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ON THE CONDITIONS FOR SPARKING AT THE BREAK OF AN INDUCTIVE CIRCUIT.

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INTRODUCTION.

LORD RAYLEIGH has discussed the relation existing between the energy given to the secondary of an induction coil when the primary current is broken and the velocity of separation of the contacts of the break of the primary.[1] He showed qualitatively, breaking the primary circuit by means of a rifle bullet, that in some cases a greater potential was produced in the secondary than when an ordinary break was used with its *optimum* capacity, and he came to the conclusion that if the break were made with sufficient abruptness, we might do away with the primary condenser altogether.

By optimum capacity is meant that capacity which, when put in parallel with the break of an inductive circuit, is sufficient, and just sufficient, to prevent a spark at the break, when the circuit is broken in a given manner.

We may sum up the chief results of experience in this connection under the following heads:

1. For a given break and primary current there is a given optimum capacity. Walter[2] has shown very clearly by means of figures obtained with a Braun tube that the fall of current in the primary, and therefore the rise of the induced potential, is most

[1] Phil. Mag. (6), 2, p. 581, 1901; Papers, Vol. IV., pp. 564–8.

[2] B. Walter, Ueber die Vorgänge im Inductionsapparat, Wied. Ann., 62, p. 300, Sept., 1897; and 66, p. 623, Nov., 1898.

rapid when the capacity in the primary has been brought to the least possible value without causing the primary break to spark. As the capacity is diminished, the induced potential rises until values are reached so large as to cause an arc to form at the break retarding the rapidity of fall of the current.

2. For larger values of the primary current the optimum capacity is larger. That is, a greater capacity is necessary to prevent a spark.[1]

3. For a given primary current a great increase of abruptness of breaking the circuit diminishes the necessary primary capacity (Lord Rayleigh).

Since the induced potential is proportional to the primary current and inversely to the square root of the capacity it is clear from the foregoing that to get the best possible secondary potentials we must have the largest possible primary currents and the least possible primary capacity, and in order to realize these conditions it is necessary to have the most abrupt possible break.

With a view to determining quantitatively the relation existing between these quantities, J. E. Ives undertook a series of measurements,[2] but he was led to experiment with values of the current so small that he did not succeed in finding the desired relation.

The present paper is an attempt to establish the relation which exists between the rapidity with which the break contacts are pulled apart, or the *velocity of break*, as we shall call it, and the constants of the system together with the primary current, *for a single circuit;* for the results may easily be extended (see supra) to induction coils by the consideration of principles laid down by B. Walter.

Theoretical.

Consider the processes going on in an inductive circuit immediately following an interruption of the current. As the lines of magnetic force due to the current disappear, an induced electro-motive force causes a spark at the break, provided the contacts have not become sufficiently separated to prevent it. This spark represents

[1] Walter, loc. cit.; T. Mizuno, On the Function of the Condenser in an Induction Coil, Phil. Mag. (5), 45, pp. 447–454, 1898. K. R. Johnson, On the Theory of the Function of the Condenser in an Induction Coil, Phil. Mag. (5), 49, pp. 216–220, 1900.

[2] Phys. Rev., Vol. XV., No. 1, p. 7, 1902.

a waste of energy and retards the fall of the current. If the velocity of separation of contacts, or the velocity of break, be great enough to prevent a spark, the current will fall to zero with a rate dependent upon its own initial value and the constants of the system.

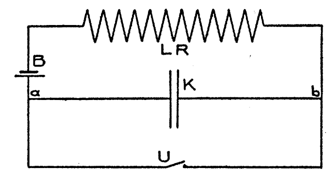

Fig. 1.

Let a circuit be connected as shown in Fig. 1. *LR* is a coil of inductance L and resistance R. It is connected in series with a battery of electro-motive force E and a break U. Across the break is connected a condenser of capacity K. We wish to find a relation, if any exist, between the velocity of break, the initial current, and the constants of the system when the conditions are just sufficient to prevent sparking at the break.

We have for the equation of the current, on the assumption that there is no spark,

$$L\frac{d^2I}{dt^2}+R\frac{dI}{dt}+\frac{I}{K}=0,$$

which, with the initial conditions,

$$t=0, \qquad I=I_0, \qquad Q=0.$$

where Q is the charge on the condenser, has for its solution

$$I=I_0\frac{2\sqrt{L}}{\sqrt{4L-R^2K}}e^{-\frac{R}{2L}t}\cos\left\{\sqrt{\frac{1}{KL}-\frac{R^2}{4L}}\cdot t-\tan^{-1}\sqrt{\frac{R^2K}{4L-R^2K}}\right\}.$$

In the experiment which we shall consider K is of the order 10^{-9} farads, L is 1 to 3 henries, and R is 50 to 200 ohms; accordingly, to an approximation having far greater accuracy than we can reach, experimentally,

$$I=I_0e^{-\frac{R}{2L}\cdot t}\cos\left(\frac{t}{\sqrt{KL}}\right).$$

This equation was used by B. Walter in his experiments with the induction coil.

We have also

$$V=E+L\frac{dI}{dt}-RI,$$

giving

$$V = E + I_0 \sqrt{\frac{L}{K}} \sin\left(\frac{t}{\sqrt{KL}}\right).$$

Since we are dealing with induced electromotive forces of from a few hundred to many thousands of volts we may neglect E, which was usually about two volts. It is clear that the spark, if it takes place at all, will occur some time before the first maximum of potential is reached. Using the above values for the constants of the circuit, the exponential factor $e^{-\frac{R}{2L} \cdot t}$ will have a value of about .99 at the end of the first quarter period. We therefore have, to an approximation of about one per cent.,

$$V = I_0 \sqrt{\frac{L}{K}} \sin\left(\frac{t}{\sqrt{KL}}\right).$$

We consider now the action of this potential in producing a spark at the break, assuming that the contact surfaces are portions of spheres of large radius in comparison with sparking distances.

In the first place, we have a rapidly varying potential. Does a varying potential follow the law which connects a constant potential with its spark length? There has been much controversy on this point. Jaumann[1] maintained that the spark length for a varying potential does not depend upon the value of the potential at the instant of sparking but upon the value of VdV/dt. As J. J. Thomson has pointed out,[2] his experiments may be brought into accord with the law for constant potentials by considering the electric waves set up in the system. Some work of Warburg[3] seems to settle the question, for he shows that within limits potential changes are without influence on the sparking constants, provided the gas is ionized.

