THE PRESSURE DUE TO RADIATION.¹

(Second Paper.)

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HE disc was calibrated for temperature in terms of the deflection for a definite sensitiveness of the galvanometer G_1 . For this purpose the disc was immersed in a kerosene bath and the galvanometer deflection measured for two different temperatures of the disc. One of these was about 18° C. above the comparatively steady temperature of the room, or calorimeter containing the standard temperature junctions (see Fig. 6), and the other about the same number of degrees below the room temperature. These two temperatures were measured by a Fuess standard thermometer divided into tenths of a degree and calibrated at the Reichsanstalt. Two calibrations of the silver disc were made some days apart. One of these series appears in full in Table V. The first three columns of the table give the zero, direct and reversed reading of the galvanometer G_1 . The fourth column gives the temperature of the bath in which the disc was immersed, and the fifth, that of the constant temperature calorimeter. The sixth column gives the deflections of G_1 . The seventh column the means of the alternate deflections. The eighth, the mean of the two columns preceding it. The last column gives the difference in temperature between the two calorimeters in degrees C. For the total temperature range in the table, 39.11°, the deflection of G_1 was 393.8 scale divisions for a sensitiveness of $G_1 = 996$. A range of one degree would thus give a deflection of 10.03 divisions for a sensitiveness of $G_1 = 1,000$. The mean of two separate calibrations was 9.96 scale divisions for one degree temperature difference.

Before beginning a series of intensity measurements the disc was suspended in an air-chamber containing phosphoric anhydride and ¹Concluded from July number.

TABLE V.

Calibration of Silver Disc.

			Co	ld Bath.				
(G ₁ Reading	s.	-	Room	Deflec-	Means of Alter-	G,	T 0_T 0
Rev.	Zero.	Direct.	Disc T ₁ ^o	cal. T_2°	of G ₁	Deflec- tions.	Means.	12 11
	221.2							
	221.0	35.2	1°.58	20°.05	185.8	1		
	220.9							
402.0	221.0		1°.60	20°.07	ן 181.0	}185.7	183.4	18.47
	221.2							
	221.2	35.7	1°.57	20°.10	185.5 }	J 181.3	183.4	18.53
(02.1	221.2		10 50	000.14	101 0	105 0	102 7	10 70
403.1	221.5		1°.50	20°.14	181.0	185.8	183.7	18.58
	221.9	35.0	10 54	200 16	186 1	182.2	184 1	18 62
	222.1	03.7		20.10	100.1	102.2	101.1	10.02
405.0	222.2		1°.52	20°.20	182.8	186.3	184.5	18.68
	222.4							
	222.7	36.2	1°.57	20°.22	186.5	182.7 ך	184.6	18.65
	223.0							
405.8	223.1		1°.60	20°.26	182.7	186.7	184.7	18.66
	223.2							
	223.3	36.3	1°.59	20°.30	187.0	J	104.0	100.00
	223.5	l Linn to T	00.00		l	l	184.0	180.60
	Correc	tion to I_1	$= 0^{\circ}.00.$					
			1	Varm Bat	th.			
	217.3	424.0	410 45	000 40	015 6			
	218.0	434.2	41°.45	20°.40	215.0]		
2.0	219.9		410 35	20° 42	218 5	2137	216.1	20.93
2.0	220.0		+1 .05	20 .42	210.5		210.1	20.95
	222.4	434.2	41°.25	20°.44	211.8	216.4	214.1	20.81
	223.7							
10.0	224.4		41°.08	20°.50	214.4	209.1	211.7	20.58
	225.1							
	225.7	432.1	40°.90	20°.55	206.4	212.4	209.4	20.35

40°.80

40°.67

40°.55

40°.43

431.0

429.4

Correction to $T_1 = 0^{\circ}.10$.

