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THE DISTRIBUTION OF ENERGY IN THE SPECTRUM  
OF THE ACETYLENE FLAME.

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THE published work on the distribution of energy in flame spectra is not very extensive, and some of it is difficult to find, occurring merely as a minor part of a problem considered. As might be expected, the methods and apparatus of the different investigators are quite similar. Lens and mirror spectrometers and also gratings have been used for the production of spectra, and linear thermopiles and bolometers for the measurement of the radiant heat.

Tyndall<sup>1</sup> explored the spectra of illuminating gas and hydrogen flames. He moved a thermopile backward and forward in the spectrum of each flame, and determined the point of maximum intensity, which was found to be farther toward the long wave-lengths with the hydrogen than with the gas flame. Magnus,<sup>2</sup> while working upon the radiation of platinum, found the distribution of energy in the spectrum of a bunsen flame, but his results are not in accord with those of subsequent observers. Langley<sup>3</sup> found the distribution of energy in the flame of illuminating gas, and, from the curve, calculated the radiant efficiency to be 0.024. In a later article<sup>4</sup> he mentions the fact of having also investigated the candle flame, but gives neither curves nor data. Julius<sup>5</sup> obtained the spec-

<sup>1</sup> Tyndall, *Annal. der Phys.*, 124, 1865, p. 36.

<sup>2</sup> Magnus, *Annal. der Phys.*, 124, 1865, p. 476.

<sup>3</sup> Langley, *Science*, June 1, 1883.

<sup>4</sup> Langley, *Phil. Mag.*, 30, 1897, p. 260.

<sup>5</sup> Julius, *Licht- und Wärmestrahlung verbr. Gase*; Berlin, Leonh. Simion, 1890.

tral energy curves for flames of hydrogen, carbonic oxide, illuminating gas in bunsen and the ordinary lava tip burners, carbon bisulphide, sulphur, sulphuretted hydrogen, cyanogen, carbonic oxide in oxygen, mixtures of hydrogen and chlorine and hydrogen and bromine, and phosphuretted hydrogen. He also took observations on several parts of the illuminating gas flame. In work upon the emission of gases, Paschen<sup>1</sup> obtained the distribution of energy in the flames of the bunsen and oxyhydrogen flames. He made most accurate determinations of the positions of the bands of CO<sub>2</sub> and H<sub>2</sub>O, and these results will be discussed in connection with the work herein described. The results of Rubens and Aschkinass<sup>2</sup> upon the bunsen flame are in agreement with those of Paschen. Lummer and Pringsheim<sup>3</sup> determined the wave-length of the maximum intensity of radiation of several sources of light, and among them the candle and argand burner flames. As it was necessary for them to take observations over merely a small part of the spectra, the only data given are these values of wave-length.

The great value of acetylene, not only for illuminating purposes, but also in experimental research, makes a knowledge of the distribution of energy in its spectrum of considerable importance. In fact, the problem was suggested by a demand which has already arisen in experimental work conducted in this laboratory. In the work here described, a mirror spectrometer and the radiometer of Nichols<sup>4</sup> were used.

## I. APPARATUS AND ADJUSTMENT.

### 1. *The Spectrometer.*

The spectrometer available was manufactured by Franz, Schmidt & Haensch, Berlin. The mirrors, made by Brashear, had a focal length of 55.7 cm. and an aperture of 9 cm. As it was necessary to keep the radiometer stationary, the mirror-prism device of Wadsworth was used.<sup>5</sup> Another advantage of this arrangement is that

<sup>1</sup> Paschen, *Annal. der Phys.*, 51, 1894, p. 1.

<sup>2</sup> Rubens and Aschkinass, *Annal. der Phys.*, 64, 1894, p. 584.

<sup>3</sup> Lummer and Pringsheim, *Verh. der Deutsch. Phys. Gesell.*, 1899, p. 215.

<sup>4</sup> Ernest Nichols, *PHYS. REV.*, IV., 1897, p. 297.

<sup>5</sup> Wadsworth, *Phil. Mag.*, 38, p. 337.

any wave-length measured is always at the angle of minimum deviation.

The fluorite prism used throughout the work was a very clear specimen of the purple variety. It was  $2.5 \times 3$  cm. and had an angle of  $38^\circ 59' 55''$ .

The arrangement of the spectrometer and radiometer is shown in Fig. 1.  $R$  is the radiometer;  $M_1$  and  $M_2$  the spectrometer mirrors;

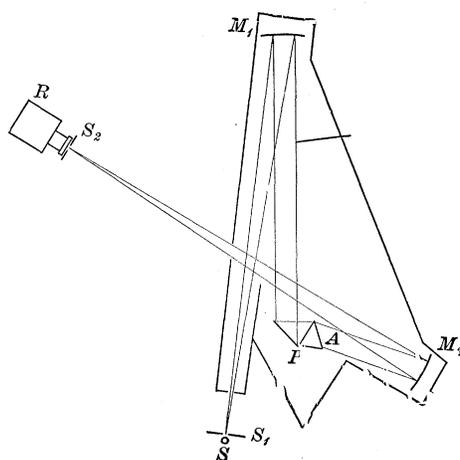


Fig. 1.

$P$  the mirror-prism system;  $S$  the source; and  $S_1$  and  $S_2$  two slits of the same width, 0.477 mm.  $S_2$  is placed directly in front of the radiometer and in the focus of  $M_2$ , thus avoiding the use of any lens whatever. In order to prevent stray radiation, numerous shields of blackened tin were constructed, but only those about the spectrometer are shown in the figure. As will be shown later, the arrangement was found quite satisfactory. The shutter operated to give deflections and zero readings was placed at  $A$ . The radiometer could not be conveniently mounted on the arm of the spectrometer, and although they stood on the same support, there were changes in their relative positions which caused considerable annoyance.

The refractive indices used in the computations are due to Paschen.<sup>1</sup>

<sup>1</sup> Paschen, *Annal. der Phys.*, 56, 1895, p. 765.

*Adjustment on the Sodium Line.*—It was necessary to have an accurate method of causing the sodium line, or any other known part of the spectrum, to fall upon the slit  $S_2$ . This method had to be not only accurate, but also simple, for, owing to slight changes in the relative positions of the radiometer and spectrometer, it was necessary that the adjustment be repeated quite frequently.

