EXPERIMENTS ON RESONANCE IN WIRELESS TELEGRAPH CIRCUITS. PART IV.

By George W. Pierce.

X. The Direct Coupled Type of Sending Circuit.

IN the previous parts[1] of this research an account was given of experiments on the resonance conditions in wireless telegraph circuits of the "electromagnetically-connected" type. It is proposed in the present paper to present the results of similar investigations with other types of circuits.

The sending station in the set of experiments here described was kept constant throughout, and is represented diagrammatically in Fig. 30. It is seen to consist of an autotransformer instead of the tesla arrangement of the previous experiments. The high frequency oscillations in the sending antenna are produced by the discharge of a condenser through some of the turns of an inductance in the mast circuit.

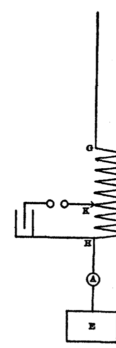

Fig. 30.

This form of sending circuit is employed in several of the commercial systems of wireless telegraphy at present in operation. Itseems to have been first proposed by Professor Ferdinand Braun of Strassburg, and is covered by German Patent No. 111,578, issued October 14, 1898, to the Gesellschaft für drahtlose Telegraphie. Braun's claim of priority in the use of this circuit is substantiated by Righi and Dessau,[2] Max Wien,[3] Fleming and various other writers on wireless telegraphy. For example, Fleming[4] says, "The second method, due to Braun, consists of attaching the aerial to some point on an oscillation circuit consisting of a condenser, an

[1] Physical Review, Vol. XIX., p. 196, 1904; Vol. XX., p. 220, and Vol. XXI., p. 367, 1905.

[2] Righi and Dessau, Telegraphie Ohne Draht, p. 416, 1903.

[3] Wien, Ann. d. Phys., 8, p. 686, 1902.

[4] Fleming, Pop. Sci. Monthly, June, 1903.

inductance coil and a spark gap, in series with one another, and charging and discharging the condenser across the spark gap so as to create alternations of potential at some point on the oscillation circuit. The length of the aerial must then be so proportioned * * * that it is resonant to this frequency."

Adjustment of Sending Circuit. — Evidently in order to produce a strong radiation of electric waves the different parts of the sending station must be properly proportioned. This adjustment may be made in several ways; by variation of the capacity of antenna, the capacity of the condenser, the whole inductance *GH*, or the part of the inductance in the condenser circuit *KH*.

For the present experiments the first three of these quantities were arbitrarily chosen and kept constant, and adjustment was made by moving the connector *K* to various points on the inductance. At each position the key in the primary of the charging transformer was closed and the current between the antenna and the ground was read on a hot-wire ammeter at *A*. The connector *K* was finally set at the position that gave the greatest current in the ammeter. This adjustment makes the sending station into a system with a strong oscillation of current in the antenna, which is the condition for a strong radiation of energy.

The dimensions of the parts of the sending station in their final adjustment had the following values:

The sending antenna, shown in Fig. 13 of Part II., extended to a height of 15.8 meters above the mast inductance. For the greater part of its length it consisted of four wires .208 cm. in diameter and 59 cm. apart.

The coil *GH*, Fig. 30, consisted of five turns of wire .208 cm. in diameter wound in a spiral 46 cm. in diameter with a pitch of 5.08 cm. The total inductance of this coil, measured on a Rayleigh's bridge, is 1.56×10^{-5} henries.

The part of the coil between the leads to the condenser, *KH*, consisted of 1.2 turns and had an inductance of $.151 \times 10^{-5}$ henries.

The condenser was made up of sheets of copper separated by miconite plates. Its capacity was not measured.

XI. Resonance Curves with the Direct Coupled Sending Station and Various Forms of Receiving Circuit.

Experiment XII. Tuning the Receiving Circuit with a Variable Inductance in the Antenna. — In many of the wireless telegraph plants in commercial operation the receiving station is tuned by varying an inductance placed above the instrument in series with the antenna. This is the method of the present experiment. Fig. 31 is a diagram of the receiving circuit. The receiving instrument represented at I is the high frequency dynamometer used in the previous experiments. The coil of the dynamometer has a resistance of 1.33 ohms and an inductance of 1.17×10^{-5} henries.

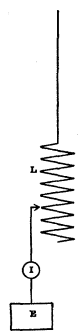

Fig. 31.

The tuning coil L is a variable inductance and consists of 51 turns of copper wire .208 cm. in diameter, wound on a vulcanite drum 13 cm. in diameter. The pitch of the windings is .42 cm. Variation of its inductance is made by a wheel contact moving along the wire as the drum is rotated. Values of the inductance for various positions of the movable contact are known from a calibration of the tuning coil by the aid of a Rayleigh's bridge. The inductance of the whole of the tuning coil is 16.5×10^{-5} henries. Any fraction of this inductance can be used.

The experiment consists in finding what inductance is required to be added by the tuning coil L in order to bring the receiving station to resonance with the sending station when various lengths of antenna are used at the receiving station. Quantitative readings were taken so as to give also the shape of the resonance curves and the intensity of the signals received.

A set of observations with this method of tuning are shown in the curves of Fig. 32. The different curves were obtained with different heights of the receiving antenna, the heights being designated by the numbers at the vertices of the curves. The abscissas of the curves are the inductance of the tuning coil; the ordinates are the deflections in centimeters all reduced to one scale, namely, the deflection that would be had if the receiving instrument should be set at one fiftieth its best sensitiveness.