Further, there is the question of the lag effect on the spark length. Jaumann[4] showed that there is an interval between the time of application of the potential and the time of sparking, during which some

[1] Wied. Ann., 55; p. 656, 1895.
[2] Conduction of Electricity through Gases, p. 351.
[3] Wied. Ann., 62, p. 385, 1897; Ann. d. Phys., 5, p. 818, 1901.
[4] Loc. cit.

process goes on in the gas which changes it into a conductor. J. J. Thomson applied without causing a spark a potential several times the sparking value to a spark gap in carefully dried air. The complete determination of the lag effect would require more knowledge than we have at present of the growth of ionization produced by an electric field. According to J. J. Thomson's theory of the spark discharge[1] when a sufficiently great difference of potential is applied to a spark gap, ions initially present are driven with great enough velocity to produce fresh ions by collision with molecules of the gas. These new ions produce ions in their turn and the ionization and accordingly the conductivity of the gas will increase in geometric progression until the spark takes place. The duration of the lag will depend on the number of ions initially present. The lag is much diminished by any ionizing agency, such as ultra violet light, or the spark itself.

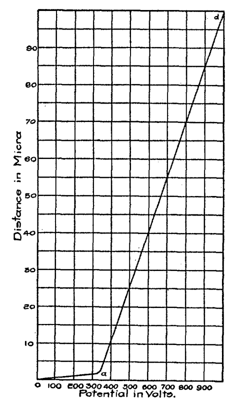

Fig. 2.

Earhart[2] has obtained values of the sparking potential for small distances between spherical surfaces in air. The relation is shown in Fig. 2 taken from Earhart. For distances from zero to .003 mm. the potential is proportioned to the distance (*oa*, Fig. 2). Here the spark potential, $V_s = gx$, where g is about 1.2×10^6 volts per cm. The curve changes abruptly between $x = .003$ mm. and $x = .01$ mm. after which it follows the law $V_s = ax + b$, where a is about 70,000 volts per cm. and b is less than the minimum spark potential, 351 volts. In this paper we shall deal only with the branch of the curve *ad*, Fig. 2, where $V_s = ax + b$. We shall assume the equation to hold regardless of variations of potential, and endeavor to find by means of deviations from it some indication of the magnitude of the lag effect in the cases under consideration.

We have then, as the limiting conditions for sparking,

$$V_s = V$$

and

[1] Phil. Mag. (5), 50, p. 278, 1900.

[2] Earhart, Phil. Mag., VI., 1, p. 147, 1901.

$$\frac{dV_s}{dt}=\frac{dV}{dt},$$

also, $x=vt$, where v is the velocity of break. Therefore,

$$avt+b=I\sqrt{\frac{L}{K}}\sin\frac{t}{\sqrt{KL}},$$

and

$$av=\frac{I}{K}\cos\frac{t}{\sqrt{KL}}$$

where I is the initial current. Eliminating t,

$$av\sqrt{KL}\cos^{-1}\frac{avK}{I}+b=I\sqrt{\frac{L}{K}}\sqrt{1-\left(\frac{avK}{I}\right)^2}.$$

In the present experiments $avK/I=Z$ was seldom as much as .4, that being one of the largest values recorded.

We have

$$\cos^{-1}Z=\tfrac{1}{2}\pi-Z-\frac{Z^3}{6}\cdots,$$

$$(1-Z^2)^{\frac{1}{2}}=1-\tfrac{1}{2}Z^2-\tfrac{1}{8}Z^4\cdots.$$

Neglect of higher powers of Z than the second will cause an error under one per cent. Accordingly

$$av\sqrt{KL}\left(\frac{\pi}{2}-\frac{avK}{I}\right)+b=I\sqrt{\frac{L}{K}}\left[1-\frac{1}{2}\left(\frac{avK}{I}\right)^2\right],$$

or

$$\frac{\pi}{2}\cdot\frac{avK}{I}-\frac{1}{2}\left(\frac{avK}{I}\right)^2+\frac{b}{I}\sqrt{\frac{K}{L}}=1.$$

As a first approximation

$$\frac{\pi}{2}\cdot\frac{avK}{I}+\frac{b}{I}\sqrt{\frac{K}{L}}=1.$$

Solving for avK/I and substituting its square in the first equation,

$$\frac{\pi}{2}\left(\frac{avK}{I}\right)=1-\frac{b}{I}\sqrt{\frac{K}{L}}+\frac{2}{\pi^2}\left(1-\frac{b}{I}\sqrt{\frac{K}{L}}\right)^2$$

whence

$$v = \frac{2I}{\pi a K}\left(1 + \frac{2}{\pi^2}\right) - \frac{2b}{\pi a\sqrt{KL}}\left(1 + \frac{4}{\pi^2}\right) + \frac{4b^2}{\pi^3 a I L}.$$

or

$$(1) \qquad v = .7656\frac{I}{aK} - .8946\frac{b}{a\sqrt{KL}} + .1290\frac{b^2}{aLI},$$

the equation connecting the velocity of break with the constants of the system and the initial current, which we shall use in connection with the experiments.

We may write the first approximate equation thus:

$$v = \frac{2}{a\pi\sqrt{KL}}\left(I\sqrt{\frac{L}{K}} - b\right),$$

showing that the velocity of break necessary to prevent a spark is roughly proportional to the frequency and to the excess of the maximum potential over the minimum spark potential.

Experimental Arrangements.

The apparatus was usually arranged in the following manner: An inductance, LR, storage cell, B, interrupter U, and box resistance R_b, were connected in series as shown in Fig. 3. A condenser K of variable capacity is joined in parallel with the box resistance and break. The purpose of this resistance is to permit variations of current without affecting thedamping factor. A quadrant electrometer is connected idiostatically with the terminals of the condenser. The inductance used has been described by J. E. Ives.[1] It was wound in a series of six coils in a large flat wooden spool. The total inductance is 3.01 henries. About two miles of wire was used, having a resistance of 62.18 ohms. A bridge circuit, not shown, was arranged for the purpose of measuring the resistance of various parts of the circuit, or of all together, at will. The terminal of the condenser next the battery was kept to earth at E. The capacity of the system is that of K with one of its plates to earth, together

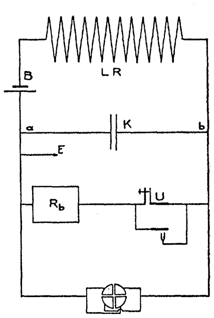

Fig. 3.

[1] J. E. Ives, Phys. Rev., XIV., p. 298, 1902.

with the capacity of its leads, of the break with its contacts apart and its leads, and the unknown, and in these experiments unmeasured capacity of the coil due to the oscillations of the potential. The capacity of the coil is probably negligibly small in comparison with the external capacity, the more so, as Drude[1] has shown that the capacity of a coil vibrating in its natural period is lessened with the addition of end capacity.