The radiometer was adjusted so that its fluorite window was not quite perpendicular to the incident beam, the reflected light returning in the neighborhood of  $M_2$ . The apparatus  $C$  in Fig. 2, consisting of a long focus lens and two mirrors, could be lowered into the spectrometer in the region of the reflected beam. The light from the fluorite window would then pass through the lens and be reflected from the mirrors into the microscopic eyepiece  $D$ , Fig. 2.

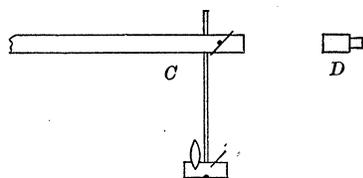


Fig. 2.

Whenever an adjustment on the sodium line was desired, sodium light was produced in front of the spectrometer slit, the prism table was turned until in the proper position for this wave-length, and  $C$  was lowered into the spectrometer. The position of the spectrometer was now

shifted until the eye at  $D$  saw that the image of  $S_1$  coincided with  $S_2$ . The radiometer slit was then upon the sodium-line. The error introduced by this method was about 3 seconds of arc as read on the spectrometer, or less than  $0.002\mu$  at this point in the spectrum.

## 2. The Radiometer.

(a) *Construction.*—There was at my disposal a radiometer which had been constructed according to the description of Nichols.<sup>1</sup>

This instrument, modified to suit the work undertaken, is shown in Fig. 3. The bronze block was  $5 \times 5 \times 10$  cm., and the vertical boring 3 cm. in diameter. There were two lateral borings; the one in front of the vanes was 2 cm. in diameter, and the one in front of the mirror, not shown in the figure, was elliptical and longer on the outside. From the crossbar resting on a shelf on the interior just

<sup>1</sup> Ernest Nichols, *PHYS. REV.*, IV., 1897, p. 297.

below the top, was hung the vane and mirror suspension. The crossbar fitted loosely in the boring so that the suspension could be shifted horizontally. In front of the vanes was a thin sheet of mica supported by a brass cylinder which fitted in the lateral boring. The window which had been at *a* was discarded, and in place was soldered a brass tube closed at one end, but with an aperture as shown in the figure. A mixture of beeswax and tallow was rubbed upon the ground surface of this partially closed end. A fluorite plate, 4.5 mm. in thickness, was then pressed firmly into place. A piece of mirror glass was sealed over the other window in a similar manner. If the metal surfaces were first slightly warmed, then, when the fluorite and glass were put into place, there resulted joints which were, so far as the writer could observe, perfectly air-tight. For a top, the neck of a round bottle was utilized. Into this neck was ground a glass tube which led off to a stopcock, and thence through two rubber-mercury seals to the mercury pump. The same mixture of beeswax and tallow was used at all four joints at the radiometer. Melted beeswax was poured into the bottle-neck to prevent that joint being disturbed when the top was removed.

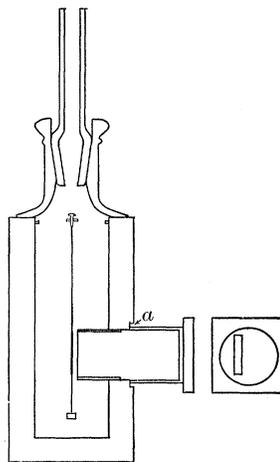


Fig. 3.

The deflections of the mirror were read by the telescope and scale method.

Two different suspensions were used. In both the vanes were approximately  $2 \times 15$  mm., and were fastened together by glass fibers, the distance between them being about 1.5 mm. The mirrors were about  $2 \times 3$  mm. The vanes, mirror and fibers were fastened by shellac.

In the first suspension, which was used in exploring the visible spectrum, the vanes were of thin mica coated with lampblack. A firm black coating was obtained by first dipping the mica into a very dilute alcoholic solution of shellac, and then holding it above a piece

of burning camphor. The loose particles were then wiped off gently, and the operation repeated until a coating sufficiently thick was obtained. The weight of this suspension was 4.2 mg. The quartz fiber, by which it was suspended, was so fine that even at the point of maximum sensitiveness the system was aperiodic, and one had to wait five minutes in order to take one reading.

In the second suspension platinum-black was used. While lamp-black and platinum-black are far from being perfect absorbers of long rays, yet the absorption of the latter covers a much wider range than that of the former.<sup>1</sup> According to Kurlbaum,<sup>2</sup> platinum-black is preferable for all experiments, not only because of its better absorption, but also because of its greater conductivity, and above all because a uniform layer is deposited and the degree of absorption can be defined by the data of electrolysis. By determining the ratio of emission of different thicknesses of these two absorbers to that of an ideally black body and assuming Kirchoff's Law, he has found the percentage of radiation which will be absorbed by layers of different thickness. He shows that a layer of platinum-black  $190 \frac{mg}{dm^2}$  absorbs almost 97 per cent. and fully as much as a layer of twice that thickness.

The composition of the electrolyte as given by Kurlbaum and as used in this case, is 1 part platinum chloride, 30 parts water, and 0.008 part lead acetate. An attempt was made to deposit the platinum-black upon aluminium foil; but it failed because the aluminium was attacked by the solution. This would not have been wholly unexpected had the writer been aware that what is commonly called platinum chloride is really chlorplatinic acid. Platinum foil then seemed to be the most practicable material for the vanes. Pieces were rolled to the thickness of about 0.01 mm., a thin coating of shellac placed on one side to prevent electrolysis, and the platinum-black deposited. The layer obtained was about  $250 \frac{mg}{dm^2}$ . The weight of this suspension was about 15 mg. The fiber was not so fine as in suspension number one, and at the point of maximum sensitiveness the time required for a deflection was about 40 seconds.

<sup>1</sup> Rubens and Nichols, *PHYS. REV.*, IV., 1894, p. 314.

<sup>2</sup> Kurlbaum, *Annal. der Phys.*, 67, 1899, p. 846.

(b) *Sensitiveness*.—As is well known, the sensitiveness of the radiometer does not continue to increase with higher vacua, but reaches a maximum. The following curve was taken to show this point, the pressures being read with a McLeod gauge.

