THE STUDY OF THE DISTRIBUTION OF STRAINS BY POLARIZED LIGHT.

By A. Marston.

A FEW months since the writer became interested in the determination of the safe loads for a certain class of members of bridges. There was no good theory as to the distribution of the strains in those members, and it was suggested to him to attempt its study by means of polarized light. The method to be described in this paper was worked out in the course of the investigation then undertaken.

The fact that this method, which is mainly of interest to the physicist, was thus the result of the study of an engineering problem is an interesting illustration of the interdependence of the various professions and sciences. It is this interdependence which makes the writer willing to venture to present the results of his experiments in a physical journal, although he is aware that the apparatus at his command was crude, and that his methods of research were approximate. He is the more willing to do this because the interpretation of the results of these experiments depends wholly upon a few elementary principles of optics, well known to all scientists.

When any transparent isotropic substance, such as glass, is subjected to strain, the relative positions of its particles are changed, and, corresponding to the ellipsoid of elasticity of a bi-refractive crystal, we have the ellipsoid of strain. Hence, we should expect that when light falls upon it, only those components of the vibrations would be transmitted which are parallel to the axes of the diametrical section of the ellipsoid of strain made by a plane perpendicular to the ray of light, just as in the case of the bi-refractive crystal.
As a matter of fact this property of glass has long been known. When a plate of it is subjected to strain, and interposed between the polarizer and analyzer brilliant color effects are produced, just as with thin crystalline plates.

This property of glass can be taken advantage of to determine exactly the intensity and direction of the strain at any given point in it. Since glass is an isotropic body, whose elastic constants are known, we are able to test any theoretical solution of the general problem of determining the distribution of the strain in an isotropic body by comparing the results of theory with the strain in a similar body of glass, subjected to the action of similar external forces. In this line something has already been done. Wertheim, whose work is referred to in Tyndall's "Lectures on Light," used the colors of polarization in the study of the elasticity of glass. He established the fact that the retardation of the ray is proportional to the load. He described a device, which he designated the "chromatic dynamometer," for the study of stress uniformly applied. In the Transactions of the American Society of Civil Engineers, Vol. III., Nickerson has published an account of some experiments to determine the position of the neutral axis in beams by this method. The theory on which he based the interpretation of his results was, however, incorrect, and it seems strange that his work was not criticized at once.

There are two things to determine in order to specify the strain at any point in a solid; one, the directions of its axes, and the other, the intensities of the strains along those axes. Corresponding to these two things we have two sets of effects when a glass body, strained so that one of the axes of strain has a known constant direction for all points of the body, is placed between two Nicol's prisms and examined by polarized light passed through it parallel to the known axis. First, if the body be strained in such a way that the lines of strain vary in direction from point to point, we shall find dark bands traversing the glass body, which will vary in position as the prisms rotate relative to the glass body or to each other. In the second place, we shall observe color bands

varying in quality and position, with similar rotation of the prisms. The dark bands depend upon the directions of the lines of strain, while the color effects are due to the relative intensities of those two principal strains whose axes are perpendicular to the direction of light. For the sake of clearness we will consider these effects separately in the discussion which follows, and separate them in the figures illustrating the appearances observed in actual cases.

THE DIRECTIONS OF THE LINES OF STRAIN.

We will take up first, then, the study of the directions of the lines of strain throughout the body. In this case, the interpretation of the meaning of the dark bands observed in the field of view of the analyzer will be much simplified if we keep the prisms crossed. Let the polarizing plane of the polarizer be set at any given angle with the vertical. Then at all points where the lines of strain are parallel or perpendicular to the plane of polarization, the light will not have the direction of its vibrations changed in passing through the glass, and consequently cannot pass through the analyzer. Hence at those points the surface of the glass will remain dark. At any other point, however, the plane of polarization will be rotated so as to contain a tangent to the line of strain. Light from that point will, therefore, be able to pass more or less completely through the analyzer.

The result will be that the surface of the glass, as seen through the analyzer, will appear to be crossed by one or more dark bands, which will give the loci of the points where the lines of strain are parallel or perpendicular to the plane of polarization of the polarizing prism.

By setting the plane of polarization at any desired angle with the vertical, we may thus determine the direction of the lines of strain at a number of points in the glass body, and may draw elementary arcs through those points. By varying the angle of inclination of the polarizing plane and taking a series of observations, we may obtain the elementary arcs at points as close together as desired. By connecting these elementary arcs, we shall then have an approximate sketch of the lines of strain throughout
the body, and sometimes may be able to deduce at once the true theory of their location.

