Observation of beam-size blowup due to half-integer resonance in a synchrotron

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Beam blowup due to a half-integer resonance was observed in the HIMAC synchrotron with a nondestructive two-dimensional beam-profile monitor. As the betatron tune approached a half-integer, the vertical beam size became larger by about 13%. The measured rms beam size is in good agreement with a space-charge-included numerical simulation.

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I. INTRODUCTION

In a high-intensity synchrotron, one of the most serious problems is beam loss, which causes the activation of accelerator components. One of its major sources is betatron resonance. However, the resonance behavior is very complicated in the presence of a space-charge field, and the detailed mechanism is not clearly understood.

One familiar model [1,2] shows that the betatron tune of a particle is reduced by a space-charge field in proportion to the beam intensity, and the oscillation resonates with a periodic external field. This effect limits the maximum beam density. However, this model is not self-consistent because it assumes that both the space-charge field and the tune of a particle are constant. In an actual beam, however, the tune of a particle is shifted when the space-charge field is decreased by a beam-size blowup. Therefore, when the amplitude of a particle oscillation is increased due to a resonance, the tune of a particle becomes higher and the resonance condition is changed. This is called detuning effect, which should be taken into account in a self-consistent model.

One method used to analyze the betatron resonance in a high-intensity synchrotron is to employ the Vlasov-Poisson equation. Another is to use an envelope equation, which can be applied only to half-integer resonance. The envelope equation for a uniform beam density was first derived by Sacherer [3] to describe a half-integer resonance in terms of the coherent quadrupole-mode oscillation. The envelope equation was generalized to the rms envelope equation [4] for a beam with a general distribution. With the rms envelope equation, the beam size, as far as its rms value, can be analyzed self-consistently.

One of the remarkable results from the envelope equation is that the half-integer resonance occurs at coherent quadrupole-mode tune, instead of incoherent tune. A number of numerical studies [5–7] were conducted to study the half-integer resonance of a high-intensity synchrotron, and verified that the half-integer resonance occurs at coherent quadrupole-mode tune and thus the intensity limit of a beam under a half-integer resonance is related to the coherent quadrupole-mode tune.

It has been experimentally observed that the half-integer resonance occurs depending on the beam intensity [8]. In 2000, an experimental study of the half-integer resonance was conducted with the Heavy Ion Medical Accelerator in Chiba (HIMAC) synchrotron [9] in the National Institute of Radiological Sciences (NIRS). In the experiment, the decrease in the beam intensity was measured when the betatron tune was swept across a half-integer value. As a result, it was found that the threshold of the bare tune needed to avoid beam loss was shifted higher when the initial beam intensity was high. Also, it was found that the beam was lost gradually when a half-integer resonance was crossed from above with decreasing tune, while rapidly from below. The gradual beam-loss behavior can be explained by the fact that the tune shift depends on the beam density. Since the beam loss decreased the space-charge density, the depressed tune was increased away from the half-integer value when the bare tune was above the half-integer. That effect compensated for the bare tune approaching the resonance, and thus the beam intensity gradually decreased.

The next subject is how far the bare tune should be moved away from the half-integer. It is obvious that the beam loss begins when the beam full size reaches the aperture of a vacuum chamber. Therefore, it is essential to know the matched beam size, which is defined by the closed envelope trajectory in one revolution, for a given bare tune, beam intensity, emittance, and periodic external-field error. According to a numerical simulation [8], the matched rms envelope and the beta function with a fixed emittance gradually increases when the bare tune approaches a half-integer from above, and rapidly from

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TABLE I. Parameters of the HIMAC synchrotron at injection energy.

Parameter	Value
Circumference	$2\pi R = 129.6$ m
Lattice structure	12 FODO, superperiod is six
Beam energy	$K_{\mathrm{inj}} = 6.0$ MeV/u ($\gamma = 1.06$, $\beta = 0.113$)
Revolution frequency	261.4 kHz
Aperture limit	± 123 mm (H), ± 32 mm (V) at quadrupole magnet

below. Consequently, above a half-integer tune, the threshold of the bare tune needed to avoid beam loss directly depends on the aperture limit, as well as the beam intensity and the strength of the harmonic component of the external-field error.