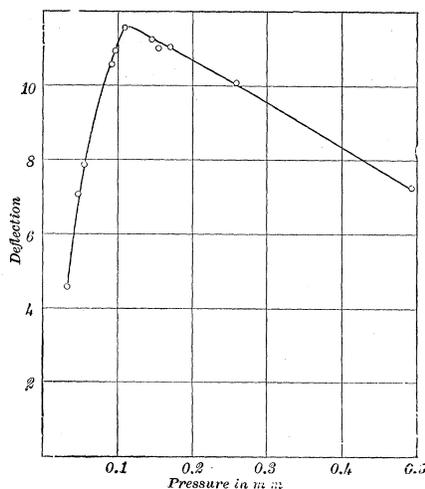


Fig. 4.

Ernest Nichols has found this maximum point to be about 0.05 mm., while our determination is about 0.1 mm. It is difficult to see how this difference can be accounted for in the construction of the radiometers. On the other hand, such a discrepancy might easily be due to the gauges themselves.

During the greater part of the experiment, I had no gauge at my disposal. This was of very little inconvenience as the period of vibration, and also the action of the valves in the pump served fairly well as methods of ascertaining the vacuum.

With suspension number two, an ordinary candle at a distance of 300 cm. gave a deflection of 103 mm. on a scale 63 cm. from the radiometer vanes. The instrument used by Nichols<sup>1</sup> in his recent work on star radiation, gave a deflection of 11 mm. on a scale 183 cm. from the mirror, the candle being 811 cm. distant. The radiomicrometer used by Boys in attempting to measure the radiation

<sup>1</sup> Ernest Nichols, *Astrophys. Jl.*, Mch., 1901, p. 101.

from the stars, gave a deflection of 1.7 cm. under similar conditions. Computing our deflections under these same conditions, we have for number two 42 mm., and for number one, which was about four times as sensitive, 170 mm. But comparison should be made considering like areas of the surface exposed to radiation. When this is done, the sensitiveness of each instrument and suspension is as follows: Boys 0.9, Nichols 11, suspension number two 4.2, and suspension number one 17.

It must be remembered that it is not a very difficult matter to make a radiometer suspension more sensitive than any of these, but the difficulty lies in its use.

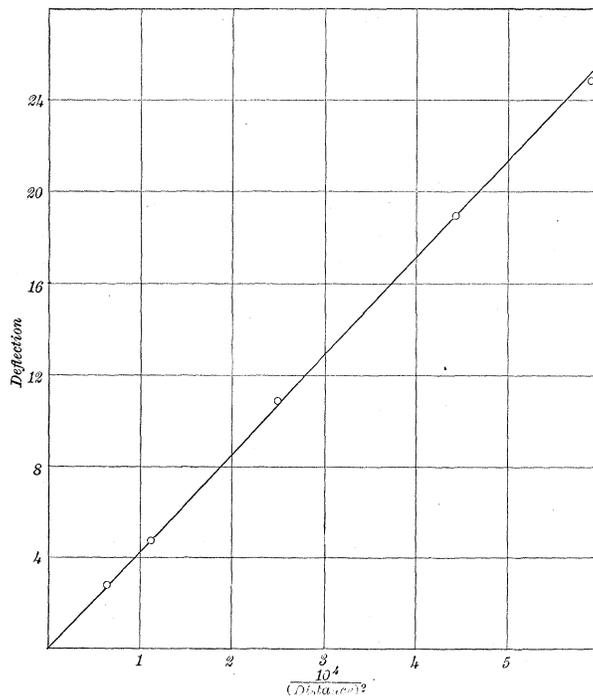


Fig. 5.

(c) *Accuracy.*—That the deflections of the radiometer are proportional to the energy when the vanes are 2 or 3 mm. from the window, has already been proven by Nichols. However, it

seemed desirable to make some sort of a test. I used the convenient method of observing the deflections produced by a constant source, placed at different distances. At first, results obtained in this way did not follow the inverse square law, varying as much as 15 or 20 per cent. The source, an acetylene flame, was constant to within much less than 1 per cent., and the large errors were due to the reflections from the walls of the room. The accompanying curve shows a series of observations made after various precautions were taken.

It will be noticed that, with one exception, the deviations from the mean are less than 2 per cent., and the character of these deviations show that with sufficient care a curve could be obtained which would agree with the inverse square law to within less than 1 per cent.

(d) *Difficulties.*—The advantage which the radiometer possesses over other devices for measuring radiant energy are well known, but as the instrument is somewhat new, the difficulties encountered with these sensitive adjustments will be mentioned in considerable detail.

1. The zero reading drifted almost constantly. The changes of temperature in the room, the lack of proper protection against the radiations from different parts of the room, the air draughts, and the lack of perfect symmetry in the construction of the vanes, tended to produce this effect.

The first of these causes could not be completely removed as the apparatus could not be conveniently set up in a constant temperature room.

The glass and fluorite windows were carefully shielded from all sources of radiation. It was found necessary to place a shutter in front of the glass window, and, with the more sensitive suspension, this shutter was opened only when taking a reading. Radiation from the room entering the top of the instrument caused no disturbances. In fact, a gas light several feet away and shining obliquely into the top seemed to have very little or no effect.

In the construction of the vanes care was taken to make them alike, but how nearly this was accomplished could not be learned as the construction of the instrument prevented such a test being made.

This drift of the zero of course affected the accuracy of the readings, but by taking the reading of the zero before and after each deflection, the error was less than one small scale division, and quite often less than one-fifth of this.

2. The annoyance which could be the least easily controlled, was the vibration of the suspension due to the tremor in its support. The entire apparatus was set upon a stone pier and this effect minimized as far as practicable. The tremor did not prevent the observer from reading the scale, but caused the suspension to vibrate over several small scale divisions. This effect seemed to be due to the large area and small inertia of the vanes, and the viscosity of the residual gas.

3. Throughout the experiment the radiometer was kept connected to the pump so that the vacuum could be changed at any time. Only twice did any trouble from leakage occur, and in both cases the trouble was located in the mercury-rubber seals. The radiometer itself would stand for weeks with practically no leakage. Even when in connection with the pump, at the end of a week the change in the vacuum would be less than 0.01 mm.

4. If the air were allowed to rush in or out of the radiometer too rapidly, the vanes would strike the mica window and become electrified. To prevent this, the rush of air was controlled by the pump stopcock, while the movement of the vanes was watched through the telescope.

Several methods of discharging the vanes were tried. The simplest and most successful was to permit the instrument to stand several hours in the open air, after first freeing the clinging vanes from the window. This electrification could be prevented by a little care, but, when it did occur, was a source of considerable annoyance.

