THE VELOCITY OF SOUND.

BY THOS. C. HEBB.

ASSUMING that the velocity of sound is an important constant, no apology is necessary for a redetermination of its value, in as much as the results obtained by different observers differ very much from one another, and in many cases the determinations of a single observer differ considerably among themselves. In the following experiment the aim has been to make the result accurate to at least one tenth of one per cent. With what success this has been attended will appear in the course of the paper.

In order to see what errors enter into the different methods of determination of this constant, the methods will be divided into (1) direct and (2) indirect. (1) can be further divided into (*a*) long distance, and (*b*) short distance.

There are several objections to the long distance method:

1. Very intense sounds — as the discharge of a cannon — are used, and it is very doubtful whether the velocity near the source is the same as that at some distance away.

2. It is practically impossible to correct for wind, temperature, and humidity with any great degree of accuracy.

3. The method of performing the experiment involves the "personal equation" of either an observer, or of a recording device as used by Regnault.

In the case of short distance methods, objections (1) and (2) are overcome, and (3) will depend on the method employed. The only method used with any success has been that proposed by Bosscha[1] and put into execution by Szathmari.[2] Although Szathmari's final result seems to be very near the true value, his individual results differ from the mean by 1 per cent.

From the great number of direct determinations of the velocity

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[1] Pogg. Ann., 92, 1854, p. 485.

[2] Wied. Ann., 2, 1877, p. 418.

of sound, Wüllner[1] chooses the results of Regnault, Moll and Van Beck, Bravais and Martin, and the French Academy as the most accurate.

In Regnault's experiments a gun was used as the source of sound. In order to free the experiment from the personal equation of an observer, he had it so arranged that both the discharge of the gun at the one station, and the arrival of the sound wave at the other station, were recorded by an electrical device. Although this frees the experiment from the "personal equation" of an observer, still there is the "personal equation" of the recording device to be taken into account. In order to eliminate wind effect, simultaneous firing from both stations was used. The distance employed was between one and two miles.

In the case of the other sets of experimenters, cannons were used as the source of sound. The time between the flash of light and the arrival of the sound wave was recorded by observers. Wind effect was eliminated as in the case of Regnault. The French Academy used a distance of eleven miles, Moll and Van Beck a distance of ten miles, and Bravais and Martin nearly two miles.

In long distances like these, it is impossible to suppose that the true corrections for temperature, wind and humidity have been made. These sources of error, coupled with the errors due to personal equation and different intensities of the source are sufficient to explain the variation between the different results.

In the case of indirect observations, the wave-length is determined in a tube, making use of Kirchoft's formula connecting the wave-length in a tube with that in free air. If λ is the wave-length in air, L the wave-length in the tube, ε a constant, γ the radius of the tube, and n the frequency of the source, then

$$L = \lambda\left[1 - \frac{\varepsilon}{2\gamma\sqrt{n\pi}}\right].$$

The objection to this method of determining λ lies in the fact that there is good reason to believe that ε is not a constant as defined by Kirchoff.[2] If the size of the tube used could be so great that the

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[1] Experimental Physik, Vol. I., p. 939.

[2] J. Muller, Drude, Vol. 4, 1903, p. 331.

effect of the walls was negligible, or if the walls could be taken away entirely, the above objection would disappear and $\lambda = L$.

In the following experiment proposed by Professor Michelson, the attempt has been made to free the method from all of the above objections. The method is this :

Two paraboloids A_{I} and A_{II} (see cut) are placed coaxially so that waves sent out from the focus S of A_{I} are collected at the focus F_{II} of A_{II}. In the one focus, S, is placed a source of sound and near it a telephone transmitter, T_{I}, while in the other focus is placed another transmitter, T_{II}. Each transmitter is connected in series with a battery (B_{I} and B_{II} in figure), and one primary of an induction coil, I, made with two primaries. A telephone receiver, R, is in series with the secondary of the coil.

Now, suppose waves are given out by the source S. Some pass directly to the transmitter, T_{I}, placed near it and set it in vibration with a definite phase relation depending on its distance from the source S; others being reflected to the other paraboloid, A_{II}, are collected at its focus, where they act on the second transmitter, T_{II}, and set it in vibration with a definite phase relation depending on the distance between the two paraboloids. The vector sum of these two *effects* is given in the receiver. Assuming that the intensities of these effects are the same, it is possible, by moving one paraboloid parallel to itself, to change the relative phase of the two effects so that they will alternately annul and reinforce one another. This affords a method of getting the wave-length, for having determined the position of the paraboloid for two different minima, and knowing the number of waves, and the distance between these two positions, the value of the wave-length follows immediately. If now the paraboloid can be moved one hundred waves, and if the minima can be located to one tenth of a wave, the value of the wave-length so obtained should be accurate to one part in one thousand. This assumes that the difference of phase is only a function of the distance between the paraboloids. In order to realize this, the waves reflected from the paraboloids must be plane.