For a break was used one of the contact levers of Dr. Webster's drop chronograph.[2] The lever is so adjusted that when the break is closed the end which is struck by the falling weight is on a level with the pivot, ensuring that the velocity of the end when struck, in the direction perpendicular to the radius is equal to the velocity of the falling weight. The velocity of break is then equal to $r\sqrt{2gh}$ where $\sqrt{2gh}$ is the velocity of the projectile and r is the ratio of the length of the arm carrying the contact to the length of the arm struck having the value .479.

Measurement of Capacity.

The most trouble encountered in the experiments was in connection with the measurements of capacity. The capacity of the system is of the order 10^3 cm. which is difficult to measure electrostatically with a great degree of accuracy. The method of measurement adopted was suggested by Dr. Webster. K_2 (Fig. 4) is a standard condenser. K_1 is the system the capacity of which is to be measured. The part of the system which was connected to earth in the oscillation experiments is the upper part of K_1, which together with the upper side of K_2 may be connected to earth at E_1, at a, but always connected with the electrometer. A steady current flows from a large number of storage cells B through R_1 and R_2 between which is an earth connection, E_3. A commutator connects 2 with

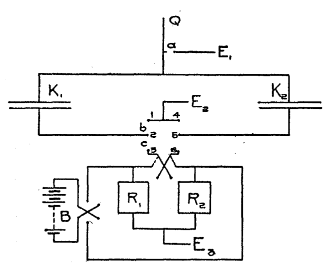

Fig. 4.

[1] Drude, Ann. d. Phys., 314, p. 293, 1902.

[2] A. G. Webster, An Experimental Determination of the Period of Electrical Oscillations, Phys. Rev., VI., p. 300, 1898.

3, and 5 with 6, at c. The lower plates of K_1 and K_2 are thus at potentials proportional to R_1 and R_2. Equal and opposite charges are induced on the opposite branches of the system. If, now, the earth connection at a be broken and the commutator rocked so as to connect 1 with 2 and 4 with 5 at b, the charges on the lower plates run to earth at E_2 and, if the charges on the upper plates be equal, no deflection in the electrometer will result. Then will

$$\frac{K_1}{K_2} = \frac{V_2}{V_1} = \frac{R_2}{R_1}.$$

All possible precautions were taken to insure perfect insulation. All connections were of copper, as it was found that the use of mercury connections introduced serious error, as from heating by a nearby lamp, or mechanical agitation, producing thermo-electric effects. It was found necessary to cement all binding posts to glass with sealing wax and to have them sufficiently removed from other conductors to make their capacity negligible. The switches and commutator were made of small metallic parts embedded in beeswax or paraffine. On dry days it was found possible and easy to measure capacities of 10^3 cm. to 1 cm. The resistances were reversed, the direction of current was reversed and various points of connection were shifted without causing changes greater than those observed when the conditions were kept the same. On damp days it was found possible to measure capacities only after all connections had been carefully gone over with a jet of air which had been passed through a quantity of concentrated H_2SO_4. To test the method different lengths of parallel wires were connected to K_1 and the increase in capacity thus produced agreed with that calculated to 1 cm.

The variable capacity consisted of two plates, the upper one being in the form of a small plate with a guard ring which were connected together so as to act as one plate with a diameter of 18 inches. This plate was supported at the edge upon ebonite pillars which rested upon a rigid iron frame work. Beneath this was the other plate, 16.5 inches in diameter, supported by a large screw which, by means of a nut at the base of the instrument, could be raised or lowered without rotation. The screw threads were 20 to the inch.

One revolution of the nut from the position which gave contact (not taken as the zero) gave 820 cm. capacity.

The standard condenser has been fully described by Dr. Webster.[1] The lower plate was supported upon three rigid, ebonite tipped legs. On top of the plate, directly over its points of support, were placed three small glass cylinders, of accurately known length, and upon these was set the top plate. The condenser was kept throughout at some distance from the other apparatus, and at about 5.5 feet distance from the floor and eight feet from the nearest wall. On the approach of a person as closely as possible without touching, the condenser showed a change of capacity of about one fourth per cent.

It is of course necessary to keep the form of all circuits, connections, and apparatus the same during measurements of capacity as they are kept when experiments are made in which use is to be made of capacity measurements. There must be no electrometer deflection when the earth connection at a is broken, whether K_1 or K_2 is connected in alone, or both together. This was the test for leakage in the capacities. We are thus able to measure with considerable accuracy all the capacity effective in the oscillation experiments with the exception of that due to the coil. The variable condenser was calibrated. On the addition of the electrometer with its leads the increase of capacity was found to be 13 cm. On the addition of the leads to the break the increase was 12 cm. In the oscillation experiments, accordingly, the reading of the condenser screw was taken, the capacity found from the calibration curve, and 25 cm. added. This was occasionally verified by the measurement of all together.[2]

An example is here given of the magnitude of the changes produced in the capacity measurements when the currents and resis-

[1] Loc. cit.

[2] A slight modification of the above method enables us to measure the capacity of a small body with reference to all surrounding bodies taken together. Put a at E_1 permanently to earth, and make the electrometer connection at E_2. Then when the commutator is rocked after disconnecting E_2 from earth, if the charge on the lower parts of K_1 and K_2 are equal no deflection will result. A spherical acoustical resonator of 23 cm. diameter, having a slight projection on one side, was supported at a distance of three feet from all other conductors. It was put in connection with the lower side of K_1. The observed increase in capacity was $12 \pm .25$ cm.

tances were reversed. The nut of the variable capacity was turned .588 revolution from zero position. Twenty storage cells were used in the battery.

	R_1	R_2	R_2/R_1
	640.5	1,000.0	1.5613
Current reversed.	639.5	1,000.0	1.5637
	1,000.0	1,565.5	1.5655
Current reversed.	1,000.0	1,565.5	1.5655

$$K_2 = 826.5 \text{ cm.} \quad K_1 = \frac{R_2}{R_1} K_2 = 1{,}293 \text{ cm.}$$

The following experiments do not call for this degree of accuracy and it might seem that the large amount of time spent upon the capacity measurements was lost, but in such work a very small amount of leakage causes indefinitely large errors and care had to be taken that everything of the kind should be eliminated.