20°.60

20°.61

20°.63

 $20^{\circ}.65$

210.4

203.2

206.7

200.4

204.8

208.5

201.8

207.6

205.8

204.2

209.8

 $Corrected,\ 20.51$

20.20

20.07

19.92

20°.41

226.3

226.7

227.2 227.8

228.4

228.5 228.7 229.0

229.3

16.3

21.8

surrounded by a jacket of ice and salt. The disc was thus lowered to a temperature of about zero degrees and was then quickly transferred to the chamber C (Fig. 6), and the beam was directed upon it. When its temperature had risen to within five or six degrees of that of the chamber C, galvanometer readings were made at intervals of five seconds until the disc was heated to a temperature several degrees above its surroundings. The temperature of the chamber C was determined by removing the disc and cooling it to a point near the room temperature, then replacing it and observing its rate of temperature change for several minutes.

The note-book record of one series of observations showing the heating of the disc by the light beam is given in full in Table VI. It will be seen from the table that the temperature of the disc passed

August 16. Energy Measurements. Through Air. Series 4. Zero of Galvanometer G_1 (closed circuit) Determined by Method of Cooling = 216.8 = Reading at Room Temperature.

Time.	G_1	Time.	G_1	ΔG_1	Δt	$\frac{\Delta G_1}{\Delta t}$ (in mm. per sec.)
0 secs.	174.5	60 secs.	253.2	78.7	60 secs.	1.312
5 ''	182.0	55 ''	247.3	65.3	50 ''	1.306
10 ''	189.0	50 "	241.3	52.3	40 ''	1.308
15 ''	196.2	45 ''	235.2	39.0	30 ''	1.300
20 ''	203.0	40 ''	229.1	26.1	20 ''	1.305
25 ''	209.7	35 ''	222.8	13.1	10 "	1.310
30 ''	216.4				Avera	ge 1.307

The lamp reading (G_2) was 924.

The sensitiveness of \overline{G}_2 was 667, and of G_1 was 996.

 $\frac{\Delta G_1}{\Delta t}$ reduced to standard conditions becomes

 $\mathbf{I}.307 \times 667 \times 996 \div (924 \times \mathbf{I},000) = 0.943 \text{ mm. per sec.}$

that of the chamber thirty seconds after the beginning of the series. The readings of G_1 at equal time intervals on either side of the zero are on horizontal lines. The last column of the table contains the rate at which the galvanometer deflection was changing when the disc and its surroundings were at the same temperature.

Energy series were made "through air," "through red glass," and "through water cell," as in the pressure measurements. During the experiment the black coatings were frequently cleaned off from the disc and new ones deposited. The final result therefore does not correspond to an individual, but to an average coating.

To correct for any inequality between the two disc thermo-junctions or any lack of symmetry in their positions, referred to the central plane of the disc, which might prevent the mean temperature of the two junctions from representing the mean temperature of the mass, series of observations were made on each face of the disc. The

						Front 1	Face.					
			Thro	ugh A	ir.		Throu	gh R	ed Glass.	Throu	gh W	Vater Cell.
Dat	te.	$\frac{\delta G_1}{\delta t}$	<i>G</i> ₂ (Lamp).	S1	S2	$\frac{\frac{\delta G_1}{\delta t}}{\text{Reduced}}$ Standard	$\frac{\delta G_1}{\delta t}$	G2	$\frac{\frac{\delta G_1}{\delta t}}{\text{Reduced}}$ Standard	$\frac{\delta G_1}{\delta t}$	G2	$\frac{\frac{\delta G_1}{\delta t}}{\text{Reduced}}$
Aug.	10	1.387	980	990	689	.965				.437	345	.864
	"	1.263	920	990	689	.936				.400	311	.877
" "	• •									.369	279	.902
	11	1.244	866	986	701	.992	.750	546	.950	.412	315	.905
"	" "	1.455	1010	986	701	.995	.750	546	.950	.510	382	.922
" "	"	1.505	1047	986	701	.994				.516	381	.935
" "	16	1.447	1022	996	669	.942	.736	529	.927	.416	327	.873
" "	" "	1.284	886	996	669	.966	.740	527	.936	.451	352	.853
"	"	1.316	925	996	669	.948	.797	550	.965	.502	382	.875
" "	" "	1.307	924	996	669	.943						
"	18	1.598	1110	995	667	.955	.738	515	.952	.449	333	.895
"	"	1.550	1047	995	667	.984	.732	518	.940	.445	342	.865
" "	"'	1.548	1031	995	667	.995	.730	518	.938	.451	346	.867
" "	"	1.410	957	995	667	.977						
• •	"	1.330	898	995	667	.983						
"	19	1.241	862	1001	675	.975	.760	532	.965	.451	343	.892
"	"	1.360	934	1001	675	.985	.728	512	.960	.452	338	.904
"	""	1.324	905	1001	675	.990	.738	525	.950	.466	351	.898
" "	"	1.364	934	1001	675	.988						
			A	verag	;e =	0.973 ⊨0.003	Aver	rage =	0.948 ⊨0.002	Ave	rage H	0.888 = 0.004