Each curve has two distinct maxima, showing that the direct coupled sending circuit like the electromagnetically coupled circuit has two distinct periods of oscillation, as has been frequently pointed out.

Sharpness of Resonance. — The curves of Fig. 32 are seen to be very obtuse in comparison with those obtained with the arrangement of apparatus in the previous papers. For example, the first curve of the present series has a maximum for 1.8×10^{-5} henries of added inductance, which together with the inductance of the instrument makes the total inductance 2.97×10^{-5}. The deflection drops to half for a decrease of 26 per cent. or an increase of 60 per cent. of this value. This result compares very unfavorably with

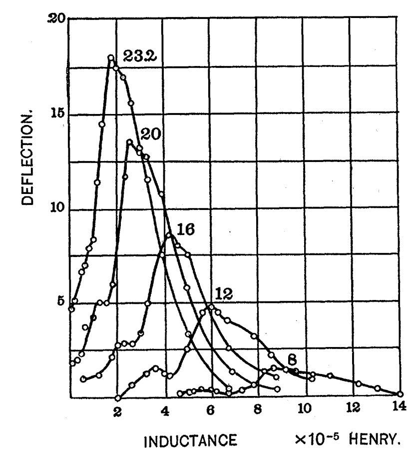

Fig. 32.

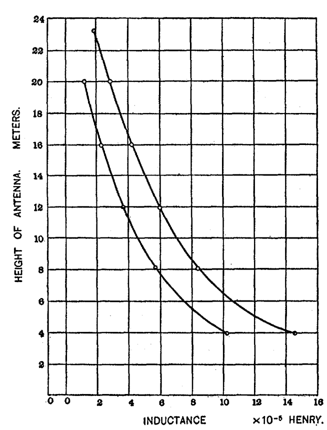

Fig. 33.

some of the curves of the previous research with the electro-magnetically coupled system, where a corresponding change of 5 per cent. or even 2.5 per cent. has caused the energy received to drop to one half. This loss in sharpness of resonance does not seem to be adequately compensated for by a gain in intensity of signals, for the deflections in the present experiment are only 1.4 times as great as those that were obtained in the case that gave the fall of energy to one half for a 5 per cent. departure from resonance. The power expended at the sending station was about the same in the two cases.

Relation Between Height of Antenna and Resonant Inductance in the Tuning Coil. — As the length of antenna is diminished in the above experiment, the inductance in the mast circuit must be increased in order to obtain maximum deflection with a fixed frequency of incident waves.

TABLE XIV.

Showing Inductance Required for Resonance. Height Varied. Incident Waves Constant.

Height, Meters	L' Henry.	L'' Henry.	l' Meters.	l'' Meters.
23.2	1.8×10^{-5}		4.08	
20	2.65	1.2×10^{-5}	5.30	2.86
16	4.2	2.35	7.35	5.30
12	5.0	3.6	9.60	6.50
8	8.5	5.6	12.25	9.00
4	14.6	10.3	18.80	14.30

Table XIV., taken from the curves of Fig. 32, shows the values of the inductance of the tuning coil required with various heights of antenna in order to obtain the maximum deflection. L' is the inductance that produces the strong maximum, and L'' the weaker maximum; l' and l'' are the length of wire in these two inductances respectively. The two maxima arise in the existence of two periods of the incident waves.

The values of L' and L'', recorded in Table XIV., are plotted in Fig. 33, in which the two curves relate to the two periods. These are empirical curves. The distance OO' is the inductance of the instrument. The fact that the curves of Fig. 33 are not straight lines shows that the problem of the propagation of electric waves in wire wound in a coil is less simple than the problem of its propagation in a straight wire. In the case of the non-magnetic straight wire in air the self inductance per unit of length is inversely proportional to the capacity per unit length, so that the velocity of propagation in the wire, as derived from the wave equation, is constant and is equal to the velocity of light. This deduction from the theory of electric waves has been amply verified by experiments. In some other simple cases, *e. g.*, the case of two parallel wires or two coaxial cylinders, the same relation of inductance to capacity holds and the velocity of the waves in such conductors is the velocity of light. On the other hand, in the case of the solenoid here used as a tuning coil, in order to give curves of the shape of those of Fig. 33, the velocity of the waves is different from the velocity of light and furthermore the velocity of the waves in the solenoid is different in different parts of the solenoid. This is evident if we recall that with a resonant receiving circuit the time for the wave to travel from the top of the receiving antenna to the ground is one fourth the period of the sending circuit. When we shorten this time by cutting off antenna we must increase it equally by adding inductance if the circuit is to remain resonant.

It is seen from the curves that for *equal* decrements of the length of antenna, which are proportional to decrements in time, we need to make *unequal* increments of the inductance, which are, therefore, not proportional to the time. In a similar way, by plotting length of antenna against length of wire in the solenoid (curves not here given), it may be seen that lengths on the solenoid are not proportional to the time for waves to traverse them ; that is the velocity of the waves on the wire of one part of the solenoid is different from their velocity on the wire of another part of the solenoid. In terms of the theory of electric waves in a conductor this is equivalent to saying that the inductance per unit of length of the solenoid is not inversely proportional to the capacity per unit of length of solenoid. This same result has been reached by Drude,[1] in a masterly theoretical and experimental research on the natural period of electric oscillation of solenoids with antenna attached.

