THE PRESSURE DUE TO RADIATION.

(Second Paper.)

By E. F. Nichols and G. F. Hull.

IN a preliminary communication to the American Physical Society, meeting with Section B of the American Association for the Advancement of Science at Denver, August, 1901, the writers presented the results of observations they had made on the pressure due to radiation. An abstract of that paper appeared in Science, October, 1901, and the full paper in this journal in November of the same year. In the present paper the writers wish to present a more complete historical statement than appeared in the preliminary paper, alterations which were made in the earlier methods, and more recent observations and data under conditions permitting greater accuracy in measurement.

As early as 1619 Kepler[1] announced his belief that the solar repulsion of the finely divided matter of comets' tails was due to the outward pressure of light. On the corpuscular theory of light Newton[2] considered Kepler's idea as plausible enough, but he was of the opinion that the phenomenon was analogous to the rising of smoke in our own atmosphere. In the first half of the eighteenth century DeMairan and DuFay[3] contrived elaborate experiments to test this pressure of light theory in the laboratory, but, because of the disturbing action of the gases surrounding the illuminated bodies employed in the measurements, they obtained wholly confusing and contradictory results. Later in the same ceutury the Rev. A. Ben-

[1] De Mairan, Traité physique et historique de l'Aurore Boréale (seconde edition), pp. 357–358, Paris, 1754.

[2] Isaaci Newtoni, Opera quae Existent Omnia. Samuel Horsley, LL.D., R.S.S., Tom. III., pag. 156, Londinium, 1782.

[3] De Marian, l. c., p. 371. This treatise contains also the accounts of still earlier experiments by Hartsoeker, p. 368, and Homberg, p. 369. The later experiments are of more historic than intrinsic interest.

net[1] performed further experiments but could find no repulsive force not traceable to convection currents in the gas surrounding the body upon which the light was projected, due in his opinion to the heating effect of the rays. Finding no pressure due to radiation, he made the following unique suggestion in support of the wave theory of light: "Perhaps sensible heat and light may not be caused by the influx or rectilinear projection of fine particles, but by the vibrations made in the universally diffused caloric or matter of heat or fluid of light. I think modern discoveries, especially those of electricity, favor the latter hypothesis." In the meantime Euler,[2] accepting Kepler's theory attributing the phenomenon of comets' tails to light pressure, had hastened to the support of the wave theory by showing theoretically that a longitudinal wave motion might produce a pressure in the direction of its propagation upon a body which checked its progress. In 1825 Fresnel[3] made a series of experiments, but arrived at no more definite conclusion than that the repulsive and attractive forces observed were not of magnetic nor electric origin.

Crookes[4] believed in 1873 that he had found the true radiation pressure in his newly invented radiometer and cautiously suggested that his experiments might have some bearing on the prevailing theory of the nature of light. Crookes' later experiments and Zöllner's[5] measurements of radiometric repulsions showed that the radiometric forces were in some cases 100,000 times greater than the light pressure forces with which they had been temporarily confused. Zöllner's experiments are among the most ingenious ever tried in this field of work, and he missed the discovery of the true radiation pressure by only the narrowest margin. An excellent bibliography of the whole radiometric literature is given by Graetz,[6] and an account of some of the older experiments not mentioned above is given by Crookes.[7]

[1] A. Bennet, Phil. Trans., p. 81, 1792.
[2] L. Euler, Histoire de l'Academie Royale de Berlin (2), p. 121, 1746.
[3] A. Fresnel, Ann. Chem. et Phys., XXIX., pp. 57, 107, 1825.
[4] W. Crookes, Phil. Trans., p. 501, 1873.
[5] F. Zöllner, Pogg. Ann., CLX., pp. 156, 296, 459, 1877.
[6] L. Graetz, Winkelmann's Handbuch der Physik, 2 b, p. 262. Breslau, 1896.
[7] W. Crookes, l. c., p. 501.

In 1873 Maxwell,[1] on the basis of the electromagnetic theory, showed that if light were an electromagnetic phenomenon, pressure should result from the absorption or reflection of a beam of light. After a discussion of the equations involved, he says: "Hence in a medium in which waves are propagated there is a pressure in the direction normal to the waves and numerically equal to the energy in unit volume." Maxwell computed the pressure exerted by the sun on the illuminated surface of the earth and added: "It is probable that a much greater energy of radiation might be obtained by means of the concentrated rays from an electric lamp. Such rays falling on a thin metallic disc, delicately suspended in a vacuum, might perhaps produce an observable mechanical effect."

Apparently independent of Maxwell, Bartoli[2] announced in 1876 that the Second Law of Thermodynamics required the existence of a pressure due to radiation numerically equal in amount to that derived by Maxwell. Bartoli's reasoning holds for all forms of energy streams in space and is of more general application than Maxwell's equations. Bartoli contrived elaborate experiments to verify this theory, but was balked in the search, as all before him had been, by the complicated character of the gas action which he found no way of eliminating from his experiments.

After Bartoli's work the subject was dealt with theoretically by Boltzmann,[3] Galitzine,[4] Guillaume,[5] Heaviside,[6] and more recently by Goldhammer.[7] Fitzgerald,[8] Lebedew,[9] and Hull[0] have discussed the bearing of radiation pressure upon the Newtonian law of gravitation with special reference to the repulsion of comets' tails by the sun.

[1] J. C. Maxwell, A Treatise on Electricity and Magnetism (1st edition), II., p. 391, Oxford, 1873.

[2] A. Bartoli, Sopra i movementi prodotti della luce e dal calorie, Florence, Le Monnier (1876), also Nuovo Cimento, XV., p. 193, 1884.

[3] L. Boltzmann, Wied. Ann., XXII., pp. 31, 291, 1884.

[4] B. Galitzine, Wied. Ann., XLVII., p. 479, 1892.

[5] Ch. Ed. Guillaume, Arch. de Gen. (3), XXXI., p. 121, 1894.

[6] O. Heaviside, Electromagnetic Theory, I., p. 334, London, 1893.

[7] D. A. Goldhammer, Ann. Phys., IV., p. 834, 1901.

[8] G. F. Fitzgerald, Proc. Roy. Soc. Dub., 1884.

[9] P. Lebedew, Wied. Ann., XLV., p. 292, 1892. Astrophys. Jour., XIV., p. 155, 1902.

[0] G. F. Hull, Trans. Astron. Soc. Toronto, p. 123, 1901.