TABLE VII.

black coating was always cleaned off from the face of the disc away from the light. All of the series of energy measurements are gathered together in Tables VII. and VIII. In the tables, under the head "through air," the first column contains the observed rate of increase in the galvanometer deflection G_1 , when the disc and its surroundings were at the same temperature; the second column,

Table VIII.

Rear Face.

			Thr	ough	Air.		Thre	ough I	Red Glass.	Throu	Through Water Cell.			
Da	ate.	$\frac{\delta G_1}{\delta t}$	G2 (Lamp).	\$1	S_2	$\frac{\delta G_1}{\delta t}$ Reduced to Standard.	$\frac{\delta G_1}{\delta t}$	G_2	$\frac{\delta G_1}{\delta t}$ Reduced to Standard.	$\frac{\delta \dot{G_1}}{\delta t}$	G.2	$\frac{\frac{\delta G_1}{\delta t}}{\text{Reduced}}$ Standard		
Aug.	12.	1.374	960	991	684	.970	.808	578	.949	.495	370	.906		
"	12.	1.331	932	991	684	.968	.740	536	.935	.434	320	.919		
"	12.	1.284	900	991	684	.967	.765	542	.957	.489	371	.895		
"	15.	1.428	992	996	670	.960	.703	506	.926	.490	368	.890		
"	15.	1.428	984	996	670	.968	.742	526	.941	.466	352	.885		
" "	15.	1.531	1068	996	670	.962	.765	551	.926	.440	337	.873		
""	20.	1.477	1047	996	685	.961	.703	522	.918	.458	375	.833		
"	20.	1.520	1090	996	685	.951	.760	537	.965	.497	400	.848		
"	20.	1.576	1130	996	685	.951	.781	570	.935	.507	408	.848		
"	20.	1.568	1124	996	685	.950								
" "	21.	1.783	1224	995	668	.970	.846	604	.932	.503	393	.852		
	21.	1.773	1232	995	668	.957	.790	575	.915	.481	377	.850		
" "	21.	1.705	1190	995	668	.953	.803	575	.930	.483	373	.862		
""	21.	1.452	1019	995	668	.948								
				Avera	age	0.960	Average 0.936			Ave	Average 0.872			
± 0.0014							± 0.003 ± 0.005				0.005			
Aver	age of	front a	nd rear fa	ice, 0.	966 ±	- 0.0034	0.942 ± 0.0036 0.880 ± 0.000				- 0.0064			

the corresponding mean lamp deflections of galvanometer G_2 . The third and fourth columns contain the sensitiveness of galvanometers G_1 and G_2 , respectively, and the last column the values of the first column reduced to standard lamp and standard sensitiveness of both instruments. The series on the two faces of the disc are recorded and averaged separately, then combined with their probable errors in the general average at the end of Table VIII.

Tables VII. and VIII. give the following results: The average increase in the reading of G_1 for standard conditions is 0.966 mm. per second. From the thermal calibration, a deflection of 9.96 divisions corresponds to a temperature difference of I° C. Consequently the rise in temperature of the silver disc per second when the light passed:

(a) through air = $0.966 \div 9.96 = (0^{\circ}.0970 \pm 0^{\circ}.00034 \text{ C.}).$

(b) through red glass = $0.942 \div 9.96 = (0^{\circ}.0946 \pm 0^{\circ}.00036 \text{ C}.)$.

(c) through water cell = $0.880 \div 9.96 = (0^{\circ}.0884 \pm 0^{\circ}.00064 \text{ C}.)$.

