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THE MAGNETIC SUSCEPTIBILITY OF WATER.

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THE magnetic susceptibility of water at a given temperature, such as 20° C., is regarded as one of the constants of nature, but there is no general agreement as to its numerical value. Table I. contains the results published by a number of careful observers. In writing the susceptibility (K) no correction has been made for the susceptibility of air. The temperature coefficient of (K) is of the order of .002, hence reduction to a common temperature could not bring the results into agreement.

TABLE I.

Determinations of (K).

Date.	Observer.	Temperature.	$K \cdot 10^6$.
1885	Quincke[1]	20° C.	−.84
1888	DuBois[1]	15	−.86
1892	Henrichsen[2]		−.75
1895	Curie[3]	18	−.79
1896	Townsend[4]	10	−.77
1898 } 1901 }	Königsberger[5]	22	−.80
1899	Jäger and Meyer[6]	18	−.66

[1] Wied. Ann., Vol. 35, p. 137, 1888.
[2] Wied. Ann., Vol. 45, p. 38, 1892.
[3] Journal de Physique, p. 206, 1895.
[4] Proc. Roy. Soc. London, 60, p. 186, 1896–7.
[5] Wied. Ann., Vol. 66, p. 698, 1898; Drude's Ann., Vol. 6, p. 506, 1901.
[6] Wied. Ann., Vol. 67, p. 712, 1899; Drude's Ann., Vol. 6, p. 870, 1901.

With the assistance of Mr. Angus L. Cavanagh, a student of physics in Stanford University, I have made a new determination.

We have used that general method in which a right cylinder of water of cross-section (q) is set perpendicular to a magnetic field whose strength at one end of the cylinder is equal to (H) and at the other end is negligible. (K) is given by the equation,

$$K = -\frac{2gp}{H^2 q},$$

where (p) is the numerical value of the magnetic repulsion in grams on the cylinder in the direction of its axis. The value of (g) in this locality is 980.

In determining (p) we used a balance as Jäger[1] and Meyer did in their third determination. A glass tube was suspended from one end of a non-magnetic balance. The lower end was sealed abruptly and reached the center of the air space between the parallel pole-pieces of a large electromagnet. In this position the change in the weight of the tube due to exciting the magnet was determined by several successive weighings. The tube was then filled with water to a height at which the field strength was negligible and the weighings were repeated. Table II. contains the record of the weighings and the computed value of (p) for the four different tubes that were used. The changes of weight indicated by (p') and (p'') in Table II. were made by the use of a single rider whose mass was determined by each of us separately by comparison with a standard mass. Our results differed by .05 per cent.

The cross-section (q) of the cylinder of water contained in each tube was determined by the usual method of calibrating tubes by means of mercury. The height of the mercury column was read by each of two micrometer microscopes furnished by the Geneva Society, one of them belonging to the "comparateur" furnished by this society. The mercury was weighed on an ordinary analytical balance. Explorations of the magnetic field showed that the cross-section (q) should be measured between the heights of 1.5 cm. and 4.5 cm. above the lower end of the tube. Above 4.5 cm. the field strength was so small relatively that slight variations in (q) could

[1] Wied. Ann., vol. 67, p. 707, 1899.

produce no appreciable effect, and below 1.5 cm. no lack of uniformity in the field was found. The values found for (q) are given in Table II.

The water used was distilled from rain water caught in a glass vessel as it fell from a tile roof after copious showers had cleaned the roof. The water was condensed in tubes Nos. 1, 2 and 3 directly from the still. That in tube No. 4 had been in another glass vessel for some months. Experiment showed, however, that even water from the University water system gave the same values for (K) as the nearly pure water used.

TABLE II.

Values of (p), (q) *and* $\left(\frac{p}{q}\right)$.

Tube No. 1. (q) = 1.282 sq. cm.		
Temperature.	**Current of Electromagnet.**	**Repulsion.**
	Tube empty.	
(t)	(c)	(p')
21	12	−.0039
23	12	−.0038
23	12	−.0038
24	12	−.0039
24	12	−.0038
24	12	−.00385
22	12	−.0039
24	12	−.0039
Mean 23	12	−.00385
	Tube filled with water.	
(t)	(c)	(p'')
22	12	.0290
22	12	.0290
22	12	.0290
24	12	.0291
Mean 23	12	.02902
	Water alone (calculated).	
(t)	(c)	(p) = ($p'' - p'$)
23	12	.03287

TABLE II.—*Continued.*

Tube No. 2. $(q) = 1.279$.

