Development of axisymmetric rf focusing effects for an ion linac

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High intensity ion beam dynamics in an axisymmetric rf focusing acceleration structure is considered. New equations of particles motion in polyharmonic rf field are devised by means of smooth approximation. Analysis of these equations allows one to formulate the conditions of beam stability. A new approach to rf focusing acceleration channel parameters choice is suggested. The relationship between 4D phase space volume of the bunch and transmission is studied by means of Hamiltonian analysis. The influence of the space charge on rf focusing conditions, transmission, longitudinal and transverse emittances, and other acceleration system characteristics is investigated by computer simulation. The "superparticles" method is being applied. The efficiency of the approach is confirmed by numerical simulation.

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I. INTRODUCTION

Simultaneous longitudinal and transverse stability of the beam motion in linacs is known to be ensured either by using external focusing elements or by applying special configurations for accelerating rf fields (rf focusing); for low energy ion linacs, the latter is more effective. The following types of such focusing are currently known: alternating phase focusing (APF), radio frequency quadrupoles (RFQ), and undulator rf focusing (RFU). Only axisymmetric systems are discussed here. The main principles of the one wave-approach for APF were described in Refs. [1–3]. It was shown that alternating the sign of the equilibrium particle phase in each acceleration gap can result in focusing effect. In later studies, the model with two traveling waves was taken as the basis of APF description [4]. In this model, a beam is accelerated by the field of a synchronous wave with a fixed sign of synchronous particle phase. Here, the transverse focusing results from the nonsynchronous harmonic action. Attempts to prove the equivalence of these two approaches has been done [5]. Further analysis [6] has shown that in some cases two nonsynchronous field harmonics should be considered as well. The case of the large number of standing wave space harmonics in the short gaps approximation was discussed in Ref. [7]. The ordinary Wideroe-type structure was used in the majority of previous works. The focusing influence of higher harmonics is insignificant since the amplitudes of these harmonics decrease fast compared to their number in such structures. Increasing equilibrium particle phase to avoid the transverse defocusing leads to a decrease of longitudinal acceptance. Attempts to apply the basic harmonic as a focusing wave and the higher harmonic as an accelerating wave were unsuccessful. Additionally, the formulated APF theory has other significant shortcomings. No relation between longitudinal and transverse motion was considered. This concerns the influence of rapid longitudinal oscillations as well. The averaging technique which was used for rf focusing analysis in [5] is not well defined. Therefore, the developed APF theory does not allow one to make use of all rf focusing potentials. There has been a great deal of interest in increasing transmission for low energy linacs. In the scope of ordinary APF description, it is impossible to achieve a large value of longitudinal acceptance and output current. A decrease of interest in axisymmetric rf focusing (ARF) in recent years is accounted for by these circumstances. Smooth approximation use was taken as a basis of the rf focusing systems considered in [8]. In such an approach, the Hamiltonian analysis can be used for a complete 3D description of beam motion. This method in a single particle model with the use of harmonic expansion for rf fields was further developed in [9,10] without taking into account the Coulomb interaction of particles.

In this work, the version of the ARF system based on using the modified Wideroe-type structure is suggested. Methods to increase the transmission coefficient are offered. The influence of the Coulomb interaction is studied entirely by means of computer simulation.

II. MOTION EQUATION

First, we study the single particle approach taking no account of space charge field. The acceleration cavity rf field is given by

$$\begin{aligned} E_z &= \sum_{n=0}^{\infty} E_n I_0(h_n r)\cos\Big(\int h_n\,dz\Big)\cos(\omega t)\,, \\ E_r &= \sum_{n=0}^{\infty} E_n I_1(h_n r)\sin\Big(\int h_n\,dz\Big)\cos(\omega t)\,, \end{aligned} \tag{1}$$

where E_n is the rf field harmonic amplitude on the axis, $h_n = h_0 + 2\pi n/D$, $h_0 = \mu/D$, μ is the phase advance per period of structure D, and I_0, I_1 are modified Bessel functions. The particle trajectory in the polyharmonic field (1) can be presented as a sum of slowly varying coordinate $\mathbf{r}^{\text{slow}}$ and rapidly oscillating coordinate $\mathbf{r}^{\text{rapid}}$. We obtain the motion equation in the form

$$\frac{d^2\mathbf{R}}{d\tau^2} = -\frac{\partial}{\partial \mathbf{R}} U_{\mathrm{eff}}\,, \qquad (2)$$