To illustrate the application of this method let us take the case shown in Plate I. Here a one-inch cube of optical glass was pressed by a one-inch roller of optical glass loaded with 400 lbs. The cube had two opposite faces polished, and the others carefully ground. The polished faces were the ones observed, and were set at right angles to the rays of polarized light. The roller was one inch in diameter and one inch long, and was carefully ground cylindrical. Its ends were polished. It was placed with its axis parallel to the rays of light, and the line of contact was a medial line of the upper surface of the cube. The cube rested on the upper face of a piece of plate glass \( \frac{3}{4} '' \) thick, \( 1 '' \) wide in the direction of the rays of light, and \( 1 \frac{1}{2} '' \) long. The plate glass rested on a block of hard wood. The applied force was measured by spring balances.

The dark bands shown in Figs. 1 to 8 were sketched by dividing the observed face of the cube into \( \frac{1}{10} '' \) squares by placing against it a reticule of fine threads stretched in a brass frame. The sketches were made on one-inch squares similarly divided, as shown in the figures.

The polarizer was a Nicol’s prism having the distance between parallel sides about \( 1 \frac{1}{2} '' \). The analyzer was a similar prism having about \( 1 '' \) between parallel sides. These prisms were set in brass tubes in the usual way, and were mounted on wooden stands in such a way that the plane of polarization of each could be set at any desired angle with the vertical, and the angle measured within a degree or two.

When the strained cube was placed between the polarizer and analyzer, with the plane of polarization inclined to the right of the vertical, the amount indicated on each figure, the dark bands shown on the figures were observed. It is to be noted that in addition to these dark bands, color effects were also observed similar to those shown on Plate II. These are omitted from the figures for reasons already given.

In interpreting the results of this experiment, we notice first that there are two regions, in the right and left upper corners.
DETERMINATION OF LINES OF STRAIN BY POLARIZED LIGHT.

1 Inch Cube Pressed by 1 Inch Roller Loaded with 400 Lbs. The Color Bands are Omitted from the Figures. Prisms Crossed.
PLATE II.

DIFFERENCE OF PRINCIPAL STRAINS BY POLARIZED LIGHT.

1 inch Cube Pressed by 1 inch Roller Loaded with 400 Lbs. Prisms Parallel.
respectively, which were dark for nearly all positions of the prisms. In those figures in which these regions are not dark, the light may have been reflected from the sides of the prism in such a way as to produce the appearances seen. It seems probable that the stress was there too feeble to give the glass any bi-refractive qualities.

Aside from this, certain of the dark bands indicate the regions where the lines of strain are inclined in the direction of the planes of polarization, and the others the regions where the lines of strain are perpendicular to those planes.

Where these dark bands, and the dark regions already mentioned in the upper right and left corners overlap, we have the curious appearances seen in Figs. 4, 5, and 6.

By drawing elementary arcs, inclined at the proper angles to the vertical, through the points indicated by the dark bands, and connecting them, we obtain the sketch of the lines of strain shown in Fig. 9.

The dark places along the bottom of the cube, seen in Figs. 1, 2, and 8, indicate points of local intensity of pressure and distortion of the lines of strain, due to the unevenness of the bearing surface. By comparing closely with Fig. 7, Plate II., we can see how the lines of strain converge from each side to these points of local intensity of pressure.

In this case the equations of the lines of strain have not been worked out, but in a series of experiments, which the writer has in progress, he was able to decide at once upon the true theory of their directions. In making a complete study of the theory of the strain in the cube under consideration, it would be necessary to make a set of experiments similar to the one illustrated, for different loads and for different dimensions of rollers and cubes.

The Intensity of the Strain.

Let us consider next the study of the intensity of the strain by means of the color-effects already referred to. It may be said, in advance, that no such complete solution of this problem
can be obtained as of that of the directions of the lines of strain. Nevertheless, we get very interesting and instructive results.

In what follows, it must be understood that the discussion applies only to cases where one of the axes of strain has a known direction, constant for all points of the glass, and that the polarized light is passed through the glass parallel to this known axis.

Reasoning by analogy from bi-refractive crystals, we should expect that the velocities of the two rays of light in the case under consideration would be inversely proportional to those two axes of the strain ellipsoid which are at right angles to the direction of the polarized light, i.e. to the intensities of the principal strains in those directions. Actual experiments, whose results will be given hereafter, approximately verify this assumption, at least for small strains. Hence the relative retardation will depend upon the relative intensities of the principal strains named, and in the colors observed in the strained glass we have a measure of the difference between those principal strains at each point of the body.