The wave-length here used is determined in the next experiment in which the conditions prove to be simpler.

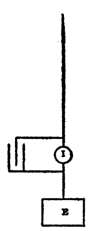
Fig. 34.

Experiment XIII. Tuning the Receiving Circuit by a Variable Capacity Shunted About the Instrument.—The form of receving circuit employed in this experiment is shown in Fig. 34. When the antenna has not sufficient capacity to be in resonance with the incident waves, additional capacity is introduced in the form of a variable condenser shunted about the coil of the receiving instrument. The condenser used is one of the calibrated air condensers described in Part I.

Evidently as the length of antenna (a single wire .208 cm. in diameter) is diminished, the shunted capacity must be increased. A

[1] Drude, Ann. der Phys., 9, p. 293, 1902, and 11, p. 957, 1903.

set of the observations obtained is presented in the curves of Figs. 35 and 36. (In order to bring out details the curves of Fig. 36 have their ordinates magnified [1] twenty-five times with reference to Fig. 35.) The numbers at the vertices of the curves designate

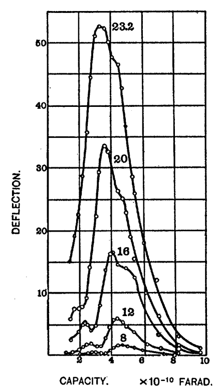

Fig. 35.

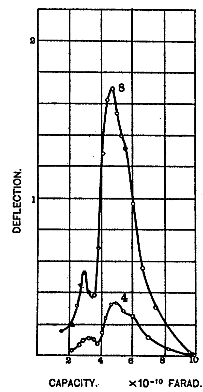

Fig. 36.

heights of antenna in meters. The abscissas are capacity of shunt, and the ordinates are the deflections, reduced to centimeters at one fiftieth the best sensitiveness of the receiving instrument.

These curves bear a marked resemblance in form to the curves of Fig. 32. The irregularities near the summits, possessed in common by the two sets of curves, evidently belong to the wave produced at the sending station and are not characteristic of the receiving station.

Other experiments have shown that these irregularities may be eliminated by better adjustment of the parts of the sending circuit to resonance with each other.

Sharpness of Resonance. Hysteresis Loss in Dielectrics Other Than Air. — As to sharpness of resonance the curves obtained by tuning with the aid of the shunted capacity are about equivalent to those obtained by tuning with inductance in series. It might be remarked in passing that glass condensers and mica condensers, when used in place of the air condenser at the receiving station, gave about the same sharpness of resonance and nearly the same intensity of signals as the air condensers.

[1] Small readings are made with as great accuracy as large readings, because the small readings are taken as large deflections with increased sensitiveness of the instrument.

To show the relative merit of the two dielectrics the following readings of deflections, Table XV., are taken at random from among the observations in which both glass and air condensers were used as shunts about the receiving instrument.

TABLE XV.

Showing Relative Merit of Glass and Air as Dielectrics.

	Air.	Glass.	Glass / Air.
Deflection.	7.0	6.6	.95
	8.6	7.9	.94
	11.3	10.6	.95
	39.0	37.7	.97
	54.0	49.5	.91
			Mean .94

Whence it is seen that in comparison with air the loss of energy by hysteresis or leakage when glass is used as dielectric in the condenser at the receiving station amounts to about six per cent.

Experiments were also made with mica and waxed silk as dielectrics, but the comparison with the air condensers in those cases were not direct, so the numerical data are not reported. It appeared, however, that with the present form of receiving circuit Fig. 34, condensers of either of these substances or of glass may be used in place of air condensers at the receiving station without great loss of energy. An exhaustive study has not yet been made with a variety of dielectrics, and the result here given should be regarded merely in the nature of a preliminary note.

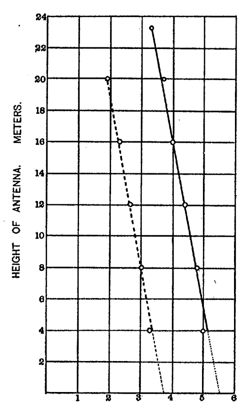

Fig. 37.

Relation between the Length of Antenna and the Resonant Capacity of the Condenser Shunt. — In Fig. 37, height of antenna is plotted against the shunt capacity required for resonance. The relation is more simple than the relation of resonant inductance to height in the previous experiment, and is represented by two straight lines, corresponding respectively to the two periods of the incident waves.

The straight line relation is taken to indicate that as capacity is cut off from the antenna equal capacity must be added in the condenser about the instrument. If we proceed on this theory it is easy to determine the two wave-lengths sent out from the sending station.