Tube empty.

	(t)	(c)	(p')
	25	12	−.0038
	22	12	−.0039
Mean	23	12	−.00385

Tube filled with water.

	(t)	(c)	(p'')
	24	12	.0291
	22	12	.0291
	22	12	.0290
	20	12	.0290
Mean	22	12	.02905

Water alone (calculated).

(t)	(c)	$(p) = (p'' - p')$
23	12	.0329

Tube No. 3. $(q) = 1.333$.

Tube empty.

	(t)	(c)	(p')
	25	12	−.0068
		12	−.0066
Mean	25	12	−.0067

Tube filled with water.

	(t)	(c)	(p'')
	23	12	.0275
	23	12	.0275
	23	12	.0274
Mean	23	12	.02747

Water alone (calculated).

(t)	(c)	$(p) = (p'' - p')$
23	12	.03417

TABLE II.—*Continued.*

Tube No. 4. $(q) = .469$.

Tube empty.

	(t)	(c)	(p′)
	21	11.7	−.0011
	21	11.6	−.0012
	24	11.3	−.00115
	22	12	−.0012
Mean	22	11.7	−.00116

Tube filled with water.

	(t)	(c)	(p″)
	20	12	.0109
	20	12	.0109
	22	11.6	.0108
Mean	21	11.9	.01087

Water alone (calculated).

(t)	(c)	$(p) = (p'' - p')$
21	12	.01205

Values of (p/q).

No. of Tube.	Temperature.	(p/q)
1	23	256.4
2	23	257.2
3	23	256.3
4	21	256.9
Mean	23	256.7

The close agreement of the values of (p/q) would seem to show that the mean value cannot be in error by more than .5 per cent.

In determining the field strength (H) three test coils were used. Coil No. 1 was mounted so that it could be turned through 180° in the field (H). Coils No. 2 and 3 were mounted on handles so that they could be drawn out of the same field. No. 2 and No. 3 were of nearly the same diameter and were wound with nine turns of wire each. Coil No. 1 was of smaller diameter and was wound with six turns of wire. Its effective area was almost exactly the same as that of the other two, its actual area being doubled on account of its turning through 180°. An astatic mirror galvanometer of long period placed at a sufficient distance from the electromagnet was connected in series with the three coils, and the galvanometer deflections due to operating in the field (H) with the coils were observed. After continued practice a comparison of the three coils was made and the results are given in Table III.

TABLE III.

Comparison of the test coils in the field (H).

No. of Coil.	Area (A).	Deflection (a).	Current.	a/A
3	16.84	212	11.8	1.259
1	16.92	213	11.8	1.259
2	16.88	212	11.7	1.256
3		212	11.7	
1		213	11.7	
2		212	11.7	
3		212	11.7	
1		213	11.7	
2		212	11.7	

This agreement being satisfactory coils No. 1 and No. 3 were selected for further use.

As a standard instrument a current inductor of the pattern described on page 224 of Henderson's Practical Electricity and Magnetism was used. The formula for the number of lines of force cut by the test coil of such a current inductor when the current through its solenoid is broken is

$$N_1 = \frac{4\pi n n_1 CA}{10}\left(1 - \frac{1}{2}\frac{r^2}{l^2}\right).$$

Measurements of the constants of this instrument gave, for the value of (N_1),

$$N_1 = 210 \cdot 10^3 C.$$

(C) was measured by Weston direct-reading portable ammeter No. 10364 whose indications are certified to by the company as correct to .25 per cent.

To test the accuracy of the measured constant of the current inductor, this inductor was connected in series with an earth inductor and the galvanometer, and the deflections of the two inductors were compared. The result is given in Table IV.

TABLE IV.

Comparison of earth inductor and current inductor.

Inductor Operated.	Deflection.	Current in Solenoid of Current Inductor.
Current	237	.717
Earth	235	
Current	236.5	.715
Earth	234.5	
Current	234	.710
Earth	234.5	
Current	234.5	.712
Earth	235	
Current	234.5	.712
Earth	235	
Mean { Current	235.3	.713
Mean { Earth	234.8	
Calculated { Current	234.8	.712
Calculated { Earth	234.8	

The formula for the earth indicator is

$$N_2 = 2A'H',$$

where (H') is the horizontal component of the earth's magnetic field.