A proof of the relative correctness of the calibration of the variable condenser is found in the fact that the calibration curve is a very smooth one and follows the formula

$$K = \frac{c}{d + e}$$

over a long range, c being a constant, d is the reading of the screw from zero position and e is a small correction to be added to d for the reason that the plates, owing to imperfect surfaces and lack of parallelism did not everywhere touch when $d = 0$. Part of the calibration is here given to show the agreement between measured values of K and those calculated from the above formula, using $c = 863.5$ and $e = .080$.

d	K Meas.	K Calc.	Diff.
.610	1,252	1,252	0
.660	1,170	1,167	−3
.720	1,079	1,079	0
.800	980	982	+2
.890	891	890	−1
1.000	799	799	0

The formula takes no account of the thickness of the plates or of the metallic frame work.

Method of Taking Readings.

In finding the values of velocity of break just sufficient to prevent sparking, the apparatus was connected as in Fig. 3. When the circuit is broken at U, there will result oscillations of potential in the condenser and at the break which are measured by the electrometer, connected as shown. The drop, in continuing its fall, after breaking the circuit closes it again by means of another lever, indicated in the figure, after the potential oscillations have died away, but in so short a time as to eliminate the effect of the battery e.m.f. on the readings of the electrometer. The latter are then proportional to the time integral of the square of the induced potential. In all the following experiments except the first, the constants of the circuit were kept the same in each set while the current was varied. The electrometer readings, provided there is no spark, will then be proportional to the square of the initial current.

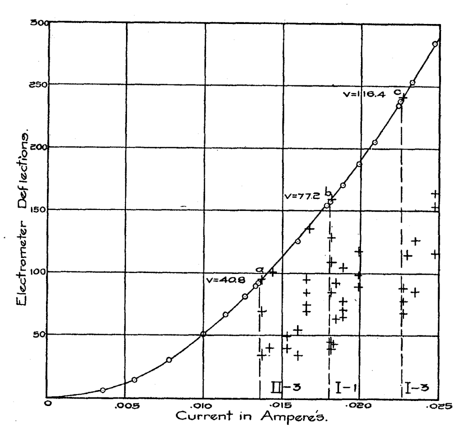

Fig. 5.

Imagine the projectile of the drop chronograph hung at a distance h above the contact lever of the break. Each interruption of the current caused by the fall will be made with the same velocity of break. Make R_b very large so as to cause a small current to flow through the inductance when the circuit is closed. If the drop is allowed to fall, a definite deflection will result, and will be the same for every fall if there is no spark. Let smaller values of R_b be chosen, so as to increase the initial current. There will be continuously larger electrometer deflections, and the curve plotted from initial currents and electrometer readings (see Fig. 5) will be parabolic in form as long as no sparking takes place. As we go on increasing the initial current we at length come to a point (at a, for instance) beyond which the electrometer readings fall short of the curve and differ among themselves by several hundred per cent. An examination of the break reveals the fact that a small spark takes place at each interruption. The region of this discontinuity was carefully explored. The limiting value of the current is taken as the largest value of the current which will give five successive electrometer readings on the curve when the circuit is broken. This rule was adopted both to eliminate personal judgment from the results, and because it was often found possible by careful polishing of the break contacts to get sparks after two or three readings had been taken which lay on the curve, showing that the critical value had not been reached. Points somewhat separated were chosen for Fig. 5, so as to show the regions explored without overlapping.

The following table for the exploration of the neighborhood of a point of discontinuity (c, Fig. 5) will show in what manner the limiting values of the current corresponding to velocities of break were chosen. The last two rows of figures give the number of readings on and off the curve, respectively,

$K = 585$ *cm.* $L = 3.01$ *henry.* $R = 62.2$ *ohms.* $h = 30.02$ *cm.* $v = 116.4$ *cm./sec.*
$E = 2.029$ *volts.*

R_b	10	20	24	26	27	27.5	28
I.	.02816	.02471	.02354	.02302	.02278	.02263	.02253
No spark.	0	0	0	0	3	5	5
Spark.	2	3	2	1	3	0	0

giving $I = .02272$ ampere, corresponding to $R_b 27.3 \pm 2$ (see supra, series 1–3). The velocities corresponding to a and b of the curve were respectively 40.8 (series II. — 3), and 77.2 cm./sec. (series I. — 1).

Values obtained in this way were supplemented by direct observations at the spark gap and were always verified. It was thus possible to determine the limiting values of the current to one per cent. The electrodes of the break were carefully polished with a silk cloth after each reading. The electromotive force of the battery was measured whenever a limiting value of the current had been reached, and the resistance of the entire circuit including the break, closed, was found at the same time. Occasional verifications of the magnitude of the capacity were made. The switch for releasing the drop and the telescope for reading the electrometer, as well as the commutator for the purpose of bringing its needle to rest were placed on a table at a distance of several feet from the oscillation circuit.

PRELIMINARY EXPERIMENT.

Connection Between Optimum Capacity and Velocity of Break.

An experiment was first undertaken to ascertain suitable values of the quantities involved for the purpose of later work. The system was set up as in Fig. 3 omitting R_b. The variable capacity was made large and gradually diminished until the electrometer readings on breaking the circuit at a definite rate ceased to be a continuous function of the capacity. The region of discontinuity was explored, much as described in the previous section. The optimum capacity was chosen as the least capacity which gave five successive readings which lay on the capacity-deflection curve. The experiment was undertaken before much progress had been made with the measurements of capacity and the accuracy of the latter is probably not much greater than five per cent. The following table gives the values of the optimum capacity with the corresponding velocities of break:

$L = 3.01$ *henry.* $R = 63$ *ohms.* $R_b = 0.$ $E = 2.02$ *volts.* $v = {}^r\sqrt{2gh} = 21.2\sqrt{h}.$

h cm.	v cm./sec.	K cm.
69.1	176.2	670
15.5	83.5	1,160
4.8	46.4	1,780
1.11	22.3	4,170

The curve got by plotting K and v is almost hyperbolic in form (Fig. 6) showing that for a given current the optimum capacity is nearly inversely as the velocity of break.

If we put for equation (1) when K is in farads

$$v = \frac{A}{K} - \frac{B}{\sqrt{K}} + C$$

we get from the above data,

$$A = 238{,}000.$$
$$B = 6{,}270.$$
$$C = 66.4.$$

Where

$$A = .7656\frac{I}{a}; \quad B = .8946\frac{b}{a\sqrt{KL}}; \quad C = .1290\frac{b^2}{aL}.$$

From A,

$$a = 92{,}800 \text{ volts/cm}.$$

from B and a,

$$b = 1{,}190 \text{ volts}.$$

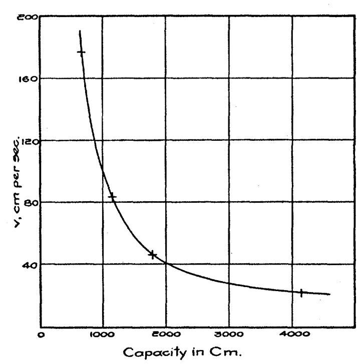

Fig. 6.