APPARATUS.

The paraboloids used were 5 feet in aperture and 15 inches in focal length. They were made of plaster of paris and were accurate of at least one tenth of an inch. They were mounted on strong trames with their axes horizontal and were provided with wheels for locomotion.

The transmitters used were disk-shaped, 3 inches in diameter and 1.3 inch thick. The aperture through which the sound entered was .3 inch in diameter.

The source of sound consisted of a tube .75 inch in diameter, closed at one end and having a stream of air blowing across the other. It was so arranged that as few overtones as possible were present. The air pressure was supplied by the pressure tubes of the laboratory. This pressure is generated at the University power-house, 600 feet away, and hence has a very constant value in the laboratory pipes. In order to further increase its constancy, the air was passed through two large gas tanks and then through cotton wool and lime. This also removed the dust and moisture. In this manner it was possible to keep the pitch of the whistle constant to one part in five thousand for a time sufficient to make an observation.

One paraboloid was movable on a track. This track was built in sections and was taken up after each experiment. The rails were 2.5 feet apart and prevented any side motion of the paraboloid.

The induction coil used with the transmitters had two primaries wound with No. 22 Cu wire, and having a resistance of .3 ohms. The secondary was also of No. 22 Cu wire and had five times the number of turns of either primary.

The experiment was performed in one of the halls of Ryerson Physical Laboratory. The hall is 120 feet long, 10 feet wide, and 14 feet high. It should be remarked that the experiment need not be restricted to this distance. In this place no wind was encountered, and the temperature remained quite constant. Six thermometers corrected for zero point were placed at equal intervals on alternate sides of the hall and the mean of their readings taken as the temperature at which the experiment was performed.

Arrangement of the apparatus: The two paraboloids A_{I} and A_{II} were arranged coaxially (see cut), one stationary at the end of the hall, and the other movable on a track extending from one end of the hall to the other. The stationary paraboloid A_{I} had the whistle at its focus and near it a transmitter T_{I}. Wires led from this transmitter through a battery B_{I}, to the one primary of the induction coil I placed at the back of the paraboloid A_{II}. In the focus of A_{II} was the other transmitter T_{II} whose wires led through a battery B_{II}, to the other primary of the induction coil. Having a receiver R in series with the secondary of the coil, it was possible to listen in the telephone, and move the paraboloid A_{II} at the same time. In each of the primary circuits was placed a slide wire resistance, so that the intensities of the effects in the receiver could be kept equal and constant.

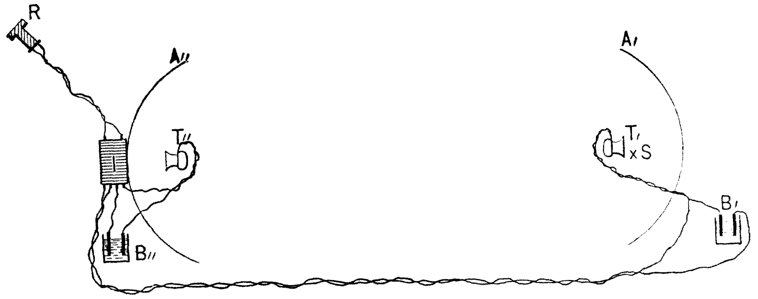

Fig. 1.

To perform an experiment the whistle was brought into unison with a tuning fork and the movable paraboloid having been brought to within fifteen feet of the fixed one, a position was found at which the sound in the telephone receiver was a minimum. Trouble was encountered here for the pitch of the whistle depends on the position of the movable paraboloid. This is due to the reflected wave changing the pressure at the mouth of the whistle and is overcome by changing the pressure in the air supply. This reflected wave would also have the effect of changing the phase of vibration of the transmitter near the whistle. This was reduced to a minimum by making the reflected wave as weak as possible. Other causes of change of phase in the transmitter of the movable paraboloid are the direct action of the whistle, and the action of the waves reflected from the sides of the hall. These however were very small as could be tested by putting the whistle before an open door at the end of the hall. The position of the paraboloid for which the telephone gave minimum intensity could be determined to .5 inches. Two of these positions were found and hence the wave-length of the fork could be found.

Experimenting with a 2560 V. S. fork, the following values of the wave-length for one night were obtained :

TABLE I.