Hereafter, unless otherwise specified, by principal strains will be meant those two principal strains at right angles to the ray of light.

When the difference between the principal strains at any point in the glass gradually increases from 0, the first color waves in the polarized light to be suppressed will be the shortest ones, i.e. those in the vicinity of violet. We should therefore expect that the first color to make its appearance would be the complementary color, yellow, and that following this we would have red, and then blue. When the difference becomes equal to one and one-half times that causing yellow we will again have yellow, the relative retardation in this case being one and one-half times the wave-length of violet. Following this second yellow will come second bands of red and blue, and so on.

To show how completely this theory is borne out by experiment, we give the results obtained by subjecting the glass cube, already experimented with, to strains of known character. In an ordinary Olsen testing machine of 50,000 lbs. capacity, the cube was sub-
jected to a series of uniformly distributed vertical compressive stresses, gradually increasing from 0 to 7050 lbs. The appearance when polarized light was passed through it was observed by means of the Nicol's prisms, already described. The prisms were set with their polarizing planes parallel, and inclined at an angle of 45° to the vertical. The resulting colors are shown in the following table:

**Table No. 1.**

**SHOWING COLORS OBSERVED IN 1" CUBE OF OPTICAL GLASS WHEN SUBJECTED TO SIMPLE VERTICAL COMPRRESSIVE STRESSES.**

<table>
<thead>
<tr>
<th>Load</th>
<th>Color</th>
<th>Load</th>
<th>Color</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 lbs.</td>
<td>Clear</td>
<td>2100 lbs.</td>
<td>Greenish-yellow</td>
</tr>
<tr>
<td>200 &quot;</td>
<td>Cloudy</td>
<td>2550 &quot;</td>
<td>3d Red</td>
</tr>
<tr>
<td>300 &quot;</td>
<td>Faint yellow</td>
<td>3000 &quot;</td>
<td>Green</td>
</tr>
<tr>
<td>400 &quot;</td>
<td>Bright yellow</td>
<td>3650 &quot;</td>
<td>4th Red</td>
</tr>
<tr>
<td>525 &quot;</td>
<td>1st Red</td>
<td>4050 &quot;</td>
<td>Green</td>
</tr>
<tr>
<td>600 to 900 &quot;</td>
<td>Blue</td>
<td>4750 &quot;</td>
<td>5th Red</td>
</tr>
<tr>
<td>1200 &quot;</td>
<td>Yellow</td>
<td>5250 &quot;</td>
<td>Green</td>
</tr>
<tr>
<td>1550 &quot;</td>
<td>2d Red</td>
<td>5900 &quot;</td>
<td>6th Red</td>
</tr>
<tr>
<td>1800 &quot;</td>
<td>Greenish-blue</td>
<td>6450 &quot;</td>
<td>Green</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7050 &quot;</td>
<td>7th Red</td>
</tr>
</tbody>
</table>

In making this experiment it was found that the delicacy of the test was such that it was very difficult to get rid of the effects of local pressures, due to slight inequalities in the bearing surfaces. However, by using steel bearing blocks one inch square, having the bearing surfaces carefully surfaced, and by interposing a few thicknesses of paper between them and the cube, the resulting colors were so evenly distributed over the face of the cube that no doubt was left in the mind of the observer that such slight unevenness as still remained was due entirely to inequalities in the distribution of the applied load.

In this experiment, since both of the principal stresses other than the vertical were 0, the difference between the principal
strains is proportional to the load. The results may be regarded as a calibration of the observed colors for the glass cube when it is subjected to strain.

It was observed that the range of the loads corresponding to any particular band of red was less than for either of the other colors. As near as could be observed the loads corresponding to the middle lines of the different bands of red were as follows:—1st, 525 lbs.; 2d, 1550 lbs.; 3d, 2550 lbs.; 4th, 3550 lbs.; 5th, 4550 lbs.; 6th, 5500 lbs.; 7th, 7050 lbs.