The value of a as obtained by Earhart is, as we have seen, about 70,000 volts/cm., while that of b is 351 volts. The agreement for a is perhaps as good as could be expected in view of the small number of readings, the errors of capacity measurements, and the neglect of the lag effect.

Experimental Results.

The following results were obtained by the method described in the last article but one. The capacities were measured with an accuracy of from one tenth to one third per cent., not including the capacity of the coil. The limiting values of the current correspond-

Series I.

$K = 585$ *cm.* $L = 3.01$ *henry.* $R = 62.2$ *ohm.*

	R_b	±	E	I	h	v
1	50.0	2.0	2.032	.01810	13.29	77.2
2	37.7	.2	2.032	.02032	22.08	99.6
3	27.3	.2	2.029	.02272	30.02	116.4
4	19.2	.2	2.029	.02496	40.89	135.4
5	18.5	.5	2.030	.02517	40.89	135.4
6	10.2	.2	2.030	.02806	56.01	158.8
7	75.0	5.0	2.030	.01482	6.59	54.5

ing to the velocity of break were measured within one per cent. except where specified. The inductance and velocity of break are known to a much greater degree of accuracy. All quantities are in the usual units except the capacity, kept in cm. for convenience.

The total resistance, from which I is calculated, is $R_b + R$ plus the small resistance of the leads and break, which was found to be negligible in the above results. The accuracy of the results 2–6

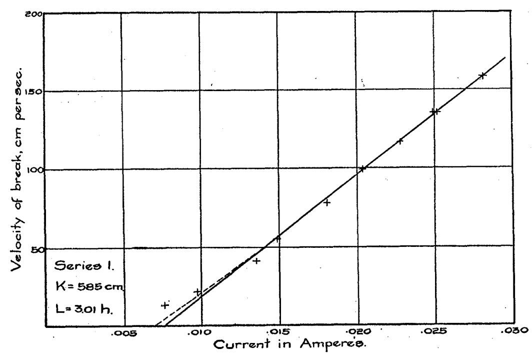

Fig. 7.

is thus seen to be well within one per cent. The currents and velocities of break are plotted as abscissæ and ordinates respectively, being all but the lowest three points of Fig. 7 which belong to the next series. It will be seen that the relation is approximately a linear one.

We may write equation (1) thus:

$$(1) \qquad v = AI - B + \frac{C}{I},$$

where C has the value $.1290b^2/aL$. For the moment let us assume for a and b the values obtained by Earhart. Taking representative value of the current, .025 am. say, the third term on the right of (1) has the value 3 cm./sec. It will differ from this by only one or two for any other value of the current in the range. The velocities run from 50 to 160 cm./sec. Accordingly the C term in the above equation is in the nature of a small correction to the equation.

$$(2) \qquad v = AI - B,$$

the asymptote of (1). Selecting the values 2–6 in the above results as being much more accurately determined than the other two, let the straight line determined by them be

(3) $$v = A'I - B'.$$

Then

$$A = 7{,}751\,; \quad B' = 58.8.$$

We have, from values of A, B and C, already given,

$$C = \frac{.1290 \times .7656}{(.8946)^2} \frac{B^2}{A}$$

Substituting the approximate values A' and B',

$$C = .055$$

The slope of the curve (3) is A' and is sensibly equal to the slope of the curve (1), especially for larger currents. We have then, for the current $I_1 = .025$ am. say,

$$A = A' + C/I_1^2 = 7{,}839,$$

which will differ by one per cent. or less if we choose any other current value in 2–6 as the basis of calculation. Similarly,

$$B = AI_1 - v_1 + C/I_1 = 63.1$$

where v_1 is taken from the curve (3) to correspond to I_1.

The curve $v = 7{,}839I - 63.1 + .055/I$ is shown deviating from the straight line to the left in Fig. 7. What we have done in getting the constants is equivalent to assuming that the curve (1) is tangent to (3) at the point (I_1, v_1). The justification lies in the fact that the curve (1) satisfies the points in the range of observation 2–6 as well as the line (3).

Since

$$A = 7{,}839 = \frac{.7656}{aK}, \quad a = 150{,}000 \text{ volts/cm.},$$

and since

$$B = 63.1 = \frac{.84966}{a\sqrt{KL}}, \quad b = 474 \text{ volts}.$$

The constant a comes out about double the value found by Earhart and b comes out one third greater. We shall see that this increase is clearly explained by the lag effect.

In showing the order of magnitude of the C term in (1) we used Earhart's values for a and b. If we use our own values it will come even smaller.

It is clear from this experiment that the lag effect is independent of the initial current, that is, of the magnitude of the induced potential, when the constants of the circuit are kept the same, since we get an approximately linear relation for v and I in agreement with the theory. Jaumann's hypothesis that the spark lengths for rapidly varying potential depend upon a definite value of VdV/dt is thus not borne out.

An attempt was next made to get values of v for very small initial currents. We should expect, when we approach induced potentials of the order of the minimum spark potential, to get a change in the spark constants, as in Earhart's results (Fig. 2), where, for $V < 351$ volts, we have $V = gx$, where g is about 1.2×10^6 volts /cm. For this branch of the curve the relation between the velocity of break, current, and constants of the circuit may be found by putting $b = o$ and $a = g$ in eq. (I). Then we have $v = .7656\ I/gK$. Accordingly, for smaller and smaller currents, we should at length come to an abrupt deviation from the straight line intersecting the I axis as in Fig. 7 and the points would rapidly approach the origin.

The readings for such small currents were very difficult to obtain owing both to the small readings of the electrometer, and to the minuteness of the spark, which rendered it difficult to determine whether the spark had or had not taken place. The following values were obtained:

Series II.

Constants as in Series I.

	R_b	±	E	I	h	v
1	205	5	2.028	.00760	.390	13.2
2	146	1	2.025	.00973	1.05	21.7
3	87	2	2.025	.01357	3.70	40.8

giving the lowest three points in Fig. 7. We have perhaps an indication of the change in the first two points. For the first the maximum induced potential is 517 volts.

Experiments were next tried to find whether the spark constants, not affected by changes of current, would be affected by change of capacity.

Series III.

