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ON THE REFLECTION AND TRANSMISSION OF ELECTRIC WAVES BY SCREENS OF RESONATORS AND BY GRIDS.[1]

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I. INTRODUCTION.

EXPERIMENTS on electrical resonance for linear strips of metal have been conducted in a number of ways. Garbasso[2] worked with the reflection of electrical waves from wooden walls covered with linear resonators, parallel and uniformly distributed. He found that there was a selective absorption of the waves, getting high reflectivity when the resonators were in tune with the vibrator and then only. Garbasso and Aschkinass[3] measured the deviation of electrical waves through a prism built up of linear resonators on glass plates, the transmitted wave falling on a small parabolic mirror, itself constructed of resonators. They observed that the deviation depended upon the length of the resonators constituting the receiving mirror. Thus they found that both the refraction and the absorption decreased with increasing wave-length. Aschkinass and Schaefer[4] measured the transmission of electric waves through screens of resonators both when in air and when immersed in certain liquid dielectrics. In this way they obtained the refractive indices of the dielectrics for the wave employed.

[1] Read before the American Physical Society, Oct. 28, 1905.

[2] Garbasso, Atti Acc. di Torino, Vol. XXVIII., pp. 470 and 816, 1893. Journal de Physique, Vol. 22, p. 259, 1893.

[3] Garbasso and Aschkinass, Ann. der Physik, Vol. 53, p. 534, 1894.

[4] Aschkinass and Schaefer, Ann. der Physik, Vol. 5, p. 489, 1901.

The present experiments were undertaken at the suggestion of Professor E. F. Nichols with the view of studying more completely than hitherto the passage of electric waves through systems of linear resonators and, if possible, to throw further light on the question of the relation between the length of the wave and its corresponding resonator-length.

2. Apparatus and Wave-length.

Vibrator. — The vibrator was of the customary three-gap or Righi type placed in the focal line of a cylindrically parabolic mirror. It consisted of two brass balls (*BB*, Fig. 1), each 0.952 cm. in diameter, sparking in olive oil, the lead-in copper wires forming the air gaps *AA*. The two auxiliary spheres were two knobs of 0.15 cm. diameter fused on the ends of the lead-in wires. They were made small in order to disturb as little as possible the characteristic vibration of the two principal spheres. By means of sealing-wax the two balls *B*, *B*, were attached to glass tubes, through which the lead-in wires passed.

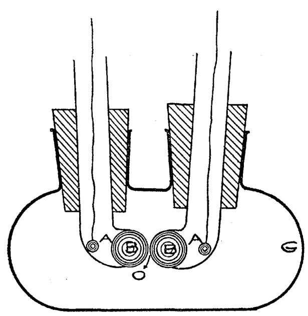

Fig. 1.

To avoid line effects these wires were drawn through an opening in the mirror directly behind the vibrator and brought out as close together as non-sparking would allow. The air-gaps *A* and *A*, were usually about 0.4 cm. long. The whole was surrounded with a glass reservoir *G* for oil, provided with suitable inlet and outlet tubes (not shown) and fitted with rubber corks through which the glass tubes led. Back of the mirror these tubes were attached to a thumb-screw arrangement by means of which the length of the spark-gap *O* in oil could be altered at will. The efficiency of the vibrator was found to be a maximum when the balls were all but touching, *i. e.*, when the length of *O* was a very small fraction of a millimeter.

The source of energy was a 10-inch induction coil, with interrupter and condenser removed, connected to a 110-volt 60-cycle alternating-current circuit. In this way the use of an interrupter, so frequently unsatisfactory, was avoided. The current in the primary was reduced to 7 amperes by suitable resistance in circuit.

Early in the work it was found that the oil surrounding the spark gap hindered the escape of the decomposition gases due to the spark. The axis of the vibrator was therefore changed from the vertical to a horizontal position and a constant stream of oil was led through the chamber *G*, entering below the spheres *BB*.

Receiver. — The wave detector was of the Klemenčič type, differing from the form used by Willard and Woodman[1] only in minor details. The diameter of the iron and constantan wires used was 0.0025 cm. These wires were soldered to copper strips 2 mm. wide fixed on wood and then they were crossed and joined to copper wires of larger size leading to the galvanometer. These larger wires were drawn away from the receiver immediately back of the thermal junction, so that no wire was exposed to the waves beyond the width of the copper strips. The total length of both the iron and constantan wires was thus not more than 3 mm. each. All junctions were soldered, as little solder as possible being used on the thermal junction itself so as not to lower too greatly its sensitiveness. This receiver was placed with its length horizontal in the focal line of a second parabolic mirror. The two mirrors were of sheet zinc and of equal aperture 70 cm. across by 63 cm. along the axis. The focal distance was 7.5 cm. A second, or check, receiver was placed directly in front of the sending mirror at a distance of 23 cm. from the vibrator and connected with a second galvanometer. This receiver will be spoken of as the check receiver, and the one in the receiving mirror as the main receiver. The resistance of each was about 2 ohms. As all junctions were soldered the resistance of neither receiver varied appreciably. The receiver resistances were always measured before and after a curve was taken. When, as happened occasionally, anomalous results were obtained in the midst of a curve, checking up the receiver resistances generally revealed an unsoldered or broken junction.