5. In constructing a radiometer, the difficulties in using a suspension of long period should be borne in mind. In such a vibrating system, the moment of inertia is of small importance as compared with the viscosity of the gas, and so an effort must be made to make the area of the vanes as small as the work undertaken will permit. An improvement could have been made in our instrument by placing the slit closer to the vanes and making the vanes smaller. As in

many instruments, the accuracy is not proportional to its sensitiveness. For example, the sensitiveness of suspension number one was four times as great as suspension number two, yet its accuracy was not nearly so great in proportion, due to the fact that changes of temperature, air draughts, etc., produced a greater effect.

## II. OBSERVATIONS IN THE VISIBLE SPECTRUM.

The visible spectrum was explored previously to the employment of the device for the accurate setting of the sodium line upon the radiometer slit. The method here used consisted merely of adjusting until the reflection from the fluorite surface appeared the color of sodium light.

It was not supposed that this would give an accurate, but merely a consistent setting. Indeed, the average deviation from the mean of several adjustments was not more than  $5''$  on the spectrometer, or about  $0.003\mu$ . The later method showed that the adjustments made in this part of the experiment were in error by about  $45''$  toward the violet, or about  $0.025\mu$ . It was then a simple matter to correct the wave-lengths, no account being taken of the fact that the light did not pass through the prism at minimum deviation.

The source of light was a cylindrical acetylene flame from a single tip burner. This type was used because the intensity per unit area was greater than in the flat flame.

On account of the drift of the vanes, the zero was read before and after each deflection and the average taken in the computations. The time required for one deflection being five minutes, ten minutes was consumed in taking one reading.

The table below gives the results of the observations in the visible spectrum. The deflections recorded are averages of a large number of observations, taken with different degrees of sensitiveness, but reduced to similar conditions. They are about the actual size of the deflections which would be obtained with the most sensitive adjustment of suspension number one. The first column indicates the rotation of the prism table, the positive sign meaning a rotation toward longer waves. The points  $0.800\mu$  and  $0.665\mu$  were obtained from observations given under III.

TABLE I.

Angular Rotation.	Wave-Length.	Def.	Angular Rotation.	Wave-Length.	Def.
—	0.800 $\mu$	64.80	— 5'	0.460 $\mu$	.60
+4'	.710	28.60	— 8'	.415	.44
—	.665	16.25	—12'	.374	.44
+3'	.662	16.35	—15'	.352	.25
+2'	.620	8.90	—18'	—	.25
0'	.552	3.00	—21'	—	.25
—1'	.527	1.92	—25'	—	.17
—3'	.489	.97			

These data are plotted to a different scale in Fig. 6, curve 1. The deflections recorded between 0.3 $\mu$  and 0.4 $\mu$  are thought to be produced by diffused light. In the subsequent use of this curve, the

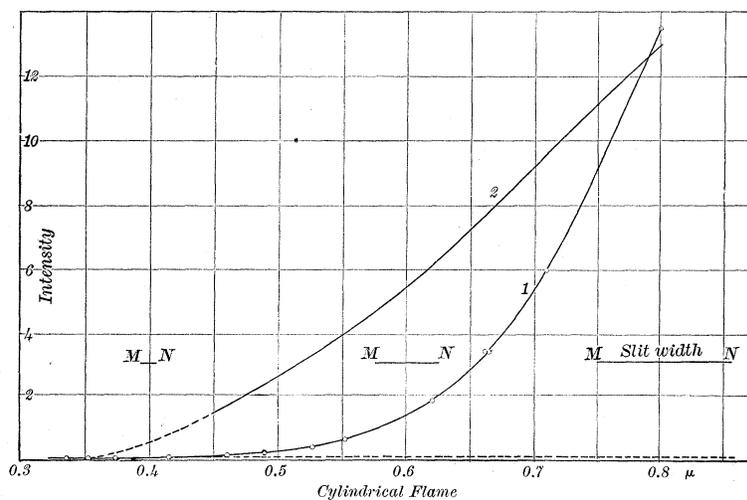


Fig. 6.

ordinate of the dotted horizontal line was considered as representing zero intensity.

Curve 2 is the result when proper corrections are made for slit widths. This method of correction will be explained under IV. The relation between the ordinates of any wave-length in the two curves has no significance, both being plotted to a convenient scale.

## III. OBSERVATIONS IN THE INFRA-RED.

I. *Cylindrical Flame.*

Owing to the difference in the dispersion of the prism, a slight change in the relative positions of the spectrometer and radiometer was of greater consequence in the infra-red than in the visible portion of the spectrum.

Correct results could be obtained only by making a series of observations in a small part of the spectrum and then checking back upon the sodium line. Many results had to be discarded because the relative position of the instruments had changed by an appreciable amount.

TABLE II.

	Wave Length.	Zero.	Reading.	Zero.	Def.
2'	0.665 $\mu$	41.75	39.50	41.61	2.18
4'	.775	41.28	33.92	41.20	7.32
5'	.855	40.80	29.40	40.69	11.35
6'	.952	40.50	24.81	40.38	15.63
7'	1.062	40.20	20.05	40.12	20.11
8'	1.196	39.85	19.00	39.80	20.82
9'	1.351	39.70	19.79	39.70	19.91
10'	1.504	39.70	21.31	39.80	18.44
1 <sup>1</sup>					
10'	1.504	39.97	21.59	40.00	18.40
11'	1.676	40.10	24.00	40.00	16.05
12'	1.847	39.90	26.02	39.90	13.88
13'	2.000	39.90	28.20	39.90	11.70
14'	2.140	39.90	30.20	39.90	9.70
15'	2.285	39.82	31.82	39.82	8.00
16'	2.419	39.70	33.69	39.80	6.06
17'	2.550	39.80	34.50	39.81	5.31
18'	2.664	39.81	34.53	39.79	5.27
19'	2.785	39.78	35.38	39.80	4.41
20'	2.904	39.80	36.10	39.80	3.70
2 <sup>2</sup>					
10'	1.504	39.78	21.50	39.71	18.25
3 <sup>3</sup>					
10'	1.504	39.50	21.05	39.30	18.35
20'	2.904	39.00	35.22	38.90	3.63
21'	3.012	38.80	35.80	38.70	2.95

<sup>1</sup> Here spectrometer was re-set on Na, and found 5'' in error.<sup>2</sup> Returned to 10'.<sup>3</sup> Spectrometer re-set on Na, and found 4'' in error.