March 24.

No. of Waves Beginning at 15 Feet from Source.	Distance Between Minima in Inches.	Temp. C. °.	λ_t	λ_0
10	106.62	22.4	10.662	10.249
20	213.05	22.5	10.652	10.237
30	319.23	22.6	10.641	10.225
41	436.78	22.5	10.653	10.238
51	541.58	22.4	10.619	10.207
61	647.70	22.7	10.618	10.202
71	754.19	22.7	10.622	10.205
81	861.12	22.9	10.631	10.210
119	1,263.57	22.9	10.618	10.199

In these results no correction has been made for humidity, but this would not change for one night. As can be seen from the table, the values of the wave-length decrease as we proceed from the source up to a certain point, and then remain constant. This decrease cannot be explained by the experimental error, for similar results were obtained at different times.

The following table will show that even under different circumstances the value of the wave-length in the case where the whole length of the hall is used is a constant:

TABLE II.

Date.	No. of Waves.	Distance Between Minima in Inches.	Temp. C.°.	λ_t	λ_0	δ
Mar. 22	80	848.81	22.2	10.610	10.203	+.008
24	119	1,263.57	22.9	10.618	10.199	+.004
25	117	1,240.82	22.0	10.604	10.200	+.005
27	119	1,255.25	20.0	10.548	10.181	−.014
Apr. 3	119	1,255.87	19.7	10.554	10.192	−.003
				Mean	10.195	

The probable error of the mean is .003. There is however in these results a constant error due to the fact that the humidity has not been taken into account. When these observations were taken however, the weather was still cold so that the dew point was probably below zero; and hence the above values are possibly .1 per cent. too great. It will be of interest as a check to see how the velocity of sound derived from this wave-length with the aid of the marked vibration number of the tuning fork agrees with the accepted value. The tuning fork had been kindly loaned to me by Professor Angell, Head of the Psychology Dept., and he assured me when I got it that it was not in error more than one-half a beat. Accepting these two values then we get for the velocity of sound at zero degrees:

$$U = \lambda_0 n = \frac{10.195 \times 1280}{39.37} \text{ meters} = 331.46 \text{ meters.}$$

This is about .1 per cent. less than the accepted value. It would appear from this that the decrease in wave-length as shown in Table I. is not very serious. Believing that the decrease was due to the fact that the wave-length was too long to be properly reflected, a wave six inches in length was tried. The following table shows that the trouble has been overcome:

Table III.

Date.	No. of Waves Beginning 15 Feet from Origin.	Temp. C.°.	Distance Between Minima in Inches.	λ_t	λ_0
April 17	25	20.5	142.61	5.7044	5.5013
"	75	20.9	428.20	5.7093	5.5022
"	100	21.1	570.50	5.7050	5.4962
19	25	19.3	142.36	5.6944	5.5029
"	50	19.4	284.35	5.6870	5.4947
"	75	19.8	427.21	5.6961	5.4996
"	100	19.7	568.94	5.6894	5.4942
"	126	19.5	717.05	5.6909	5.4976
"	151	19.4	859.16	5.6898	5.4975
"	176	19.1	1,001.27	5.6890	5.4995
"	220	19.2	1,251.87	5.6903	5.4999

It is seen from this table that there is no tendency for the wave-length to decrease as was the case with the longer wave-length. What variations there are, are within the experimental error. This seemed to show that all constant errors were eliminated and hence the observations given in the following table of observations were made. Unfortunately there are not more data to offer due (1) to the fact that the experiment is very trying on the nerves and (2) to lack of time. These observations were made however under different atmospheric conditions, and as the apparatus was taken down after each experiment there is little likelihood of there being a constant error of method. It might also be added that no observations have been rejected.

TABLE IV.

Date.	No of Waves Used.	Distance Between Minima in Inches.	Temp. C.°.	Dew Point	Press. mm.	λ_t	λ_0	δ
Apr. 23	217	1,244.00	22.5	14°	746.5	5.7328	5.4940	+.0046
"	216	1,237.87	22.5	14	746.5	5.7309	5.4921	+.0027
24	217	1,238.12	21.3	4	746.5	5.7056	5.4866	−.0028
26	218	1,240.00	18.8	1	752.0	5.6888	5.4956	+.0062
30	219	1,241.37	17.5	4	747.5	5.6684	5.4855	−.0039
May 1	216	1,224.37	17.4	5	750.0	5.6684	5.4865	−.0029
2	218	1,235.50	17.4	4	750.8	5.6674	5.4855	−.0039
							Mean 5.4894	

The probable error of the mean value is .0010. λ_0 has been calculated by aid of the following formula:

$$\lambda_0 = \lambda_t \sqrt{\frac{\rho}{\rho_0}}$$

where λ_0 = wave-length at zero for dry air, λ_t is wave-length at time of experiment, ρ is the density of the air at the time of the experiment, and ρ_0 is the density of dry air at 0° and pressure equal to that at time of experiment.

