Do students use and understand free-body diagrams?

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Physics education literature recommends using multiple representations to help students understand concepts and solve problems. However, there is little research concerning why students use the representations and whether those who use them are more successful. This study addresses these questions using free-body diagrams (diagrammatic representations used in problems involving forces) as a type of representation. We conducted a two-year quantitative and qualitative study of students' use of free-body diagrams while solving physics problems. We found that when students are in a course that consistently emphasizes the use of free-body diagrams, the majority of them do use diagrams on their own to help solve exam problems even when they receive no credit for drawing the diagrams. We also found that students who draw diagrams correctly are significantly more successful in obtaining the right answer for the problem. Lastly, we interviewed students to uncover their reasons for using free-body diagrams. We found that high achieving students used the diagrams to help solve the problems and as a tool to evaluate their work while low achieving students only used representations as aids in the problem-solving process.

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I. INTRODUCTION AND PURPOSE OF THE RESEARCH

The conceptual knowledge in physics courses is often presented in an abstract symbolic form. The symbols have precise meanings and are combined with rules that must be used correctly. In contrast, the human mind relates best to picture-like representations that emphasize qualitative features but not detailed precise information.[1] There have now been a great many studies on physics learning indicating that students taught with an emphasis primarily on using mathematics to develop and apply concepts fail tests with seemingly simple conceptual questions that measure understanding. In these courses they learn to use formula-centered problem-solving methods with little understanding.[2]

If we want students to understand and learn to use the symbolic representations that are part of the practice of science (for example, the mathematical descriptions of processes), we have to link these abstract ways of describing the world to more concrete descriptions. A main question in this paper is to decide if a learning system with considerable emphasis on describing processes in concrete and in abstract ways and in building links between these different representations enhances student learning and problem-solving ability.

Students in courses that incorporate multiple representations have been very successful on such tests as the force concept inventory (FCI),[2] mechanics baseline test (MBT),[3] and conceptual survey of electrostatics and magnetism (CSEM) (Refs. 4–6), and in hands-on tasks.[7] But there is no literature concerning the effects of the quality of the multiple representations students construct to help with their quantitative problem solving and what they actually do while solving those problems. In this paper we provide a detailed study of student use of one of the representations, specifically a free-body diagram (FBD). This study investigates three questions:

(a) If students are in a course where they consistently use free-body diagrams to construct and test concepts in mechanics and in electricity and magnetism and to solve problems during the class, do they draw free-body diagrams on their own when solving multiple-choice problems on tests?

(b) Are students who use free-body diagrams to solve problems on tests more successful than those who do not?

(c) How do students use free-body diagrams when solving problems?

The answers to these questions will provide insights concerning the importance of multiple representations (specifically free-body diagrams) in student learning, thinking, and problem solving.

II. CONCEPTUAL FRAMEWORK

A. Problem solving: Expert versus novice

There are multiple differences in the approaches of experts and novices to problem solving.[8] Novices choose a strategy to solve a problem based on the superficial surface features while experts choose strategies based on concepts that are relevant to the problem.[8] Experts also utilize a larger number of heuristics or experimentally derived cognitive "rules of thumb.[9]" When experts use heuristics, they "chunk" the information together while novices look at the problem in pieces.[10] Novices also differ from experts in their search techniques during problem solving.[11] Novices typically first write down the known and unknown variables. Next, they use a backward inference technique—a search for equations involving variables they think they can use. Experts use a forward inference technique. A summary of the main differences between experts and novices[12] in problem solving is listed in Table I and can be found in Ref. 12.

In addition to these differences between experts and novices when they solve problems, researchers have documented

TABLE I. Differences in problem solving between experts and novices (Ref. 12).

Expert	Novice
Conceptual knowledge affects problem solving.	Problem solving largely independent of concepts.
Often performs qualitative analysis, especially when stuck.	Usually manipulates equations.
Uses forward looking concept-based strategies.	Uses backward looking means-end techniques.
Has a variety of methods for getting unstuck.	Cannot usually get unstuck without outside help.
Is able to think about problem solving while problem solving.	Problem solving uses all available mental resources.
Is able to check answer using an alternative method.	Often has only one way of solving a problem.

some differences between experts and novices when they construct representations to help them solve problems.[13–19] For example, in genetics, experts and novices differ in terms of the level of sophistication of their constructed diagrams and how they reason with their diagrams. As Ref. 16 states; "the experts displayed a variety of diagram-related reasoning behaviors such as knowledge-dependent representational variability, fine-tuning of the diagrams to the immediate reasoning task, and systematic use of fine-tuned diagrams as *tools to think with* while reasoning.[18]" Novices showed either very little or no evidence of these abilities. Similar differences were found in mathematics.[17]

Although both experts and novices in mathematics made visual representations while solving problems, novices typically did not use their visual representations to help solve the problems. Experts not only constructed visual representations more frequently but used them to explore the problem space, develop a better understanding of the situation, and to help solve the problem.[17]

Another issue is that novices may not always use the most effective representation when they attempt to solve a problem.[20,21] Usually novices proceed from an abstract verbal problem statement to an even more abstract mathematical representation[22] while an expert in the same situation would have an intermediate representation such as a picture, a graph, a force diagram, etc. Thus, it is important to create a representation-rich learning environment which helps students learn how to use different representations. Part of this environment includes helping students see how to use representations when solving sample problems and then transferring those representations to isomorphic target problems, which students can solve successfully.[23] Another integral part of the representation-rich environment is to have students solve real-world type problems via different representations while collaborating with others. Giving students a representation with a problem is not enough to help them become successful in learning and becoming confident with the content.[24] This environment can either focus on the role of representations explicitly or implicitly.[25]

B. Do multiple representations help students master physics?

In the early 1990s Van Heuvelen developed a curriculum that was based on the use of representations in his Overview, Case Study Physics (OCS).[26,27] This learning system was based on research by Larkin *et al.*,[1] Heller and Reif,[28] and others. In the OCS curriculum the instructor uses representations such as pictures, words, diagrams, and graphs to help students understand a concept and then students use these representations to solve quantitative problems based on this concept. Students' learning gains on a diagnostic test from the OCS course were 15% higher than those in a traditional class, and the OCS students were also able to retain information longer.[26]

Gautreau and Novemsky[29] reported on an adaptation of the OCS course that emphasized multiple representations and active student participation in the learning. They found that these OCS students scored significantly higher on problem-solving hour exams and on a final exam than traditionally taught students in three other sections of the same course. The exams were written by the professors in the three traditionally taught sections and taken at the same time by students from all four sections.