$K = 789$ *cm.* $L = 3.01$ *henries.* $R = 62.2$ *ohms.*

	Total R	E	I	h	v
1	101.0	2.043	.02020	9.65	65.8
2	73.0	"	.02801	30.45	116.9
3	71.5	"	.02856	31.67	119.3
4	71.2	"	.02870	34.30	124.1
5	69.0	"	.02964	38.56	131.6
6	81.2	"	.02516	21.07	97.2
7	172.6	4.093	.02372	17.10	87.6
8	154.8	4.088	.02642	24.99	106.0
9	228.8	4.085	.01786	6.41	53.7
10	197.8	"	.02066	11.79	72.8
11	165.8	"	.02464	18.49	91.2

The straight line given by (I, v) is plotted in Fig. 8, No. III.

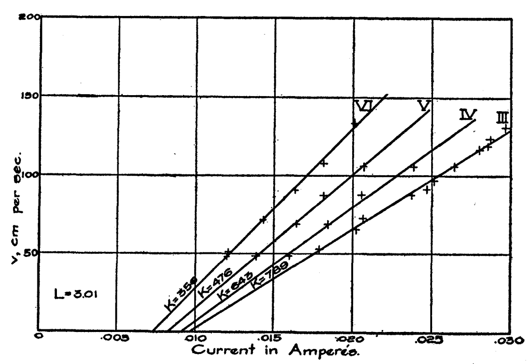

Fig. 8.

Using the same method as before for obtaining A, B, and C, we obtain

$$A = 6{,}510; \quad B = 67.9; \quad C = .074.$$

Whence

$$a = 134{,}100 \text{ volts/cm.}$$

$$b = 523 \text{ volts.}$$

SERIES IV. (Fig. 9, *II.*)

$K = 643$ *cm.* *L and R as in III.*

	Total R	E	I	h	v
1	256.3	4.088	.01594	5.16	48.1
2	222.6	4.093	.01838	10.44	68.5
3	199.6	4.093	.02052	17.10	87.6
4	171.8	4.088	.02380	24.99	106.0

From which

$$A = 7{,}578\,; \quad B = 75.3\,; \quad C = .077.$$

and

$$a = 141{,}400 \text{ volts/cm.}$$

$$b = 552 \text{ volts.}$$

SERIES V. (Fig. 9, *III.*)

$K = 476$ *cm.* *L and R as before.*

	Total R	E	I	h	v
1	294.7	4.088	.01386	5.16	48.1
2	249.6	4.093	.01641	10.44	68.5
3	225.8	4.088	.01812	17.10	87.6
4	197.8	4.088	.02066	24.99	106.0

giving

$$A = 8{,}748\,; \quad B = 77.7\,; \quad C = 0.71.$$

$$a = 165{,}500 \text{ volts/cm.}$$

$$b = 573 \text{ volts.}$$

SERIES VI. (Fig. 9, *IV.*)

$K = 356$ *cm.* *R and L as before.*

	Total R	E	I	h	v
1	338.7	4.088	.01206	5.16	48.1
2	336.8	4.073	.01210	5.96	51.7
3	284.8	4.073	.01431	11.40	71.6
4	249.8	4.073	.01634	18.31	90.7
5	224.8	4.070	.01811	25.85	107.8
6	202.8	4.070	.02006	39.63	133.4

giving

$$A = 10{,}390\,; \quad B = 81.5\,; \quad C = .066.$$

and

$$a = 186,200 \text{ volts/cm.}$$

$$b = 585 \text{ volts.}$$

From Fig. 9 it is seen that for diminishing capacity, the curves (I, v) progress toward the origin and increase in slope, agreeing with equation (1). But the increase in slope is not so great as would be expected, for the quantity a steadily increases, as we may see from the following summary of series I., III.–VI.

Series.	K cm.	a	b
III.	789	134,100	523
IV.	643	141,400	552
I.	585	150,000	474
V.	476	165,500	573
VI.	356	186,200	585

The quantity a and the quantity b, with one exception, are seen to increase with diminishing capacity. Without, for the present, considering the cause, we pass to experiments in which the capacity was kept constant while the inductance was changed.

Series VII.

$K = 789$ *cm.* $L = 1.97$ *henries.* $R = 50.9$ *ohms.*

	Total R	E	I	h	v
1	188.9	4.073	.02159	6.62	54.6
2	156.9	"	.02600	13.41	77.7
3	128.9	"	.03164	27.42	111.0
4	186.9	4.079	.02184	6.12	52.4
5	159.0	"	.02570	12.95	76.3
6	136.9	"	.02982	22.97	101.6

From these

$$A = 5{,}967\,; \quad B = 81.1\,; \quad C = .113,$$

giving

$$a = 146,300 \text{ volts/cm.}$$

$$b = 551 \text{ volts.}$$

SERIES VIII.

$K = 789$ *cm.* $L = 1.32$ *henries.* $R = 38.9$ *ohms.*

	Total R	E	I	h	v
1	150	4.073	.02721	6.69	54.8
2	119	"	.03430	16.50	86.2
3	105	"	.03888	26.60	109.2
4	128	"	.03188	11.01	70.4

From which

$$A = 4{,}845\,; \quad B = 84.6\,; \quad C = .152,$$

giving

$$a = 180{,}200 \text{ volt/cm.}$$
$$b = 580 \text{ volts.}$$

These two series with III., in which the capacity was the same, are shown in Fig. 9. We notice progression away from the origin

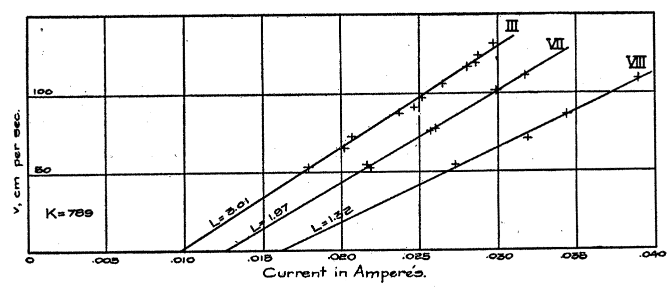

Fig. 9.

with diminishing inductance, as we should expect from the theory. But, instead of the slope being practically constant, it diminishes with the inductance. We show the results together for change of inductance.

$K = 789$ *cm.*

Series.	L	a	b
III.	3.01	134,100	523
VII.	1.97	146,300	551
VIII.	1.32	180,200	580

We notice increase of both quantities a and b with diminishing inductance, as with diminishing capacity.

We close with one more series, in which the period is less than that in any previous set.

Series IX.