Galvanometers. — The galvanometers were of the du Bois-Rubens two coil type, triply ironclad. Owing to the proximity of all sorts of heavy street traffic and varying electric currents from tram and

[1] Willard and Woodman, Phys. Rev., Vol. 18, p. 3, 1904.

lighting circuits it was found necessary to hang the galvanometers from wall brackets by Julius suspensions and to add additional armor. For the latter purpose sheet iron was employed, five layers being wound spirally, separated from one another by strips of cork-rubber matting. In this way the magnetic disturbances were generally not more than five scale divisions for galvanometer C (attached to check receiver) and three for galvanometer M (main receiver). As the readings for the curves were generally large, some 200 or 300 scale divisions, the error due to these extraneous disturbances was small. The sensitiveness of M was 2.5×10^{-9} amperes, C 2.2×10^{-9} amperes. With coils connected in multiple the resistance of each galvanometer was about 2.5 ohms.

A rather elaborate system of mercury-cup connections was inserted as a key-board between the receivers and the galvanometers to admit of cross connections for reducing the effect of magnetic disturbances by allowing the two galvanometers to be read always in the same direction, and the insertion of suitable shunts to reduce the larger readings to scale limits. The two galvanometers were accurately adjusted to the same period, four seconds for a complete swing. Having once adjusted them to this period they remained so steady it became unnecessary to touch them throughout the course of the work. Readings were obtained by noting the zero position and the first throw. The drift of the zero point was usually slow and so small as to make the reading of return swings unnecessary.

Tuning of the Receivers. — For this purpose the two mirrors were placed facing each other in the usual way, their focal lines being horizontal, 1.3 meters above the floor and 2.4 meters apart. To prevent air currents affecting the thermal junctions the apertures of both mirrors were covered with thin card-board and each receiver was surrounded by a small paper box.

By a preliminary experiment the check receiver had been shortened to a length of 3.5 cm. and it was kept constant at this length during the shortening of the main receiver. The tuning of the latter is plotted as a curve in Fig. 2. Abscissas are total lengths of the receiver in centimeters ; ordinates, the ratios M/C of the galvanometer deflections. It is to be noted that the curve gives multiple tones of the receiver, minima at even, and maxima at odd multiples of the fundamental length. This is as it should be, for the current in the receiver is a maximum when there are loops of potential difference at its ends. So far as it was carried the curve agrees with curves 1 and 2 obtained by Willard and Woodman.[1] Their curves show the harmonics of the vibrator but not the multiple tones of the receiver, for they did not start with sufficient lengths to show these.

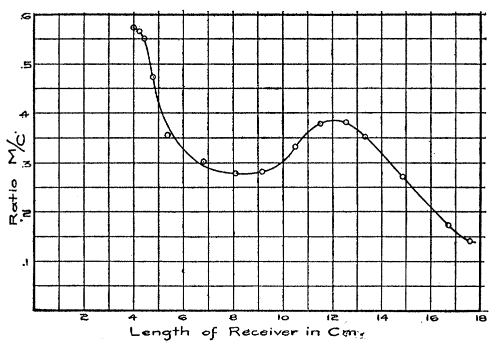

Fig. 2.

By trial it was found immaterial whether the check receiver was tuned — for some of the work it was kept at the length 3.5 cm.; later it was changed to the proper resonance length, 4.0 cm.

Deterioration of the Vibrator. — Although a check receiver was employed to take account of the irregular action of the vibrator, it was found that, starting with the vibrator in a new and fresh condition, the ratio M/C gradually lessened as the spark gap deteriorated. In addition to the constant eating away of the material of the spheres a filament of carbon from the decomposed oil would gradually bridge the gap, both causes tending to lessen the radiation. By making and breaking the circuit a few times this bridge could be torn away and the galvanometer readings immediately following its disappearance were invariably very large. Before this unsatisfactory condition of the vibrator was reached, however, it was generally renovated. To do this it had to be taken all apart, because the construction and the small size of the balls prevented the possibility of turning new portions of the spheres into position

[1] Willard and Woodman, l. c.

for sparking by merely rotating the glass tubes to which they were attached. In all cases the vibrator was not touched during the progress of any one set of readings although tests were made which showed that any measurement could be duplicated to within 5 per cent. after the vibrator had been removed, readjusted and replaced. The ordinary duplication for any curve-point, obtained by simply repeating readings was usually within 2 per cent.

The following sample series of figures illustrates the decrease in the ratio M/C as the condition of the vibrator grew poorer. Each figure is the mean of seven readings, successively taken under constant conditions; 0.804, 0.802, 0.798, 0.784, 0.790, 0.784, 0.776, 0.764, 0.777, 0.768, 0.754, 0.750, 0.741, 0.740, 0.732, giving a total variation of 9 per cent. Moreover, the series shows that the falling off is somewhat irregular. Now it was found that an increase in the length of the spark gap O invariably produced a decrease in the energy radiated and a corresponding decrease in the ratio M/C. In general, it would seem that the violent action to which the vibrator was subjected, together with the constant wearing away of the brass spheres gradually lengthened the gap; and yet at any one instant a projecting particle of brass might easily have shortened it for the moment, thus accounting for the irregularity in the decrease of the ratio.