In order to verify that the beta function gradually depends on the bare tune above a half-integer value, we measured the beam size in the HIMAC synchrotron near a half-integer tune. The measured rms beam size as a function of the bare tune agreed with both the matched rms envelope with a fixed emittance and that of a multiparticle simulation, which is described in Sec. III. We compare the results to a similar experiment done by Cousineau *et al.* [10], also in Sec. III.

II. EXPERIMENT

A. Synchrotron

The experiment was carried out in the HIMAC synchrotron with a coasting C^{6+} beam at an energy of 6 MeV/u. The main parameters of the synchrotron are tabulated in Table I. The amplitude of betatron oscillation of a particle is limited by the aperture of the vacuum chambers at quadrupole magnets, which is ± 123 mm in the horizontal direction and ± 32 mm in the vertical direction.

The circulating-beam current was monitored with a dc current transformer (DCCT). The betatron tune was measured with an electrostatic quadrupole pickup, or by the rf-knockout method. The betatron tune of a low-intensity beam (ν_{0x}, ν_{0y}, bare tunes) depends linearly on the currents of a vertical focusing magnet [horizontal defocusing quadrupole (QD)] and a horizontal one [horizontal focusing quadrupole (QF)]. The coefficients were measured to be [8]

$$\begin{bmatrix} \delta\nu_{0x} \\ \delta\nu_{0y} \end{bmatrix} = \begin{bmatrix} +0.060 \pm 0.001 & -0.00766 \pm 0.0001 \\ -0.0098 \pm 0.0005 & +0.05966 \pm 0.00001 \end{bmatrix} \times \begin{bmatrix} \delta I_{\mathrm{QF}} \\ \delta I_{\mathrm{QD}} \end{bmatrix}, \tag{1}$$

where the currents of the quadrupoles (I_{QF}, I_{QD}) were measured in the unit of ampere. Those coefficients were used to evaluate the bare tune, when the currents were varied.

The synchrotron naturally has a harmonic component of the gradient-field error, which resonates with betatron oscillation near a half-integer. The $\nu_y = n/2$ half-integer stop band width is given by

$$\frac{\delta\nu_y}{2} = \left| \frac{1}{4\pi} \oint \beta_y(s) K(s) \exp[in\phi_y(s)] ds \right|, \tag{2}$$

where $\beta_y(s)$ is the betatron amplitude function, $\phi_y(s)$ the betatron phase advance, $K(s)$ the field gradient error, defined by $K = (\partial B_x/\partial y)/B_0\rho$, and $B_0\rho$ the magnetic rigidity of a beam. In the HIMAC synchrotron, the stop band width of $\nu_y = 3.5$ half-integer resonance was measured by observing the beam intensity as a function of tune [8]. There, a pair of additional quadrupoles (QDS) at exactly opposite sides of the synchrotron was excited in counterphase in order to determine the absolute phase of the field-error harmonics. As a result, the contribution of gradient-field error was estimated to be

$$\frac{\delta\nu_y}{2} = |(7.1 \pm 0.5) + i(3.1 \pm 0.8)| \times 10^{-3}, \tag{3}$$

where the longitudinal coordinate is taken so that $\phi_y(s) = 0$ at one of the QDS magnets.

B. Nondestructive beam-profile monitor

In our experiment, a gas-sheet beam-profile monitor (SBPM) [11,12] was employed to measure the beam profiles. One of the advantages of the monitor is that a very short-time measurement is possible. For example, a measurement within a few μs is possible in our experimental condition, where the number of circulating particles is more than 10^8. Another advantage is that the transverse space-charge field can be derived, because the monitor can measure a two-dimensional profile in real spaces.

The SBPM is composed of a gas-sheet beam generator and a multichannel plate (MCP) profile monitor. A pulse of an O_2 gas-sheet beam is introduced inside the vacuum chamber to make a screen of the gas target (Fig. 1). Because the gas target is very thin (1.3 mm), the emittance blowup of a beam due to the interaction with it is negligible. Secondary ions produced in the gas target are collected to the MCP by a collection field. The effects of the field on a circulating beam are compensated by two correction electrodes on both ends of the monitor. Since the gas-sheet plane is inclined by 45° with respect to the circulating-beam axis, the image of the secondary ions on the MCP shows a two-dimensional beam profile in transverse real space.