$K = 356 cm.$ $L = 1.97$ *henries.* $R = 50.9$ *ohms.*

	Total R	E	I	h	v
1	216	4.079	.01882	13.50	77.9
2	189	"	.02161	23.98	103.8
3	171	"	.02390	34.98	125.3
4	203	"	.02014	17.91	89.7

giving

$$A = 9{,}550; \quad B = 108.3; \quad C = .125.$$

$$a = 202{,}700 \text{ volt/cm.}$$

$$b = 685 \text{ volt.}$$

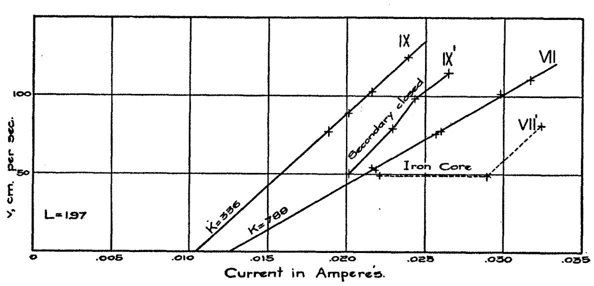

Fig. 10.

These points, with which those of series VII., are plotted in Fig. 10 for purposes of comparison.

Discussion of Results.

It might appear at first sight that the unknown capacity of the coil would explain the variation of the quantities a and b from the accepted values. If we assume $a = 70{,}000$ volt/cm. and calculate from the constants A and the capacities made necessary by the assumption, we shall find that they are all the way from two to three times the capacity which was measured. Moreover, the excess of apparent capacity over the measured capacity in series I.–VI. will be found to diminish as the latter diminishes, which is contrary to what we should expect if we were to attribute the apparent excess to the coil. Again, in the preliminary experiment, no perceptible difference was noticed, whether the battery side of the circuit was or was not earthed. We may therefore assume that the whole of the coil is at practically a node of potential and that its capacity is negligible.

We may very simply explain the results on the basis of any theory which takes into account the fact that an appreciable time must elapse after the application of a potential to a spark gap before a spark can take place, some process going on which converts the gas between the electrodes from an insulator into a conductor.

Such a theory has been developed by J. J. Thomson.[1] He obtains for the equation for the production of ions by an electric field between two parallel plates

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nu) = \frac{nu}{\lambda}\{f(Xe\lambda) - \gamma\}.$$

where n is the number of ions per cubic centimeter, X is the electric force u, λ, e, and $Xe\lambda$ are respectively the average velocity, mean free path, charge and mean kinetic energy of an ion. γ is a constant.

The complete solution of the above equation for n will be an exponential function of the time. The density of ionization and, accordingly, the conductivity of the gas will increase in geometric progression until they reach the required value for sparking, when $\partial n/\partial t$ becomes zero. With the electric force and spark length kept constant, the condition that the gas has reached the steady state, gives for the spark potential, according to J. J. Thomson,

$$V = ad + b,$$

for spark lengths, d, greater than the critical spark length, a and b being constants.

For the purposes of this paper we require the solution of the differential equation for the following condition: Given a spark length d, what will be the potential necessary to produce a spark at the time t? It seems probable that it will be of the form

[1] J. J. Thomson, Phil. Mag. (5), 50, p. 278, 1900. See also Conduction of Electricity through Gases, pp. 231, 341, 379.

$$V = (a + m)d + b + n.$$

When m and n are such functions of the time that when t approaches zero they become indefinitely large and, conversely, when t becomes large they will become zero and V will have its normal value. m and n are thus measures of the lag effect.

Although we have V and x, the spark length, as functions of the time, we may show that the conditions of our experiments are such that the electric force will be only dependent upon the time to the second order. This will be evident without proof if we consider that the conditions are such that the limiting condition for sparking requires that the induced potential and the spark gap increase at roughly the same rate. The problem is thus reduced approximately to the one stated above.

We may show also that the electric force, under the experimental conditions, is such as to be independent of the current for larger values of the latter, for we found by experiment that $v = AI - B + C/I$. The spark length is proportional to v. The electric force, then, will contain I only in the denominators of the small terms. This gives us justification for calculating a and b from A and B of the previous series, for that calculation involves the assumption that the lag effect is constant for a given period regardless of the initial current.

The constants m and n will depend upon the time at which the spark takes place. This time, as given in the first part of this paper is

$$t = \sqrt{KL}\cos^{-1}\left(\frac{a'vK}{I}\right) = \sqrt{KL}\left(\frac{\pi}{2} - \frac{a'vK}{I}\cdots\right)$$

where $a' = a + m$. We have seen that $a'vK/I$ is small in the limits of our experiment, being from .2 to .4. We may therefore consider the time required to be roughly proportional to the period, and for purposes of illustration we may take m and n as such functions of the period that they will vanish when the latter is infinitely large, and become themselves infinite when the period approaches zero. This will be true in general for m at least, whatever theory of ionization we adopt. The simplest functions which will satisfy the relation will be

$$a' = a + m = a\,(1 + \alpha/\sqrt{KL})$$
$$b' = b + m = b\,(1 + \beta/\sqrt{KL})$$

where a' and b' are the quantities previously called a and b in the tables, and α and β may be considered as time constants of the gas.

The following results from series I.–IX., with the preliminary experiment P, show the dependence of a' and b' upon the period. Three significant figures are kept.

Series.	$\sqrt{KL} \times 10^5$.	$t \times 10^5$.	a'	b'
IX.	2.79	3.30	203,000	685
VIII.	3.40	3.88	180,000	580
VI.	3.45	3.86	186,000	585
V.	3.99	4.52	165,000	573
VII.	4.16	4.82	146,000	551
I.	4.42	4.59	150,000	474
IV.	4.64	5.43	141,000	552
III.	5.14	5.76	134,000	523
P.	8.05	10.14	93,000	

The time t is calculated from the formula just given, using the mean value of $a'vk/I$ in series I.–IX., and in the preliminary ex-

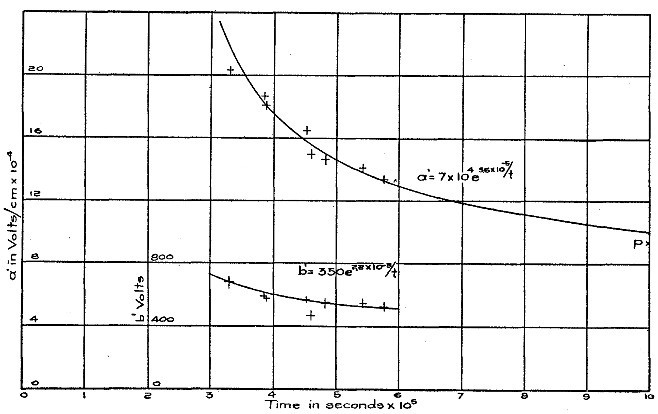

Fig. 11.

periment using the mean value of K. The plots of a' and b' with the times as abscissæ are shown in Fig. 11.