But however irregular the vibrator's action, due to *any* cause, one would expect the ratio M/C to hold constant, provided both receivers wese sufficiently far removed from the vibrator as to be affected by *pure* radiation only. The observed decrease in this ratio indicates that the check receiver, distant from the vibrator 23 cm. or a little more than two wave-lengths, was close enough to receive some of the energy that periodically flowed back into the vibrator. Since these two portions (the *pure* and the *impure*) of the radiation which reaches the check receiver do not vary proportionally, for a small amount of energy emitted by the vibrator the denominator of the fraction would be relatively too large.

Instead of removing the check receiver to a greater distance it was easier to eliminate as far as possible all irregularity in the vibrator's action in the following way: Whatever varying system was being measured, readings of a second constant system were alternately taken, giving a second ratio M'/C'. The quotient $\frac{M/C}{M'/C'}$ should not contain any effect of irregularity in the vibrator's action. For example, suppose one was measuring the transmission of energy through a system of resonators as their length was gradually shortened. He could take alternate readings of the transmission through air, obtaining a second ratio M'/C', and thus eliminate various irregularities.

Measurements of the Wave-length. — For measuring the length of the principal wave emitted by the vibrator the interferometer method[1] was employed. The separating surface was a grid of fine copper wire (B. & S., No. 34) mounted on a wooden frame 125 by 100 cm. The distance between the wires was 3 cm. The fixed mirror was zinc, 117 by 95 cm.; the movable mirror was a silvered

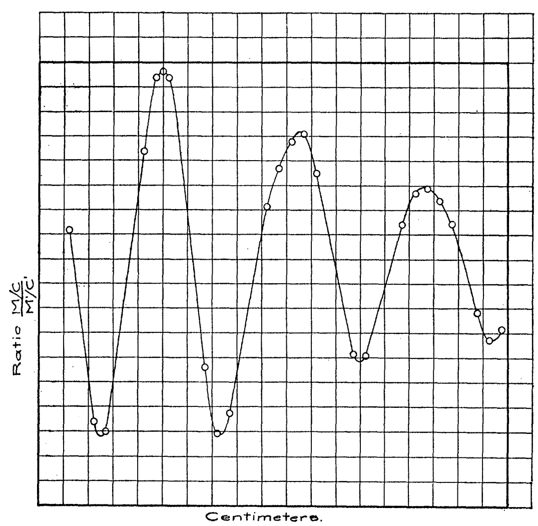

Fig. 3.

surface on the front face of a piece of plate glass 122 by 96.5 cm. This silver mirror was supported in one of two equal compartments of a sliding framework, itself encased in another framework arranged to move in the direction of the wave along a set of ways. The

[1] For the advantage of this method see G. F. Hull, PHYS. REV., Vol. 5, 1897, and Willard and Woodman, l. c.

sliding device, by allowing the removal of the silver mirror from the path of the wave, afforded alternate readings of the effect of reflection from a third metal mirror fixed in position back of the movable one. Thus the ratio M'/C' was obtained. The interference curve is shown in Fig. 3, the ordinates being the ratio $\frac{M/C}{M'/C'}$. The complex character of the radiation is shown by the unequal spacing of successive maxima and minima. For estimating the wave-length from this curve only the principal maximum and its two adjacent minima were considered, for it is in the immediate neighborhood of this maximum, where the two plane mirrors are equally distant from the separating surface, that the heads of the two wave-trains combine most strongly. In this way the wave-length came out 9.9 cm. An almost identical curve was obtained by using a separating surface with the wires 1.5 cm. apart. Had the spacing been 2.2 cm. the sharpest maxima and minima possible would have been obtained.

3. Resonators in Air.

The energy transmitted and reflected by various systems of resonators in a plane both in air and on glass was measured as their lengths were shortened and the results are plotted as curves in Figs. 6, 8, 9 and 10. All of the useful details as to the distribution of the resonators for the curves shown are collected into Table I. Tin-

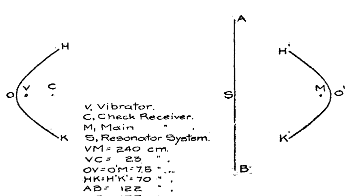

Fig. 4.

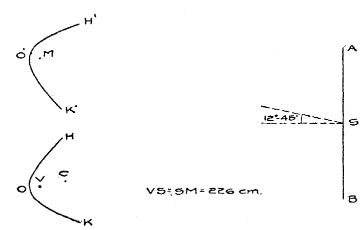

Fig. 5.

foil strips 2 mm. wide were employed throughout the work, their length being always parallel to the electric force of the wave, and in its plane for the transmission measurements. The arrangement of apparatus for transmission is shown in Fig. 4 and for reflection in Fig. 5; both diagrams are drawn to scale. The reflecting angle was 12° 45′, with a cosine of 1 approximately. To avoid diffuse

Table I.

Distributions of Resonators.

Designation of Curves.	Number of Columns.	Number of Resonators in a Column.	Distance between Columns (Adjacent Edges).		Distance between Resonators Sidewise (Axially).	Length of Resonators.		
			At Beginning.	For Resonance.		At Beginning.	For Resonance.	
Fig. 6, T and R.	7	39	1.0	5.7	3	10.0	5.3	Air.
" 6, T_1.	13	20	0.7	1.1	6	5.3	4.9	Air.
" 6, T_2.	13	12	1.0	2.6	10	6.0	4.4	Air.
" 6, T_3.	15	5	1.0	1.6	20	5.0	4.4	Air.
" 8, T and R.	13	39	0.5	3.5	3	6.0	3.0	Sheet Glass.
" 8, T_1.	7	39	1.0	8.0	3	10.0	3.0	Sheet Glass.
" 8, T_2.	13	23	0.5	3.2	5	5.5	2.8	Sheet Glass.
" 9, T and R.	13	39	1.0	4.45	3	6.0	2.55	Plate Glass.
" 10, T and R.	7	117	0.5	6.0	1	10.5	5.0	Sheet Glass.
" 10, T_1.	7	75	1.0	7.0	1.5	10.0	4.0	Sheet Glass.
" 10, T_2 and R_2.	7	59	0.5	7.7	2	10.5	3.3	Sheet Glass.
" 10, T_3.	7	12	0.5	8.5	10	10.5	2.5	Sheet Glass.

reflection from the walls of the room the apparatus was arranged diagonally with respect to them.