The mean of four determinations of (H') by means of a Kew magnetometer set up where the earth inductor was used gave

$$(H') = .2482.$$

Hence

$$N_2 = 2.301 \cdot 10^3 \cdot .248 = 149300.$$

For N', using for (C) the value .712 taken from Table IV., we have

$$N' = 210 \cdot 10^3 \cdot .712 = 149500.$$

This good agreement seemed to justify the use of the current inductor as an absolute standard.

To determine (H) by comparison with the field of the current inductor, coil No. 1 was connected in series with the test coil of the inductor and with the galvanometer, and the deflections due to rotating coil No. 1 in the field (H) and to breaking the current through the solenoid of the current inductor were observed. Coil No. 1 was then replaced by coil No. 3 and two more series were made on different days. The results are given in Table V.

TABLE V.

Value of (H).

Instrument Operated.	Deflection.	Current in Solenoid.	Current in Electro-magnet.
First series.			
Current inductor.	233	.669	
Coil No. 1.	233		12
Current inductor.	233	.668	
Coil No. 1.	234		12
Current inductor.	234	.670	
Coil No. 1.	233		12
Current inductor.	234	.670	
Coil No. 1.	234		12
Mean { Current inductor.	233.5	.669	
Mean { Coil No. 1.	233.5		12

Area of coil No. 1, 16.92.

$$H = \frac{210 \cdot 10^3 \cdot .669}{16.92} = 8300.$$

Instrument Operated.	Deflection.	Current in Solenoid.	Current in Electro-magnet.
Second series.			
Current inductor.	222	.666	
	220	.666	
	220	.664	
	220	.663	
	219	.663	
Coil No. 3.	219		11.7
	219		11.7
Mean { Current inductor.	220	.665	
Mean { Coil No. 3.	219		11.7
Calculated { Current inductor.	220	.665	
	220		12

TABLE V.—*Continued.*

Third series.

	Current inductor.	222	.678	
		220	.668	
		220	.668	
		223	.679	
	Coil No. 3.	220		12
	Current inductor.	219	.666	
	Coil No. 3	219		12
	Current inductor.	219	.665	
	Coil No. 3.	219		12
Mean	Current inductor.	220.5	.6706	
	Coil No. 3.	219.3		12
Calculated	Current inductor.	219.3	.667	
	Coil No. 3.	219.3		12

Mean current in solenoid in second and third series .666
Area of coil No. 3 16.84

$$H = \frac{210 \cdot 10^3 \cdot .666}{16.84} = 8300.$$

In view of the agreement between the current inductor and the earth inductor and also among the test coils, a greater error than 1 per cent. in the value of (H) seems very improbable.

Careful explorations of the magnetic field showed that the residual field and the field at the top of the water cylinder were negligible. Hence it is correct to use as the formula for (K)

$$K = -\frac{2gp}{H^2q}.$$

The first four lines of Table VI. contain the values for K calculated from the values of (p/q) in Table II. and from the value of (H) in Table V.

The poles of the electromagnet were moved closer together and another determination of (K) was made by the same method and with the same apparatus. This value appears in the fifth line of Table VI.

The poles were brought back to their original position and additional determinations were made, the field strength being varied by changes in the exciting current of the electromagnet. The results are given in the last three lines of Table VI.

TABLE VI.

Values of (K).

Temperature.	(*H*)	($K \cdot 10^6$)
23	8300	−.730
23	8300	−.732
23	8300	−.730
21	8300	−.731
22	9700	−.734
22	8150	−.736
22	8030	−.735
21	7470	−.738
Mean 22		−.733

Admitting a possible error of 1 per cent. in the measurements of (H) and of .5 per cent. in the measurements of $\left(\frac{p}{q}\right)$ the maximum possible error in the result would be 2.5 per cent., and it would seem certain that the true value of (K) lies between the values $-.715 \cdot 10^{-6}$ and $-.750 \cdot 10^{-6}$ at the temperature of 22° C.

Opposed to this conclusion stand the results of all the observers quoted in Table I. except that of Henrichsen. The situation is well expressed by Jäger and Meyer,[1] who say, " Eine ausreichende Erklärung für die starken Abweichungen der Resultate der verschiedenen Beobachter steht sonach noch aus."

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[1] Drude's Ann., Vol. 6, p. 870, 1901