The theory of radiation pressure combined with the known properties of negative electrons has recently been more or less speculatively applied by Arrhenius [1] to the explanation of many cosmical and terrestrial phenomena among which the following may be mentioned: The solar corona, zodiacal light, gegenschein, comets, origin of cometary and meteoric material in space, the emission of gaseous nebulæ, the peculiar changes observed in the nebula surrounding Nova Persei, the northern light, the variations in atmospheric electricity and terrestrial magnetism and in the barometric pressure. Swartzschild [2] computed from radiation pressure on small spherical conductors the size of bodies of unit density for which the ratio of radiation pressure to gravitational attraction would be a maximum.

Before the Congrès International de Physique in 1900, Professor Lebedew,[3] of the University of Moscow, described an arrangement of apparatus which he was using at that time for the measurement of light pressure. He summarizes the results already obtained as follows: "Les résultats des mesures que j'ai faites jusqu'ici peuvent se résumer ainsi: L'expérience montre qu'un faisceau lumineux incident exerce sur les surfaces planes absorbantes et réfléchissantes des pressions qui, aux erreurs près d'observation, sont égales aux valeurs calculées par Maxwell et Bartoli." No estimate of the "errors of observation" was given in the paper nor other numerical data. Unfortunately the proceedings of the Paris Congress did not reach the writers nor any intimation of the methods or results of Professor Lebedew's work until after the publication of their own preliminary experiments.

In the preliminary paper already referred to, the writers presented the results they had obtained by measurement of radiation pressure at various gas pressures. The main arguments underlying the method of measurement of the radiation pressure follow.

In the experiments of earlier investigators every approach to the experimental solution of the problem of radiation pressure had been

[1] S. A. Arrhenius, Lehrbuch der Cosmischen Physik, Leipzig, 1903, pp. 149–158, 200–208, 226, 920–925.

[2] K. Swartschild, Kgl. Bayer. Akademie d'Wissenschaften, XXXI., 293, 1901.

[3] P. Lebedew, Rapports présentés au Congrès International de Physique (2), p. 133, Paris, 1900.

balked by the disturbing action of gases which it is impossible to remove entirely from the space surrounding the body upon which the radiation falls. The forces of attraction or repulsion, due to the action of gas molecules, are functions, first, of the temperature difference between the body and its surroundings, caused by the absorption by the body of a portion of the rays which fall upon it; and second, of the pressure of the gas surrounding the illuminated body. In the particular form of apparatus used in the present study the latter function appears very complicated, and certain peculiarities of the gas action remain inexplicable upon the basis of any simple group of assumptions which the writers have so far been able to make.

Since we can neither do away entirely with the gas nor calculate its effect under varying conditions, the only hopeful approach which remains is to devise apparatus and methods of observation which will reduce the errors due to gas action to a minimum. The following considerations led to a method by which the elimination of the gas action was practically accomplished in the present experiments.

1. The surfaces which receive the radiation, the pressure of which is to be measured, should be as perfect reflectors as possible. This will reduce the gas action by making the rise of temperature due to absorption small, while the radiation pressure will be increased; the theory requiring that a beam, totally reflected, exert twice the pressure of an equal beam, completely absorbed.

2. By studying the action of a beam of constant intensity upon the same surface surrounded by air at different pressures, certain pressures may be found where the gas action is less than at others.

3. The apparatus — some sort of torsion balance — should carry two surfaces symmetrically placed with reference to the rotation axis, and the surfaces on the two arms should be as nearly equal as possibly in every respect. The surfaces or vanes should be so constructed that if the forces due to gas action (whether suction or pressure on the warmer surface) and radiation pressure have the same sign in one case, a reversal of the suspension should reverse the gas action and bring the two forces into opposition. In this way a mean of the forces on the two faces of the suspension should be, in part at least, free from gas action.

4. Radiation pressure, from its nature, must reach its maximum value instantly, while observation has shown that gas action begins at zero and increases with length of exposure, rising rapidly at first, then more slowly to its maximum effect, which, in many of the cases observed, was not reached until the exposure had lasted from two and a half to three minutes. For large gas pressures, an even longer exposure was necessary to reach stationary conditions. The gas action may be thus still further reduced by a ballistic or semi-ballistic method of measurement.

Ballistic observations of radiation pressure were made at air pressures ranging from 96.3 mm. to 0.06 mm. of mercury. The average radiation pressure of the standard beam was found to be 1.05×10^{-4} dyne with a probable error of 6 per cent. To compare this value of the pressure with the theoretical value as given by the Maxwell-Bartoli formula,

$$p = \frac{E(1 + \rho)}{V},$$

it was necessary to measure E, the energy of the beam. This was done by measuring the resistance of a platinum disc P (Fig. 1) when it was exposed to the radiation and by determining the heat generated in P by an electrical current across AB when the resistance of P was equal to the value it possessed when it was exposed to the beam. Using the value of the energy thus obtained and 0.92[1] as the reflection coefficient of the silver surfaces, it was found that the pressure directly observed was about 20 per cent. lower than that computed from Maxwell's formula.

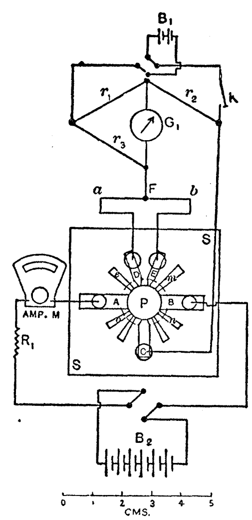

Fig. 1.

After the publication of the paper it was discovered that in dissolving the silver from the platinum when the bolometer was made, the acid had eaten away the silver from the strips A and B for a

[1] This value was obtained from the measurements of Langley, Rubens, Nichols and Paschen for the assumed mean wave-length of the energy of the beam.

distance of nearly a millimeter under the asphalt coating. The resistance 0.278 ohm given for the disc was thus too high. It was impossible to redetermine the resistance by the direct method because of an accident to the bolometer by which the disc was nearly severed from the strip B. The disc was therefore carefully torn away from its supports, mounted on a glass plate and cut on a dividing engine into strips, 1 and 2 mm. wide, parallel to AB. The resistance along these strips was measured by the fall of potential method. The resistance was found to vary slightly in different parts of the disc due to lack of uniformity in the thickness of the metal. After many measurements, an average value was reached and the resistance of the disc computed theoretically as follows: —