More recently, De Leone and Gire[30] investigated whether the use of multiple representations in courses affected student problem solving. They studied how many representations students used when solving open-ended problems on quizzes and tests. The course under study was taught via interactive engagement strategies with frequent use of different representations. De Leone and Gire found that the majority of the students used many representations such as pictures, free-body diagrams, graphs, etc. while solving the open ended problems. Also, they found that all of the students who correctly solved the majority of the problems were high multiple representations users. In all but one of the coded problems, students who used representations had a higher success rate in solving the problem. Their research suggests that if students learn physics in an environment that emphasizes the use of multiple representations, students will use them to help solve open-ended problems. However, De Leone and Gire did not assess the quality of the representations students constructed.

Students often recognize that constructing a representation is not a task in itself, but rather that a representation might help them solve the problem.[31] Van Heuvelen and Zou[31] found that students learn better if they understand the reason behind different pedagogical strategies such as using bar charts to solve problems involving energy. Student understanding and problem solving is enhanced if students

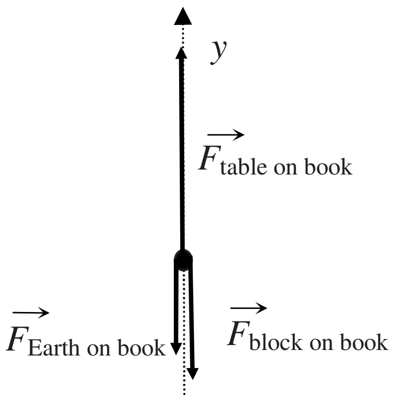


FIG. 1. An Example FBD.

learn to move back and forth in any direction between different representations[29,31] However, according to Kohl *et al.*,[25] students use representations whether they are taught explicitly or implicitly. Regardless, as we stated previously, if we want our students to become expert problem solvers, we must help them learn to construct different representations and to use them for problem solving.

C. Multiple representation example (free-body diagrams)

Much of the research described in Secs. II A and II B dealt with how experts and novices solve problems and use representations to learn. Since our study focuses on one representation in particular, the free-body diagram, we focus the rest of this section specifically on free-body diagrams (FBDs). An FBD is a diagrammatic representation in which one focuses only on an object of interest and on the forces exerted on it by other objects. Figure 1 is an example of an FBD for a book on a table with a block on top of it.

Instructors can teach FBDs in many different ways[26,28,32–38] but they all have the same goal, to help students solve problems involving forces. The method the instructor used in this study was developed by Etkina and Van Heuvelen[32] but is based upon the work of Heller and Reif.[28] We will explain this method in depth later in Sec. III.

As there are many different ways of drawing FBDs, it is important to highlight some of the differences.[34–39] Some researchers recommend special labeling techniques[35] while other researchers[38,39] have special placement of the vectors in the diagram with the forces drawn in a specific way. Some suggest that students should include the angles in the diagram.[35] Another approach involves students drawing a system schema[28,40,41] before they construct an FBD. A system schema is a pictorial representation showing the object of interest and how it interacts (via direct contact or at a distance) with other objects.

Regardless of how the free-body diagram is constructed, it helps students identify all of the forces exerted on an object of interest by other objects and then allows them to correctly apply Newton's second law in component form to determine the magnitude of the object's acceleration, or if the acceleration is known to determine the magnitude of an unknown force. This step is how FBDs play an integral part in the problem-solving process—as a transition from a concrete physical situation to an abstract mathematical equation(s).

The various ways a person can construct a free-body diagram has been well documented. However, none of the above mentioned studies discuss the relationship between the quality of the diagrams students construct and their success when they use the diagrams. One study analyzed the quality of free-body diagrams and how many students use them to solve the problems but did not relate the quality of the diagrams to student success on the problems.[42] This study investigates whether students who learn physics in an environment that explicitly focuses on multiple representations, and specifically on free-body diagrams, use free-body diagrams to help them solve problems, and whether the quality of the diagrams that students draw is related to their problem-solving success.

III. METHOD

A. Context

This study was conducted in two consecutive years in a two-semester large-enrollment (about 500 students in each of the two years) algebra-based physics course for science majors with the same instructor. The instructor of the representation-rich course followed the Investigative Science Learning Environment (ISLE) format[6,43]—a guided inquiry learning system that engages students in the active construction of knowledge mirroring the processes used by physicists to acquire knowledge. Since one of the processes that physicists use to solve problems and communicate information is representing knowledge in multiple ways,[22,26] the ISLE curriculum emphasizes the use of multiple representations. This emphasis is reflected in the course materials. *The Physics Active Learning Guide* (*ALG*) (Ref. 32) included, among other things, special innovative multiple representation tasks as separate problems. These tasks ask the students to represent the same phenomenon in different ways or to construct a new representation of a phenomenon using some other representation without having the students calculate a numerical answer. An example of a typical multiple representation task is provided in Appendix A.

During the large-room meetings, the instructor discussed with the students how to represent a process in a particular way and how to use one type of representation to help construct another. The instructor helped his students learn how to use pictorial and physical representations (motion diagrams, free-body diagrams, energy, and momentum bar charts) to reason about physical processes and to solve problems. The instructor used the following strategy to help students learn how to draw FBDs.[44] The steps listed in Fig. 2 (Ref. 45) are for a box being pulled across the floor.

(1) Sketch the situation described in the problem.

(2) Circle an object (objects) of interest in the sketch—we call this the system.

(3) Model the system as a particle (if possible). Place at the side of the sketch a "particle" dot to represent the system.

(4) Look for objects outside the system (external objects) that interact with the system.

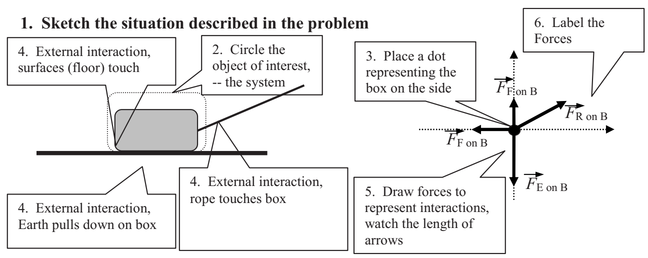


FIG. 2. Picture, free-body diagram and steps in how to construct FBDs (Ref. 45).