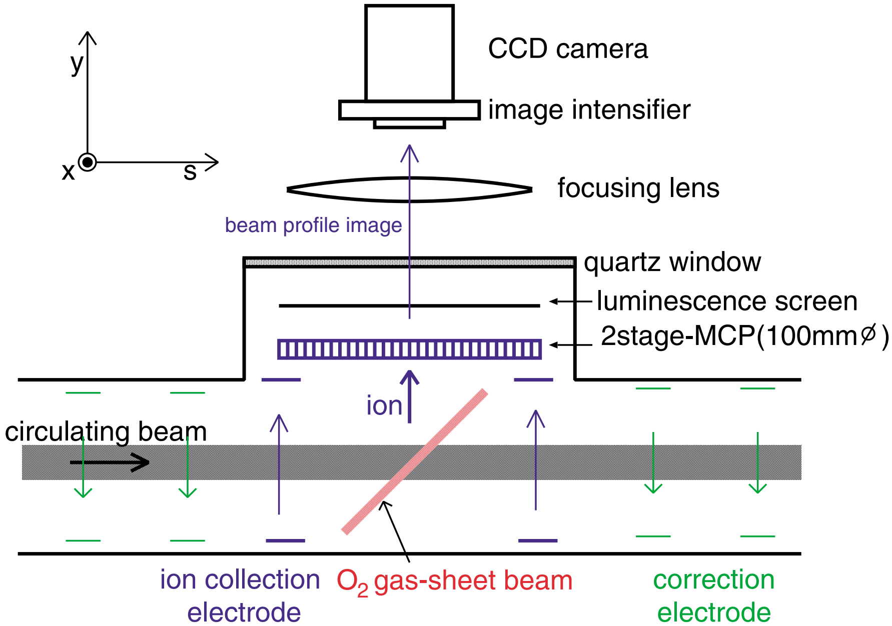

FIG. 1. (Color) Layout of the gas-sheet beam-profile monitor.

The resolution of the monitor was measured with a very small cooled beam. As the beam was cooled, the measured beam profile became smaller under the low intensity, and the final beam sizes were 0.76 mm (horizontal) and 1.51 mm (vertical), respectively, in the full width of half maximum (FWHM). The vertical resolution was worse than the horizontal one because it was affected by the thickness of the gas target.

The sensitivity of the MCP was not uniform among the locations. According to a measurement with an ultraviolet (UV) light source, the sensitivity decreased near the center of the MCP. In the beam-profile measurement, the nonuniformity of the MCP was corrected with the sensitivity map measured with UV light with 2 mm $\times$ 2 mm step size.

C. Experiments

The time dependence of the two-dimensional beam profile was measured with sweeping the vertical bare tune across 3.5 downward. The beam intensity was measured simultaneously with the DCCT. The bare tune was controlled by changing the strength of the QD, while the QF was kept constant. Figure 2 shows the excitation pattern of the QD. The bare tune at the starting point was measured by the rf-knockout method; a monochromatic rf field in the horizontal and vertical directions was applied to a low-intensity beam and searched the frequencies where the beam was to be lost. The tunes were found to be (3.204, 3.575) with an accuracy of ± 0.002. The bare tune at an arbitrary time was evaluated with the current of QD and the coefficients in Eq. (1).