In interpreting these results it is to be noticed that the loads given are approximate, because each is for a band of color, which corresponds to a certain range of loads. Moreover, the testing machine used was not very sensitive to increments of load less than 100 lbs., and no means were available for testing its accuracy. To remove as far as possible inaccuracies due to the testing machine, a similar series of observations was made on a piece of optical glass one inch thick by two inches square, finished similarly to the glass cube before tested. The loads for the same strains and colors as in the cube should have been twice those already given. The results were as follows:—1st, 1100 lbs.; 2d, 3175 lbs.; 3d, 5300 lbs.; 4th, 7500 lbs.; 5th, 9600 lbs.; 6th, 12,000 lbs.; 7th, 14,100 lbs.

These results establish the law that, as far as the experiments extended, the retardation varies approximately directly with the strain. They indicate that the stress necessary to produce a retardation of one wave-length of greenish blue, the complementary color of red, is about 1065 lbs.

The experiments could not be carried further because, as the loads increased, the colors became pale, and after the seventh series were hardly distinguishable from each other or from white light. After the second series we have green in place of the yellow and blue. This increasing paleness of the colors is due to the well-known fact that, for retardations of several half wave-lengths, the interference effects in the case of white light become indistinct. This difficulty could be overcome by using monochromatic light.

To determine the effect of compound stress upon the colors, a
small hydraulic press was constructed, by which horizontal pressures as high as 300 lbs. could be applied to the inch cube at the same time that it was subjected to vertical pressure in the Olsen machine. The following will serve as an illustration of the results:

**Table No. 2.**

**SHOWING COLORS OBSERVED WHEN POLARIZED LIGHT IS PASSED THROUGH A 1" CUBE OF OPTICAL GLASS Subjected TO COMPOUND STRESSES.**

<table>
<thead>
<tr>
<th>Pressures</th>
<th>Color</th>
<th>Pressures</th>
<th>Color</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical</td>
<td>Horizontal</td>
<td>Vertical</td>
<td>Horizontal</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>400</td>
<td>300</td>
</tr>
<tr>
<td>100</td>
<td>0</td>
<td>500</td>
<td>0</td>
</tr>
<tr>
<td>200</td>
<td>0</td>
<td>500</td>
<td>100</td>
</tr>
<tr>
<td>300</td>
<td>0</td>
<td>500</td>
<td>200</td>
</tr>
<tr>
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<td>100</td>
<td>500</td>
<td>300</td>
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<tr>
<td>300</td>
<td>200</td>
<td>600</td>
<td>0</td>
</tr>
<tr>
<td>300</td>
<td>300</td>
<td>600</td>
<td>100</td>
</tr>
<tr>
<td>400</td>
<td>0</td>
<td>600</td>
<td>200</td>
</tr>
<tr>
<td>400</td>
<td>100</td>
<td>600</td>
<td>300</td>
</tr>
<tr>
<td>400</td>
<td>200</td>
<td>600</td>
<td>300</td>
</tr>
</tbody>
</table>

Similar observations were made for a series of vertical loads varying up to 4400 lbs. In the above experiments the lack of uniformity in the coloring in different parts of the field indicates some irregularity in the distribution of the loads. Allowing for this, however, the results show just what our theory would lead us to expect, viz. that the colors observed correspond to those caused by simple stresses equal to the difference between the components of the compound stress.

To further determine the effect of compound stress, a few experiments were made with a slip of plate glass $1_{\frac{3}{8}}'' \times 1_{\frac{5}{16}}'' \times 3_{\frac{1}{4}}''$. 
which was subjected to vertical tensions in the direction of its length of from 0 to 250 lbs. in the Olsen machine, and at the same time to horizontal pressures by the hydraulic press. The horizontal pressures were applied to the middle of the edges of the slip, over a space about one inch long, and varied from 0 to 300 lbs. The results showed in general that the resulting colors corresponded to simple stresses, equal to the numerical sum of the components of the compound stresses.

The preceding experiments show that, for small strains at least, the colors seen in a strained glass body, when polarized light is passed through it in a direction parallel to one of the axes of strain, are measures of the algebraic differences of the intensities of those two principal strains whose directions are perpendicular to the direction of the polarized light.

Since the axes of the principal stresses coincide with those of the principal strains, and since the difference of any two principal strains is directly proportional to the difference of the corresponding principal stresses, it follows that the word stress might have been substituted for strain throughout the preceding statement.

Let us now apply this method of studying the intensity of the stresses to the case of the one-inch cube of optical glass, for which the lines of stress have already been determined. [See Plate I. and the accompanying discussion.]