The straight lines in Fig. 37, when produced, cut the axis of abscissas at the points $C' = 5.5 \times 10^{-10}$ and $C'' = 3.7 \times 10^{-10}$ farads. These quantities give respectively the values of capacity that shunted about the instrument *with zero antenna attached* form the two cases of closed circuits in unison with the sending station. The period of such closed circuits is given by the formula

$$T = 2\pi\sqrt{LC}. \tag{1}$$

Knowing L, the inductance of the instrument, and the two values of C (C' and C'') we may obtain the two periods of the sending circuit by substituting these quantities in equation (1).

$$T' = 2\pi\sqrt{5.5 \times 1.17 \times 10^{-15}} = 51 \times 10^{-8} \text{ sec.},$$

and

$$T'' = 2\pi\sqrt{3.7 \times 1.17 \times 10^{-15}} = 43 \times 10^{-8} \text{ sec.}$$

Whence the two wave-lengths of the sending system are

$$\lambda' = vT' = 153 \text{ meters}, \quad \text{and} \quad \lambda'' = vT'' = 129 \text{ meters}.$$

Special Case of the Electromagnetically-connected Receiving Circuit. — The straight-line relation shown in Fig. 37 may be derived theoretically by treating the circuit with capacity shunted about the receiving instrument, Fig. 34, as a special case of the electromagnetically-connected receiving circuit given in Fig. 10, Part II. In Part II. (cf. eq. (4), p. 246, Vol. XX.) it is shown experimentally and theoretically that the condition for resonance is

$$(L_3L_4 - M^2)\omega^4 - \left(\frac{L_3}{C_4} + \frac{L_4}{C_3}\right)\omega^2 + \frac{1}{C_3C_4} = 0. \tag{2}$$

With the arrangement of apparatus in the present experiment

$$L_3 = L_4 = M.$$

Whence, the condition for resonance becomes

$$\omega^2\left(\frac{L_3}{C_4}+\frac{L_3}{C_3}\right)=\frac{1}{C_3C_4} \tag{3}$$

or

$$C_3+C_4=\frac{1}{L_3\omega^2} \tag{4}$$

In this equation C_3 is the capacity of the receiving antenna and is approximately proportional to its length; C_4, the capacity of the condenser shunted about the instrument; L_3, the inductance of the instrument; and ω the angular velocity of the incident waves. Equation (4) is the linear relation shown in Fig. 37 and discussed in the preceding section.

If we let $C_3=0$, and make use of the corresponding values of C_4, we obtain equation (1) and confirm the above calculation of wave-lengths.

This remark is here inserted as an example of the manner in which the whole theory of the direct-coupled system of wireless telegraphy may be regarded as a special case of the electromagnetically-connected system studied in Parts I., II. and III. of this research.

Capacity of Receiving Antenna. — In Fig. 37 the linear relation between the height of antenna and the capacity that must be shunted about the instrument to produce resonance gives a simple method of obtaining approximately the capacity of a given length of the antenna. For example, the right hand straight line shows that 23.2 meters of a single wire .208 cm. in diameter with an added condenser capacity of 3.4×10^{-10} farads gives the same period of oscillation as 4 meters of wire with an added capacity of 5.0×10^{-10} farads. That is, when we cut off 19.2 meters of antenna, we must add 1.6×10^{-10} farads as a compensation, and if we neglect the fact that the capacity of the wire is distributed instead of localized, we may say that 19.2 meters of this particular wire has a capacity of 1.6×10^{-10} farads. This would give the value 8.35×10^{-12} farads as the average capacity of one meter of the antenna, a value that agrees satisfactorily with the value 8.2×10^{-12} computed from the usual formula for the capacity per unit length of a straight conductor

$$K=\frac{l}{2\ln l/r},$$

in which r is the radius of the wire in centimeters, $l=100$ cm., and ln means natural logarithm.

The parallelism of the two straight lines of Fig. 37 arises in the fact that both lines must give the same value for the capacity of a given length of antenna.

Relation of Integral Current Received to Height of Receiving Antenna as Determined by Experiments XII. and XIII. — As the height of the receiving antenna is diminished the amount of energy received falls off rapidly. The curves of Fig. 38 give the deflection plotted against height of antenna. The values on the lower curve *A* were obtained when the receiving circuit is brought to resonance by putting proper inductance in the mast circuit. The values on the upper curve *B* are the corresponding deflections when the receiving circuit is tuned by the capacity in shunt. All the deflections are on the same scale and were taken as alternate readings so as to eliminate uncertainties arising from any slow changes that might occur in the sending station. It is seen that the latter method of tuning gives larger deflections. The amount of the advantage the shunt capacity has over the tuning coil is different for different heights. The advantage increases with increasing height, and in the particular case studied, when the height of antenna is 23.2 meters, the deflections with the capacity in shunt are three times as great as the deflections with the tuning coil in series.

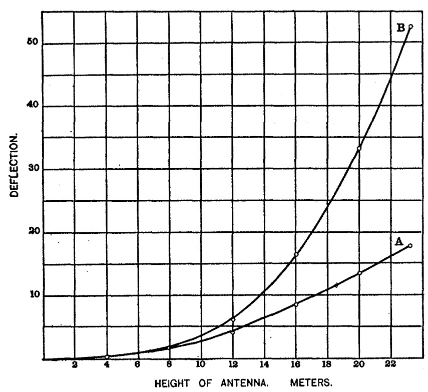

Fig. 38.

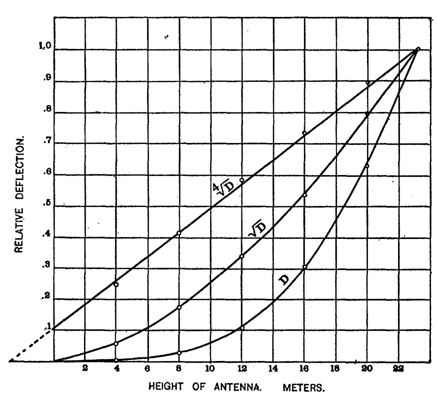

Fig. 39.