In order to measure the beam size precisely, it is important to eliminate any coherent-mode betatron oscillations, which may cause a spread in the beam profile. A flat region of the QD was put at the beginning (Fig. 2) in order to stabilize any coherent oscillation arising from an injection mismatch. A time of 60 ms was sufficient for that purpose. It was verified with a fixed QD that the two-dimensional beam profile remained constant after a few

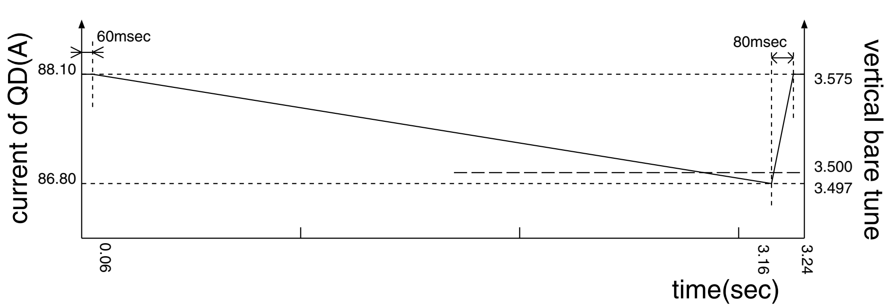

FIG. 2. Operation pattern of the vertical bare tune in resonance crossing experiments.

times of 10 ms. It is expected that the change of tune in the operation shown in Fig. 2 does not excite a betatron mismatch because a tune variation of 0.01×10^{-5} per revolution is sufficiently adiabatic.

In the experiment, no harmonic field was excited artificially as a driving force of the half-integer resonance, and the natural gradient-field error [Eq. (3)] was used.

Each beam profile was measured in different machine cycles. The reproducibility of the vertical beam profile was very good within a few percent.

D. Experimental results

Figure 3 shows that the beam intensity decreased as the bare tune approached the half-integer value. The two-dimensional beam profiles were measured at the points plotted by open circles in Fig. 3. Each profile showed an elliptic cross section in the transverse real space (Fig. 4). Because of the multiturn beam injection in horizontal space, the horizontal emittance was much larger than the vertical one. In Fig. 5, the two-dimensional profiles are projected onto the horizontal and vertical spaces, respectively. Until the beam began to be lost, the horizontal profile was kept constant with an rms size ($\tilde{x}$) of 17 ± 1 mm. On the other hand, the rms size of the vertical profile ($\tilde{y}$) was around 4 mm in the beginning, and became gradually larger as the bare tune approached the half-integer value [Fig. 7(b)]. The expected tune shift is 0.0086 for quadrupole mode and 0.0057 for a particle of an rms-equivalent uniform beam.

Similar to the gradual beam-loss characteristic, the gradual dependence of the beam size on the bare tune can be explained by the detuning effect. When the beam size became larger, the depressed tune gets higher and is no more on the half-integer stop band. This mechanism stabilized the resonant blowup of the beam at a finite beam size for a given bare tune.

In order to evaluate the increase of the vertical beam size, the vertical beam profiles $n(y)$ were fitted with overlapped Gaussians,

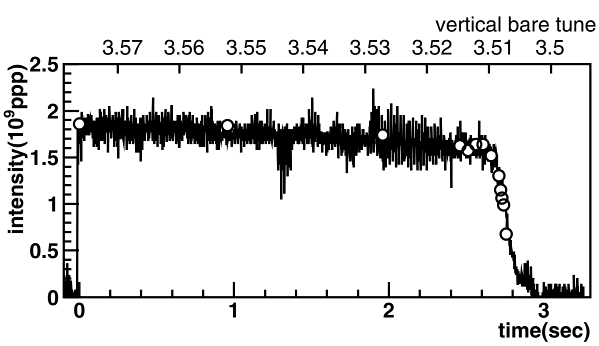

FIG. 3. Beam-loss behavior when the resonance line was crossed downward. The open circles show that the two-dimensional beam profiles were measured at those points.