using averaging over rapid oscillations (as was done in [8,9]). Here, $U_{\mathrm{eff}} = U_0 + U_1 + U_2 + U_3$ is the effective potential function

$$\begin{aligned}
U_0 &= \frac{1}{2} e_s \beta_c [I_0(\eta/\beta_c)\sin(\psi + \chi/\beta_c) - (\chi/\beta_c)\cos\psi]\,,\\
U_1 &= \frac{1}{16} \sum_{n\neq s} \frac{e_n^2}{\Delta_{s,n}^{-2}} g_{s,n}(\eta) + \frac{1}{16} \sum_{n} \frac{e_n^2}{\Delta_{s,n}^{+2}} g_{s,n}(\eta)\,,\\
U_2 &= \frac{1}{16} \sum_{\substack{n\neq s\\ n+p=2s}} \frac{e_n e_p}{\Delta_{s,n}^{-2}} [f_{s,n,p}^{(1)}(\eta)\cos(2\psi + 2\chi/\beta_c) + 2(\chi/\beta_c)\sin(2\psi)]\,,\\
U_3 &= \frac{1}{8} \sum_{n\neq s} \frac{e_n e_p}{\Delta_{s,n}^{-2}} [f_{s,n,p}^{(2)}(\eta)\cos(2\psi + 2\chi/\beta_c) + 2(\chi/\beta_c)\sin(2\psi)], \qquad h_n - h_p = 2h_s\,,
\end{aligned} \qquad (3)$$

where $e_n = e\lambda E_n/2\pi mc^2$, $\mathbf{R} = [\chi, \eta]$, $\chi = 2\pi(z^{\mathrm{slow}} - z_c^{\mathrm{slow}})/\lambda$, $\eta = 2\pi r^{\mathrm{slow}}/\lambda$, $\tau = \omega t$, $\Delta_{s,n}^{\pm} = (h_n \pm h_s)/h_s$,

$$\begin{aligned}
g_{s,n}(\eta) &= I_0^2(\tfrac{h_n}{h_s}\tfrac{\eta}{\beta_c}) + I_1^2(\tfrac{h_n}{h_s}\tfrac{\eta}{\beta_c}) - 1\,,\\
f_{s,n,p}^{(1)}(\eta) &= I_0(\tfrac{h_n}{h_s}\tfrac{\eta}{\beta_c})I_0(\tfrac{h_p}{h_s}\tfrac{\eta}{\beta_c}) - I_1(\tfrac{h_n}{h_s}\tfrac{\eta}{\beta_c})I_1(\tfrac{h_p}{h_s}\tfrac{\eta}{\beta_c})\,,\\
f_{s,n,p}^{(2)}(\eta) &= I_0(\tfrac{h_n}{h_s}\tfrac{\eta}{\beta_c})I_0(\tfrac{h_p}{h_s}\tfrac{\eta}{\beta_c}) + I_1(\tfrac{h_n}{h_s}\tfrac{\eta}{\beta_c})I_1(\tfrac{h_p}{h_s}\tfrac{\eta}{\beta_c})\,.
\end{aligned}$$

ψ, z_c, and $\beta_c = \omega/h_s c$ are phase, coordinate, and velocity of the synchronous particle, respectively, and s is the synchronous harmonic number.

The effective potential function U_{eff} describes 3D particle dynamics completely and defines the relationship between longitudinal and transverse motions. It also determines the system Hamiltonian

$$H = \frac{1}{2}\left(\frac{d\mathbf{R}}{d\tau}\right)^2 + U_{\mathrm{eff}}\,. \qquad (4)$$

By using the Hamiltonian (4) and analyzing the bunch form in the 4D phase space, one can find the relationship between the defined longitudinal acceptance and the limit value of transverse emittance, which provides the maximum transmission coefficient. In general, the averaging procedure can be carried out with regard to the Coulomb field.

The terms comprising U_{eff} are determined by the rf field harmonic structure. Summand U_0 describes the interaction of particles with single synchronous harmonic accelerating and defocusing of the beam. The term U_1 evaluates transverse focusing only and is independent of the synchronous wave phase. These two summands correspond to the so-called two-waves approach, when synchronous and single nonsynchronous harmonics exist in the cavity. The terms U_2, U_3 describe the influence of higher harmonics on beam motion. If condition $n + p = 2s$ is satisfied, $U_2 \neq 0$. This term affects both transverse and longitudinal motion. Finally, in the case of $h_n - h_p = 2h_s, n \neq s$, summand U_3 also appears. This term influences phase and radial dynamics, but in some cases of the two-wave approach it can differ from zero.