By a carefully conducted series of tests with a grid of fine wires 5 mm. apart the wave emitted by the vibrator was found to be 100 per cent. plane-polarized. Accordingly it was deemed unnecessary to insert, after the method of Aschkinass and Schaefer,[1] such a grid in the path of the waves. Moreover, a screen to cut off diffraction effects was not used for it was thought that better comparative results for transmission and reflection would be obtained without its use, since in the one case such a screen could not be used. However, to make sure that the diffracted energy was small, a plane metal mirror the size of the resonator system was inserted at *S*, Fig. 4, and the galvanometers read. The effect at *M* was less than 1.5 per cent. of the incident energy. In Fig. 5 such a mirror was inserted in a position close to the vibrator mirror but so inclined as to reflect none of the energy into the receiving mirror. A small effect, 1.4 per cent., was still felt at *M*. In both of these cases the phase relation that this diffracted energy bore to the main portion of the energy was unknown and so it was not corrected for in any of the curves. It shows itself especially in curves *R*, Fig. 6, length 0 cm., and *R*, Fig. 12, distance 8 cm.

In the transmission work, to avoid the possibility of interference effects at the check receiver *C* the resonator system *S*, Fig. 4, was placed relatively close to the receiving mirror. By choosing a resonator system that reflected the larger proportion and yet transmitted some of the energy and by making careful tests as it was moved along the line *VM* (Fig. 4), no effects due to interference could be detected at *C*.

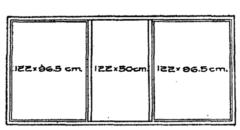

Fig. 5a.

The resonator system *S* was fitted into one of two equal compartments of a sliding framework (Fig. 5, *a*). For the work on transmission through resonators in air the other compartment was left empty; for resonators on glass it was either empty or fitted with a bare glass plate, the duplicate of the one supporting the resonators. For the work on reflection the second compartment

[1] Aschkinass and Schaefer, l. c.

held a plane metal mirror, the silver mirror used in measuring the wave-length serving as the check reflector. By sliding S back and forth across the path of the waves readings were alternated between the two compartments, giving the ratio $\frac{M/C}{M'/C'}$. Thus the reflection or transmission was measured.

As a rule only five readings constituted a set for any given length of strips, three of the resonator system under observation and two of the check system; for during the progress of the work it had been found that better results were obtained for a small than for a large number of readings, since in this way excessive deterioration of the vibrator was avoided.

Of the curves in Fig. 6 for resonators in air T was taken with the

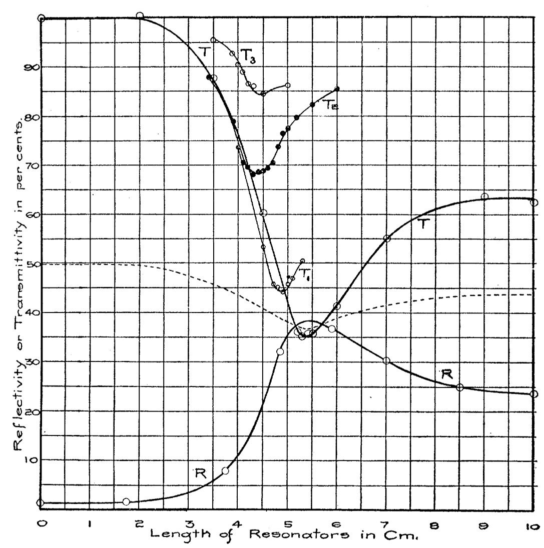

Fig. 6.

strips attached to cross section paper and R, T_1, T_2 and T_3 to tracing cloth. By a previous test both of these substances were found to have an index of refraction of 1 for the wave employed. R was the first one of the curves taken and shows that the resonance-length was passed unexpectedly. It was carefully located, however, at 5.3 cm. in the T curve. As the mean of T and R, the dotted curve shows that the scattered radiation is a maximum at the resonating length. This agrees with expectation on the assumption that none of the energy suffers degradation. It is seen that when the strips are shorter than 2 cm. or $\frac{1}{5}\lambda$ there is practically no absorption.[1]

It was thought worth while to see what would be the maximum reflection obtainable from a set of resonators of the above length, 5.3 cm., using a more dense distribution. In the work with resonators on glass (to be mentioned later) it had been found that a change in the side-on distance between resonators altered the length for maximum resonance while a change in the end-on distribution, within the limits investigated at least, did not affect it. Accordingly, keeping the side-on distance the same as before, 3 cm., the number of columns was increased from 7 to 13, making the distance between adjacent edges of adjoining columns 0.7 cm. The coefficient of reflection of this set was 79.0 per cent.