The resistance of a conducting sheet of infinite extent, when the current enters and leaves the sheet by electrodes[1] of relatively great conductivity, is $\sigma/4\pi C$, where σ is the resistance of any square of the sheet, and C is the electrostatic capacity of the two electrodes. If the electrodes are cylinders, the lines of flow are circles orthogonal to them. When the sheet, in place of being infinite, is bounded by one of these circular lines of flow, the resistance is $\sigma/2\pi C$. In particular if the electrodes of radii r are on a diameter of this circular sheet of radius R, then the resistance can be shown to be

$$\frac{\sigma}{\pi}\log_e \frac{2r^2 + R^2 + R\sqrt{4r^2 + R^2}}{2r^2}.$$

Assuming for the moment that the leading-in strips of the bolometer (Fig. 1) were of great conductivity compared to that of the thin platinum sheet and that they terminated in circular arcs orthogonal to this circular sheet, the resistance would be, giving to r the value of 2.79 mm. and to R the value 11.25 mm., $0.922 \times \sigma$. But the leading-in strips terminated on the boundary of the large circle. The resistance was therefore altered by two facts — the lines of flow were changed and the distance between the electrodes was increased. The latter is the important item. It is necessary therefore to find approximately the resistance of these gibbous portions of the large disc previously considered as electrodes. This may be done by estimating the area of these parts and by considering the average

[1] J. J. Thomson, Electricity and Magnetism, 2d edition, p. 314, Cambridge, 1897.

equipotential line as midway between the cord and arc of the cylindrical electrode. It results that the amount to be added on account of this calculation is $0.471 \times \sigma$. Hence the resistance between the electrodes is now $(0.922 + 0.471)\,\sigma = 1.393 \times \sigma$. The value of σ as found by the fall of potential method was 0.148 at 19° C. When corrected for the temperature of the disc exposed to the lamp, σ becomes 0.160. Hence the resistance of the disc when hot was $1.393 \times 0.160 = 0.221$ ohm.[1] Substituting this computed value of the resistance in place of the one used, the energy of the standard beam becomes $0.221 \times 0.75 \times 10^7$ ergs-seconds and

$$p = \frac{1.92 \times 0.221 \times 0.75 \times 10^7}{3 \times 10^{10}} = 1.05 \times 10^{-4} \text{ dynes.}$$

This result is in accidental agreement with the observed pressure. If necessary corrections, determined by later experiment, had been applied, the difference between the observed pressure and the pressure computed from the energy measurements would have been about three per cent. Moreover the probable error of the final result was roughly double this amount.

In the November number of the Annalen der Physik for 1901 Professor Lebedew[2] published the results of a more varied series of measurements of radiation pressure than the early measurements of the present writers. The principal difference between the methods employed by him and by the writers for determining the pressure was that he used very thin metallic vanes surrounded by gas at extremely low pressures, thus following Maxwell's suggestion literally, while the writers used silvered glass vanes and worked at large gas pressures for which the gas action had been carefully and exhaustively studied and found to be negligibly small for short exposures. From our knowledge of the variation of gas action in different vacua, we feel sure that our method would not have been successful in high vacua because of the relatively large gas action. Professor Lebedew's own results, with blackened vanes of lower heat conductivity, show that his success in eliminating gas distur-

[1] The resistance of a trial disc was measured experimentally with the result that the experimental value differed from the theoretical by about one per cent.

[2] P. Lebedew, Ann. Phys., VI., 433, 1901.

bance was due to the high heat conductivity of thin vanes rather than to the high vacua employed.

Professor Lebedew's[1] estimate of the accuracy of his work is such as to admit of possible errors of twenty per cent. in his final results. An analysis of Professor Lebedew's paper and comparison with our preliminary experiments seems to show that his accidental errors were larger than ours, but through the undiscovered false resistance in the bolometer our final results were somewhat further from the theory than his. Either of the above researches would have been sufficient to establish the *existence* of a pressure due to radiation, but neither research offered, in our judgment, a satisfactory *quantitative* confirmation of the Maxwell-Bartoli theory.

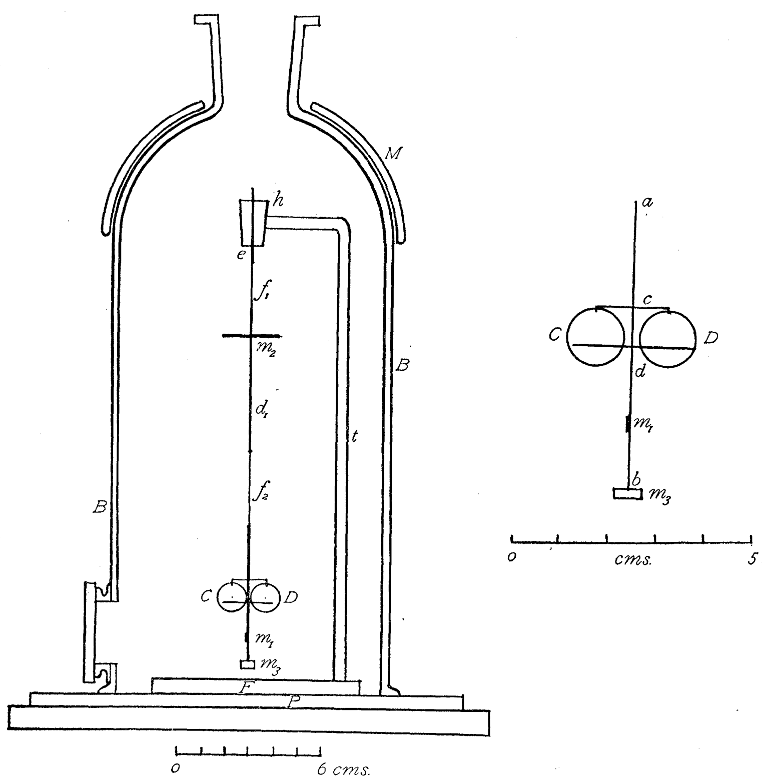

Fig. 2.

[1] P. Lebedew, Ann. Phys., VI., 457, 1901.

Later Pressure Measurements.

Description of Apparatus.—The Torsion Balance.

The form of suspension of the torsion balance, used to measure radiation pressure in the present study, is seen in Fig. 2 and is described in detail in our preliminary paper. Its essential parts are a fine glass rod *ab* to which was attached a weight m_3, a scale mirror m_1, a cross arm *c* holding the mirrors *C* and *D*; a quartz fiber f_2; a glass rod d_1 to which was attached a small magnet m_2 and a silk fiber f_1. The cover glasses *C* and *D* which served as vanes were silvered and brilliantly polished and were so hung on the small hooks that both silver or both glass faces were presented to the light. A semicircular magnet *M*, fitted to the vertical curvature of the bell-jar, was used to direct the suspended magnet m_2 and thus to control the zero position of the torsion balance. By turning *M* through 180°, the opposite faces of the vanes *C* and *D* could be presented to the light.