DETERMINATION OF THE PITCH OF THE TUNING FORK.

This consisted of three parts (1) the comparison of the high fork with one marked 512 V.D., (2) the comparison of this latter fork with a pendulum, and (3) the comparison of the pendulum with the clock. The comparisons in (1) and (2) were made by means of traces on a smoked glass disk.

1. In this case it was found that the vibrations of the high pitch fork were of too short a duration to admit of an accurate comparison by means of a trace from it. It was therefore found necessary

to use the trace given by a stile fastened to the telephone receiver diaphragm, when the latter was actuated by the whistle in unison with the fork. In order to make the trace of sufficient amplitude to be distinctly seen with a microscope, it was necessary to bring the two paraboloids close together so that the intensity of the sound at the mouth of the transmitter used was rather great. This together with the use of a large current through the transmitter gave sufficient amplitude of the receiver diaphragm to make its trace seen with a low power microscope. As stated above, the unison of the whistle and tuning fork could be obtained to one part in five thousand. The stiles whose traces were being compared were arranged close together so that the traces were in the field of view of a microscope, and so that a variation of the speed of rotation of the disk did not affect the results. The unison of the telephone and tuning fork was tested before and after an experiment.

The following three determinations were made :

Table V.

Date.	Temp. C °.	n_1	n_2	n_1/n_2	δ
May 9	19.5	1,041	224	4.6473	.0007
12	19.3	1,008	217	4.6452	.0014
13	19.0	781	168.05	4.6474	.0008
			Mean	4.6466	.00097

n_1 is the number of waves of the telephone diaphragm which corresponded to n_2 of the 512 V. D. fork. The above numbers were obtained by counting the number of waves between two points at which the crests of the two waves coincided. These positions could be determined to .03 of a 512 wave.

2. The pendulum used in this case was made to close a current which acted through an electromagnet bearing a stile. A complete period of the pendulum was about 2 sec. Using the usual precautions with the mercury contact, etc., the following two values for the number of waves of the tuning fork between the two breaks of a complete oscillation of the pendulum were obtained.

May 17,	Temp. 18.3.
First observation gave	1,017.7
Second observation gave	1,017.7
Mean	1,017.7

The position of the break of the electromagnet could be located to .1 of a wave.

3. The pendulum was compared with the Howard astronomical clock of the laboratory. In two separate comparisons when the number of swings of the pendulum in one hour was obtained, the following values for a half swing were found :

$T = .99498$

$T = .99485$

Mean $T = .99492$.

Therefore the pitch of the high tuning fork at 18.3 is

$$N_{18.3} = \frac{4.6466 \times 1017.7}{2 \times .99492} = 2{,}376.5.$$

Using McLeod and Clarke's formula for the change in pitch of a tuning fork due to change in temperature, viz. :

$$n_t = n_0\,(-\,.00011t)$$

the following table which is derived from Table IV. is obtained :

TABLE VI.

Date.	Temp. C.°.	λ_0	n_t	$v_0 = n_t\lambda_0$	δ
April 23	22.5	5.4940	2,375.4	331.48	+.19
23	22.5	5.4921	2,375.4	331.37	+.08
24	21.3	5.4866	2,375.8	330.95	−.34
26	18.8	5.4956	2,376.4	331.72	+.43
30	17.5	5.4855	2,376.7	331.15	−.14
May 1	17.4	5.4865	2,376.7	331.21	−.09
2	17.4	5.4855	2,376.7	331.15	−.14
				Mean 331.29	.20

The probable error of the mean is .04.

This value lies a little below the theoretical value obtained by assuming the ratio of the specific heats to be that obtained by Röntgen, viz., 1.405. Using this value of γ, 331.8 is obtained for the velocity of sound. It also lies below the mean of the four best determinations which according to Wüllner[1] are :

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[1] Experimental Physik, Vol. I., p. 939.

Regnault,	330.7
Moll and Van Beck, etc.,	332.7
Bravais and Martin,	332.4
French Academy,	331.2
Mean,	331.75

In conclusion I wish to thank Professor Michelsen for proposing the problem, and for his valuable advice at all times freely given.

I also wish to thank Professor Millikan for his kind assistance.

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