The mass of the silver disc was 4.80 grams, its specific heat¹ at 18° C. = 0.0556; the mechanical equivalent of heat at 18° C. = 4.272 × 10^{7} ergs.² Consequently the energy of the standard radiation is

(a) through air, $0.0970 \times 4.80 \times 0.0556 \times 4.272 \times 10^7$

or $E_a = (1.108 \pm 0.004) \times 10^6$ ergs per second.

- (b) through red glass, $E_g = (1.078 \pm 0.004) \times 10^6$ " " "
- (c) through water cell, $E_{a} = (1.008 \pm 0.007) \times 10^6$ """""

Reflecting Power of the Surfaces Used.

According to Maxwell and Bartoli, the pressure in dynes per square centimeter for normal incidence is equal to the energy in ergs in unit volume of the medium. The energy in unit volume is made up of both the direct and reflected beams. If E is the intensity of the incident beam and ρ the reflection coefficient, the pressure $p = \frac{E(1 + \rho)}{V}$, where V is the velocity of light. The methods for measuring ρ and E have already been described. The determination of ρ for both sides of the vanes C and D was made as follows. The supports of the torsion balance were replaced by the divided circular plate A (Fig. 7), of a force table which could be rotated about a central, vertical axis. The rod about which the plate turned passed up through the plate and at its top the mirror holder bb was fastened. The vanes were freshly silvered and mounted on a plate glass carrier aa, which was held by a clamp against the back face of bb. The beam was directed on the vanes by the lens L_3 (Figs. 3 and 7) exactly as it had been in the pressure observations. After reflection from the vane the beam fell on a concave mirror M which projected an image of the vane upon a simple sheet bolometer B, forming the unknown resistance of a post-officebox bridge. The current was supplied from storage cells and the galvanometer was the same as used in the energy determinations, but fitted with low resistance coils. The bolometer was covered by the

¹U. Behn, Ann. Phys., IV., p. 266, 1900.

² Mean of Rowland's and Griffith's values, Phil. Trans., V., pp. 184, 496, 1893.

bell-jar used earlier. The mirror M, the bell-jar and bolometer were attached to the plate of the force table. The full line diagram shows the arrangement for reflection. The dotted figure shows the position for a measurement of the direct beam. All measurements of direct reflection were made for an angle of incidence of $12^{\circ}.5$.

The method of observing will be seen from the note-book record of a single series of measurements given in Table IX. In the table,



D and R indicate direct and reflected beams, respectively. The first and second columns contain the zero points and end of swings of the galvanometer G_1 , and the third column, the deflection. The remaining columns, in order, contain the lamp galvanometer deflection; the deflection of G_1 reduced to constant lamp; the means of each pair of D or R values; the means of alternate readings; and the final column, the quotients of the two preceding columns which are the reflection coefficients sought. In all, three series of measurements were made on the silver, and two series on the glass-silver

G1		Deflect		G1		Alternote	Deflection
Zero.	Turning Point.	G ₁	Lamp.	Reduced to Standard.	Averages.	Averages.	Coefficient.
R 350.0	159.5	190.5	132.6	143.8	1		
349.0	152.0	197.0	139.3	141.3	$\int 142.5$		
D 349.5	100.5	249.0	136.8	182.1		140.0	550
350.0	111.5	238.5	130.5	183.0	182.5	142.0	.779
R 346.0	177.0	169.0	119.3	141.2		100 7	F FF
347.0	171.0	176.0	124.4	141.7	j 141.5 j	182.5	.775
D 348.5	123.0	225.5	124.0	181.8	100 4	141 1	500
348.5	120.0	228.5	125.0	183.0	182.4	141.1	.773
R 345.0	172.0	173.0	122.6	141.0	1 100	100 7	880
345.0	171.0	174.0	124.0	140.3	f 140.6 J	182.7	.770
D 346.0	132.0	214.0	115.5?		1 102 0	141.0	550
346.0	124.0	221.0	120.7	183.0	\$ 183.0	141.2	.773
R 344.0	173.0	171.0	120.7	141.8	1 1 41 17	100 1	770
344.5	171.0	173.5	122.6	141.6	\$ 141.7	182.1	.778
D 346.0	119.0	227.0	125.3	181.0	1 101 0	141.0	701
346.0	117.5	228.5	126.0	181.3	5 181.2	141.0	./81
R 342.0	174.0	168.0	118.0	142.3	147 6	101 0	700
342.0	170.5	171.5	122.0	140.8	\$ 141.5	181.8	.780
D 347.0	130.0	217.0	119.0	182.3	100 5	747.4	777
347.0	134.0	213.0	116.7	182.7	\$ 182.5	141.4	.//5
R 341.5	174.5	167.0	118.0	141.3	1 141 0		
341.0	173.0	168.0	119.0	141.1	$\int 141.2$		
						Average	0.776