The Arrangement of Apparatus.

A horizontal section of the apparatus through the axis of the light beam is shown in Fig. 3. An image of the aperture d_3, very brightly illuminated by the white-hot carbon S_1 and the lenses L_1 and L_2, was formed by the lens L_3 in the plane B_1. A shutter S_2 controlled by a magnetic escapement operated by the seconds contact of a standard clock permitted the exposure of the beam upon the vanes for any whole number of seconds. The stops S_3 and S_4 were so arranged that when the lens L_3 was against either stop the image of d_3 was central upon one of the vanes. A glass plate inclined at an angle of 45° to the axis of the beam and a lens L_5 gave an image of d_3 upon one arm of the bolometer at *R*. This bolometer was of sheet platinum 0.001 mm. thick rolled in silver. The strip was cut out in the form shown in Fig. 4, and mounted on a thin sheet of slate *S*. Two windows had been cut in the slate behind the strips at *AB*, *CD* where the silver had been removed, leaving the thin platinum. The platinum surfaces were blackened by Kurlbaum's process. The image from L_5 (Fig. 3) fell at *D*. The silver ends between *A* and *C* were connected with *E* and *F* respec-

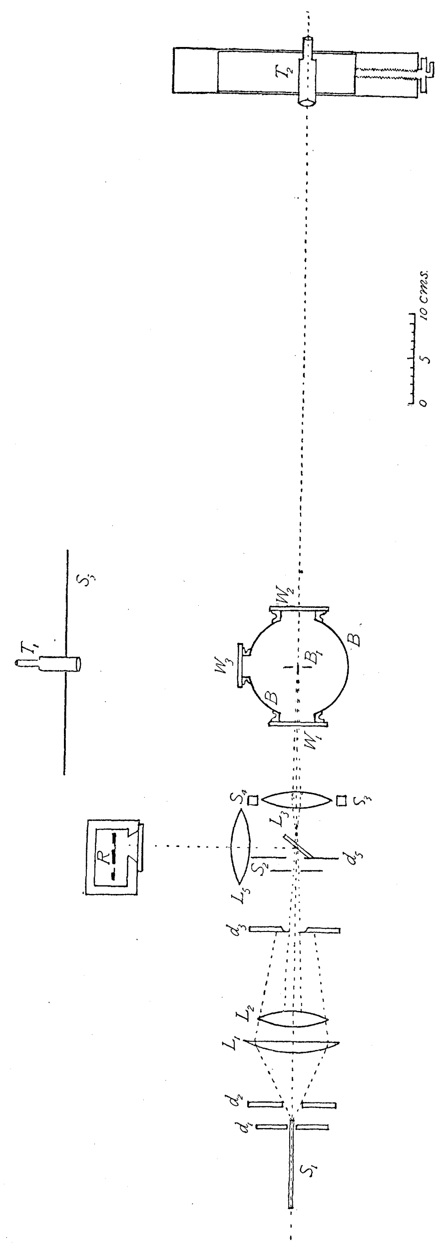

Fig. 3.

tively. On the heavy wire *EF* a sliding contact *c* served to balance the bridge, all four arms of which are shown in the figure.

Methods of Observation.

The observations leading to the results given later were of three different kinds: (1) The calibration of the torsion balance; (2) the measurement of the pressure of radiation of the law in terms of the constant of the balance; and (3) the measurement of the energy of the same beam in erg-seconds by the rate of temperature rise of a blackened silver disc, of known mass and specific heat.

1. The determination of the constant of the torsion balance was made by removing the vanes *C* and *D* and accurately measuring the period of vibration. Its moment of inertia was easily computed from the masses and distribution of the various parts about the axis of rotation. The moment of torsion

for 1 mm. deflection on a scale 105 cm. distant was 0.363×10^{-5} dyne $\times$ cm. This value divided by one half the distance between the centers of the light spots on the two vanes gave the force in dynes per scale division deflection. As the light spots were circles

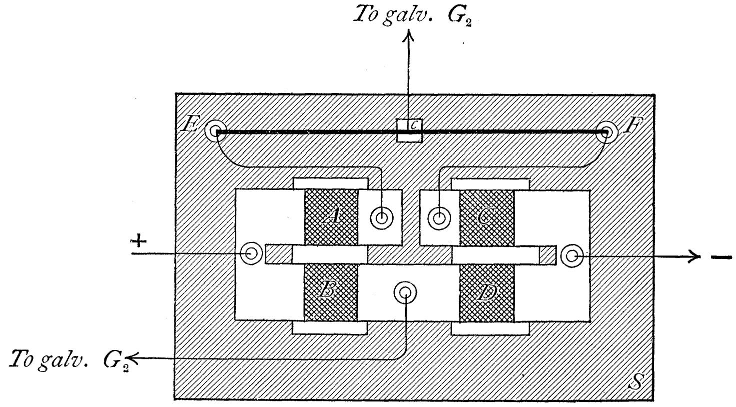

Fig. 4.

11.25 mm. in diameter the area of the image was very nearly 1 $(\text{cm.})^2$, hence the above procedure gave roughly the pressure in dynes per square centimeter.

2. In the measurements of radiation pressure, it was easier to refer the intensity of the beam at each exposure to some arbitrary standard which could be kept constant than to try to hold the lamp as steady as would otherwise have been necessary. For this purpose, the bolometer at R (Fig. 3) was introduced, and simultaneous observations were made of the relative intensity of the reflected beam by the deflection of the galvanometer G_2, and the pressure due to the transmitted beam by the deflection of the torsion balance. The actual deflection of the balance was then reduced to a deflection corresponding to a galvanometer deflection of 100 scale divisions. The galvanometer sensitiveness was carefully tested at the beginning and end of each evening's work. All observations of pressure were thus reduced to the pressure due to a beam of fixed intensity.

At each series of radiation pressure measurements, two sets of observations were made. In one of these sets, static conditions were observed, and in the other, the deflections of the balance due to short exposures were measured. In the static observations, each vane of the balance was exposed in turn to the beam from the lamp, the exposures lasting until the turning points of the swings showed that stationary conditions had been reached. The moment of pressure of radiation and gas action combined would thus be equal to the product of the static deflection and the constant of the balance. The torsion system was then turned through 180° by rotating the outside magnet, and similar observations were made on the reverse side of the vanes. All turning points of the swinging balance in these observations were recorded. From the data thus obtained the resultant of the combined radiation and gas forces could be determined for the time of every turning point. Every value was divided by the deflection at standard sensitiveness of the galvanometer G_2 read at the same time and was thus reduced to a standard lamp. Results thus obtained, together with the ballistic measurements, showed the direction and extent of the gas action as well as its variation with length of exposure.