The necessary condition for simultaneous transverse and longitudinal focusing is the existence of the total minimum of U_{eff}. In this case the effective potential function is 3D potential well in the beam frame. Expanding the U_{eff} near equilibrium particle coordinate ($\chi \to 0, \eta \to 0$) we can formulate these conditions as

$$\omega_\chi^2(e_s, e_n, \ldots) > 0, \qquad \omega_\eta^2(e_s, e_n, \ldots) > 0\,, \qquad (5)$$

where ω_χ, ω_η are frequencies of small longitudinal and transverse oscillations.

Another important restriction on the choice of space harmonic amplitudes can be obtained from the condition of nonoverlapping for different waves resonances in the phase space. This restriction defines limits of the applicability of the averaging method. Moreover, it is the necessary condition of longitudinal stability.

III. CAVITY PARAMETERS CHOICE

A. rf field harmonic structure

Let us consider the acceleration system in the two-waves approach (synchronous and one nonsynchronous harmonics of the rf field). Such a system is effective for acceleration of low energy nonbunched beams. In this case $U_{\mathrm{eff}} = U_0 + U_1$; i.e., the focusing effect is achieved by using one nonsynchronous harmonic and longitudinal motion is determined by the only synchronous wave. In the bunching part of the structure, the influence of terms U_2, U_3 can bring some undesirable effects, such as phase capture decrease, second bunch formation, and others [10]. In the two-waves approach the transverse stability condition for all particle phases gives

$$-e_s\beta_c \sin\left(\psi + \frac{\chi}{\beta_c}\right) - \frac{3}{32}e_s^2 < \frac{3}{8}e_n^2\left(\frac{1}{\Delta_{s,n}^{+2}} + \frac{1}{\Delta_{s,n}^{-2}}\right)\left(\frac{h_n}{h_s}\right)^2. \tag{6}$$

It can be seen from Eq. (6) that ARF efficiency depends significantly on the beam velocity. The nonsynchronous wave moves faster or slower than the synchronous wave. The particle is acted on by two transverse forces: the defocusing force of the synchronous wave and the focusing force of the nonsynchronous wave. In the bunch frame, the second one affects the particle similar to the periodicical sequence of electrostatic lenses. This is an analogy of either the electrostatic focusing or the undulator focusing mechanism. It is more effective in the case of low beam velocity. Choosing the rf field harmonic structure it is necessary to satisfy Eq. (6) along all acceleration channels. Thus, the rf focusing effectiveness is limited above by the beam energy. Equation (6) also shows that transversal focusing with arbitrary synchronous phase for the considered range of beam energy (see below) can be achieved only if the amplitude of the nonsynchronous wave is larger than the amplitude of the synchronous wave.

In the simplest case of APF mentioned in [2], the acceleration wave is faster than the focusing wave ($s < n$) and harmonic amplitudes decrease with the growth of their number ($e_s > e_n$) for the ordinary structure period. Thus the transverse focusing condition can be satisfied only in the case of $|\sin\psi| \ll 1$. It leads to small longitudinal acceptance. In one of the cases of ARF we considered, it is also supposed that $n > s$. But Eq. (6) is satisfied with $|\sin\psi| \sim 1$, i.e., with a large longitudinal emittance. This is possible if the amplitude of focusing harmonic e_n is greater than the amplitude of the acceleration harmonic e_s. The acceleration gradient $dW_s/dz = 0.5E_s\cos\psi$ is proportional to e_s. It sets bounds for the parameter e_s. This version of the ARF can be created by special construction of a structure period containing two or more acceleration gaps. Equation (6) also shows that systems with $s > n$ are ineligible because of insufficient transverse focusing. Structures with a large harmonic number are not effective. The value of the field amplitude which corresponds to separatrixes overlapping decreases fast compared to the growth of the harmonic number. Realization of a structure with $n > 2$ is hardly possible since it is necessary to set many acceleration gaps per period.

One can calculate the transverse oscillation frequency ω_η at fixed e_s, e_n, ψ to select the rf field harmonic structure. The set $\{\mu, s, n\}$ providing the most effective transverse focusing is to be chosen. Arguments noted above should be taken into account. For the cavity parameters we consider (see below), the acceleration structure $\{\mu = \pi, s = 0, n = 1\}$ is to be regarded as the best.