The preceding table, giving a series of observations covering a large part of the spectrum, shows the method used. The angles given in the first column signify the rotation of the prism table from the starting point, *i. e.*, the sodium line.

Curve 1, Fig. 7, obtained from this and other data, shows the distribution of energy as observed throughout the spectrum. It was useless to go any further than  $6.0 \mu$ , for readings beyond this region are perhaps false on account of diffused light. If all the observations taken were plotted, all the points would not lie upon this line, but the variation would not be 1 per cent. As an example of this variation, the maximum point, which is somewhat indefinite anyway, was found to shift only about  $0.03 \mu$ , from  $1.14 \mu$  to  $1.17 \mu$ . In an experiment of this sort, where the error due to the setting is so large in comparison with the errors in radiometer deflections, it is more satisfactory to draw a curve representing one series of observations carefully taken, than to draw an average curve. Curve 2 is the corrected curve which will be discussed under IV.

The most striking characteristic of curve 1 is the elevations which are due to the emission bands of the gases in the flame. If the acetylene is pure, the gases to be expected are  $\text{CO}_2$  and  $\text{H}_2\text{O}$ .

Paschen<sup>1</sup> has made a study of the emission bands of both of these and has found the wave-lengths of the maxima under different conditions and temperature. He gives the following data for the bands found in the radiation from a bunsen burner.

#### $\text{H}_2\text{O}$ SPECTRUM.

Maximum points,  $1.46\mu$ ,  $1.90\mu$  and  $2.83\mu$ .

#### $\text{CO}_2$ SPECTRUM.

Maximum points,  $2.71\mu$  ( $2.68\mu$  when  $\text{CO}_2$  is not dry) and  $4.40\mu$ .

Although these values are computed from a dispersion curve different than the one used in our computations, yet the difference at the points considered would not be  $0.01\mu$ .

The energy curve shows elevations whose maxima agree very closely with these values, with the exception of the band  $2.83\mu$

<sup>1</sup> Paschen, *Annal. der Phys.*, 53, 1894, p. 334.

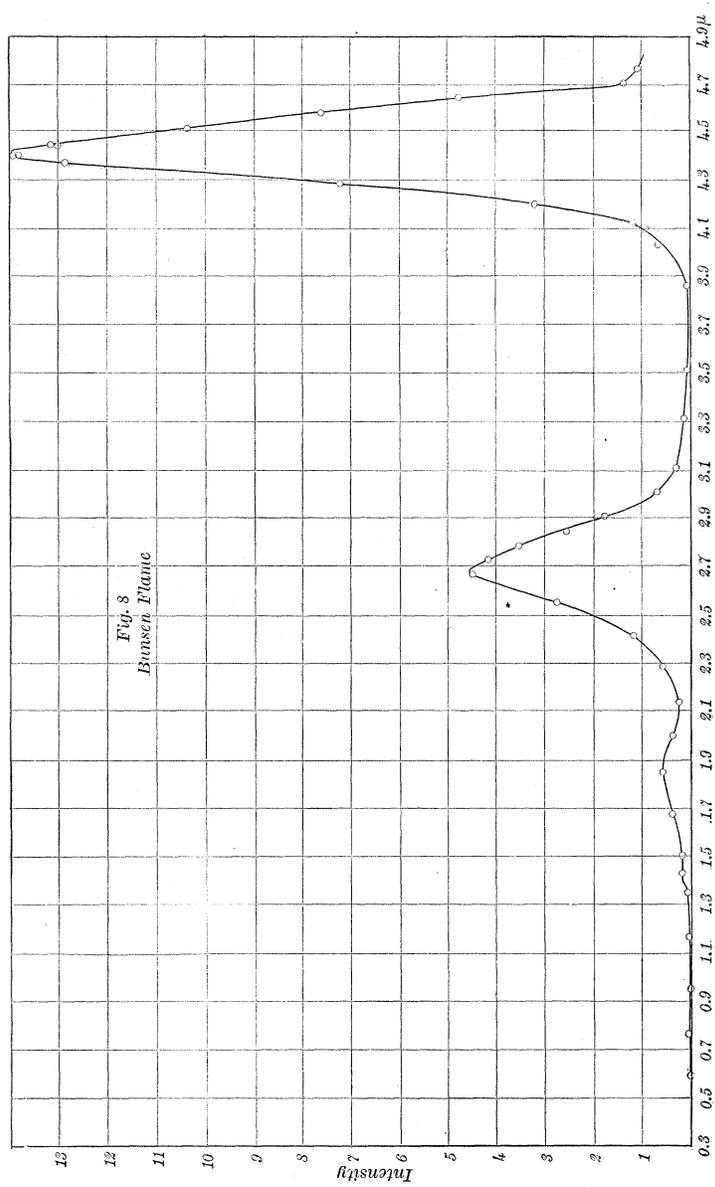


Fig. 8.

which is perhaps too small. By means of these points, we have an easy and accurate method of testing the spectrometer adjustment. The slight elevation at  $2.3\mu$  was obtained several times, and may be due to the emission of some impurity in the acetylene.

### 2. Bunsen Flame.

The curve for the acetylene bunsen flame is shown in Fig. 8. In this curve all five emission bands appear, and the values of the wavelength at the maximum points agree with the values quoted above. The bands being more distinct than in Fig. 7 we have still a better test for the adjustment of the apparatus. If we compare the bunsen and cylindrical flame curves, we see that they do not agree in all respects. The elevations due to the emission of the  $H_2O$  gas have not the same relative values in the two cases, but this discrepancy disappears when both curves are corrected for slit width. The band at  $2.3\mu$  does not appear in the bunsen flame, and this fact makes its presence in the other somewhat in doubt. That the apparatus permits only a very small amount of stray radiation, is well shown by this bunsen flame curve. In the neighborhood of  $3.7\mu$  the deflection is practically zero, while at  $4.4\mu$  it is very large.

### 3. Flat Flame.

Observations were next taken upon the flame of a two-jet Naphey burner. The relative positions of the flame and the spectrometer slit are shown in Fig. 9. The work upon the flat flame is not yet finished, the results obtained up to the present time, shown in Fig. 10, curve 1, being quite unsatisfactory. The lack of consistency in the observations is, in all probability, due to an unsteadiness in the flame itself. This might be due to the fact that the layer of burning gas is thin enough to be affected easily by air draughts.