The reasons for reversing the suspension follow: The beam from the lamp, before reaching the balance, passed through three thick glass lenses and two glass plates. All wave-lengths destructively absorbed by the glass were thus sifted out of the beam by the time it reached the balance vanes. The silver coatings on the vanes absorbed therefore more than the glass. The radiation pressure was always away from the source irrespective of the way the vanes were turned, while the gas action would be exerted mainly on the silvered sides of the vanes.

At the close of the pressure and energy measurements, when the reflecting power of the silver faces of the vanes was compared with that of the glass-silver faces, the reflection from the silver faces was found very much higher than that for the glass faces backed by silver. This result was the more surprising because the absorption of the unsilvered vanes was found by measurement to be negligibly small.[1] This unexpected difference in reflecting power of the two faces of the mirrors prevented the elimination of the gas action, by the method described, from being as complete as had

[1] Lord Rayleigh records a similar difference between the reflection from air-silver and glass-silver surfaces. Scientific Papers, Cambridge, II., pp. 538–539, 1900.

been hoped for. But by choosing a gas pressure where the gas action after long exposure is small, the whole gas effect during the time of a ballistic exposure may be so reduced as to be of little consequence in any case.

By exposing each of the vanes in turn and by reversing the suspension and averaging results, nearly all errors due to lack of symmetry in the balance or in the position of the light images with reference to the rotation axis, or errors due to lack of uniformity in the distribution of intensity in different parts of the image, could be eliminated.

The changing character of the gas action, both with time of exposure and gas pressure surrounding the balance vanes is well illustrated in eight series of static observations in which the glass faces of both vanes were exposed.[1] The results obtained on the two vanes were averaged and plotted as curves in Fig. 5, where static deflections due to combined radiation pressure and gas action are shown as ordinates and duration of exposure, in seconds, as abscissæ.[2] A horizontal line through the diagram gives the mean value of the moment of radiation pressure computed from the data in Table II. Decrease of the deflection with time indicates gas repulsion on the warmed silver faces and increase in deflection, gas suction. It will be seen from the curves that beginning at a gas pressure of 66 mm. of mercury, the gas action was repulsion changing to suction in passing from 19.8 to 11.2 mm. In the last two cases the total gas action is small. For lower pressures the suction increases to 0.05 mm. At a gas pressure of 0.02 mm. the gas action is again a strong repulsion.

The curves indicate the existence of two gas pressures, at which the gas action in our arrangement of apparatus should be zero, one between 19.8 and 11.2 mm. and the other between 0.05 and 0.02 mm.[3] The former region was chosen for the ballistic measurements

[1] Observations were also made on the silver faces, but the gas action when the glass faces were exposed was considerably greater that for the silver faces, so the least favorable case is shown.

[2] Ordinates of the curves are proportional to moments.

[3] Crookes in his work with the radiometer discovered certain gas pressures for which the combined gas and radiation forces neutralized, but as he did not discriminate between forces due to radiation and gas forces his results were apparently capricious and his reasoning somewhat confused. See Phil. Trans., p. 519, 1875.

and nearly all of the observations were made at a gas pressure of approximately 16 mm. Even for the two pressures where the decrease in the static deflection was most rapid, *i. e.*, at gas pres-

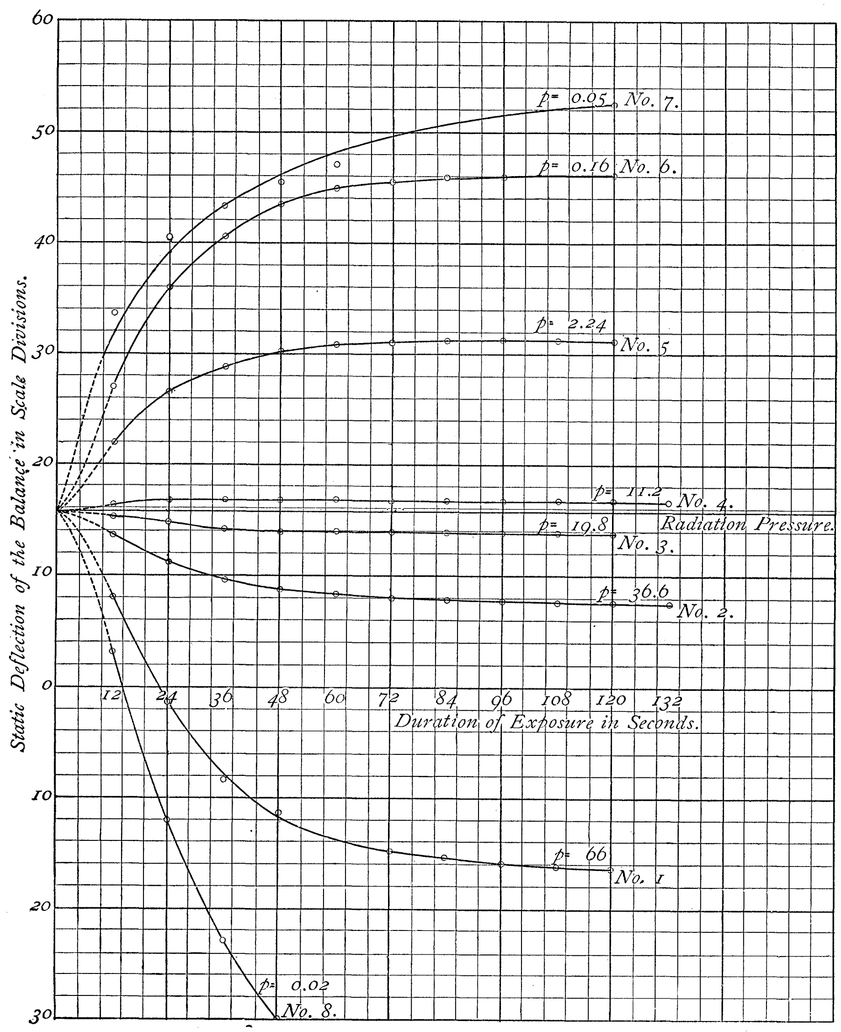

Fig. 5.

sures of 66 and 0.02 mm., the first throw was always in the direction of radiation pressure. The gas action is strongly influenced by very slight changes in the inclination of the plane of the vanes to the vertical and also by any object introduced under the bell-jar anywhere near the vanes. For instance, a very considerable effect was observed when a small vessel of phosphoric anhydride was placed under the jar behind the vanes, though the nearest wall of the vessel was separated from the vanes by a distance of at least 3 cm.