(5) Draw force arrows that represent the external interactions that affect the behavior of the system object. Draw the tails of these force arrows beginning on the particle dot. Draw the lengths of the arrows to represent the relative magnitudes of the forces.

(6) Label the forces in the diagram with two subscripts identifying two interacting objects. Note: The forces in the diagram should represent the force that some object outside the system exerts on the object inside the system. To start, identify the external object that causes each force and also the object on which the force is exerted.

It is important to note that all parts of the course were representation oriented: the interactive lectures or large room meetings, the problem-solving sessions, the homework assignments, and the instructional laboratory activities. Special multiple representation tasks (Appendix A) occupied 40% of the problem-solving sessions and several homework problems. These tasks helped students learn how to construct and evaluate their representations. After students submitted their homework, problem solutions provided to the students modeled the desired approach.

In the instructional laboratory sections students had to use representations when analyzing the data they collected or to help make predictions about the outcomes of the experiments. For example, several of the instructional laboratories required students to conduct an experiment whose outcome they had to predict in advance using prior knowledge or a hypothesis proposed in the instructional laboratory handout. The students needed to construct a representation, often a free-body diagram, which modeled that specific situation in order to make their prediction.

It is important to note that in problem-solving sessions and instructional laboratories, students worked in groups of 3–4 and in the interactive lectures they worked in groups of 2. Problem-solving session and laboratory instructors provided oral and written feedback to the students; in the interactive lectures the feedback was provided by the professor and via an electronic personal response system.[46]

We also conducted a separate study in the same course to serve as a control but taught at a different time. This course was taught by a different instructor who did not use the ISLE curriculum or The Physics Active Learning Guide, although his lectures were interactive, engaging, and also relied on students using an electronic personal response system. The instructor did not treat free-body diagrams nor emphasize how to convert from one type of representation to another in the fashion that the instructor from the two-year study did: but the other instructor paid a great deal of attention on helping students learn how to approach problem solving. We assume that the student population from the control group was similar to that in our two-year study.

B. Quantitative study

1. Sample

The data for the study came from exam problems (quantitative part of the study) and interviews (qualitative part of the study). For the quantitative part of the study in the first year, we used the data from 125 students chosen at random; in the second year we used the data from 120 students. This sample size was about 25% of their respective classes for both years. To make sure that the grade breakdown for the students for both years was almost identical to the breakdown for the class, we used a Kruskal-Wallace test and found no significant differences in the grades between the students in our sample and the students in the class. Thus, we believe that the sample was representative of the student population. The students for the qualitative study were selected from the second year students (the details are provided later in the paper). For our control group, we used fewer problems than the two year study (however, the problems that were used were exactly the same). To address this limitation, we increased our sample size by analyzing the work from all of the students in our control group (479).

2. Instruments and data collection procedures

The data came from student work on selected problems on multiple-choice exams. These problems were chosen because they were difficult to solve without a free-body diagram. We examined the FBDs that students drew on the exam sheets (either near the problem or on scrap paper if we had it) for certain problems. The problem statements did not specifically ask for or hint at a diagram. Students did not receive any partial credit for work or diagrams; their work was graded with Scantron sheets. In the first year, we chose five problems from four exams; in the second year we chose seven problems from four exams; in the control group we chose two problems from the first year study. Four sample problems are shown in Appendix B. All 12 problems can be found in Ref. 47. We collected and photocopied all of the work from the students in our sample. We then coded the diagrams using the rubric in Table II. The rubric was developed as a part of a larger study described elsewhere.[48] Two different researchers had an inter-rater reliability of 90% for a subset of the coded problems with this rubric.

There is one important fact we must discuss for students whose diagrams were coded as 0 –“No evidence of.” This zero code does not imply that students did not construct a diagram; it only means that we did not have any evidence of a constructed diagram. The students may have constructed a diagram in their head (as was stated during some interviews that will be described later), on scrap paper that we did not collect, or perhaps did not construct one. This limitation leads to a certain level of undercounting of the number of students who used an FBD to help solve problems.

TABLE II. Rubric for coding free-body diagrams.

Codes			
0	1	2	3
No evidence of	Inadequate	Needs improvement	Adequate
No representation is constructed	FBD is constructed but contains major errors such as missing or extra forces. Forces may be pointed in the wrong direction.	FBD contains no errors such as missing or extra forces but lacks a key feature such as labels or forces are mislabeled or do not contain a labeled axis if appropriate. Lengths of force arrows could be incorrect.	The diagram contains no errors in terms of the number of forces, the direction, length of force vectors, and the direction of axes. Each force is labeled so that it is clear what it represents.

3. Findings

To find how many students drew FBDs to solve problems, we counted the number of free-body diagrams that students drew while solving the chosen exam problems regardless of the quality of the diagrams. The results are shown in Table III. On average, 58% of the students from the representation-rich course drew free-body diagrams to help them solve the problems (this average includes electrostatics problems). It is difficult to say with certainty whether it is a big number as we cannot compare these numbers to students taught differently if they solved all 12 of the same problems. For our control study, we were only able to analyze two of the problems. We included those results in Table III (and which two of the original 12 problems the coded problems were). For these two problems 17% of the students from the traditional course constructed a diagram compared to 68% from the representation-rich course.

This number agrees with what was reported by Van Heuvelen[22] that only 10–20 % (Ref. 49) of students from traditionally taught courses construct an FBD for multiple-choice exam problems. However this reported 10–20 % is not for the same problems as those in our study and it is for a different population of students. Though we do not have control data for all 12 problems and this is a limitation for our study, the average number of students from the representation-rich course who constructed an FBD with these two problems is much higher than the students who constructed an FBD from the control group. Furthermore, the one problem, Mechanics Year 1 Exam 1 MC 1, was specifically chosen because it has the highest percentage of students from the representation-rich environment constructing an FBD out of all 12 problems (80.8%). The percentage of students in the traditional course who constructed a diagram for the same problem was much lower (22.5%). In fact, this 22.5% is lower than any of the percentages from the 12 problems in the representation-rich environment. These high percentage rates extend beyond just this course. Other studies where students are in a representation-rich environment show a high percentage of students also constructing representations to help them solve problems.[25,30]

To find whether students who used FBDs to solve these exam problems were more successful than those students who did not, we introduced a measure called "success rate." The success rate of a group of students is the percentage of the students in the group who solved the problem correctly. We divided the students into four groups for each problem based upon the quality of their free-body diagram as assessed by the rubric (Table II). Examples of the differences in success rate for the four groups and the average results are provided in Table IV.