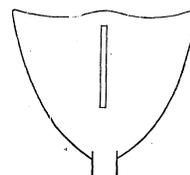


Fig. 9.

The fact that the maximum with the Naphey burner is further toward the violet, is in agreement with the observations of Hartman,<sup>1</sup> who showed photometrically that the Naphey burner flame was

<sup>1</sup> E. L. Nichols, Journ. Frank. Inst., Nov., 1900.

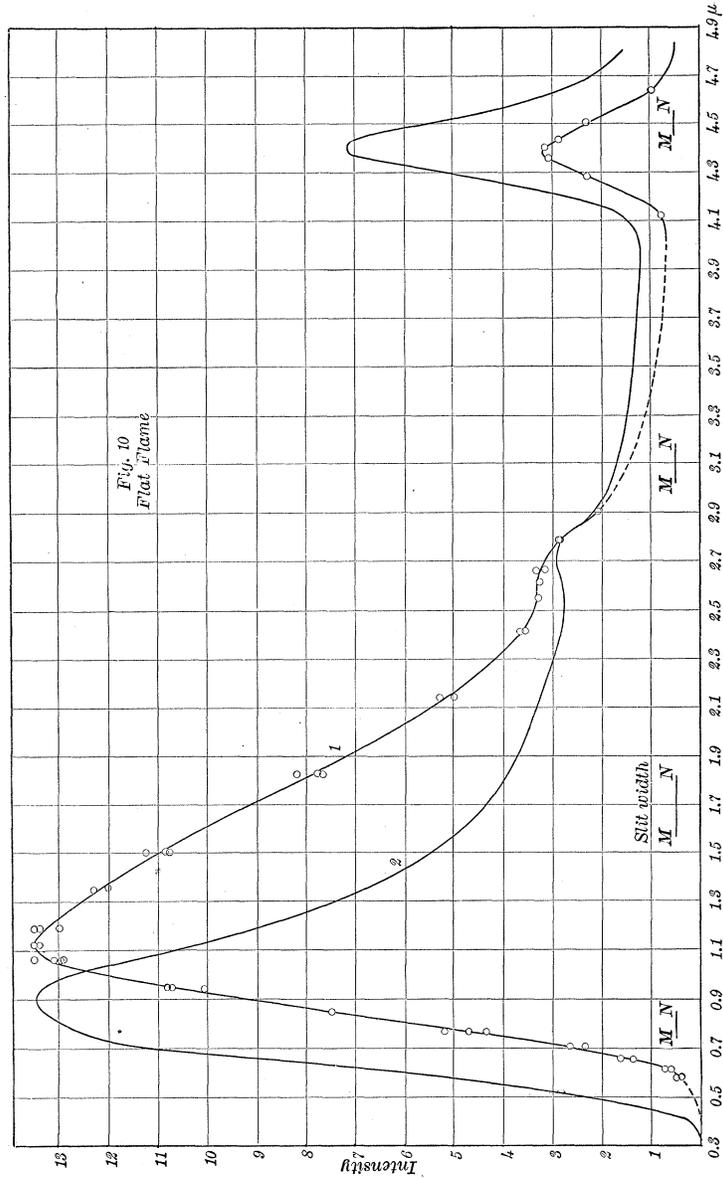


Fig. 10.

much richer in the violet. Curve 2 is the corrected curve and will be discussed later.

#### IV. DISTRIBUTION IN THE PURE NORMAL SPECTRUM.

##### *I. Slit Width Correction.*

The curves of observations do not represent the actual distribution of energy in the pure spectrum. No matter how narrow the slit, a correction must always be made, for in such measurements we are concerned not so much with the actual as with the relative widths in the different parts of the spectrum. The method of this correction will be discussed in considerable detail, as there does not seem to be uniformity among the various writers who have had occasion to make such a correction.

When I first considered the matter, the following seemed to be an approximately correct method. Consider the spectrum falling upon the radiometer slit, or the spectrometer spectrum. In different parts of this spectrum the slit has different widths when expressed in wave-lengths, and if  $b$  and  $b'$  are the intensities in the spectrometer spectrum and  $a$  and  $a'$  the corresponding slit widths in wave-lengths, then the deflection obtained when the radiometer is at  $b$  is proportional to  $ab$ , and when at  $b'$ , to  $a'b'$ . To find the distribution of energy in the spectrometer spectrum, we would then need only to divide the ordinates of the observed curve by the slit widths expressed in wave-lengths. But this spectrum is really composed of an infinite number of overlapping images of the spectrometer slit. Consider each wave-length to give such an image. Then at any point the number of overlapping images would be proportional to the width of the spectrometer slit expressed in wave-lengths. Hence, the intensity at any point in the spectrometer spectrum is proportional to the intensity in the pure spectrum and also to  $a$ , where  $a$  is the width of the spectrometer slit expressed in wave-lengths. Then to find the pure spectrum when the spectrometer spectrum is known, one has only to divide the intensities of the latter by  $a$ ,  $a'$ , etc. Combining these two steps, the slits being of equal width, the distribution of energy in the pure spectrum is obtained from the observed distribution by dividing the intensities of the latter by the square of the slit widths ex-

pressed in wave-lengths. This method is accurate only in so far as the pure spectrum curve may be considered a straight line for a length corresponding to a slit width.

Langley's<sup>1</sup> method of correcting for the bolometer strip width is, in effect, dividing the observed energy by the strip width in wave-lengths, and does not seem to take consideration of the spectrometer slit width.

Paschen<sup>2</sup> has given an accurate method of correction, taking into account both the strip width and spectrometer slit width. An approximation of his equation shows that he divides only by the slit or strip width, while on the other hand, Runge<sup>3</sup> in a later article, divides by the square of the slit width.

In recent articles on the radiation of black bodies, I find references to the method of Paschen and to the method of Runge, and one article refers to both as though they were the same. This is rather confusing. In my opinion Paschen is in error, and I shall repeat here part of his derivation of the equation to be used, at the same time making a correction.