The function $a' = a(1 + \alpha/t)$ seems to satisfy the results of series I.–IX. as well as any function which could be chosen. But it comes very wide of the point P, obtained from the preliminary experiment, which, while given no weight in the calculations, is supposed to be determined within five per cent. The spark constant a, calculated from the results will be about 5×10^4 volt/cm. in place of Earhart's value of about 7×10^4. Though our data are not sufficiently extensive to enable us to tell definitely, it would seem that the function m must be some function of the time which diminishes more rapidly as the time increases than does $a\alpha/t$.

If we put $a' = ae^{\alpha/t}$, and assume $a = 70{,}000$ volts/cm., the mean value of α from the nine series is 3.6×10^{-5} sec. The curve thus obtained is the upper one in Fig. 11. It gives a value within 8 per cent. of the a' given by the preliminary experiment.

The curve obtained through the observations b' is obtained by putting $b = 350$ volts and $\beta = 2.2 \times 10^{-5}$ sec. in the equation $b' = be^{B/t}$.

Addenda.

Though the aim of the work here described was to study the conditions for sparking at the break of the primary circuit, some experiments were made with the view of finding what the condition would be for the primary of the induction coil with iron core and with closed secondary. The presence of an open secondary was found to be without perceptible effect on the necessary velocity of break for a primary circuit.

Secondary Closed. — In Series IX. the inductance used consisted of the first five parts of the coil described by Dr. J. E. Ives,[1] the sixth part being disconnected and open. When the latter was closed a falling off was found in the necessary velocity of break for given current. This is shown by the crooked line under IX. in Fig. 10. This is due to the absorption of energy by the secondary as was shown by the diminished electrometer readings for the primary oscillations.

When the secondary was connected to an electrometer and readings taken simultaneously with those of the primary electrometer, the maxima of electrometer readings for the two circuits were found to occur at the same time; *i. e.*, the induced potential in the secondary is a maximum, when the current in the primary has its greatest

[1] Loc. cit.

possible value consistent with no sparking at the break. This is equivalent to the well known fact that the secondary potential is greatest when the optimum capacity is used with the primary. Moreover, it was found that the greater the velocity of break the greater the possible secondary potential, because it enables the primary current to be made larger, or the capacity to be made less. This is in direct agreement with Lord Rayleigh's result.

Core. — Heavy cores of brass were used without perceptible effect on the necessary velocity of break. After Series VII. was obtained, bundles of iron wire were introduced into the core of the coil, and caused a marked falling off of the necessary velocity of break, as shown by the dotted line in Fig. 10. There was also a great falling off in the electrometer readings, probably due to the hysteresis loss and the lengthening of the period. According to a principle laid down by B. Walter[1] "the maximum potential for the secondary, excepting for losses of electric energy through break sparks, magnetic and dielectric hysteresis, and Joule's heat, is directly proportional to the maximum value of the potential of the primary." When the secondary is open, the velocity of break necessary to prevent sparking will be determined by the constants of the primary circuit and by the value of the initial current; and this velocity of break will permit of the highest possible secondary potential. When the secondary is closed, energy is absorbed from the primary, the necessary velocity of break will be less than for open secondary. The investigation will be complicated in this case by the introduction of a damping factor in the primary circuit.

Mercury Break. — In some early experiments a study of the mercury break was made. The break consisted of a blunt needle which was withdrawn at a definite rate from a well of mercury. Photographs were taken of the break, the illumination being that of its own spark. It was found that a column of mercury followed the needle for a short distance above the surface before separation took place. The velocity of break is thus not as great as could be obtained with a mechanical break going at the same rate, showing the decided advantage of the latter. No dependence of optimum capacity and velocity of break could be found except for very small

[1] B. Walter, Wied. Ann., 62, p. 300, 1897.

induced potentials. This we should expect, since a given point has a fixed discharge potential such that for higher values discharge will occur independently of the distance.

It was found that the capacity necessary to prevent a spark at the break was about one half as much when the needle was connected to the positive pole of the battery as was required when it was connected to the negative pole, so that much greater induced potentials could be reached in the former case than in the latter. This agrees with results which have been published by Dr. J. E. Ives.[1] It is probably due to the fact that the discharge potential for a negatively electrified point is less than for the same point positively electrified[2] and is a good point to be remembered by those using the mercury break with an induction coil.

Summary.

This work was undertaken in the hope of finding relations which were indicated in some experiments by Lord Rayleigh, existing between the constants of an oscillatory electric circuit, such as the primary of an induction coil, the initial current, and the velocity with which the contacts of the break are separated, when the conditions are just sufficient to prevent sparking at the break.

By neglecting the "lag" effect in the spark, a theoretical relation was found, and may be roughly expressed by saying that the velocity of break necessary to prevent a spark will be proportional to the frequency of oscillation and a linear function of the maximum induced potential, the latter implying a linear relation between velocity of break and initial current.

The necessary velocity of break was found experimentally to be:

1. A linear function of the initial current (see Figs. 7–10).

2. For given current, increasing in value in a manner consistent with the equation when the capacity is diminished (Figs. 6, 8, 10).

3. Diminishing according to the equation when the inductance is diminished (Fig. 9.)

The assumption that the lag of the spark might be neglected did not hold true. It was found that the spark constants, *a* and *b*,

[1] J. E. Ives, On the Asymmetry of the Mercury Break, Phys. Rev., 17, p. 175, 1903.

[2] Tamm, Annalen der Physik, 6, p. 259, 1901.

were too large, but diminished systematically as the time between the application of the potential and the passage of the spark increased, and would apparently reduce to the ordinarily accepted values if the time were great enough to allow the gas to become sufficiently ionized.

The results enable us to place a limit on the velocity of break necessary to prevent a spark at the break of an inductive circuit, if we know the inductance, capacity and initial current. The necessary velocity will be lessened by any condition which causes the absorption of energy from the primary, such as a closed secondary, hysteresis, heat losses, etc.

The method here developed seems to be well adapted to the study of the lag effect in the electric spark, or of the ionization produced by an electric field. A method in which use is made of constant potentials is likely to present great mechanical difficulties, especially as these results seem to show that the potential necessary to produce a spark across a given distance within the time t will have decreased from infinity to only e times its normal value in about 3.6×10^{-5} sec. The mechanical difficulties are reduced to a minimum in the present method, the main trouble to be expected lying in the interpretation of results. I hope soon to carry out similar experiments with the method much improved and with results covering a wider range for the purpose of getting data on the ionization produced by an electric field.

I wish to express my best thanks to Professor Arthur G. Webster, who has constantly aided me with suggestions, and to the authorities of Clark University for the facilities placed at my disposal.

July, 1904.