In order to examine further the relation of height of antenna to current in the receiving instrument the curve B of Fig. 38 is plotted in Fig. 39 with the ordinates all changed by a constant multiplier chosen to make the largest deflection unity. Together with this the square roots and the fourth roots of the deflections are also plotted against height of antenna. It is seen that the relation of the fourth root of the deflections to the height of antenna is represented approximately by a straight line, which cuts the axis of ordinates at 2.75 meters. The heights are here measured from the coil of the receiving instrument. If the straight line relation permits of slight extension, the negative intercept 2.75 should indicate the height of the instrument above the point of effective ground. This interpretation seems not unlikely when we take into account the fact that the wire lead from the instrument to the surface of the earth is about two meters long; this two meters is also exposed to the waves.

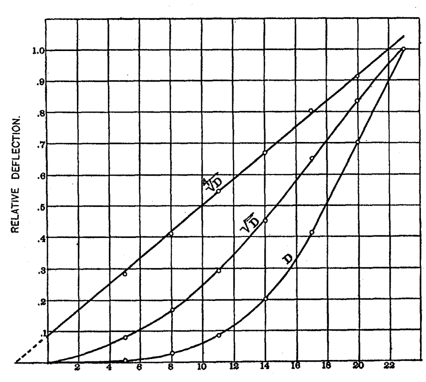

Fig. 40.

A search among earlier data in the observer's notebook disclosed that a previous experiment under different conditions, but with the same method of tuning, gives confirmatory data, which are plotted in Fig. 40 and which show also the fourth root of the deflections proportional to height. The intercept two meters in Fig. 40 is slightly less than that of Fig. 39, but the difference is not much greater than the error in determining the intercepts by lines drawn through the points.

In Part II. it was deduced theoretically and proved experimentally that the deflections of the instrument are proportional to the square of the current through it; therefore, the relation given by the curves of Fig. 39 and Fig. 40 may be stated as follows:

I. *With a constant sending circuit and a constant source of waves the current received in a circuit of the form of Fig. 34, brought to resonance by a capacity in shunt, is proportional to the square of the height of the receiving antenna measured from the "effective ground."*

In applying this law the point of effective ground may be determined graphically if the current received from *two* heights is known. As an approximation the effective ground may be taken as coincident with the surface of the earth.

The law applies only to an approximately vertical receiving antenna, and to the capacity-shunt method of tuning. Its application to that case seems rational, because the current through the instrument is proportional to the impressed electromotive force and to the capacity of the antenna; both the capacity of attenna and electromotive force are proportional to the height of antenna: therefore, current is proportional to the square of the height of antenna.

When the series inductance is used in tuning, a different relation is obtained. The relative deflections in this case, taken from curve *A*, Fig. 38, and plotted with the largest deflection put equal to unity, are shown in Fig. 41. In this figure the square roots and fourth roots of the deflections are also plotted against height of antenna. Instead of the fourth roots of the deflections falling on a straight line as they did in the previous case, the square roots with this method of tuning are approximately in linear relation to the heights of antenna. That is

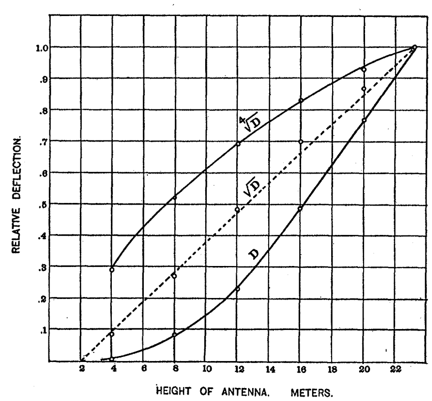

Fig. 41.

II. *The current in the receiving instrument is proportional to the height of antenna*, when the circuit is brought to resonance by added inductance in series with the antenna.

This statement is probably only an approximation, as seen from the fact that the points obtained as square roots of the deflections, instead of lying well on the dotted straight line in Fig. 41, apparently follow a wavy line about it. Also the dotted line cuts the x-axis at 2 meters, so that in applying the rule, 2 meters must be subtracted from the height of antenna measured from the receiving instrument.

The relations I. and II. here stated for the dependence of the current in receiving instrument upon height of receiving antenna may fail of verification when tested with greater height[1] of antenna, but the results are interesting in that they show entirely different laws for the two methods of tuning.

Experiment XIV. Tuning with the Instrument in a Direct Coupled Side Circuit at the Receiving Station. — Measurements were made with the form of receiving circuit shown in Fig. 42. The points *M* and *D* are contacts movable along the tuning coil used in Experiment XIII. The inductance between *M* and *B* is referred to as the inductance in the mast circuit. The inductance in the part of the coil between *D* and *B* is the added inductance in the instrument circuit. The results obtained are shown in the curves of Fig. 43.

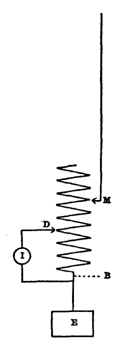

Fig. 42.

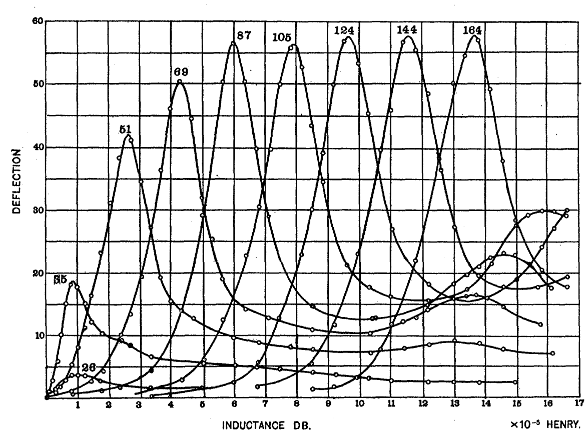

Fig. 43.