B. Acceleration channel parameters

Let the acceleration cavity consist of two subsections: the gentle buncher subsection and the acceleration subsection. In the gentle buncher, the synchronous particle phase increases linearly from $\psi = -\pi/2$ to some nominal value, and the rf field amplitude increases as a fair curve. In the acceleration subsection these parameters are fixed. Such separation allows one to enlarge the phase capture significantly. The rf field amplitude versus longitudinal coordinate in the gentle buncher is to be defined. The goal is to make transmission as high as possible.

It is convenient to define the rf field harmonics amplitudes as $E_n(z)$, $E_s(z) = \alpha E_n(z)$. To find the optimal function $E_n(z)$ let us start from the 1D model. Then the system Hamiltonian on the axis can be presented as

$$\overline{H} = \frac{P_\varphi^2}{2m_\varphi} + V_{\text{ext}}(\varphi) + V_c\,. \tag{7}$$

Here $\{P_\varphi, \varphi\}$ are the canonical momentum $P_\varphi = W - W_c$ and the canonical coordinate $\varphi = \frac{2\pi}{\lambda}\int dz^{\text{slow}}(1/\beta - 1/\beta_c)$, $m_\varphi = mc^2\beta_c^2/\omega$, $V_{\text{ext}} = (2\pi mc^3\beta_3/\lambda)U_0(\varphi, \eta = 0)$, where V_c is part of the potential function related to the space charge. The distribution function may be chosen in the form

$$f = f_0\sqrt{H_0 - \overline{H}}, \qquad \text{if } \overline{H} \le H_0\,,$$
$$f = 0, \qquad \text{if } \overline{H} > H_0\,. \tag{8}$$

Such distribution allows the neutral equilibrium to be realized. The total potential function $V_{\text{ext}} + V_c$ has a rectangular shape well under the limit current I_n. This parameter is a function of longitudinal coordinate

$$I_n(z) \propto \beta_c^2(z)E_n(z)\Phi(\psi)\,, \tag{9}$$

where $\Phi = \sin(\frac{\Delta}{2} + \psi)(\sin\frac{\Delta}{2} - \frac{\Delta}{2}\cos\frac{\Delta}{2})$, Δ is the separatrix width. It can be shown that the limit current I_n is approximately proportional to the longitudinal acceptance. To provide a high transmission the longitudinal acceptance should be a nondecreasing function. This statement allows one to find the function $E_n(z)$. Let us assume that in the gentle buncher section

$$I_n(z) = I_n(0)F(z)\,, \tag{10}$$

where $I_n(0)$ is the initial value of the limit current and $F(z)$ is some increasing function. Equations (2), (9), and (10) determine the relationship between the acceleration cavity parameters $E_n(z)$, $\beta_c(z)$, and $\psi(z)$. The unknown function $F(z)$ is to be chosen as a sufficiently fast growing function. At the same time, the maximum field amplitude is restricted by cavity breakdown voltage. Thus, function $F(z)$ is limited above. The field amplitude function $E_n(z)$ found by this approach may be improved using numerical optimization.

C. Accelerating resonator

The system we consider can be realized as some variety of interdigital H-type structure [11,12]. It consists of a cavity, two vanes, and a number of drift tubes alternatively connected to the vanes, thus forming accelerating gaps. The required distribution of the field amplitude $E_n(z)$ along the cavity (ramp by a factor of 6–10) may be achieved in such a structure [11,12]. By forming the structure period configuration, it is possible to obtain the rf field we need: $\{\mu = \pi, s = 0, n = 1\}$ with $e_s < e_n$. In this case the potential of the rf field has three loops per period. One can set three electrodes (drift tubes) here to provide such distribution of field potential (Fig. 1). The main electrode's geometry parameters are length, internal radius, and corner radii of each drift tube. There are two types of tubes (Fig. 1). By varying the ratio between their internal radii r_2/r_1 it is possible to control the ratio between harmonics amplitudes α. One finds that when $r_2/r_1 = 1$ only one nonsynchronous harmonic is presented. It means that $\alpha = 0$ and there is no acceleration under a good transverse focusing. On the contrary, if the value r_2/r_1 is large, one gets a great acceleration gradient but transverse motion is defocused. The ratio r_2/r_1 is an analogy of the modulation parameter m in RFQ. The problem is that it influences both the longitudinal and the transverse motion. In our structure we choose $r_2/r_1 \sim 1.1-1.6$ (see Fig. 2) in order that $\alpha = 0.1$. By varying the length and the corner radii of electrodes one can depress the higher harmonics and obtain only two waves in the structure. The second important parameter which allows us to control the radial focusing is $E_n(z)$. The required distribution of field amplitude $E_n(z)$ may be obtained by varying radius r_1 by analogy with RFQ. For example, in a conventional four-vane RFQ resonator the field potential on the vanes is constant as a function of longitudinal coordinate z. Here the required field amplitude distribution on the axis can be realized only by varying parameter a (minimum distance from axis to vane). But it is not so for some other types of resonators. For instance, in a simple interdigital resonator the field potential on the tubes is distributed as an increasing function in the initial part of the cavity. There are some special methods which allow one to modify such structures and to realize the field amplitude distribution increase in the initial part of the cavity and uniformity in the main part. Actually, the required distribution $E_n(z)$ may be realized even if radius r_1 is constant along the cavity [11,12]. The distances from the center of one gap to the center of the next gap inside the period are defined by the nonsynchronous harmonic phase velocity and the parameter α. Here, the length of the electrodes is about $D/5$ to $D/7$ and the corner radii are about $D/10$ to $D/15$. The problem of calculating the field potential distribution can be solved in the electrostatic approach by use of the POISSON code. It has been proven that by choosing the tube's geometry parameters it is possible to