In a pure spectrum, let  $f(x)$  be the intensity per unit wave-length at the point  $x$ , where  $x$  is measured in wave-lengths. Let the radiometer slit or bolometer strip, and likewise the spectrometer slit, have a width  $a$ , where  $a$  is expressed in wave-lengths. If the radiometer slit is at  $x$ , then an image of the spectrometer slit due to the wave-length  $x$  will fall entirely within the radiometer slit, and the intensity of this image is  $af(x)$ . The intensity of the wave-length  $x + v$  falling upon the radiometer slit at  $x$  is  $(a - v)f(x + v)$ . If  $F(x)$  represents the total energy falling upon the radiometer slit, we then have,

$$F(x) = \int_0^a (a - v)[f(x + v) + f(x - v)] dv.$$

This differs from Paschen by the factor  $a$ . He says that the energy in the image of  $x$  is  $f(x)$  and in the image of  $x + v$  is  $\frac{a - v}{a}f(x + v)$ , and so his equation is

$$F(x) = \int_0^a \frac{a - v}{a} [f(x + v) + f(x - v)] dv.$$

<sup>1</sup> Langley, *Phil. Mag.*, 17, 1884, p. 184.

<sup>2</sup> Paschen, *Annal. der Phys.*, 60, 1897, p. 712.

<sup>3</sup> Runge, *Zeitschr. für Math. und Phys.*, 42, 1897, p. 205.

Making the change in his work, we have as a final equation,

$$a^2f(x) = F(x) - \frac{1}{6}F_1(x) + \frac{2}{45}F_2(x) \dots$$

where

$$F_1(x) = \frac{F(x+a) + F(x-a)}{2} - F(x)$$

and

$$F_2(x) = \frac{F_1(x+a) + F_1(x-a)}{2} - F_1(x).$$

I have used this equation in correcting the curves obtained from observations.  $F_1(x)$  and  $F_2(x)$  can be found graphically as explained by Paschen.  $F(x)$  is observed,  $F_1(x)$  determined from the curve of  $F(x)$ , and  $F_2(x)$  determined from the curve of  $F_1(x)$ .

In my curves, the correction due to  $F_2(x)$  was too small to be of any consequence, and so  $f(x)$  was found simply by dividing  $F(x) - \frac{1}{6}F_1(x)$  by the square of the slit width in wave-lengths. This slit width was determined from a curve showing the relation between wave-lengths and angular rotations of the prism table, and for the sake of greater accuracy, the values obtained were plotted with wave-lengths and a smooth curve drawn through these points.

### 2. The Corrected Curves.

This correction for slit width and impurity of the spectrometer spectrum would not be of great importance in parts of the spectrum where the dispersive power of the prism changes slightly, but in my curves, particularly in the visible portion of the spectrum, the corrected curve differs greatly from that observed. The absorption of the  $\text{CO}_2$  and  $\text{H}_2\text{O}$  in the air between the source and the radiometer, make the observed curves very much in error at the points where we obtained emission bands. So the accurate method of correction was applied only from  $0.4\mu$  to  $1.9\mu$  and the remainder of the curve was obtained by dividing by the square of the slit width in wave-lengths.

In Fig. 7, it will be noticed that the maximum point has shifted from about  $1.16\mu$  on the observed curve, to about  $0.93\mu$  on the corrected one. Likewise in Fig. 10, the change has been from  $1.12\mu$  to  $0.90\mu$ . The observed maximum depends upon the values

of the energy in the pure spectrum and upon the slit widths in wave-lengths and should lie between the maxima of these two variables. This is verified, for the slit width has its maximum value at about  $1.6\mu$ .

A discrepancy in the relative intensities of the  $H_2O$  emission bands,  $1.46\mu$  and  $1.90\mu$ , in the curves of the bunsen and cylindrical flames, has already been mentioned. The bunsen shows that the band at  $1.90\mu$  is the more intense, and this is in agreement with the work done by Paschen. In the observed curve of the cylindrical flame, Fig. 7, curve 1, the opposite seems to be the case. However in the corrected curve, the band at  $1.46\mu$  scarcely shows, while the one at  $1.90\mu$  is quite distinct. The change brought about in the correction is due to the fact that the maximum value of the slit width is at about  $1.6\mu$ , and this has a tendency to exaggerate the elevation due to the band at  $1.46\mu$ .

### 3. *Temperature of the Acetylene Flame.*

The change in position of the maximum intensity in the spectrum of a black body with changes of temperature, is in agreement with the equation

$$\lambda_m T = A,$$

where  $\lambda_m$  is the wave-length of the maximum intensity,  $T$  the absolute temperature, and  $A$  a constant.

Lummer and Pringsheim<sup>1</sup> have found for  $A$  the value of 2940 and this agrees approximately with the results obtained by Paschen<sup>2</sup> and Mendenhall and Saunders.<sup>3</sup> Lummer and Pringsheim have also found the value of  $A$  to be 2630 in the case of bright platinum. They are of the opinion that the temperature of flames can be calculated from the formula, and that the true value for the temperature lies between the values obtained by assuming the two values for  $A$ .

Applying the formula, we obtain the following values :

#### CYLINDRICAL FLAME.

$$\lambda_m = 0.93 \mu, \quad A = 2940, \quad T = 3160^\circ, \text{ or about } 2890^\circ \text{ C.}$$

$$A = 2630, \quad T = 2830^\circ, \quad \text{“ “ } 2560^\circ \text{ C.}$$

<sup>1</sup>Lummer and Pringsheim, Verh. Deutsch. Phys. Gesell., 1899, p. 214.

<sup>2</sup>Paschen, Sitz. Kgl. Preuss. Akad. der Wiss. Ber., 1899, p. 959.

<sup>3</sup>Mendenhall and Saunders, Astrophys. Jl., Jan., 1901, p. 25.

## FLAT FLAME,

$$\lambda_m = 0.90 \mu, \quad A = 2940, \quad T = 3270^\circ, \text{ or about } 3000^\circ \text{ C.}$$

$$A = 2630, \quad T = 2920^\circ, \quad \text{“ “ } 2650^\circ \text{ C.}$$

The temperature of the acetylene flame as determined by Nichols,<sup>1</sup> is about 1900° C. I am at a loss to understand the cause of the discrepancy in the application of this formula. The temperatures of different sources of light, calculated by Lummer and Pringsheim (loc. cit.) in a similar manner, are certainly more nearly within range of the correct values. A difference might arise in the method of correcting the observed curve. They say they reduce their curves to the normal spectrum, but fail to give the method. In a later article,<sup>2</sup> they state that when their curves were corrected according to the method of Runge, the resulting change was not more than 5 per cent.