In taking any one of the curves the position of *M* was kept fixed at the values written at the vertices of the curves, which when multiplied by 10^{-6} give the inductance in the mast circuit. Deflections were then taken for various positions of the contact *D*, that is for various values of the inductance in the instrument circuit.

The height of the receiving antenna, 23.2 meters, remained unchanged during the experiment.

[1] In the experiments the height of antenna extended from 4 meters to 23.2 meters, that is to a little more than half of the quarter wave-length.

The meaning of the curves of Fig. 43 is made clear by a reference to Fig. 44, in which inductance in the mast circuit is plotted against the total inductance required in the instrument circuit to produce resonance. The former values are the numbers written at the vertices of the curves; the latter are the ordinates belonging to those respective vertices plus the inductance of the instrument.

Fig. 44 is a straight line and the corresponding values of the inductance in the mast circuit and the inductance in the instrument circuit have a common difference of 2.60×10^{-5} henries as given by the intercept on the axis of ordinates. This difference is evidently the inductance between the point *M* and the earth by the path *MDIE*, Fig. 42. The experiment shows that for resonance the inductance of this path must have the fixed value 2.60×10^{-5} henries. By a separate experiment the value 2.62×10^{-5} henries was found for the inductance of this path when an open circuit was made at the point *B*.

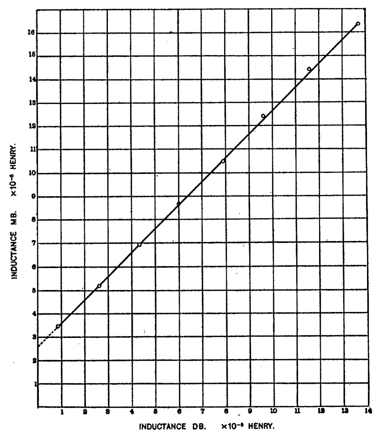

Fig. 44.

We have in this last case, however, the simple form of circuit examined in Experiment XII.

From these facts it appears, as should be seen by a glance at the circuit of Fig. 42, that the present experiment is dealing with the simple circuit of Fig. 31, with, however, an inductive[1] shunt inserted about the instrument. *The presence of the inductive shunt does not modify the resonance conditions. It simply diminishes the amount of energy obtained in the instrument.*

After the shunt has reached the value 6.0×10^{-5} henries its effect in this respect is small, so that the curves to the right of the curve with maximum ordinate 6.0, Fig. 43, have approximately constant amplitude, which is about 90 per cent. as great as the deflection obtained with the open circuit at *B*.

[1] The inductive shunt is subjected to inductive action from the current in the rest of the coil.

XXII. Experiments to Test the Image Theory of the Action of the Ground.

Grounded and Ungrounded Circuits. — The preceding experiments have all made use of grounded circuits at the sending and receiving stations. A number of wireless telegraph plants are in operation in which the circuits are not grounded. I am not aware that quantitative measurements have hitherto been made comparing the efficacy of the ungrounded circuits with the grounded.

In the experiments described below basis is afforded for at least a tentative comparison, although the experiments were undertaken primarily to illustrate the manner in which the ground acts, and the results are to be viewed chiefly as a test of the image theory of the grounded circuits.

Image Theory. — According to this theory a perfect ground introduces into the circuit a point of zero fluctuation of potential and, therefore, maximum fluctuation of current at the place where the ground is attached. The top of the antenna, on the other hand, is a point of zero current and maximum fluctuation of potential. In addition to these two end conditions the current and potential must obey certain differential equations throughout the length of the antenna. So far as concerns the part of the circuit above the earth, these conditions are the same as should obtain if the circuit instead of being grounded should be electrically mirrored in the earth. That is, the oscillation in the grounded circuit is simply the oscillation in one half of a symmetrical Hertz oscillator made up of the existing aerial system and its image, the image however contributing nothing to the radiation or reception of the waves.

To examine this view somewhat more in detail, let us confine our attention to the receiving station, and suppose we had there simply a rectilinear conductor isolated in space and placed parallel to the electric displacement of the incident waves. Let the length of the straight-line conductor be so chosen that its natural period of electric oscillation is equal to the period of the waves. The conductor would then be resonant with the incident waves. For a given strength of waves there would be a maximum current back and forth at the center of the conductor because on either side of the center there is the electrostatic capacity of half the conductor. At a point half way between the center and the end of the conductor the current back and forth would be less because out beyond this point the capacity is only the capacity of one fourth the conductor; while at the end of the conductor the current would be zero because there is no capacity beyond into which the current can flow.

We must now suppose that whatever current flows into one half of the conductor flows out of the other half; so that the potential at points in one half of the conductor rises while the potential of the other half falls. Thus the potential at the middle does not change, and the fluctuations of potential increase as we go toward either end. If we could introduce a current-reading detector into the circuit without disturbing the conditions, the instrument would give a maximum reading when placed at the center of the receiving conductor, that is, at the point where the fluctuation of potential is zero. Suppose with such an instrument in the circuit we should cut away one half the conductor; the reading would become zero. If now a capacity is attached to the instrument in place of the removed conductor, some current would flow between the straight wire and the capacity and register in the instrument.