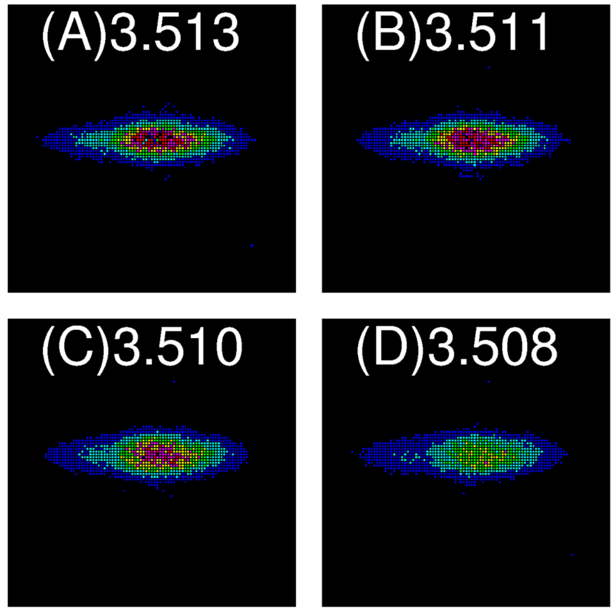

FIG. 4. (Color) Two-dimensional beam profile measured with the SBPM, for bare tunes (a) 3.513, (b) 3.511, (c) 3.510, and (d) 3.508, respectively.

$$n(y) = A_1 \exp\left[-\frac{(y-y_0)^2}{2\sigma_1^2}\right] + A_2 \exp\left[-\frac{(y-y_0)^2}{2\sigma_2^2}\right], \quad (4)$$

where $A_{1,2}$, $\sigma_{1,2}$, and y_0 are free parameters. Typical fitting functions are plotted in Fig. 6. The rms sizes were calculated by

$$\tilde{y} = \sqrt{\frac{A_1\sigma_1^2 + A_2\sigma_2^2}{A_1 + A_2} - \sigma_0^2}, \quad (5)$$

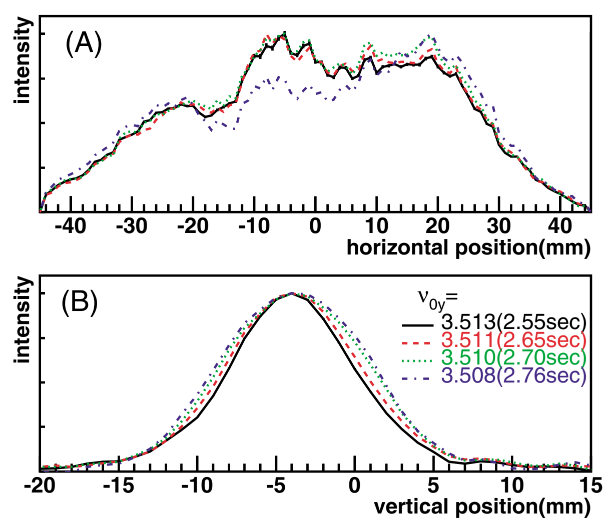

FIG. 5. (Color) Projections of the profiles on horizontal (a) and vertical (b) plane. The solid (black), dashed (red), dotted (green) and dash-dotted (blue) lines show the profile at $\nu_{0y} =$ 3.513, 3.511, 3.510 and 3.508, respectively. Those profiles are normalized with their peak values.

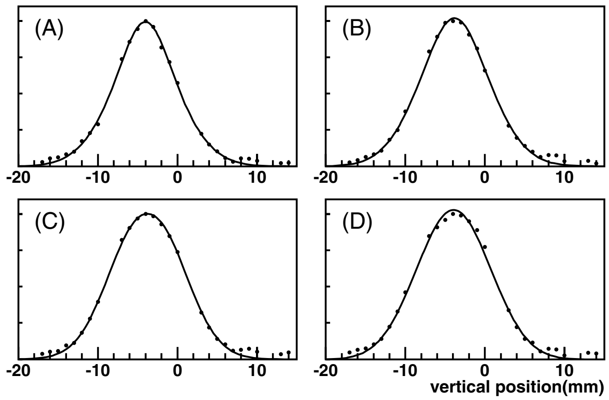

FIG. 6. Vertical beam profiles measured at $\nu_y = 3.513$ (a), 3.511 (b), 3.510 (c), and 3.508 (d). Fitting function is the superposition of two Gaussians.

where the SBPM resolution of $\sigma_0 = 0.7$ mm is taken into account. The rms beam sizes are plotted in Fig. 7. The horizontal error bars show the systematic error related to the accuracy in the initial tune measurement and the uncertainty of the coefficient in Eq. (1). On the other hand, the vertical error bars come from the fitting error including the statistical error of the projected 1D profile. The vertical error was 0.1 mm at the maximum ($\nu_y = 3.5135$).