Two other distributions of resonators in air were tried — T_1, obtained from the set just mentioned by removing alternate horizontal rows, and T_2. For T_1, with strips 6 cm. apart, resonance occurred at the length 4.9 cm.; for T_2, strips 10 cm. apart, at 4.4 cm.

To show the relation between side-on distance and resonating-length the curves in Fig. 6 are summarized in *A*, Fig. 7. It turns

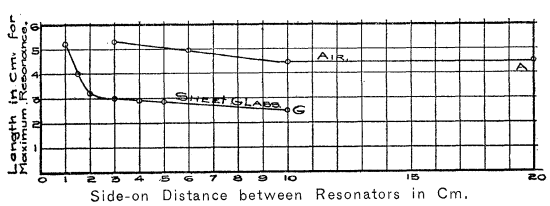

Fig. 7.

out a straight line making an appreciable angle with the axis of abscissas, and shows that the magnetic fields of the resonators do not cease to influence each other until the resonators are nearly a whole wave-length apart. That the resonance-length should decrease with increasing distance between the strips was to be expected; for since

[1] Following Aschkinass and Schaefer the word "absorption" will be used in this paper to mean the diminution in energy in passing through a resonator system; it thus includes reflection, scattering and degradation into heat if any.

at any instant the currents in the resonators are all in the same direction the effect of induction is reduced by decreasing the distance between them; hence in order to respond to a given wave the capacity of the resonators must be increased, that is, they must be lengthened.

4. Resonators on Glass.

The results obtained for resonators on glass are shown by curves in Figs. 8, 9 and 10. For the most part sheet glass 3 mm. thick was used. For the transmission work the check system for getting M'/C' was often a second plate of sheet glass bare of resonators, but as this was compared with air or empty frames the ordinates represent the actual percentages of reflection or transmission. The functions of the two glass plates were often interchanged. To facilitate the process of cutting down the resonators coördinate paper

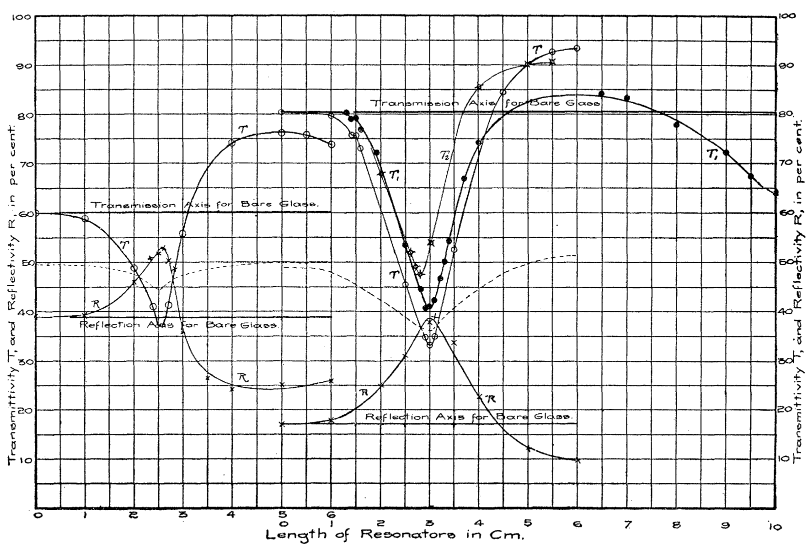

Fig. 9. Fig. 8.

was pasted on the backs of both plates, after the method of Aschkinass and Schaefer. When bare they agreed quite well with each other and with themselves from day to day, never deviating more than 2 per cent. of the incident energy from the mean, viz., 80 per cent. transmission and 17 per cent. reflection.

A study of curves T and T_1 in Fig. 8 in connection with the table shows that within the limits there specified a change in the end-on distance between resonators does not affect the resonance-length, which was 3 cm. in both curves. An attempt was made in both of these curves to locate the first upper partial of the vibrator. Though weak it was detected, as shown for the length 1.5 cm. It was found impossible to locate any upper partials beyond the first for if present they were too feeble to admit of measurement.

Perhaps the most interesting feature of the curves in Fig. 8 is that they all cut the axes for bare glass. This means that for strips on glass 3 cm. apart and of length about twice the resonating length the system transmits more and reflects less energy than for the glass plate bare. We shall refer to this phenomenon as that of *extra transmission.* It will be discussed later. It is seen that it is a maximum in this case for a length twice the resonance-length ; and as shown by T it is a rather large effect, being 13 per cent. for this 13-column distribution.

The dotted curve shows, as in Fig. 6, increased scattering at the resonance-length. This result was obtained in every case where the trouble was taken to secure both the T and corresponding R curves.

As is usual with sheet glass the pieces used were slightly warped and so to make sure that none of the effects obtained were due to lack of planeness, especially because of the finite dimensions ($12 \times 10\lambda$) of the glass, one set of curves was taken using plate glass (6 mm. thick). They are shown as curves in Fig. 9, and it is seen that the type is the same as before. The distribution was the same as for Fig. 8, T except that the end-on distance at the start was 1.0 cm. instead of 0.5 cm. The *extra transmission* is slightly greater than before, about 16 per cent. It is seen, too, that the reflection from the thicker glass is greater and the resonance-length less (2.55 cm. instead of 3 cm.) than for the sheet glass. For resonators this same distance of 3 cm. apart in air the resonating-length was 5.3 cm. It is doubtful if, following Aschkinass and Schaefer, the index of refraction of the glass could be obtained from these figures, for the thickness of the glass as well as the fact that it is on only one side of the resonators serves to make the relation a complicated one.