If the capacity attached were very large (*e. g.*, the earth), the point of zero fluctuation of potential would again be brought near the instrument and we should have the same current as when the conductor was made up of two parts symmetrical about the instrument. Since, however, with the earthed circuit the antenna exposed to the radiation is only one half the original rectilinear conductor, to make the conditions identical we should have considered the other half of the original symmetrical system to be shielded from the action of the waves.

It is by reasoning of this kind, though usually expressed in mathematical form, that the theory is developed that the action in the grounded circuit with the receiving instrument near the ground is the same as it would be if instead of being grounded it should be symmetrically duplicated below the surface of the earth.

Of course, in actual systems the grounding is not perfect, and therefore, the symmetrical image gives only approximately an equivalent system.

This theory is submitted to tests in the experiments that follow, by comparing resonance curves with various forms of grounded circuits with the corresponding curves taken with an *image circuit* in the place of the ground.

Experiment XV. The Aerial Circuit and its Image Tuned by Variable Inductance. — In this experiment the sending station was the same as that employed in the preceding experiments (Fig. 30) and was constantly grounded. The arrangement of apparatus at the receiving station is shown in Fig. 45. By means of a switch at S the ground could be thrown off and replaced by metallic parts duplicating the aerial system. The duplicate is, however, not an exact image of the aerial system, because the second antenna had to be run horizontally instead of straight down.

The horizontal wire was made equal in length to the vertical antenna, 23.2 meters, and was supported about 1 meter from the

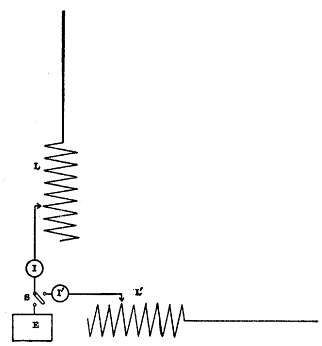

Fig. 45.

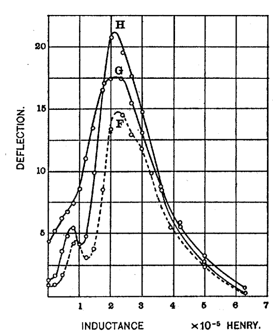

Fig. 46.

ground by cords attached to posts. In series with the horizontal wire was a variable inductance L' duplicating the tuning coil L and a small coil I' of fine wire, wound to duplicate the coil of the receiving instrument.

Curves giving the results of the experiment are shown in Fig. 46. For Curve G the grounded circuit was used, and readings of deflections were taken for various values of the inductance of the tuning coil L, Fig. 46; the deflections are plotted against values of L.

For curve H the switch was thrown so as to connect the aerial system with the horizontal system, Fig. 45, instead of with the ground, and deflections were taken for various values of L and its duplicate L' kept identical in value and varied together. In curve H the deflections are ordinates and the common values of inductance L and inductance L' are abscissas.

Discussion of Results. — The two curves G and H, Fig. 46, are seen to have their maxima for the same abscissa. That is, a given value of inductance, 2.1×10^{-5} henries, gives a maximum deflection in the grounded circuit. To obtain a maximum with the duplicated system the same inductance 2.1×10^{-5} henries must be used in both the vertical circuit and its horizontal duplicate. This result is a confirmation of the image theory.

It is interesting to note that the *deflection* (*current square*) *is about 20 per cent. larger with the duplicated system than with the grounded system* — a fact that may be accounted for by supposing a higher resistance in the grounded system than in the wholly metallic system.

The resonance curve H taken with the wholly metallic circuit is somewhat different in shape from the curve G with the grounded circuit. The two curves agree well in shape for points to the right of the maximum, but for points to the left the curve with the grounded system slopes away more gradually and does not show the secondary maximum that is so marked with the duplicated system. That there is, however, a tendency to a secondary maximum in the former curve appears in a slight bulge in the curve and also in the fact that secondary maxima are shown by other members of the same family given in Fig. 32. The greater prominence of the secondary maximum of the curve H goes with the general greater sharpness of the curve as the result of the diminution of resistance accompanying the substitution of the metallic "image" for the ground.

After a number of preliminary tests in which the horizontal circuit was not made so close a duplicate of the aerial parts, the result shown in Fig. 46 seemed a rather striking agreement with the image theory.

In the effort to find a more convenient artificial ground than that supplied by the metallic duplicate of the aerial system the curve F was obtained, with the duplicate antenna wound around the house of the receiving station instead of extending out horizontal. In this case the capacity balancing the antenna was in the form of a rectangular spiral three meters square with a pitch of forty centimeters. This was a convenient arrangement, but was accompanied by a loss of some 20 per cent. of the energy received with the grounded circuit. Other arrangements, for example, a large number of short wires in the form of a horizontal fan, supported about one meter above the earth, gave about the same energy as that obtained with the metallic duplicate of the aerial circuit, that is 20 per cent. larger than the connection to earth.

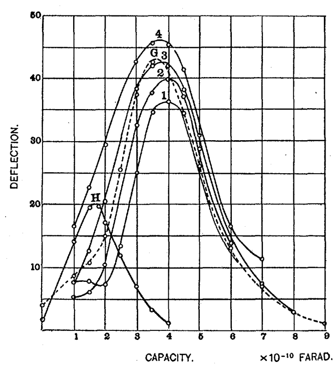

Fig. 47.