Next, we found the success rate of each group—the percent of the students in groups zero through three who chose the correct answer for each of the 12 different problems. The "whole sample success rate" indicates how difficult each problem was for all of the students in the test sample (the last column in Table IV).

We found some trends in the data from Table IV. Students who constructed a correct FBD (code 3) on the exam sheet were very likely (85%) to correctly solve the problem. Stu-

TABLE III. Number of students who drew FBDs to help solve exam problems.

	Students N year 1=125; N year 2=120; N control=479	
Problem	Number	Percentage
Mechanics year 1 exam 1 MC 1	101	80.8
Mechanics year 1 exam 1 MC 2	70	56.0
Mechanics year 1 final exam	56	44.8
Electrostatics year 1 exam 1	69	55.2
Electrostatics year 1 final exam	85	68.0
Mechanics year 2 exam 1 MC 1	76	63.3
Mechanics year 2 exam 1 MC 2	42	33.6
Mechanics year 2 final MC 1	83	69.2
Mechanics year 2 final MC 2	85	70.1
Electrostatics year 2 exam 1	68	56.6
Electrostatics year 2 final MC 1	52	43.3
Electrostatics year 2 final MC 2	59	49.2
Control: Mech. yr. 1 MC 1	108	22.5
Control: Mech. yr. 1 MC 2	52	10.9

TABLE IV. Comparison of success rate of students for all 12 problems, total of all 12 and two control problems.

	Success rate				
Exam question	FBD Score 3	FBD Score 2	FBD Score 1	FBD Score 0	Whole Sample
Mechanics year 1	40/43	26/45	4/13	14/24	84/125
Exam 1 MC 1	(93%)	(58%)	(31%)	(58%)	(67%)
Mechanics year 1	23/25	27/36	2/9	17/55	69/125
Exam 1 MC 2	(92%)	(75%)	(22%)	(31%)	(55%)
Mechanics year 1	7/10	6/25	4/21	23/69	40/125
Final exam	(70%)	(24%)	(19%)	(33%)	(32%)
Electrostatics year 1	23/26	33/37	2/6	37/56	95/125
Exam 1	(89%)	(89%)	(33%)	(66%)	(76%)
Electrostatics year 1	32/42	4/8	12/35	14/40	62/125
Final exam	(76%)	(50%)	(34%)	(35%)	(50%)
Mechanics year 2	30/37	20/30	3/9	32/44	85/120
Exam 1 MC 1	(81%)	(67%)	(33%)	(73%)	(71%)
Mechanics year 2	16/18	14/20	2/4	28/78	60/120
Exam 1 MC 2	(89%)	(70%)	(50%)	(36%)	(50%)
Mechanics year 2	20/25	40/48	4/10	19/37	83/120
Final MC 1	(80%)	(83%)	(40%)	(51%)	(69%)
Mechanics year 2	27/34	27/46	1/5	19/35	74/120
Final MC 2	(80%)	(59%)	(20%)	(54%)	(62%)
Electrostatics year 2	5/6	36/42	6/20	32/52	79/120
Exam 1	(83%)	(86%)	(30%)	(61%)	(66%)
Electrostatics year 2	11/11	14/18	15/23	33/68	73/120
Final MC 1	(100%)	(78%)	(65%)	(48%)	(61%)
Electrostatics year 2	17/18	14/15	14/26	36/61	81/120
Final MC 2	(94%)	(93%)	(54%)	(59%)	(68%)
Total of 12 questions	251/295	261/370	69/181	304/619	885/1465
(Two year study)	(85%)	(71%)	(38%)	(49%)	(60%)
Control: Mech. yr. 1	2/2	33/46	7/44	82/298	94/390
Exam 1 MC 1	(100%)	(77%)	(16%)	(28%)	(24%)
Control: Mech. yr. 1	2/2	4/12	2/25	23/281	31/320
Exam 1 MC 2	(100%)	(33%)	(8%)	(8%)	(10%)

dents who constructed an incorrect FBD (code 1) were the least likely (38%) to correctly solve the problem, even when compared to students who had no evidence of using an FBD. When we incorporate the information from the control group, we find that this trend continues with the exception that a smaller percentage of students constructed diagrams. We must note that the sample size on these two problems in the control group for this analysis is smaller than 479 students per question because we did not have their answer keys, only what they circled on the exam sheet. Not all of the students from the traditional course circled their answers on the exam, thus our N is 390 on the first question and 320 on the second.

To determine whether the differences in success rates of students in the representation-rich environment who received scores of 0, 1, 2, or 3 for their FBDs were statistically significant (in other words, whether students who drew a high quality diagram performed significantly better than those students who made mistakes in their diagram and so on), we used a chi-square analysis. We summed the number of correct and incorrect responses per code on each of the 12 questions (Table IV) to create a total value (Table V and Fig. 3). We performed the chi-square analysis on those total values.

TABLE V. Total values of correct and incorrect responses per code.

	Codes			
	0	1	2	3
Correct	304	69	261	251
Incorrect	315	112	109	44
Total	619	181	370	295
Percentage	49%	38%	71%	85%

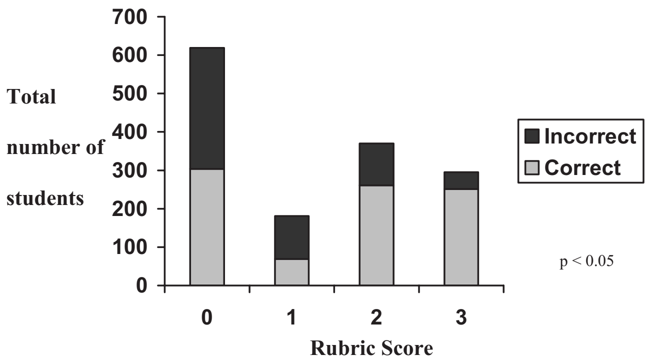


FIG. 3. Chart showing total percentages of solutions per code.