TABLE IX. October 31, 1902. Reflection Coefficient of Dg. Air.

faces of each vane. To get average coefficients which would represent the range of condition of the mirrors during the pressure measurements, the vanes were cleaned and new silver coatings deposited between each two series on the same vane. The reflection coefficients are collected in Table X. For each surface studied the diffused reflection for a beam which had traversed air was determined by setting the mirror holder for normal incidence. The diffuse energy reflected at an angle of 25° falling on the full aperture of the mirror M was measured, and the total diffuse energy for the hemisphere computed on the basis of the cosine law. If $I_{\theta}\partial A$ is the amount of diffuse radiation falling normally upon the area ∂A , distant r from the vane and at an angle θ with the incident radiation, then $I_{\theta}\partial A = I_0 \cos \theta$ ∂A . The total amount of diffuse radiation $= \int \int I_0 \cos \theta \partial A$,

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over the surface of the hemisphere $=\int_{0}^{\frac{\pi}{2}} 2\pi r^{2} I_{0} \cos \theta \sin \theta \partial \theta$ $=\pi I_{0}r^{2}$. This integral is the amount of the diffuse radiation in Table X. The force, due to radiation of intensity $I_{\theta}\partial A$, normal to the vein is $I_{\theta} \cos \theta \partial A$, and the total is equal to $\int_{0}^{\frac{\pi}{2}} 2\pi r^{2} I_{0} \cos^{2} \theta \sin \theta$ $\partial \theta = \frac{2}{3}\pi I_{0}^{2}$. It is thus seen that of the diffuse reflection, two

Г	ABLE	Χ.
~	a second second	

		С.				(\overline{f}_{g} .		
	Through Air.	Red Glass.	Water.	Diffuse.	Through Air.	Red Glass.	Water.	Diffuse.	
	92.8	94.5	88.9	0.98	77.8	75.9	80.8		
	89.8	90.8	86.0	0.92	77.6	76.6	80.0	1.6	
	90.8			1.23					
Average	91.1	92.7	87.5	1.04	77.7	76.3	80.4	1.6	
		D_{s} .			$D_{g^{\bullet}}$				
	95.0	96.3	91.5	2.2	77.6	76.5	81.0	2.8	
	92.0	94.0	90.4		76.7	75.2	79.7	2.2	
	94.8	95.0	92.3	0.8					
Average	93.9	95.1	91.4	1.5	77.2	75.9	80.4	2.5	

Reflection Coefficients in Percentages.

Air-Silver.					Glass-Silver.					
92	2.5	93.9	89.5	1.3	77.5	76.1	80.4	2.0		
		Со	rrected R	eflection (Coefficients.					

Average Reflection.

thirds is effective as light pressure. This increases the air-silver reflection coefficients by 0.9 per cent. and the glass-silver values by 1.3 per cent. The small glass rod d (Fig. 2), not present in the reflection measurements, decreased the reflecting area of the silvered surfaces in the pressure measurements by 1.54 per cent. The air-silver values are thus decreased by $0.92 \times 1.54 = 1.4$ per cent., and the glass-silver values by $0.78 \times 1.54 = 1.2$ per cent. The application of these two corrections gives the final corrected coefficients

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in Table X. The diffuse reflection of black coatings deposited by the method used in blackening the silver disc was measured and computed in the same manner as the diffused reflection from the vanes C and D. The agreement found by Angström¹ between the diffuse reflection of matte surfaces for normal incidence, and the cosine law was abundantly close for the present purpose. Five determinations of this reflection were made under different conditions and with different coatings. The values in percentages of the incident beam were 4.4 per cent., 4.5 per cent., 4.2 per cent., 4.6 per cent. and 5.2 per cent.; average, 4.6 per cent. Thus only 95.4 per cent. of the incident beam was absorbed by the black coating on the silver disc in producing the temperature increase observed. Hence the true energy of the beam is equal to the observed energy divided by 0.954.