During the observations, the polished silver coatings on the vanes deteriorated rapidly; new coatings rarely lasted for more than two evenings' work. As the balance had to be removed and the mirrors taken from the hooks, silvered, polished, and replaced a great number of times during the entire series of measurements, although great care was taken in setting the plane of the vanes vertical, it is not likely that precisely the same conditions for gas action were ever repeated. The principal value of the static results was in indicating favorable gas pressures for work, rather than affording quantitative estimates of the gas action in short exposures. The dotted parts of the curves are not based on results of observation and might perhaps have been omitted without loss.

It was plain, therefore, that further elimination of the gas action must be sought in exposures so short that the gas action would not have time to reach more than a small fraction of its stationary value. This led to the method of ballistic observations.

The Ballistic Observations.

In passing from the static to the ballistic observations it must always be possible to compute the static equivalent of the ballistic swings. Furthermore, the exposures should be made as short as possible without reducing the size of the swing below a value which can be accurately measured.

If the exposure lasts for one half the period of the balance, the deflection, if the gas action be small and the damping zero, is equal to 2θ, where θ is the angle at which the torsion of the fiber will balance the moment produced by the radiation pressure. If the duration of the exposure be one quarter of the period of the balance, the angle of deflection is $\theta\sqrt{2}$. The deflection is thus reduced by 30 per cent., but the effect of the gas action is reduced in greater proportion. It was decided therefore to expose, for six seconds, one quarter of the balance period. Neglecting the gas action, the equation[1] of motion of the balance is given by

$$\varkappa\frac{\partial^2\theta}{\partial t^2} + 2\varepsilon\frac{\partial\theta}{\partial t} = -G\theta + L$$

where $\varkappa =$ the moment of inertia of the torsion balance,

$\varepsilon =$ the damping constant,

$G =$ the moment of torsion of the fiber for $\theta = 1$ radian,

and $L =$ the moment of the radiation force.

The solution of this equation is

$$\theta = \frac{L}{G}\left\{1 - e^{-\frac{\epsilon}{\kappa}t}\cos\sqrt{\frac{G}{\varkappa} - \frac{\varepsilon^2}{\varkappa^2}}\,t\right\}$$

$$= \frac{L}{G}\left\{1 - e^{-\frac{\epsilon}{\kappa}t}\cos 2\pi\frac{t}{T}\right\} \qquad (1)$$

the constants of integration having been determined from the condition that

$$\theta = \frac{\partial\theta}{\partial t} = 0 \text{ when } t = 0.$$

When

$$t = \frac{T}{4}, \quad \theta = \frac{L}{G}$$

and

$$\frac{\partial\theta}{\partial t} = \frac{L}{G}\left(\frac{\varepsilon}{\varkappa}e^{-\frac{\epsilon}{\kappa}t}\cos 2\pi\frac{t}{T} + e^{-\frac{\epsilon}{\kappa}t}\frac{2\pi}{T}\sin 2\pi\frac{t}{T}\right). \qquad (2)$$

The light being cut off when $t = T/4$, the equation of motion becomes

$$\varkappa\frac{\partial^2\theta}{\partial t^2} + 2\varepsilon\frac{\partial\theta}{\partial t} = -G\theta \qquad (3)$$

the solution of which is $\theta = Ae^{-\frac{\epsilon}{\kappa}t}\cos\left(2\pi\frac{t}{T} + \alpha\right)$ where A and α

[1] We are justified in using quantitatively this equation, containing a damping term proportional to the velocity, because the amplitudes of the successive swings of the torsion balance, when no energy fell upon the vanes, were found experimentally to follow accurately the exponential law.

can be determined by the conditions imposed by equation (2). Neglecting very small quantities, the value of the amplitude A is expressed by the equation

$$A = \frac{L}{G}\left\{ 1 + r + \frac{2}{\pi} r^{\frac{1}{2}} \log\left(\frac{1}{r}\right)\right\}^{\frac{1}{2}}, \tag{4}$$

where r is the ratio of successive amplitudes of the damped vibrations. If $r = 1$, that is if the motion is undamped, $A = \sqrt{2}L/G$. In the partial vacuum used in the experiments (16 mm. of mercury, a value chosen from the curves in Fig. 5), r was found to be equal to 0.783 ; consequently

$$A = 1.357\frac{L}{G}. \tag{5}$$

From this it is seen that the total angle of deflection of the torsion balance in the ballistic measurements is equal to 1.357 times the angle at which the moment of the torsion of the fiber balances the moment of the radiation pressure.

The duration of exposure was always six seconds without appreciable error, but the period of the balance on account of slight accidental shifting of small additional masses upon the counterpoise weight m_3 (Fig. 2), differed from twenty-four seconds sometimes by one per cent. It is necessary therefore to find the error in the deflection due to this variation in the period. This is done by making $t = T/4 + \delta$ in equation (2) and in introducing the new conditions in equation (3). But it is simpler and sufficiently accurate to assume the motion as undamped. For this condition, the amplitude

$$A = \frac{L}{G}\left\{ 2 + 2 \sin 2\pi\frac{\delta}{T}\right\}^{\frac{1}{2}} = \sqrt{2}\frac{L}{G}\left(1 + \pi\frac{\delta}{T}\right) \text{ nearly.}$$

For $T = 23.75$ seconds, $\frac{T}{4} = 5.94$ and $\delta = 0.06$. Hence

$$A = \sqrt{2}\frac{L}{G}(1.008).$$

If $\delta = 0$, $A = \sqrt{2}\frac{L}{G}$, consequently an error of 1 per cent. in T causes an error of 0.8 per cent. in A.

To make sure that the observed radiation pressures depended only on the intensity of the beam, and were uninfluenced by the wave-length of the incident energy, the ballistic observations of pressure, the thermal measurements of intensity, and the determination of the reflection coefficients, were carried out for three entirely different wave-groups of the incident radiation. In the measurements designated "through air," no absorbing medium was introduced in the path of the beam between the lamp and the balance except the glass lenses and plates already mentioned. In the measurements "through red glass," a plate of ruby glass was put in the path of the beam between L_2 and d_3 (Fig. 3). For the observations

Table II.