We must stress here that we are not concerned with the students as the data points. Our data points are the solutions students provided to the exam questions. In the first year, we had 125 students who responded to five exam questions; in the second year, 120 students responded to seven exam questions. The number of students multiplied by the number of questions they answered gives us a total of $5 \times 125(\text{year }1) + 7 \times 120(\text{year }2) = 1465$ student solutions to exam questions which is summed in Table IV. When we performed the chi-squared analysis on the total number of correct responses to incorrect responses per category, our p value is less than 0.001. Thus, we can state that the students who constructed a correct diagram did significantly better than students who constructed an inadequate, incorrect or had no evidence of an FBD.

C. Qualitative study

1. Sample

To find out how students use free-body diagrams during the problem-solving process, we chose students of varying backgrounds for an additional qualitative study. The study was conducted in the second (spring) semester of the second year. Six students participated in this study. We chose two high achieving students (one who drew many FBDs on the exam sheets and another who did not) and two low achieving students (one who drew many FBDs and another who did not) who took the first (fall) semester in the course under study, and two students who had a different instructor in the first semester. We describe these students below.

High achieving students: Jose and Mary received an A in the first semester with this instructor who modeled the use of FBDs and other representations in the problem-solving process. Jose constructed several representations while solving exam problems in the first semester. Mary constructed fewer representations solving the same exam problems during the first semester.

Low achieving students: Anna and Eileen received a C+ and D, respectively, in their first semester with the instructor who modeled the use of FBDs and other representations in the problem-solving process. Anna used several representations while solving exam problems during the first semester while Eileen used few representations.

Students who had the first-semester physics in a more traditional environment (this professor is also well respected and usually gets high student evaluations): Krutick and Sahana were in the same traditional class together for their first semester of physics and then transferred to the representation-rich course for their second semester. These students both received a B in the first (traditional) physics class. We did not have access to their exams to compare any representations they may have constructed during that semester. These two students from a traditional course were not exposed to the same in-depth instruction on how to construct free-body diagrams in their first semester.

For the second semester, students received the following grades: Jose—A; Mary—B+; Anna—B; Eileen—C; Krutick—B; Sahana—C+.

2. Instruments and data collection procedures

We drew the data for the qualitative study from participants' exams (the same data as for the quantitative study) and from two one-on-one interviews. During the first interview students solved an open-ended problem which was a slightly reworded multiple-choice problem from one of the first year problems (Electrostatics Year 1 Final). The text of the problem is as follows:

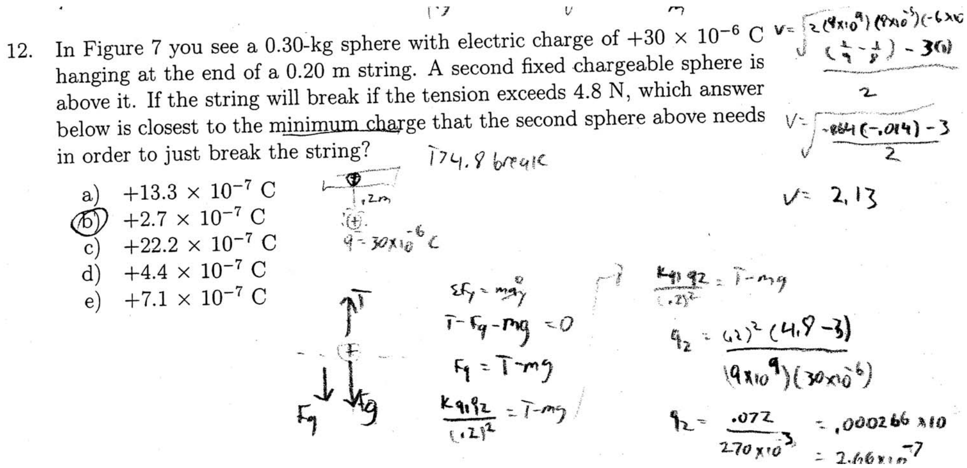
Electrostatics Year 1 Final Problem: A ball with $+2.0\ \mu C$ of charge hangs at the end of a vertical string. A second identical ball with $-2.0\ \mu C$ of charge hangs at the end of a second vertical string. The tops of the strings are brought near each other and the strings reach an equilibrium orientation (not vertical) when the balls are 3.0 cm apart. If the force that the Earth exerts on each ball is 30 N, what is the force exerted by the string on the ball?

There were no figures provided with the problem statement. The interview lasted for half an hour and was held in late January (approximately three weeks into the second semester). We did not ask students to solve the problem in any particular way, but we did ask them to comment on everything they were thinking and doing while solving the problem—a think aloud protocol.[50,51] The interviewer asked questions for clarification. In the second interview at the end of the semester, we followed up with more questions for each student about this problem and investigated how they solved the exam problems that were used in this study. We videotaped and transcribed each interview. Next, we present a description of the student problem-solving behavior.

3. Student responses

Jose: Jose started by drawing a correct picture. Jose stated that the picture allows him to make sense of the problem. [Jose] "I am just trying to make sense of it. Get a picture in my head so I can draw it down, so I can draw a picture on paper…. First thing I am going to do is draw a visual, what the words are trying to tell me. Then I am going to draw a before picture and an after picture."

From this picture, Jose constructed a correct free-body diagram. First, he singled out one ball because he noted that "the tension is going to be the same for both balls." He constructed the diagram to determine what objects are exert-



12. In Figure 7 you see a 0.30-kg sphere with electric charge of $+30 \times 10^{-6}$ C hanging at the end of a 0.20 m string. A second fixed chargeable sphere is above it. If the string will break if the tension exceeds 4.8 N, which answer below is closest to the minimum charge that the second sphere above needs in order to just break the string?

a) $+13.3 \times 10^{-7}$ C
b) $+2.7 \times 10^{-7}$ C
c) $+22.2 \times 10^{-7}$ C
d) $+4.4 \times 10^{-7}$ C
e) $+7.1 \times 10^{-7}$ C

FIG. 4. Example of Jose's work.

ing forces on the object of interest and in what directions they are exerting these forces. From the diagram, he constructed Newton's second law in component form which he used to solve for the force of the string on the ball. His first attempt at solving the problem gave him an answer of 1 N (due to two algebraic mistakes). He reasoned that this was too small a number because the force of the earth on the ball is 30 N (as he concluded looking at his free-body diagram).