The silver disc, diameter 13.3 mm., used in the energy measurements, received long waves and scattered radiation which passed round and through the light pressure vanes of diameter 12.8 mm. This amount was experimentally determined for both thin and thick silver coatings in order to approximate to the average condition of the coatings in the light pressure measurements and it was found to average (a) through air, 1.40 per cent.; (b) through red glass, 1.44 per cent.; (c) through water, 0.46 per cent. On this account the energy E of the standard radiation must be reduced by the above percentages.² Applying these corrections and the corrections due to the diffused radiation from the black coating on the silver disc, the energy of the standard radiation becomes

(a) through air,	$E_a \times \frac{0.986}{0.954};$
(b) through red glass,	$E_g \times \frac{0.986}{0.954};$
(c) through water,	$E_{w} \times \frac{0.995}{0.954}.$

¹ K. Ångström, Wied. Ann., XXVI., 271, 1885.

² As the average pitch of the cone of the incident beam was about one part in forty, no correction need be applied for inclination. Furthermore, the inside of the bell-jar was blackened and the zero of the balance was so chosen that energy reflected from the window admitting the beam could produce no pressure effects.

Hence the pressure produced by standard radiation calculated by Maxwell's formula,

$$p = \frac{E(1+\rho)}{3 \times 10^{10}},$$

since $\rho = 0.848$, becomes

(a) through air
$$p_a = E_a \times \frac{1.848}{3 \times 10^{10}} \times \frac{0.986}{0.954}$$

= 1.108 $\times \frac{1.848}{3 \times 10^{10}} \times \frac{0.986}{0.954} \times 10^6$ dynes
= (7.05 \pm 0.03) $\times 10^{-5}$ dynes;
(b) through red glass $p_g = E_g \times \frac{1.848}{3 \times 10^{10}} \times \frac{0.986}{0.954}$
= 1.078 $\times \frac{1.848}{3 \times 10^{10}} \times \frac{0.986}{0.954} \times 10^6$ dynes
= (6.86 \pm 0.03) $\times 10^{-5}$ dynes;
(c) through water $p_w = E_w \times \frac{1.848}{3 \times 10^{10}} \times \frac{0.995}{0.954}$

$$= 1.008 \times \frac{1.848}{3 \times 10^{10}} \times \frac{0.995}{0.954} \times 10^{6} \text{ dynes}$$
$$= (6.48 \pm 0.04) \times 10^{-5} \text{ dynes}.$$

A comparison of observed and computed pressures follows :

	Observed Values in 10 ⁻⁵ Dynes.	Computed Values in 10 ⁻⁵ Dynes	Obs. — comp. in Percentages
Through air,	$p_a = 7.01 \pm 0.02^1$	7.05 ± 0.03	0.6
Through red glass	$p_g = 6.94 \pm 0.02$	6.86 ± 0.03	+ 1.1
Through water,	$p_w = 6.52 \pm 0.03$	6.48 ± 0.04	0.6

¹ The pressure and energy measurements for the three different wave groups through air, red glass and water cell constitute three independent experiments. In the values for pressure, 7.01, 6.94, 6.52, equality is not to be looked for. The difference arises from the different reflecting power of the 45° glass plate (Fig. 3) for the different beams and from the fact that the indications of the lamp galvanometer G_2 connected with the bolometer R, were probably not strictly proportional to energy for throws differing as widely as 33, 60 and 100, which, roughly, were the relative intensities of the beams through water cell, red glass and air. The function of the lamp bolometer and galvanometer was purely to keep a check on the small variations of the lamp which rarely fluctuated more than 10 per cent. on either side of the mean value.

