{24} H^2 \frac{e^4}{m^3} nXT^3.$$

This indicates an increase in resistance due to the magnetic field.

Now if we suppose that a magnetic field produces a rearrangement of the molecules within a metal, it seems very reasonable to assume that the free path of a corpuscle will be altered. Without saying anything definite as to the nature of this change, let us assume that a magnetic field, H, produces a change, δT, in the time between two collisions of a corpuscle. δT will then, in general, be some function of H. With no magnetic field the current will be:

$$I = \frac{1}{2}\frac{e^2}{m} nXT. \tag{4}$$

In a transverse magnetic field the current will be, by (3):

$$I' = \frac{1}{2}\frac{e^2}{m} nXT' - \frac{1}{6}\frac{H^2e^3}{m^2}\left[\frac{1}{4}n\frac{e}{m}XT'^3 + nUT'^2\right] \tag{5}$$

where T is changed to T' the free time in the magnetic field. The effect of the magnetic field will therefore be:

$$I - I' = \frac{1}{2}\frac{e^2}{m} nX\delta T + \frac{1}{6}\frac{H^2e^3}{m^2}\left[\frac{1}{4}n\frac{e}{m}XT'^3 + nUT'^2\right]. \tag{6}$$

As we suppose the change in T produced by the magnetic field to be small, we can put:

$$T'^3 = T^3 - 3T^2\delta T,$$
$$T'^2 = T^2 - 2T\delta T.$$

Then by bringing in the assumption that the collisions between the corpuscles and atoms are like those between hard elastic spheres, we find, by the method previously employed:

$$I - I' = \frac{1}{2}\frac{e^2}{m} nX\delta T + \frac{1}{6}H^2\frac{e^3}{m^2}\left[-\frac{3}{4}n\frac{e}{m}XT^3 + \frac{1}{4}n\frac{e}{m}XT^2\delta T\right]. \tag{7}$$

The second term in the brackets will be small compared to the first, and we thus have:

$$I - I' = \frac{1}{2}\frac{e^2}{m} nX\left[\delta T - \frac{1}{4}H^2\frac{e^2}{m^2}T^3\right]. \tag{8}$$

If R is the resistance of the conductor with no magnetic field, and R' the resistance in the magnetic field, we have:

$$I - I' = \left(\frac{1}{R} - \frac{1}{R'}\right) X$$

or

$$\frac{\delta R}{R} = \frac{\delta T}{T} - \frac{1}{4} H^2 \frac{e^2}{m^2} T^2. \tag{9}$$

In order that the magnetic field may increase the resistance, it is necessary, first, that δT be positive; this means that a change in molecular configuration arising from the magnetic field produces a diminution in the time between two collisions of a corpuscle; and, second, that $\frac{\delta T}{T}$ be greater than $\frac{1}{4} H^2 \frac{e^2}{m^2} T^2$. As to the first condition, our lack of knowledge regarding molecular structure makes it difficult to decide whether it is satisfied or not. But there seems to be no evidence, at any rate, that it is not satisfied. From the second condition we can determine the order of magnitude of the change in the free time which is necessary in order to result in an increase of resistance in a magnetic field. But it is first necessary to get an estimate of the value of T.

The change in the free time of a corpuscle we have supposed due to a change in the configuration of the molecules brought about by the magnetic field. For the ordinary para- and diamagnetic metals, for which the magnetic susceptibility appears to be constant, independent of the strength of the magnetic field, this change in configuration must be supposed to increase with the magnetic field. So that if these metals show an increase in resistance for certain values of the magnetic field we would expect an increase in resistance in all magnetic fields, however great. This is the result obtained by experiment.

With the ferromagnetic metals the case is altogether different. These metals show the phenomenon of magnetic saturation. Up to a certain limit we may suppose that an increase in the magnetic force produces an increased change in the molecular configuration. After this limit has been reached, on further increasing the magnetic force no more change in the molecular configuration is produced, and consequently, no further change in the free time of a corpuscle. After a certain limit, therefore, the first term on the right of (9) remains constant, while the second term increases with H^2. It appears very probable, then, that for a certain value of H the second term on the right in (9) may become equal to the first term, and for greater values of H become larger than the first term. This would therefore result in a decrease of resistance due to the magnetic field; this is the result obtained by Grunmach experimentally.

For large values of the magnetic force, in the case of iron, for example, we can neglect the first term on the right in (9) in comparison with the second term, and obtain thus:

$$\frac{\delta R}{R} = -\tfrac{1}{4}H^2\frac{e^2}{m^2}T^2.$$

From Grunmach's results, we have, for iron:

$$H = 2 \times 10^4, -\frac{\delta R}{R} = 10^{-3}, \quad \frac{e}{m} = 10^7,$$

and hence:

$$T = 3 \times 10^{-13}.$$

This gives the order of magnitude of the free time of a corpuscle in iron, and it will probably be not very different in any other metal. We can now find the order of the magnitude of the change in the free time of a corpuscle which is necessary to get an increase of resistance in a magnetic field. For this, we have found the condition to be:

$$\frac{\delta T}{T} > \tfrac{1}{4}H^2\frac{e^2}{m^2}T^2,$$

taking

$$H = 10^3, \quad \frac{e}{m} = 10^7, \quad T = 3 \times 10^{-13},$$

we get:

$$\frac{\delta T}{T} > 2.5 \times 10^{-6}.$$

It does not seem at all improbable that a magnetic field of the strength assumed should produce a change in T of this order of magnitude.

Having found the order of magnitude of T, we can easily calculate the number of corpuscles per unit volume which are involved in the electric current. We find, by (4)

$$\frac{1}{R} = \frac{1}{2}\frac{e^2}{m}nT.$$

For iron, R is approximately 10^4; $e = 10^{-20}$ in electromagnetic units. Therefore:

$$n = 7 \times 10^{21}.$$

Taking $U = 10^7$ as a probable value of the velocity of the corpuscles inside a metal, we get for the free path:

$$L = 3 \times 10^{-6}.$$

The effect of molecular configuration on the resistance of a conductor is well illustrated in the case of a distinctly crystalline metal like bismuth. In bismuth it is known that the resistance is different in different directions; this may well be explained by the differences in the free paths of corpuscles in different directions.

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