His first attempt to correct this mistake was to evaluate his mathematical work. He found one mistake but now had an answer of 30 N. He rationalized that this is not the correct answer "because it would only be thirty Newtons if the ball and the string were directly above each other so you have the force due to gravity and you have the tension, that's when they should equal each other." While he was explaining this, he was holding a pencil in the air to represent the string and the ball and was making hand gestures showing in what direction the force of the earth and the force of the string would have to be orientated (which is straight up and down) for 30 N to be the correct answer. Since he was still not comfortable with this answer, he went back to check his free-body diagram.

Jose checked his free-body diagram to make sure that he was "taking into account all of the forces (objects exerting a force on the object of interest)." After analyzing his diagram, he made the following comment: "I feel that [his diagram] is correct. And then this part [his construction of Newton's second law] should be correct." This statement means that he was using the free-body diagram as a strong link to apply Newton's second law. He then continued to re-examine his mathematical work and found his mistake (a unit conversion) and then successfully solved the problem.

Examining Jose's answers to our questions about his work on exam problems and his approach to the first interview problem, we found that Jose consistently used the problem-solving strategy described in Sec. III A and Fig. 2. Each of his exams shows that he first wrote the given information, then drew a picture (sometimes the given data was incorporated into the picture) and an FBD and then wrote a mathematical representation. The fact that he actually followed this sequence can be seen from the solution of one of his problems (Fig. 4).

Jose made the following statement during one of the interviews: "I draw a picture, then draw a free-body diagram, then do Newton's second law to try and single out variables." Jose also commented how one representation helps in the construction of another: "It's hard for me to picture it. It's hard for me to just draw a free-body diagram, it's much more easier for me to, yes, draw the picture and with the picture I can see exactly what forces are acting on the certain thing which would help me form a free-body diagram."

When asked to clarify this thought process during the last interview, Jose said that, "I always draw a picture of the problem no matter how simple or difficult it is. I am putting it down into a picture form so it is much easier for me to digest the information and easier for me to use my logic to solve that problem." When we asked Jose about checking his work for mistakes, he said: "I am going to look at my free-body diagram to see if there are any mistakes there, and my Newton's second law."

Mary: Mary started solving the problem by drawing a picture. Her first attempt was incorrect, but her free-body diagram made her realize this and she re-evaluated her picture. When she started to draw the force exerted by the earth, she realized that "if the earth was acting then it would not be straight up [the string acting on the ball]." She was able to describe how she used her free-body diagram to evaluate her picture.

[Interviewer] What was it exactly that told you that the picture was wrong? [Mary] "I guess it would be when I was trying the free-body diagram for one of the balls…. But they were standing still. So, that means that there should be some other force counteracting the force from the charge on the other side, so if the tension were straight up there is no counteracting to that so it should move or I guess it would have some velocity or some movement and it's not moving since its standing still. The sum of the forces in the x direction should equal zero. So that means that the tension should be at an angle so there is some x component."

The free-body diagram helped her in another way; she was able to get an approximate magnitude of the size of the force that the string exerts on the ball when she stated that "So the tension for this ball would be this big." After she used the FBD to evaluate her picture and give her a rough estimate of the magnitude of her answers, she was able to go on, successfully constructing Newton's second law in component form and successfully solving the problem with just minor algebraic difficulties. When she got her answer of 50

N, she went back to her diagram to make sure it is consistent with her work. She stated that "It has to be greater than this or that [pointing to the other two arrows representing magnitudes of force on her FBD]." Mary drew and used a picture and a free-body diagram for this problem; however, for other exam problems she sometimes only used one of the two if any at all.

In the first-semester exams, Mary drew few pictures but did draw free-body diagrams for some of the more difficult questions. When asked why she drew free-body diagrams during the interview she said: "I guess it's easier to write it on another piece of paper rather than keeping it all in your head." She also added "well if it's a problem and then I drew the diagram correctly, if my answer doesn't match up I will look back up at the diagram to see where I went wrong, maybe my setup was wrong."

Finally, when asked why she didn't draw pictures or other representations for certain exam problems, she stated the following: "I think if I don't draw the picture out, I just keep it in my head and I think about it. Well I may not explicitly write that this force needs to be greater than this one, but conceptually I think if I know it well then I will know in the back of my head that this is suppose to be a certain value and this is suppose to be a certain value."

Anna: Anna drew a picture of the problem situation. However, not all of the quantities were labeled and her picture had the strings incorrectly oriented. Her free-body diagram matched the incorrect picture and the net force in both directions was not equal to zero. She used the free-body diagram to construct Newton's second law; however, initially she did not do this in component form. This mistake, combined with several other mistakes [mathematical and issues with the sign of the charge] made her come up with an incorrect answer. She was content with her answer. She had stated that if her answer was one of the choices on an exam, she would be done. She did not use any of her representations to evaluate her answer (or any of her work).

When we asked, "When you are drawing the free-body diagrams and you have the mathematical representation here, how do you go about going back and forth checking the consistency of your diagrams and mathematical or do you not?" Anna responded: "I don't think I do, I just go in order [from the problem text to picture to free-body diagram to Newton's second law as the problem-solving strategy suggested]." This approach is consistent with her exam work. She drew many pictures and FBDs but they had mistakes in force direction and were inconsistent with the mathematical work.

Eileen: Eileen was the last student in our sample who learned from the same instructor in the first semester. She started with a picture that was labeled correctly. Then she used Coulomb's law appropriately but made a unit conversion mistake. She calculated the magnitude of the force that each sphere exerts on the other before she drew the free-body diagram. However, she did not notice her mistake. Next, she drew a free-body diagram (which contained some minor mistakes). She used the diagram to help her add the forces in the x and in the y directions. Her mathematics was correct, but she continued to use the incorrect magnitude of force. Throughout the process she constantly asked for reassurance. She obtained a very large final answer (3.9×10^{13} N as opposed to 50 N) at which point she began to re-evaluate her work. She first found a mistake with a sign and another with how she put values into the calculator. She corrected those mistakes. Her next answer was also unrealistic. She searched through her mathematics and after some help she found her mistake in converting microcoulombs to coulombs. Finally, she obtained the right answer.