Experiment XVI. Tuning With Capacity in Shunt. — In further test of the image theory of the action of the ground, experiments were made with the capacity-shunt form of aerial circuit shown in Fig. 34. By throwing a switch this circuit could be grounded or completed by a metallic portion more or less nearly an image of the aerial system.

Curves are given in Fig. 47. The dotted curve G was obtained with the use of the ground; deflections are plotted against capacity about the instrument.

In taking the curve H, the ground was thrown off and a horizontal system was substituted for the ground. The horizontal system consisted of a duplicate of the coil of the instrument and a horizontal antenna 23.2 meters long. To have an exact duplicate there should be a variable condenser about the receiving instrument and an equivalent variable capacity about the duplicate of the instrument. As two such capacities were not easily available, instead of this arrangement the one variable condenser was put about both the instrument and its image. The curve H was obtained, in which deflection is again plotted against capacity of the shunt condenser. The maximum of H falls at a capacity one half the capacity for a maximum with the grounded circuit, G. This result confirms the image theory; for the condenser about the instrument and the image of this condenser in the duplicate system would be in series and therefore have together a capacity one half as great as the capacity for resonance with the grounded circuit.

In this experiment the deflections with the ungrounded circuit are a little less than half as great as the deflections with the grounded circuit. It is conjectured that this is due to the fact that our ungrounded circuit was not exactly a duplication of the original grounded system as may be seen from an examination of a diagram (not given) of the two arrangements. In the duplicated system a path is lacking for the current to go through the image capacity and then through the instrument.

Experiment XVII. Aerial System with Capacity Shunt; Image with Series Inductance. — In this experiment an aerial system with capacity shunted about the instrument (Fig. 34) could be grounded or could be connected to a horizontal system with variable inductance in series (Fig. 45). The curve *G*, Fig. 47, is a resonance curve with the grounded system. The curves 1, 2, 3 and 4 were taken with the ground replaced by the horizontal member, consisting of horizontal wire 23.2 meters long with various values of inductance in series with it, as follows:

Curve No.	Inductance in Horizontal Henries.
1	20×10^{-6}
2	27×10^{-6}
3	33×10^{-6}
4	39×10^{-6}

From the curves it is seen that the artificial ground that comes nearest to replacing the actual ground is the arrangement that gives curve 3. The horizontal portion of the circuit in this case is the same as the horizontal circuit and also the aerial circuit in experiment XV., for the common inductance in that case was 2.1×10^{-5} henries, which together with the duplicate of the instrument made a total inductance of 3.27×10^{-5} in each part of the circuit, which is in good agreement with the total inductance in the horizontal for curve 3, viz. 3.3×10^{-5} henries.

It should be remembered, however, that the curve 4 with greater inductance in the horizontal gives a larger deflection than that of curve 3 because with the greater inductance the point of maximum current is shifted into the instrument instead of being at the ground.

Experiment XVIII. Quarter Wave Ground. Determination of Wave-length. — The most interesting of the experiments confirmatory of the image theory of the action of the ground was made by replacing the ground by a straight horizontal wire of which the length could be varied. Resonance was obtained when this wire had the length of one fourth the wave-length.

Both the capacity-shunt and the inductance-series forms of aerial circuit were employed. These circuits were grounded and the capacity or inductance set for resonance. The ground was now disconnected and a long horizontal wire was attached in its place This threw the system out of resonance as shown by a small deflection. Wire was cut off the horizontal in steps and readings were taken. These readings increased as the horizontal was shortened up to a certain length and then decreased with further shortening of the horizontal. The results are plotted in Fig. 48. The curve A is with the inductance-series aerial; the curve B with the capacity-shunt aerial. The deflections were taken alternative with deflections with the circuit grounded and the ratio of the deflections with artificial ground to deflections with actual ground are plotted as ordinates against the length of horizontal as abscissas.

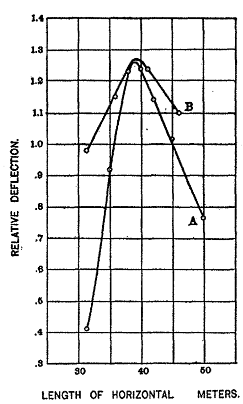

Fig. 48.

It is seen that in the experiments with both forms of aerial circuit the horizontal wire must have a length between 38 and 39 meters for a maximum deflection. If this is the quarter wave-length as the theory proposes, we should have for the wave-length a value between 152 and 156 meters, which is in excellent agreement with the value 153 meters found independently and by a different method in Experiment XIII.

Experiments XV., XVI., XVII. and XVIII. taken together show good agreement between the observations and the predictions from the image theory of the action of the ground. They also show that it is a simple matter to balance the aerial at the receiving station with an artificial ground that will increase the reception of energy (current square) by twenty per cent. or twenty-five per cent. over a good actual ground.[1]

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October 17, 1905.

[1] The ground at each station consisted of three copper plates and a piece of iron pipe. The copper plates 60 cm. square by .3 cm. thick, were embedded in the earth 270 cm. below the surface. The pipe, 3 cm. in diameter and 180 cm. long was driven in until flush with the bottom of the holes containing the plates, and therefore reached to a depth of 450 cm. below the surface.