III. DISCUSSION

Two types of numerical simulations were carried out to be compared with the experimental result: (A) We calculated the matched rms beam size with the rms envelope equation, and then (B) a multiparticle simulation was made while taking into account any change in the particle distribution.

In both simulations, the sheet beam approximation was taken here, and one-dimensional motion in the vertical space was assumed, because the horizontal emittance was more than ten times larger compared with the vertical one. A uniform distribution with an rms size of ± 17 mm was assumed in the horizontal direction. Further, the thin-lens approximation was used for the quadrupole magnets, whose effective length was 192 mm.

Two pairs of virtual quadrupole magnets were additionally used in order to simulate the harmonic component of the gradient-field error. The first set (QE1) of them was located at the same position to the QDS, and the other (QE2) was perpendicular to it in the envelope phase advance. In each pair, the normalized field strength, $\beta_y K$, has the same magnitude and opposite sign, in order not to change the betatron tune. The values of $\beta_y K$ for QE1 and QE2 are given by 2π times the real and the imaginary part of the right-hand side in Eq. (3), respectively. The modeled lattice structure is shown in Fig. 8. A simulated beam was monitored at a point corresponding to the position of the SBPM. Some details of the simulations are described separately in the following subsections, but a complete description of the methods is given in Ref. [8].

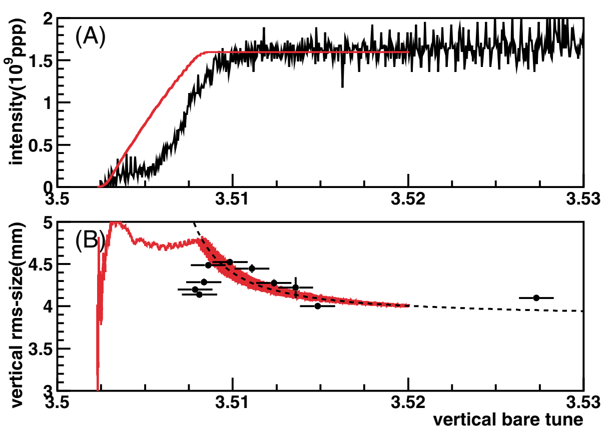

FIG. 7. (Color) Beam intensity (a) and vertical rms beam size (b) when the resonance line was crossed downward. In (a), the black and red solid lines show experimental and simulation results, respectively. The black circles and the red line in (b) show the results from an experiment and a multiparticle simulation, respectively, and the dashed line shows the matched solution of the envelope equation.

A. Matched rms envelope

The matched rms envelope was calculated as a function of the vertical bare tune for $3.5 < \nu_{0y} < 3.52$. The vertical rms emittance was fixed at 2.5π mm mrad, with which the rms beam size at the SBPM agreed with the experimental one at the initial value of the vertical bare tune (3.52).

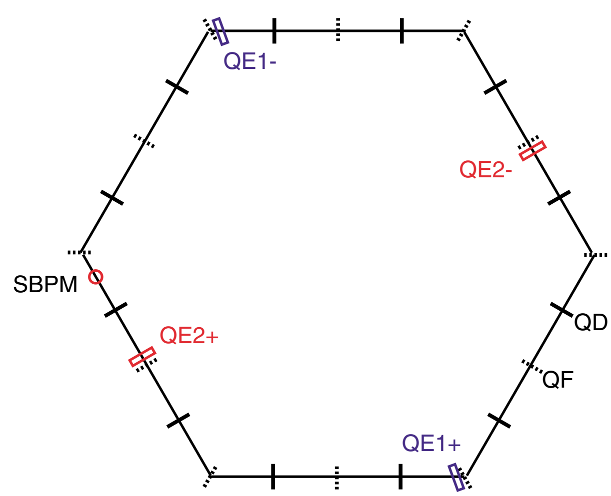

FIG. 8. (Color) Modeled lattice structure used in the numerical simulations.