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An estimate of the approximate magnitude of the gas action not eliminated by the ballistic method of observation, may be reached from the following considerations.

When radiation falls upon a vane of the torsion balance, part of it is absorbed by the silver surface. From the amounts directly and diffusely reflected, as given in Table X., the amount transmitted by the average surface (experimentally determined but not given in Table X.), the effect of the glass rod and the reflection coefficient of the glass surface, it was found that, when the silver side of the vane was toward the radiation source, the absorption coefficient for radiation through air was 6 per cent. and when the glass surface was forward, it was 18 per cent.

The total force acting on the vane is made up of two parts, that due to radiation pressure and that due to gas action. Let F_r be the force due to the first cause, assuming that all the radiation is absorbed, and F_g the effect due to the second, on the same condition. Then the total effect, when the silver side of the vane is forward and the radiation is "through air," is $1.92 F_r + 0.06 F_g$. When the glass side is forward the total effect is $1.776 F_r - 0.18$ F_g . Making these expressions equal to the reduced deflection (Table III., columns 11 and 12) on the silver and glass surfaces respectively, we have two equations by means of which the values of F_r and F_g may be obtained. Hence the effect due to gas action on each face of the vane is approximately determinate, as is also the part $(0.06F_g)$ not eliminated when we average the two columns to obtain column 13.

Applying this method to all the results of Table III. (with the exception of those results taken with poor mirrors as shown by our notes), the gas action present in the ballastic deflections "through air" is 0.8 per cent. Applying the corresponding data and equations to Table IV., the gas action present in the red glass values is I.I per cent. and in the water cell values, 0.3 per cent. The sign of F_g comes out negative, which means that the gas action was suction.

This reasoning assumes that the glass faces of the vanes during the six seconds' exposure are not warmed by absorption nor by the conduction of heat through the thin glass from the silver coating.

The effect of any such absorption or conduction would be to diminish the computed gas action. As estimated from the static observations, the gas action in the ballistic measurements is comparable in magnitude with the computed values obtained above, and of the same sign. Both results show that the uneliminated gas action by the most liberal estimate cannot have exceeded one per cent. of the radiation pressure. Because of its smallness and indefiniteness no correction for gas action has been made to the final pressure values. If corrections were applied its effect would be to slightly reduce the observed pressures.

Aside from the measurements of pressure and energy for which the probable errors are given, the percentage accuracies in the other measurements entering into the computations, and their effects upon the final result follow:

- I. Quantities which affect individual series :
 - (a) Pressure values —

(4)	Tressure values						
` /				Per cen	t.	Per	cent.
	Period of balance,	Τ,	accurate	to 0.2;	effect on	result	0.0
	Lever arm of balance,	l,	" "	0.1;	" "	" "	0.0
	Constant of galvanometer	G_2 ,	"	0.5;	" "	"	0.0
	Estimate of possible error to changing ratio of per of G_2 to length of expos	due riod sure					
	of bolometer,		"	0.4;	"	"	0.1
(b)	Energy values —						
	Constant of galvanometer	G_1 ,	accurate	to 0.1;	effect on	result	0.0
		G.,	"	0.5;	" "	" "	0.0

2. Quantities which affect final averages :

(a) Pressure values —

	Per cent.			Per cent.		
Torsion of fiber,	accurate	to $0.2;$	effect on	\mathbf{result}	0.2	
Reducing factor 1.357,	"	0.1;	""	" "	0.1	
Reducing factor 1.550 for G_2	, "	0.2;	"	"	0.2	
Reflection of surfaces of vane	es, ''	0.4;	"	""	0.2	

(b) Energy values —

Mass of silver disc,	accurate	to 0.1;	effect of	n resul	t 0.1
Thermal calibration of disc,	" "	0.5;	" "	""	0.5
Diffuse reflection black coat-					
ing,	" "	5.0;	" "	""	0.1

From the agreement, within the probable error, of the air, red glass and water values with the theory, it appears that the radiation pressure depends only upon the intensity of the radiation and is independent of the wave-length.

The Maxwell-Bartoli theory is thus quantitatively confirmed within the probable errors of observation.

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