August 28. *Light Pressure. Ballistic Measurements. Air.*

Surface.	Zero.	Throw.	Deflection.	Lamp.	Deflection (Lamp 100).	
					E. V.	*W. V.*
			mm.	mm.	mm.	mm.
EVC_s	281.4	248.5	32.9	164.3	20.0	
WVD_s	281.5	313.9	32.4	164.5		19.7
EVC_s	281.4	249.8	31.6	157.9	20.0	
WVD_s	281.5	310.5	29.0	147.0		19.8
EVC_s	281.5	252.6	28.9	144.8	20.0	
WVD_s	281.5	309.6	28.1	141.8		19.8
EVC_s	281.5	252.9	28.6	143.5	19.9	
WVD_s	281.5	309.3	27.8	140.4		19.8
				Average	19.97	19.77
				Average,	$\frac{C_s+D_s}{2}=19.87$	
			Magnet Reversed.			
EVD_g	280.1	246.0	34.1	180.4	18.92	
WVC_g	280.0	317.8	37.8	187.3		20.20
EVD_g	279.8	247.2	32.6	170.8	19.20	
WVC_g	279.4	313.7	34.3	169.4		20.25
EVD_g	279.1	248.9	30.2	161.1	18.80	
WVC_g	279.0	311.9	32.9	161.6		20.35
EVD_g	279.0	249.1	30.0	158.9	18.90	
WVC_g	278.9	311.2	33.2	164.4		20.20
				Average	18.97	20.25
				Average,	$\frac{C_g+D_g}{2}=19.61$	

Table III.

Radiation Pressure. Ballistic Measurements, through Air.

Date.	Air Pressure in mm. of Hg.	Balance Period T.	Sens. of Galvan'r. G_s (Silver).	Sens. of Galvan'r. G_g (Glass).	l = Lever Arm. cm.	$\frac{C_s+D_s}{2}=P_s$.	$\frac{C_g+D_g}{2}=P_g$.	P_s Corrected for $T=24''$.	P_g Corrected for $T=24''$.	$\frac{P_s \times G_s}{l}$.	$\frac{P_g \times G_g}{l}$.	Average.
June 19	32.5	23.75	[734]		.814	Average = 18.88		Average = 18.73		Average = 16.89		16.89
" 20	32.5	23.75	756	768	.814	19.67	16.94	19.51	16.81	18.12	15.86	17.00
" 23	37.0	23.75	700	716	.814							
July 23	16.0	23.75	682	707	.831	21.16	20.42	21.00	20.26	17.25	17.25	17.25
" 25	16.6	23.75	684	684	.815							
" 26	16.6	23.75	720	710	.815	19.34	19.98	19.18	19.82	16.93	17.28	17.10
Aug. 27	16.8	23.82	724	710	.823	20.16	19.40	20.00	19.25	17.60	16.61	17.10
" 28	15.7	23.82	721	716	.824	19.87	19.61	19.73	19.48	17.26	16.97	17.10
" 29	13.7	23.82	712	701	.824	19.68	20.07	19.53	19.92	16.90	16.97	16.94
" 31	14.0	24.00	718	713	.810	18.55	18.94	18.55	18.94	16.44	16.60	16.52
Sept. 1	16.6	24.00	692	672	.808	19.14	20.17	19.14	20.17	16.40	16.78	16.59
" 20	16.4	23.78	670	676	.812	20.96	20.02	20.81	19.87	17.17	16.54	16.86
" 23	16.4	23.78	666	684	.816	21.32	20.27	21.16	20.12	17.27	16.87	17.07
" 24	16.2	23.78	667	669	.816	20.76	19.80	20.60	19.65	16.84	16.11	16.47
									Average,	17.11	16.71	16.91±0.053

"through water cell," a 9-mm. layer of distilled water in a glass cell was placed in the path of the beam at the same point.

The separate observations entering into a single series of ballistic measurements and their treatment will appear from Table II., which is copied direct from the laboratory note-book and represents an average ballistic series. The designations EVC_s, WVD_s, EVD_g, and

TABLE IV.

Radiation Pressure. Ballistic Measurements.

Date.	$\frac{C_s + D_s}{2}$.	$\frac{C_g + D_g}{2}$.	P_s Corrected for $T = 24''$.	P_g Corrected for $T = 24''$.	$\frac{P_s \times G_s}{l}$.	$\frac{P_g \times G_g}{l}$.	P Average.
Through Water Cell.							
June 20	18.62	17.10	18.46	16.96	17.14	16.00	16.57
July 25	19.00	20.10	18.85	19.94	15.82	16.74	16.28
" 26	18.03	19.39	17.89	19.33	15.80	16.84	16.32
Aug. 27	18.63	18.66	18.50	18.53	16.29	15.99	16.14
" 29	18.25	19.02	18.10	18.87	15.68	16.06	15.87
Sept. 20	20.39	19.14	20.23	19.00	16.69	15.82	16.25
" 23	20.21	19.51	20.05	19.36	16.37	16.23	16.30
" 24	19.84	18.91	19.69	18.77	16.10	15.40	15.70
				Average.........	16.24	16.15	16.20 ±0.066
Through Red Glass.							
June 23	19.99	18.40	19.83	18.26	17.05	16.06	16.56
July 25	20.70	20.94	20.54	20.77	17.24	17.43	17.33
Aug. 27	19.97	19.25	19.82	19.10	17.46	16.46	16.96
" 28	19.99	19.42	19.84	19.28	17.36	16.75	17.05
" 29	19.99	19.92	19.84	19.77	17.14	16.82	16.98
" 31	18.98	19.14	18.98	19.14	16.82	16.84	16.83
Sept. 20	21.00	19.97	20.83	19.82	17.19	16.50	16.84
" 23	21.48	20.34	21.31	20.18	17.39	16.92	17.15
" 24	21.00	19.68	20.83	19.53	17.03	16.03	16.53
				Average.........	17.18	16.65	16.91 ±0.051

WVC_g, mean that the vane C in the first case was on the east side of the rotation axis with its silver face toward the light. The subscript g signifies that the glass face of the vane was toward the light. The second column of the table gives the zero reading of the balance before opening the shutter; the third, the end of the swing produced by a six-second exposure; the fourth, the deflection of the balance; the fifth, the ballistic deflection of the lamp galvanometer G_2. Columns six and seven give the balance deflection reduced to standard lamp.