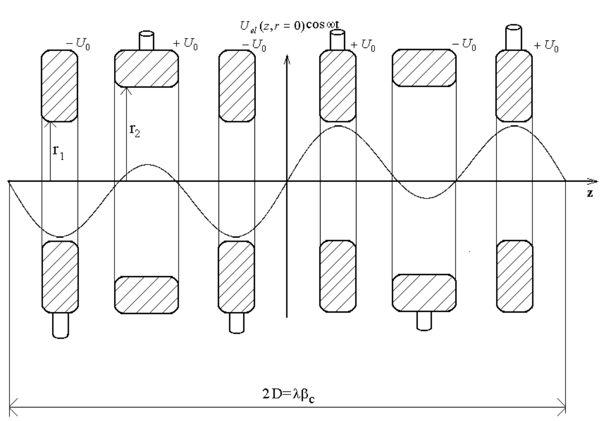

FIG. 1. Structure realization for one period of rf field potential.

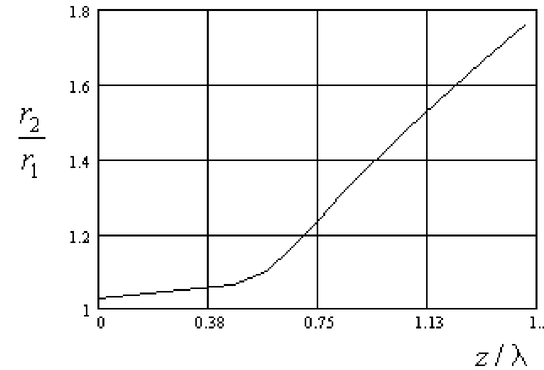

FIG. 2. Ratio r_2/r_1 versus the longitudinal coordinate.

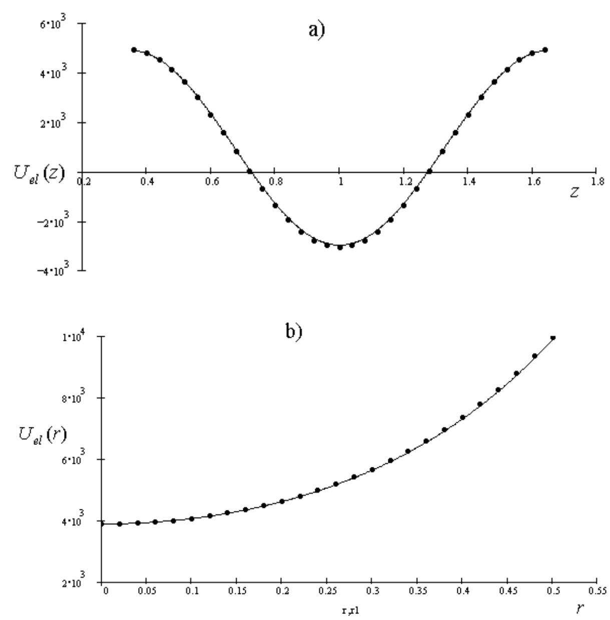

FIG. 3. Example of longitudinal (a) and transverse (b) field potential distribution.

obtain the field distribution which is described by analytic expression (1). Examples of the comparison of analytic formula (curve) and the results of computer simulation (points) can be seen in Fig. 3. Therefore, the concept presented in this paper allows one to form the structure period inside the channel which provides rf fields having the required properties.