The manner of applying the vertical load of 400 lbs. to this cube has already been described. The polarizer and analyzer for the experiment were the Nicol's prisms already described. In this case, however, their principal planes were set parallel, so that the cube in its unstrained condition was clear. By this means we get rid of all dark bands such as those shown on Plate I., and the colors make their appearance for smaller loads than do the complementary colors seen when the prisms are crossed.

When we look at the glass cube through the analyzer we see a series of bands of yellow, red, and blue, instead of the uniform tint seen in the preceding experiments. Each of these bands indicates a region where the difference between the intensities of the principal stresses falls within the range of stress shown in Table I. for the corresponding color. We observe, however, that these color
bands are not well defined in all parts of the cube for any one position of the prisms, and that when the prisms are rotated, the region in which the color bands are well defined changes.

To understand the reason for this, we must look at what takes place when the ray of polarized light passes through the glass at any point. As has already been said, it is divided into two rays, polarized in the directions of the axes of the stress. Now if one of the axes of the stress be parallel to the plane of polarization, the ray polarized in the direction of the other axis cannot pass through the analyzer at all, and consequently no interference can take place. On the other hand, when the axes of the stress make angles of 45° with the plane of polarization, the rays pass through the analyzer in equal amounts, and interference is complete. For all other angles than 45°, 0°, or 90° between the axes of stress and the plane of polarization, partial interference can take place. For this reason, in the experiments with uniform simple and compound stresses on the cube, the prisms were set at 45° to the vertical. Since the axes of strain were vertical and horizontal, the angle of 45° gave the position for complete interference. In those experiments the glass became colorless when the prisms were set with their planes of polarization vertical or horizontal, and nearly so for any angle with the vertical differing much from 45°.

In the case at present under consideration, since the lines of stress vary in direction at different points in the cube, in order to map out the color bands throughout their whole extent, it will be necessary to take a series of observations with the prisms set at different angles with the vertical, and to combine the results. This is what was done in the experiments, the results of which are given in Plate II. Fig. 1 shows where the bands were sharply defined when the prisms were set at 45° with the vertical. Evidently this will be where the lines of stress are nearly vertical or horizontal. Similarly, Fig. 2 marks the bands plainly where the lines of stress are inclined 15° or 75° to the vertical, Fig. 3 where they are inclined 30° or 60°, and so on. By comparing with Fig. 9, Plate I., it will be seen how completely the theory is borne out.

By combining the results shown in Figs. 1 to 6, we obtain
the complete outlines of the several color bands as shown in
Fig. 7. We thus obtain a map, as it were, of the difference
of the intensities of the principal stresses. By the aid of Table
I., we can assign an approximate numerical value to this difference
at any point of the cube. Thus, at the line of contact between
the roller and the cube, the difference between the horizontal
and vertical components of the stress is about 7000 lbs. per
square inch, and since both the stresses are compressive at
this point, the maximum pressure between the roller and the
cube must be greater than 7000 lbs. per square inch, though the
vertical load was only 400 lbs.

The stresses themselves are in this case probably greater at
points where their difference is greater, so that Fig. 7 gives in
a general way an idea of the distribution of the stress throughout
the cube. Thus, a quarter of an inch from the bottom we see
that the stress is quite uniformly distributed throughout the
whole cube. The fact that the color here is bright yellow, like
that caused by a uniformly distributed load of 400 lbs. [see Table
No. 1.], shows that the horizontal stresses there are nearly zero.

The small areas of red and blue along the bottom of the cube
show points of local intensity of pressure, due to inequalities
in the bearing surfaces. It is evident that, for some reason, the
pressure on the right side of the cube was greater than elsewhere.

If we were attempting to express the stress at any given point
in the cube by a formula, we could make a series of experiments
like that whose results are given in Plate II. A careful study
of all the results would give checks for any proposed formula,
or might suggest an approximate solution of the problem, which
would answer for practical purposes.

In making any such series of experiments, the writer believes
that it would be practicable, with suitable apparatus, to throw
an image of the cube, or other glass body experimented with,
upon a screen, on which the dark bands and color bands could
then be easily and accurately traced. To obtain the best results,
the experiments should be conducted with monochromatic light.

Ames, Iowa, June 12, 1893.
PLATE II.

DIFFERENCE OF PRINCIPAL STRAINS BY POLARIZED LIGHT.

Note: There were six bands of red plainly visible, with a seventh probably just on.

1 inch Cube Pressed by 1 inch Roller Loaded with 400 Lbs. Prisms Parallel.