The results of all the ballistic pressure measurements "through air," are collected in Table III. In the fourth and fifth columns two values are given for the constant of the lamp galvanometer G_2, since reversing the magnet on the balance bell-jar to reverse the suspension within affected the constant of the galvanometer slightly. The values for the silver and glass faces forward were never the same. The subscripts show to which series, silver or glass, the constant belongs. The values of the lever-arm l of the balance in the sixth column, are obtained by measuring the distance between the centers of the images when on the east and west vanes (by the dividing engine T_2, Fig. 3) and dividing by two. The columns headed $\frac{C_s + D_s}{2} = P_s$ and $\frac{C_g + D_g}{2} = P_g$ are the average moments due to pressures for the silver and glass sides of the vanes respectively toward the light. The next two columns contain these moments corrected for a period of twenty-four seconds of the torsion balance. The columns headed $\frac{P_s \times G_s}{l}$ and $\frac{P_g \times G_g}{l}$ are the corresponding forces reduced to standard sensitiveness, $G = 1{,}000$. The final column contains the averages of the two columns which precede it. Table IV. exhibits corresponding data for "red glass" and "water cell." The air pressure, period of the balance, lever arm and galvanometer constants are those given in Table III. for the same date.

In these ballistic measurements the lamp reading was the throw due to an exposure of the light upon the bolometer for six seconds, but in the energy measurements the lamp reading was a stationary deflection due to prolonged exposure. To bring the pressure values into comparison with the energy measurements it is necessary to reduce the average of the quantities in the last column to pressures in dynes by multiplying by 0.363×10^{-5}, the torsion coefficient of the quartz fiber, and to reduce not only to a static deflection of the torsion balance but also to a static deflection of the lamp galvanometer G_2. The ratio of a ballistic to a static deflection of the galvanometer G_2 was obtained from a long series of lamp exposures. This ratio was found "through air" to be $= 1.55$; "through red glass" $= 1.535$; "through water cell" $= 1.502$. These differences are probably due not solely to the damping constant of the galvanometer but to the peculiar manner in which the bolometer was warmed up to its stationary conditions by the beam from the lamp. Applying these reduction factors to the averages in Tables III. and IV., we obtain the following results. The pressure of the standard light beam which has passed

$$(a)\ \text{through air} = 16.91 \times \frac{1.55}{1.357} \times 0.363 \times 10^{-5} = (7.01 \pm 0.023) \times 10^{-5}\ \text{dynes};$$

$$(b)\ \text{through red glass} = 16.91 \times \frac{1.535}{1.357} \times 0.363 \times 10^{-5} = (6.94 \pm 0.024) \times 10^{-5}\ \text{dynes};$$

$$(c)\ \text{through water cell} = 16.20 \times \frac{1.502}{1.357} \times 0.363 \times 10^{-5} = (6.52 \pm 0.028) \times 10^{-5}\ \text{dynes}.$$

THE ENERGY MEASUREMENTS.

Before rejecting the bolometer method used in the preliminary measurement of energy, a second bolometer of slightly different construction was tried; but the lack of uniformity of resistance, already mentioned, made its indications too uncertain for the present work. The radiant intensity of the beam used in the later experiments was determined by directing it upon the blackened face of a silver disc, weighing 4.80 grams, of 13.3 mm. diameter and of 3.58 mm. thickness, and by measuring its rate of temperature rise as it passed through the temperature of its surroundings. The disc was obtained from Messrs. Tiffany & Co. and was said by them to be 99.8 per cent. fine silver. Two holes were bored through parallel diameters of the disc, one fourth of the thickness of the disc from either face. Two iron-constantan thermo-junctions, made by soldering 0.1 mm. wires of the two metals, were drawn through the holes into the center of the disc. To insulate the wires from the disc, fine drawn glass tubes were slipped over them and thrust into the holes, leaving less than 2 mm. bare wire on either side of the junctions. The wires were sealed into the tubes, and the tubes into the disc by solid shellac. The tubes projected 15 mm. or more from the disc and were bent upward in planes parallel to the faces of the disc.

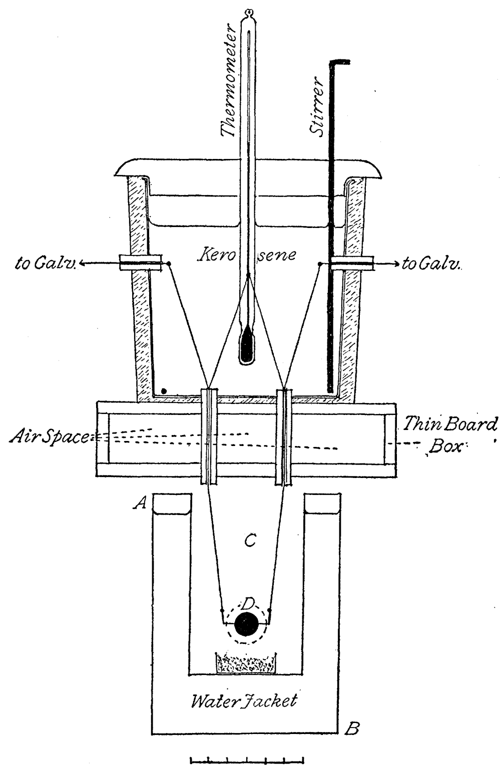

Fig. 6.

The general arrangement will be seen in Fig. 6. The disc was suspended by the four wires some distance below a small flat wooden box. On the box was fastened a calorimeter can swathed in cotton and filled with kerosene in which the constant thermo-junctions were immersed. Copper wires soldered to the two ends of the thermo-electric series were brought out of the colorimeter, and the circuit was closed through 1,000 ohms. in series with the 500 ohms. resistance of galvanometer G_1. The thermo-junctions in the disc were in series, and as each junction was midway between the central plane of the disc and either face, it was assumed that when the disc was slowly warmed by heating one face the electromotive forces obtained corresponded to the mean temperature of the disc. One face of the disc was blackened by spraying it with powdered lampblack in alcohol containing a trace of shellac. This method was suggested by Professor G. E. Hale and gives very fine and uniform dead black coatings not inferior to good smoke deposits.

For the energy measurements the bell-jar and the torsion balance were removed from the platform *P* (Fig. 2) and a double-walled copper vessel, *AB* (Fig. 6), which served as a water jacket surrounding a small air chamber *C*, was mounted in the same place. A tube 2 cm. in diameter was soldered into the front face of the jacket to admit the light beam into the chamber *C*. This opening was covered by a piece of plate glass similar to the plates forming the larger windows in the bell-jar.

The needle system in G_1, a four-coil du Bois-Rubens galvanometer was suspended in a strong magnetic field so that its period was about four seconds. The system was heavily damped by a mica air-fan of large surface. The disc junctions and galvanometer responded quickly to the radiation, as was shown by the reversal of motion of the magnet system 1.2 seconds after the light was cut off from the disc when the latter was a few degrees above the temperature of the room.

(*Concluded in next number.*)