A comparative study of the curves in Figs. 8 and 10 with reference to the table of distributions shows that a variation in the side-on distance between strips materially alters the resonance-length and in a manner similar to the case of resonators in air.

The most typical resonance curve is T_1, Fig. 8, for which the strips were 3 cm. apart. When 5 cm. apart the effect is smaller, as shown in T_2, and for 4 cm. distance the curve (not shown) lies in value between T_2 and T.

For distances apart smaller than 3 cm. (see Fig. 10) the absorption keeps increasing in amount with diminishing distance, but the sharpness of the resonance continually grows less until for a 1 cm. distance the exact location of the absorption maximum is indefinite, so flat has the curve (T) become. It was estimated at 5.2 cm., but it may easily be 2 or 3 mm. more or less than this. Thus we may say that for resonators on glass the absorption-maximum practically disappears when they are $\frac{1}{10}$ of a wave-length apart. The same thing seems probable for resonators in air.

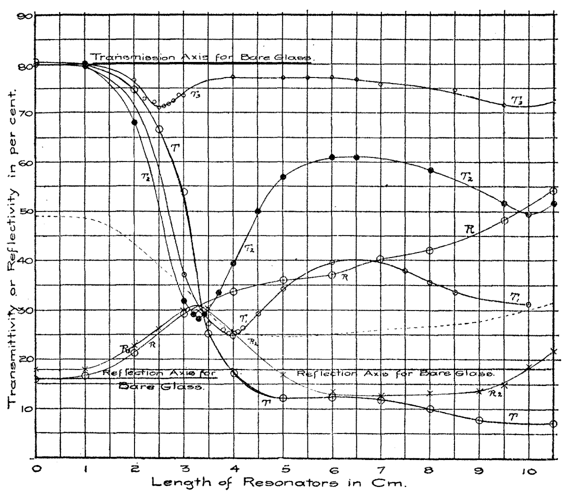

Fig. 10.

The dotted curve in Fig. 10 is the mean of R and T and shows how great the scattering becomes when the resonators are close enough together to interfere strongly with one another. The mean of R_2 and T_2 is not shown, but the fact that R_2 cuts the reflection axis for the bare glass plate while T_2 does not cut the transmission axis shows the magnitude of the scattered energy. For zero length R and R_2 differ by 2 per cent. of the incident energy, due to lack of planeness of the glass and to experimental error.

In all of the work on reflection the resonator face of the glass plate was toward the vibrator, but in the transmission work it was found immaterial which surface faced the vibrator. For instance, T_2, Fig. 8, was taken with the resonator surface of the glass toward the vibrator while T was taken with it in the opposite position. In fact, to test the idea, when the resonance length of T was reached the plate was purposely turned 180° ; there was no alteration in the transmitted energy.

As was found for the case of the main receiver (Fig. 2) one would expect for the resonators a second absorption-maximum at a length three times the fundamental length, and indeed this second maximum appears in its proper position in T_2, Fig. 10. But R_2 does not support T_2 in this respect and in T_1, Fig. 8, it is displaced 1 cm. to the right. The displacement of this second maximum is shown most strongly in T_3, Fig. 10, taken for a set of resonators 10 cm. apart. It would seem that for causes difficult to explain, the location of this second maximum for any given case cannot be successfully estimated ; to be able to predict it the law of scattering would need to be known. It is interesting to notice, however, that in all the curves, no matter what the length of maximum resonance, this second maximum occurs at the length 10 cm. It is difficult to give an unequivocal interpretation of this result, but it should be noted that at the beginning of any seven-column distribution curve the end-on distance between resonators was small (though in no case less than 5 mm.) and for such distances capacity effects due to charges on the ends of the strips might be felt. This is not borne out, however, by curve A Fig. 11 (see below), for if the charges on the ends of the strips exert an appreciable influence for end-on distances as small as 5 mm. the first point on this curve should not be on the straight line the curve shows.

Although the relation between the first and second absorption-maxima is not clear from the experiments, still the first absorption minimum always (including the case of resonators in air) occurred at the length twice the fundamental length, provided the strips were not closer together than 2 cm. Now Rubens and Nichols,[1] working with heat-waves 24 μ long reflected from fluorite, measured the reflection from silver resonators on glass for a number of different lengths varying by successive steps of 6 μ and found maxima of reflection for the lengths 12 μ and 24 μ and a minimum at 18 μ. In other words, they obtained reflection maxima for resonators whose length was a whole number of half wave-lengths, whereas we find such maxima only for odd multiples of the half wave-length. The form and distribution of the resonators on glass studied by Rubens and Nichols were so different from any case examined in the present paper that no strict comparison between their results and the present ones is possible. One assumption used in the interpretation of their results, namely, that the resonators and the glass background reflect separately and independently, seems from the present results very doubtful.