The result is plotted as the dashed line in Fig. 7(b). Until a part of the beam begins to be lost, the rms envelope agrees with the rms beam size measured in the experiment. The rms envelope gradually increases as the bare tune approaches the half-integer value.

A matched solution of the rms envelope equation exists in a bare tune larger than the stop band ($\nu_{0y} > 3.507$). Thus, the beta function remains finite even when the space-charge effect is taken into account, unless the bare tune itself enters inside the stop band. This result can be reasonably explained by the fact that the betatron tune is reduced in proportion to the space-charge density. If the depressed tune is near to a half-integer value, the beam size becomes larger due to the resonance, and the space-charge density decreases. That effect reduces the tune depression, and pushes the depressed tune upward. If the bare tune is above the half-integer, the depressed tune possibly becomes no longer inside the stop band, and the resonance is stabilized at a certain beam size. This is a detuning effect, which is neglected in the theory of betatron resonance in the single-particle model. If the rms beam size is large enough, the space-charge density is negligible, and the tune shift vanishes. Thus, the beta function had a finite value for a bare tune outside the stop band.

In a real synchrotron, however, the beam size is limited by the aperture of the vacuum chamber, so that the beam loss begins before the bare tune reaches the stop band. The threshold of the bare tune, where a part of the beam is to be lost, directly depends on the aperture limit and the ratio of full beam size to the rms, as well as the beam intensity and the strength of the harmonic component of the external field. In our experiment, the full beam size reached the chamber wall at $\nu_{0y} \sim 3.509$, where the beam began to be lost. The threshold of the beam loss should be estimated with a multiparticle simulation, which is written in the next subsection. Also, in the multiparticle simulation, the emittance is free to grow during the resonance crossing.

B. Multiparticle simulation

A multiparticle simulation was performed with 10 000 macroparticles. The particle density, and hence the space-charge potential, was assumed to be symmetric with respect to the central plane, and uniform in horizontal and longitudinal directions. The space-charge force was calculated from a 10 000 macroparticle distribution with a particle-in-cell method. A parabolic distribution in phase space was assumed as the initial state, and the rms beam size and its derivative were taken from the matched solution of the envelope equation. An aperture limit was defined at ± 20 mm throughout the circumference, assuming that there was a closed orbit distortion of 12 mm at the maximum, which effectively limited the amplitude of a particle. Any macroparticle that came over that limit was lost.

The red lines in Figs. 7(a) and 7(b) also show the result of a multiparticle simulation. The rms size agrees with the experimental result as well as the matched rms envelope. Also, the rms emittance was conserved until the beam began to be lost. This means that the variation of the bare tune occurred adiabatically along the equilibrium with finite Twiss parameters. In a multiparticle simulation with three-order faster tune variation, a quadrupole-mode oscillation was excited and the emittance grew. Such a phenomenon is reported in Ref. [10].

The simulation also supports gradual beam loss in the experiment. The rms emittance is conserved until the beam begins to be lost. During the beam loss, the rms beam size does not increase any more, as the experimental result shows.

Because the growth of the beam size has a gradual dependence on the bare tune above a half-integer, the threshold of the bare tune strongly depends on the available aperture, as well as the beam intensity and the strength of the periodic external-field error.

IV. SUMMARY

The two-dimensional beam profiles were measured while sweeping the vertical bare tune downward across a half-integer value of 3.5. The vertical beam size became larger as the tune approached the half-integer value (Sec. II). Because of the asymmetric tunes, the horizontal profile was not affected by the vertical resonance.

Until a part of the beam began to be lost, the vertical rms beam size agreed with both the multiparticle simulation and the matched rms envelope. In the simulation, the rms emittance was conserved there. Those facts mean that there was an equilibrium with finite Twiss parameters in that region of tune, and that the variation of the bare tune occurred adiabatically enough in the experiment. The presence of the finite size of the matched envelope can be explained by the detuning effect, as described in Sec. III.

Because of the detuning effect, the bare tune can be closer to a half-integer value by less than the tune shift of a particle estimated without the perturbation from the harmonic gradient error. The threshold of the bare tune, used to avoid beam loss due to half-integer resonance, depends on the aperture limit.

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