Eileen used two representations in this problem, which was uncommon for her on the exam problems. She used relatively few representations on exam problems, yet stated that she actually used representations. When asked if this is how she typically solved problems, she responded:

[Eileen] "Yeah, I actually do the picture representation and then I draw a free-body diagram. But I didn't really sit down and think what the problem was really asking. Had I drawn the free-body diagram first, then I would have known exactly what it wants you to know. So after doing this part… I should have started with a free-body diagram."

She stated how the free-body diagram would have helped her, yet on later exams she still did not use them. She explained why during the second interview. She said: "I had the formula but I didn't know how to convert it from the free-body diagram to the one using Newton's formula so I think the reason why I didn't draw it (was) because I couldn't understand the free-body diagram, how to apply it."

Krutick: Krutick started the problem by drawing a picture of the initial situation and the final situation of the problem, including key quantities. He drew arrows representing the forces directly on the picture of the final situation. Then he wrote his equations. He did not use the diagram to write equations nor did he explicitly use Newton's second law. Instead he said: "this force equals that force." He made an error which resulted in the magnitude of the force of the string on the sphere to be 1146 N. To this he responded: "That's a lot of tension. It looks, unusually large for me." He went back and re-evaluated his mathematical work. He found no mistakes which increased his confidence in the answer and he said that he would have selected that as a choice on a test. However, he did get stuck at one point in time and said that if this were a homework problem, he would stop working and then go ask the TA for help.

Krutick kept this trend of drawing partial representations on the exams throughout the second semester. Representations that he did construct on exams contained only bits and pieces of information from the problem. He stated that: "at this point, I pretty much understood what was going on. So once you start drawing it and you pretty much see what happens, you stop doing it." Although he drew representations, he did not use pictures or FBDs to write mathematical representations or to evaluate the answers.

Sahana: Sahana started to solve the problem with a picture. She did not use any obvious strategy to solve the problem other than the picture. In the picture she labeled all pieces of information. From there, she started using random ideas (needing the length of the string to find the hypotenuse to find the angle) and equations. She even stated that "usually when I work on the exam I have a formula sheet and I fool around with that." She was about ready to give up by saying "I just don't know how to do it. I just don't know how

TABLE VI. Number of comments students made.

	Students used representation to			
Student	Understand problem	Help solve problem	Evaluate work/answer	Check consistency
Jose	3	3	1	1
Mary	3	2	3	2
Anna	2	2	0	0
Eileen	2	0	0	0
Krutick	1	0	0	0
Sahana	1	0	0	0

to go about it. I just don't know which direction to go now. I just don't understand how I am going to find it." The interviewer asked a few clarification questions about her understanding of the problem. After this help, she decided to use a force approach, but did not use an FBD and did not mention Newton's second law. She was not able to solve the problem without significant help from the interviewer. When asked later about why she did not want to use an FBD, she stated that: "I don't like them, I don't make sense out of them. No seriously, I got through all of physics one (the first semester with a different instructor) without drawing any free-body diagrams." Her second semester exams indicated that she never drew an FBD to solve a multiple-choice question.

4. Analysis of responses

Using the data from the first interview we made a list of students' comments about multiple representations and then divided these comments into four different categories:

(1) Students' comments related to the use of representations to understand the problem/concept.

(2) Students' comments related to the use of representations to help solve the problem.

(3) Students' comments related to the use of representations to evaluate their work and or answers.

(4) Students' comments related to the use of representations to check for the consistency of other representations.

Table VI shows us the number of comments that fell into each of the four categories. Jose and Mary (the two high achieving students) made the most comments in all four categories. Anna is the only other student who made comments about how she used the representation (though her representations were incorrect) to help her solve the problem. All of the other students only said that they used the representation to help understand the problem. Remember that Jose and Mary both solved the problem correctly.

When we analyzed the work of the six students on the problem from the first interview, we found some trends in how the students solved the problem (Table VII). All six students drew a picture while solving the problem. Jose, Mary, Anna, and Eileen (J, M, A, and E) were part of the representation-rich first-semester class. They continued to model the same problem-solving process that they learned in that class. All of them used an FBD to construct a mathematical representation as part of the problem-solving process. However, the low achieving students only constructed the diagram as if it were part of a mechanical procedure. One of the other two students, Krutick [K], drew only a few arrows to help understand the problem statement and once he understood what was going on, he stopped drawing the diagram. The last student, Sahana [S], was adamant about not using diagrams. She explained that she would not use FBDs because she did not understand them.

To summarize the interview findings we can say that the most important result is that the high achieving students used the free-body diagrams to help evaluate their work. This evaluation included students consciously using the representations to reflect on their work and their solutions. Our findings are in agreement with the study described in Ref. 52 in which students who recognized the relationships between representations demonstrated better conceptual understanding than those students who did not recognize the relationships. The low achieving students did comment on using the diagram to help solve the problem, however they had difficulties using the FBD consistently with other representations. In fact, those students did not even check for consistency; rather they just followed the steps they learned in the classroom without having a full understanding of the importance of each step.

All four of the students who learned physics in the representation-rich environment used the free-body diagram to aid in the problem-solving process. Each of the four used the FBD to help them represent the problem situation.

IV. DISCUSSION

Recently, it has been documented that in different instructional environments that use a variety of different represen-

TABLE VII. Comparison of students.

	Drew		Used FBD	
Student	Picture(s)	FBD	To construct mathematical representation	In evaluation
Jose	√	√	√	√
Mary	√	√	√	√
Anna	√	√	√	
Eileen	√	√	√	
Krutick	√	√		
Sahana	√			

tations for concept construction and problem solving, students construct free-body diagrams on their own while solving problems.[25] Our study does not focus on differences in instructional environments, but rather researches the role of free-body diagrams as tools helping students solve problems involving forces.