Curve *G* in Fig. 7, summarized from Figs. 8 and 10, expresses the relation between the side-on distance between strips and the resonance length. It becomes a straight line for distances greater than 4 cm., and though it slopes less than *A* does, it still makes an

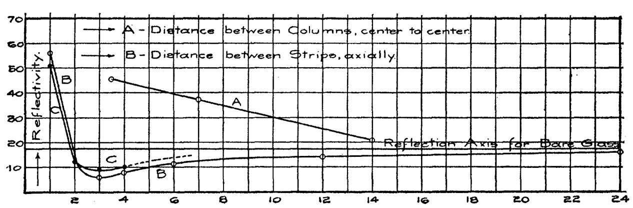

Fig. 11.

appreciable angle with the axis of abscissas. Had points on *A* been obtained for distances under 3 cm. the curve would show in all probability the relatively sharp bending that *G* does for such distances.

Starting with a dense distribution of columns of strips of the resonating-length it was thought interesting to measure the reflection as alternate columns were removed. Accordingly measure-

[1] Rubens and Nichols, PHYS. REV., Vol. V., p. 152, 1897.

ments were taken of a 27-column set of resonators 3 cm. long, 3 cm. apart sidewise and 0.5 cm. apart end-on, then for a 13-column set and finally for seven columns, care being taken, of course, always to leave the central column. Using as abscissas the distance between the axes of adjoining columns the curve (A, Fig. 11) plots a straight line.

5. LONG STRIPS ON GLASS.

In the endeavor to get more light on the observed phenomenon of *extra transmission* strips 2 mm. wide and of a length equal to the entire width of the plate were pasted on the glass at intervals of 4 cm. Such a set reflected 8 per cent. and transmitted 92.9 per cent. by measurement, the bare glass reflecting 17.5 per cent. This set being then rendered worthless by further experimentation which did not prove fruitful, another set of long strips 1 cm. apart was prepared. The coefficient of reflection having been obtained alternate strips were then removed and measurements again taken. The fact that the greatest *extra transmission* for a system of double the resonating-length had been obtained when the strips were 3 cm. apart (T, Fig. 8) had led us to suspect that 3 cm. was a critical distance, and so for the first removal of strips only every third instead of every other one was left. The results are shown in curve B, Fig. 11, while curve C shows the corresponding figures, estimated from Figs. 8 and 10 for a system of double the fundamental length. Both the reflection for 1 cm. distance and the *extra transmission* for the other distances are slightly greater for B than for C. Now curves R_2 and T_2, Fig. 10, show that while for a 2 cm. distance there is less reflection from the system of double length than from the bare glass plate, still the extra energy is scattered, not transmitted; T and R, Fig. 8, on the other hand, show that when the distance is 3 cm. the excess is actually transmitted, that the scattering is nothing. Hence it would appear that *extra transmission* occurs for long strips on glass when further apart, say than 2.5 cm. or $\frac{1}{4}\lambda$. We are inclined to think that it occurs for strips on glass any distance apart greater than $\frac{1}{4}\lambda$ and of any length greater than the fundamental length provided the length chosen is not near a secondary absorption-maximum; this idea was only partially tested however.

Perhaps the simplest explanation of the observed phenomenon of *extra transmission* and correspondingly decreased reflection which may be at present tentatively offered, is that the combination of metal strips and glass constitute a medium with a refractive index different from that of glass alone. The optical dispersion formula[1]

$$n_\lambda^2 = b^2 + \sum \frac{M}{\lambda^2 - \lambda_1^2}$$

leads directly to such a result. Thus if for the glass alone $n = b$, then when the plate is covered with resonators of a given length,

$$n_\lambda^2 = b^2 + \frac{M}{\lambda^2 - \lambda_1^2}$$

in which n_λ is the index of refraction for the wave-length λ of the incident-radiation and λ_1 is the wave-length corresponding to the natural period of the resonators ; b^2 is the dielectric constant of the medium. Hence when $\lambda_1 \gtreqless \lambda$ then $n \lesseqgtr b$. In all cases where *extra transmission* was observed $\lambda_1 > \lambda$ hence $n < b$, which is as it should be to give decreased reflection and a correspondingly increased transmission.

Seen from this point of view, the present case is analogous to that of quartz in the visible spectrum, where $n < b$, hence quartz shows smaller reflection and larger transmission for visible than for the longer electric waves. In both instances the anomalous behavior is due to the presence in the medium of structures which respond to longer waves than those present in the incident radiationl

So far as the present writers are aware this phenomenon of *extra transmission* has not hitherto been noticed for electric waves.

6. Long Strips Cut into Resonating-lengths.

The resonating-length having been determined for certain side-on distances between strips both in air and on glass, it was considered of sufficient interest to compare the reflection percentages from long strips entire and when cut into resonating lengths. Strips 77.3 cm. long and 3 cm. apart in air reflected 39.8 per cent. of the incident energy; when these were cut into 13 columns of resonators 5.3 cm. long with a 0.7 cm. end-on distance the reflection was 79.0 per cent., a two-fold increase.

[1] Drude's Optics (English translation), p. 391.

For the same distance apart on glass *extra transmission* occurred in the case of long strips, and so where strips of great length when entire reflected only 5.6 per cent. of the incident energy, when cut into 27 columns 3 cm. long and 0.5 cm. apart end-on the reflection was 46.0 per cent., an eight-fold increase.

7. REFLECTION AND TRANSMISSION OF A WIRE GRID.

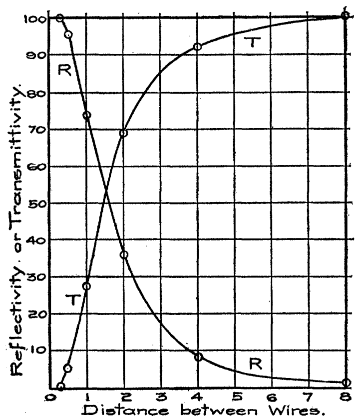

Fig. 12.