IV. NUMERICAL SIMULATION

The computer simulation of high intensity proton beam dynamics in the described axisymmetric rf focusing in a linear acceleration structure was carried out by means of the "superparticles" method. A specialized computer code AFILAC was created for this purpose. The cavity rf field is presented as a Fourier expansion (polyharmonic method). Such an approach allows one to set the acceleration structure rf field analytically. The space charge field is calculated by means of the cloud-in-cell method. Here the space charge density is computed on the grid which is set into the bunch area. The Poisson equation on the grid is solved by using a fast-Fourier transform.

The harmonic structure of the cavity was chosen using the technique described above: $\{\mu = \pi, s = 0, n = 1\}$. Figure 4 presents the cavity parameters versus longitudinal coordinate. Field amplitude $E_n(z)$ in the gentle buncher section was optimized numerically by means of the component-wise descent method. The starting field function for computer optimization was obtained by the approach described in the previous section. Table I contains characteristics of the created acceleration system. Here, amplitude $E_{\max}$ is the maximum value of function $E_n(z)$ (see Fig. 4), and the aperture radius is the minimum value of the internal radius r_1. One can see that transmission is 0.79 under acceleration gradient 0.7 MeV/m, which is a great achievement for the rf focusing structure. Transmission versus input current is shown in Fig. 5.

To test the averaging method applicability, computer simulation was carried out for both the rf field and the averaged field. All results obtained in the smooth approximation for the rf field coincide up to 5%–10%.

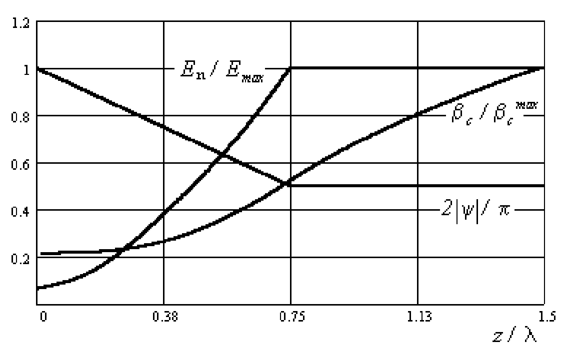

FIG. 4. Acceleration structure parameters versus longitudinal coordinate.

TABLE I. Computer simulation results.

Parameter	Value
Operating frequency (MHz)	150
Parameter α	0.1
Maximum field amplitude $E_{\max}$ (kV/cm)	300
Input/out energy (MeV)	0.1/2.2
Input/out current (mA)	100/79
Transmission	0.79
Total length (m)	3
Aperture radius (cm)	0.6

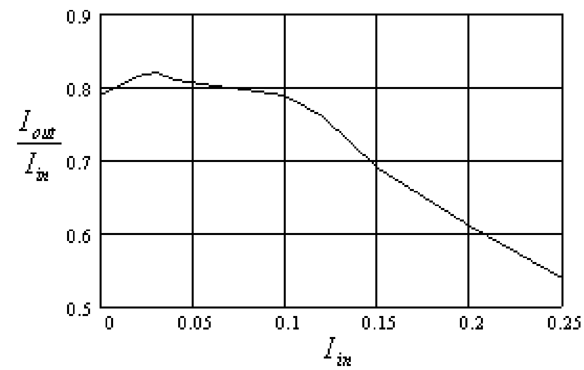

FIG. 5. Transmission versus input current *A*.

Therefore, the investigated ARF structure is sufficiently effective for acceleration of low energy ion beams. In the considered range of energy, the ARF structure can be used instead of RFQ because it is easier and cheaper to fabricate and keep in tune.

V. CONCLUSIONS

In this paper the new approach to rf focusing in the axisymmetric polyharmonic rf field of an ion linac was suggested. The method of classification for all types of axisymmetric rf focusing based on field harmonic structure analysis was presented. Because of the new ARF development, the acceleration cavity parameters choice to achieve high transmission was done. The method of ARF system realization based on the modified Wideroe-type structure was described. The computer simulation of high intensity ion beam dynamics in an ARF structure was carried out. The averaging method applicability was shown. The characteristics of the acceleration system found above prove that the rf focusing efficiencies in some cases are close to those of RFQ.

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