We found that although students received no credit for their work on the multiple-choice exam problems, an average of 58% drew free-body diagrams on their exams when in an environment that fostered the understanding and use of different representations in problem solving. In 8 of 12 problems more students did draw free-body diagrams than did not. This is much higher than what is found for the two problems from the control group (11% and 23%) and what was reported in the literature for traditionally taught courses (10–20 %).[22,49] The high percentage in our reformed course is similar to the numbers reported for another reformed course at another institution.[25]

We also found that the students in the course under study used free-body diagrams outside of pure mechanics. In year 1, just as many students used free-body diagrams in electrostatics as they did in mechanics. In year 2, although there were more diagrams drawn in mechanics than those drawn in electrostatics, there were still a large number of diagrams drawn in electrostatics.

We found that all students who drew a correct free-body diagram were much more likely to solve the problem correctly (Table IV). The other trend was that those students who drew an incorrect free-body diagram were more likely to solve the problem incorrectly than students who showed no evidence of using an FBD. Both of the above results were found to be statistically significant (Table V). It is important to note that "no evidence of an FBD" does not mean that the student did not use one. Students could have constructed one in their head, as was stated by a student during the qualitative study, or possibly on scrap paper that was not turned in to the proctors, as was stated by another student during the second interview.

The qualitative study expanded the quantitative study by adding the knowledge of how students use mathematical representations (MRs) to help them solve problems. We found that all six students, independent of their classroom instruction, spontaneously drew a picture when they started to solve a problem. However, only those that were taught explicitly to draw free-body diagrams while solving mechanics problems did draw them and used them to construct a mathematical representation. Out of those, only the high achieving students used the free-body diagrams at the end of the problem-solving process for evaluation and to check the consistency of their work, solution, and their representations.

There is another interesting fact about the six students in the qualitative study. As we previously stated, the students received the following grades in their second semester: Jose—A; Mary—B+; Anna—B; Eileen—C; Krutick—B; Sahana—C+. Jose maintained a grade of an A in both semesters. Mary, who used fewer representations, had her grade go from an A in the first semester to a B+ in the second. Anna, who was a low achieving student yet used a lot of representations went from a C+ to a B (no longer low achieving). Eileen, who was low achieving and used few representations only brought her D up to a C. Krutick used more representations than Sahana in the course and also received a higher grade, a B as compared to a C+. This limited amount of data we collected *suggests* that students who use representations will improve their grade.

Finally, it is important that we also discuss the limitations of our study. All of the quantitative data came from exam work only and the control group only had two of the 12 problems. We decided to continue this study for two years with two different groups of students to ensure the consistency of our findings and to help address the limitations. The quantitative study only tells us if the students marked the right answer not whether they actually solved the problem correctly and how they used the free-body diagram to get that answer. This is why we added the qualitative research aspect. However, qualitative research has its own limitations. We had the students solve just one problem. As the students in the interview study solved one problem, we could only check for consistency between that problem and their exam work.

V. IMPLICATIONS FOR INSTRUCTION

The students in our study used FBDs to help solve problems when no credit was given for using the diagrams. Many of the students used them not only for understanding the problem statement, but to help construct the mathematical description of the problem and to evaluate their results. We feel that these results can be attributed to several aspects of the leaning system.

(1) Students saw the value of the diagrams when in an environment where they learned how to use FBDs for concept development, for problem solving, and for conducting experimental investigations.

(2) Students acquired a habit of using the diagrams and did so automatically when in an environment when representations were used consistently in the large-room meetings, recitations, and instructional laboratories.

Learning to evaluate the consistency of different representations with respect to each other and to use them to evaluate their solutions is a very valuable ability that this learning system helped some students acquire. In short, we feel that emphasizing representation-based approaches to concept construction, problem solving, and instructional laboratory investigations results in student use of the representations for effective problem solving.

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(The students typically fill in the bottom 4 cells on the right.)

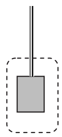
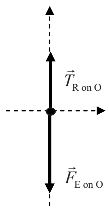
Write a word description of the situation.	Sketch the situation and circle the system object.	Free-body diagram with perpendicular axes. Label the forces if needed.	Draw a motion diagram Is it consistent with the FBD?	Write Newton's second law in component form.
An elevator is slowing down on its way up.				

FIG. 5. Example of multiple representation task.

APPENDIX A: MULTIPLE REPRESENTATION TASK

(The students typically fill in the bottom 4 cells on the right.) (Fig. 5).

APPENDIX B: SAMPLE EXAM PROBLEMS ANALYZED

Correct answers are italicized

Mechanics Year 1 Exam 1 Multiple-choice Problem 1

A 100 kg fireman starts at rest and slides down a vertical pole with a constant downward acceleration of 4.0 m/s^2. The magnitude of the friction force that the pole exerts on the fireman is closest to:

a) 1000 N
b) 1400 N
c) 400 N
d) 1600 N
e) 600 N

Electrostatics Year 1 Final Exam Multiple-choice Problem

A small metal ball with +2.0 μC of charge hangs at the end of a vertical string. A second identical ball with −2.0 μC of charge hangs at the end of a second vertical string. The tops of the strings are brought toward each other and the strings reach an equilibrium orientation (no longer vertical) when the balls are 3.0 cm apart. If the gravitational force that the Earth exerts on the each ball is 30 N, which answer below is the closet to the force that each of the strings exert on the ball?

a) 60 N
b) 50 N
c) 40 N
d) 30 N
e) 70 N

Mechanics Year 2 Final Exam Multiple-choice Problem 2

A 1000 kg elevator moving down at 4.0 m/s slows to a stop in 2.0 m. Which answer below is closest to the magnitude of the force exerted by the cable on the elevator as the elevator's speed is decreasing?

a) 16 000 N
b) 14 000 N
c) 10 000 N
d) 6000 N
e) 4000 N

Electrostatics Year 2 Final Exam Multiple-choice Problem 1

A 0.10 kg ball with a charge of $+28\times10^{-5}$ C falls vertically in a vertical constant electric field. The downward acceleration of the ball is 3.0 m/s^2. Which answer below is closest to the magnitude of the electric field? Assume that $g=10$ N/kg.

a) 2500 N/C
b) 7500 N/C
c) 17 500 N/C
d) 3250 N/C
e) 1000 N/C

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[45] A. Van Heuvelen, and E. Etkina, *Active Learning Guide, Student Edition* (Addison Wesley Longmann, San Francisco, CA, 2006), p. 1–14.
[46] A personal response system is a small electronic device that was used during formative assessments that were multiple choice. The instructor would pose a question, students would answer via their device, and then the instructor discussed the results.
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