For measuring the reflection and transmission of a wire grid fine copper wire (B. &. S. No. 34) was attached to a large (125 × 100 cm.) wooden frame at intervals of 2.5 mm., and measurements were taken as the grid-constant was successively increased by removing alternate wires. The results are shown as curves in Fig. 12. The reflection is seen to be total when the spacing is 2.5 mm. or one fortieth of a wave-length and zero when it is 8 centimeters or four fifths of a wave-length.

8. COMPARISON OF WAVE-LENGTH WITH RESONATOR-LENGTH AND WITH RECEIVER-LENGTH.

Using two different separating surfaces the principal wave-length was found to be 9.9 cm. Although in getting the curve in Fig. 2 for the tuning of the receiver no means were provided for taking account of the deterioration of the vibrator the error thus introduced in locating the tuned length of the receiver at 4.0 cm. is of the second order. This is confirmed in the curve, moreover, by the second maximum occurring quite accurately at 12.0 cm. Thus the ratio of wave-length to receiver length is 2.47, confirming MacDonald's[1] value of 2.53 over Poincaré's[2] value of 2.

Curve *A*, Fig. 7, shows that neighboring resonators no longer influence each other in a way to change the length for maximum resonance when a whole wave-length apart sidewise. Beyond this

[1] MacDonald, "Electric Waves," p. 111.

[2] Poincaré, "Les Oscillations Electriques."

distance the curve becomes a straight line, and the length of maximum resonance in a system so spaced should be the same as that for a single uniform linear resonator. For 10 cm. waves the chief resonance-length was found to be 4.4 cm. and the ratio $\frac{\text{wave-length}}{\text{resonator-length}} = 2.25$, which agrees neither with Poincaré's nor with MacDonald's theoretical value, but is midway between the two values.

The experimental evidence for this result is given in curves[1] T_2 and T_3, Fig. 6. T_2 shows the transmission as a function of the resonator length when neighboring resonators were 10 cm. apart sidewise, and T_3 shows the same function for resonators 20 cm. apart. The abscissas of the minima in both cases are fairly well marked and cannot be read as low as 4 cm., which would be necessary to establish MacDonald's ratio, nor can they on the other hand, be read as high as the 5 cm. length, required by Poincaré. The only question which can arise therefore is one concerning the accuracy of the wave-length measurement. The writers feel reasonably sure that the wave-length cannot be in error by an amount large enough to bring about even an approximate agreement with either theory.

Willard and Woodman[2] found an agreement with MacDonald's ratio for the resonance length of thermoelectric receivers, and their result is confirmed earlier in the present paper. In the thermoelectric receivers, however, a relatively high resistance occurs at the thermojunction and also a sharp constricting of the current stream lines. Such conditions undoubtedly affect the linear dimensions for maximum resonance, and it is therefore not surprising to find the resonance length for receivers different from that for uniform linear resonators.

9. Conclusions.

As a result of the present study it appears:

1. That for a system of linear resonators, the length for maximum resonance is, within fairly wide limits, independent of the distance apart of resonators in the direction parallel to the electric force, but

[1] The observations for curves T_2, T_3 were made for the writers by Messrs. Woodman and Webb after the other experiments embodied in this paper were completed.

[2] Willard and Woodman, l. c.

varies in an inverse manner with the distance between resonators in a direction perpendicular to the electric force.

2. That the mutual influence between resonators at side-on distances greater than a wave-length of the incident radiation ceases to affect the length for maximum resonance appreciably.

3. That the resonator length for maximum resonance depends upon the dielectric properties of the plate to which the resonators are attached.

4. That secondary resonance maxima occur only at odd multiples of the fundamental resonating length.

5. That neither the Hertz-Poincaré nor the MacDonald ratio for the quotient of wave-length by resonator length holds for a uniform linear resonator.

6. That certain systems made up of resonators of lengths differing with the distribution over the surface of a glass plate, transmit more energy than the same glass plate when bare, and that a simple and general explanation of this result is afforded by the theory of optical dispersion.

7. That it is possible to increase the reflection from a grid of very long strips any distance apart by cutting them up into lengths equal to the resonance length for that distance.

8. That the distance apart of fine wires in a grid to be perfectly opaque to electric waves of length λ when the wires are parallel to the electric force, should be of the order $\frac{1}{40}\lambda$. When at a distance $\frac{4}{5}\lambda$ apart the same wires transmit practically all of the incident radiation.

In closing we wish to express our indebtedness to Messrs. L. E. Woodman, H. W. Webb, F. A. Thomas and S. R. Williams for their kindly assistance rendered at various times in taking some of the observations. To Professor Hallock we are grateful for kindly interest, encouragement and friendly advice. We are especially indebted to Professor E. F. Nichols, who suggested the problem and some details of the methods for solving it.

PHOENIX PHYSICAL LABORATORIES,
COLUMBIA UNIVERSITY,
February, 1906.

NOTE. — Since the foregoing paper was written the results of an investigation by M. Pietzold, Ann. d. Phys., 19, p. 116, 1906, in some respects similar to the present one, have come to the attention of the writers.