Mendenhall and Saunders in the article already referred to say they have corrected their curves by dividing by  $\frac{d\lambda}{d\delta}$  and that corrections were also made for the impurity of the spectrum according to Runge. It seems to the writer that these two methods are at variance, the one being in effect, dividing the observed values by the slit width in wave-lengths, while Runge divides by the square of the slit width in wave-lengths.

If calculations are made from our observed curves, the resulting temperature is yet about 100° C. too high.

It is quite possible that the value for  $A$  decreases with higher temperatures. Mendenhall and Saunders find a slight decrease, while Paschen finds an increase.

#### 4. Radiant Efficiency.

In the study of devices for artificial lighting, the determination of the efficiency is of great importance, and to obtain this determination, the ratio of the radiation in the visible portion of the spectrum to the whole radiation, must be known. This ratio, or radiant efficiency, has already been determined for acetylene by Stewart

<sup>1</sup> E. L. Nichols, *Phys. Rev.*, X., 1900, p. 234.

<sup>2</sup> Lummer and Pringsheim, *Verh. Deutsch. Phys. Gesell.*, 1900, p. 163.

and Hoxie.<sup>1</sup> They used a modification of the Melloni method, and, from a large number of readings, obtained the value of 0.105 for the flat flame of a Naphey burner.

Langley was the first to utilize spectral energy curves for accurate determinations of the radiant efficiency. His method consisted in finding the ratio of the area in the visible portion of the spectrum to the area of the entire curve. My corrected curves were treated in this same manner, the visible spectrum being considered as ending at  $0.76\mu$ . The cylindrical flame curve, Fig. 7, curve 2, gave 0.100, and the flat flame curve, Fig. 10, curve 2, 0.131. I think there is no doubt but that the efficiency of the flat flame is the greater, but I do not place much reliance on the value 0.131, because the curve from which it was obtained is not satisfactory. The difference in the radiant efficiency of the two flames might be anticipated from the difference in temperature and color. If the absorption of the atmosphere were eliminated, these two values would be decreased by perhaps more than 0.01.

#### V. SOURCES OF ERROR.

1. The acetylene was made by the wet process and there was no attempt at purification. Mr. H. A. Rands made three analyses of the gas. The greater part of the acetylene was absorbed by fuming sulphuric acid, a solution of caustic potash removed the sulphur trioxide, oxygen was then removed with alkaline pyrogallol, and the remaining traces of acetylene were absorbed by cuprous chloride. He obtained 98.7, 98.4 and 99.0 as percentages of purity.

2. The gas coming from the reservoir passed through a pressure regulator devised by Prof. Moler.<sup>2</sup> Although several others were using gas at the same time, this device kept the pressure constant to within less than 1 per cent. Just how the variation in pressure would affect the energy entering the radiometer, depended not only upon the change in the flame itself, but also upon the position of the flame in regard to the spectrometer slit. It was found that if the pressure were changed 1 per cent., the change in the radiometer deflection would be only about 0.2 per cent.

<sup>1</sup> E. L. Nichols, *Phys. Rev.*, XI., 1900, p. 215.

<sup>2</sup> E. L. Nichols, *Journ. Frank. Inst.*, Nov., 1900.

3. Before reaching the radiometer vanes, the radiation from the acetylene flames passed through a fluorite prism, a fluorite plate and a very thin sheet of mica. The selective absorption of fluorite<sup>1</sup> does not begin until  $6\mu$ , and this is entirely out of our region. The mica being very thin, no appreciable absorption was to be expected. However, a test throughout the region in question was made, and this expectation verified.

4. The mirrors of the spectrometer, silvered according to Bra-shear's<sup>2</sup> method, were not highly polished. Such a surface causes diffused light and also makes the difference in percentage of reflection of the short and long waves greater than when the mirrors are highly polished. The diffused light was small, as can be seen from the visible spectrum curve, Fig. 6, and the bunsen curve, Fig. 8. Paschen<sup>3</sup> has recently shown that with a freshly silvered and highly polished mirror, the reflection at  $0.7\mu$  is 94 per cent. and at  $6\mu$  is 98 per cent., while in the case of a mirror four years old and having a distinct brownish tint, the reflection at  $0.7\mu$  is 85 per cent. and at  $6\mu$  is 95 per cent. Judging from these figures, it is probable that the reflection from our mirrors at these wave-lengths would differ by about 6 per cent. No attempt was made to correct the curves for this error.

5. The angle at which the radiation falls upon the face of the prism is different for different wave-lengths, and it is desirable to know how much error is made when this is not taken into consideration. Calculation by the Fresnel energy equation shows that when the radiometer is at  $0.6\mu$ , 3.2 per cent. is reflected, and when at  $6.0\mu$ , 2.7 per cent., a difference of only 0.5 per cent.

6. The percentage absorption of platinum-black for different wave-lengths is certainly different, and, moreover, is not known.

7. The absorption due to the presence of  $\text{CO}_2$  and  $\text{H}_2\text{O}$  in the atmosphere of the room causes the curve to be in very great error at the emission bands of these gases.

8. In correcting the curves, only two terms of the series were used, but these approximated the true result within 0.2 or 0.3 at the point of greatest variation.

<sup>1</sup> Paschen, *Annal. der Phys.*, 53, 1894, p. 812.

<sup>2</sup> Wadsworth, *Astrophys. Jl.*, Vol. I., 1895, p. 252.

<sup>3</sup> Paschen, *Annal. der Phys.*, 1901, p. 314.

If the errors 4 and 6 are not taken into consideration, I think the portion of the curve, Fig. 7, curve 1, between  $0.7\mu$  and  $2.7\mu$  is not in error by more than one per cent.

#### VI. SUMMARY.

The distribution of energy in the spectrum of the acetylene flame is found in three cases by means of the spectrometer and the radiometer.

1. The cylindrical flame of a single-tip burner, distribution observed, Fig. 7, curve 1, actual distribution, curve 2, and the visible portion of the spectrum, Fig. 6, curves 1 and 2.

2. The flat flame of a two-jet Naphey burner, Fig. 10, curves 1 and 2.

3. Bunsen burner flame, Fig. 8.

These curves show the emission bands of  $H_2O$  and  $CO_2$ , and, the position of these bands being known, a check is had upon the adjustment of the apparatus.

The radiant efficiency of the cylindrical flame is found to be 0.100.

I wish to extend my thanks to Professors E. L. Nichols and Ernest Merritt, at whose suggestion and under whose direction the experimental work was performed.

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