# Phase space tracking of coupled-bunch instabilities

S. Prabhakar,* J. D. Fox, D. Teytelman, and A. Young

*Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309*

(Received 9 June 1999; published 9 August 1999)

We describe an instability diagnostic that exploits the information contained in the angular evolution of coupled-bunch oscillations in phase space. In addition to enabling measurement of coherent tunes and bunch tunes with accuracy of a few hertz, phase space tracking allows new kinds of comparisons between instability theory and experiment. Phase space evolution of bunches participating in a low-threshold vertical instability in the high energy ring of the Stanford Linear Accelerator Center B factory (PEP-II) is used to distinguish between the fast beam-ion instability and conventional instabilities. Tracking of longitudinal instabilities at the LBNL Advanced Light Source and PEP-II is used to measure coherent tunes and gain new insights into uneven-fill instabilities.

PACS numbers: 29.27.Bd, 29.20.Dh

## I. INTRODUCTION

Accelerator performance is degraded at high currents by coupled-bunch instabilities, which arise from self-amplifying interactions between the charged particle beam and its environment. Diagnosis of the nature and cause of unstable bunch motion is the first step towards a cure. Observation of beam position monitor (BPM) power spectra under various beam conditions is the most common diagnostic. Other recently developed techniques include streak camera imaging of bunch motion and off-line analysis of digitized bunch oscillation data.

Theoretical analyses of coupled-bunch instabilities yield qualitative and quantitative predictions about bunch trajectories in phase space. Ideally, an experimenter who wants to diagnose dipole instabilities would like to be able to track the phase space positions of all bunches under various beam conditions. This would require a measurement system of bandwidth $1/2T_b$, where $T_b$ is the bunch spacing.

When viewed in the light of the bandwidth or information rate requirement, streak camera measurements are seen to be unsuitable for multibunch phase space tracking, since they suffer from update rate limitations. However, they provide excellent time resolution and are very useful in studying the bunch size and shape. By the same criterion, the traditional technique of observing BPM signals on a heterodyned spectrum analyzer is limited by the resolution bandwidth of the spectrum analyzer. To identify the instability mode number, one needs resolution bandwidths comparable to the synchrotron or betatron tune. All information outside the resolution bandwidth is lost, as is information contained in the phase of the Fourier transform of the BPM signal. For these reasons, heterodyned spectrum analyzer measurements are used mainly for steady state measurements of instability frequency and amplitude, or for narrow band detection of the phase space magnitude transient of a single coupled-bunch mode.

Phase space tracking of multibunch motion is possible only with fast digitization and storage of the oscillation coordinate of each bunch in the machine. Diagnostic techniques that satisfy this criterion suffer from loss of bunch centroid information only to the extent that measurements are never noise free. Here we use data from a programmable longitudinal feedback (LFB) system, which can digitize and store the oscillation coordinate of each bunch while simultaneously manipulating feedback parameters [1,2]. As a result, growth of unstable oscillations can be observed in the linear small-amplitude region, for which theoretical predictions exist. Another advantage of the LFB system is its ability to store downsampled data. Conventional digital oscilloscopes with data storage capabilities cannot easily use downsampling to take advantage of the fact that the beam signal contains only information near synchrotron and betatron sidebands of revolution harmonics.

This paper describes measurements of the angular evolution in phase space of longitudinal and transverse coupled-bunch instabilities at the LBNL Advanced Light Source (ALS) and Stanford Linear Accelerator Center B factory (PEP-II). Phase space tracking is used to identify unique features of uneven-fill longitudinal instabilities at the two machines. Tracking is also used to compare the signature of a vertical instability [3] in the PEP-II high energy ring (HER) to those of the fast beam-ion instability (FBII) [4,5] and conventional instabilities. We show that this method has the potential to distinguish between the two instability mechanisms.

## II. SIGNAL PROCESSING

Coupled-bunch instability data from a single BPM is often used to measure average tunes, oscillation amplitude envelopes, average phase shifts from bunch to bunch, etc. However, it is not too difficult to estimate the approximate

*Email address: shyam@slac.stanford.edu

  

phase space angle corresponding to each sample of bunch data, if the beam motion is approximately sinusoidal [6,7].

We assume that the sampled longitudinal or transverse bunch oscillation coordinate $s_n^k$ of bunch $k$ is given by

$$s_n^k = a_n^k \cos(2\pi n D \nu + \phi_n^k) = \mathrm{Re}\{u_n^k\}; \qquad (1)$$

$$u_n^k = a_n^k e^{j(2\pi n D \nu + \phi_n^k)}, \qquad (2)$$

where $n$ is the sample number, $D$ is the downsampling factor, $u_n^k$ is the analytic signal corresponding to $s_n^k$, and $a_n^k$ and $\phi_n^k$ vary slowly compared to the nominal tune $\nu$. Although there is no unique solution for the phase space magnitude $a$ and the normalized phase space angle $\phi$, we can use the approximation

$$u_n^k \approx s_n^k - j\tilde{s}_n^k, \qquad (3)$$

where $\tilde{s}_n^k$ is the Hilbert transform of $s_n^k$. The Hilbert transform is calculated by taking the discrete Fourier transform of $s_n^k$, rotating all positive-frequency components by $+90°$ and all negative-frequency components by $-90°$, and then taking the inverse Fourier transform. From Eqs. (2) and (3), we get

$$a_n^k \approx |s_n^k - j\tilde{s}_n^k|, \qquad (4)$$

$$\phi_n^k \approx \angle(s_n^k - j\tilde{s}_n^k) - 2\pi n D \nu. \qquad (5)$$

This approach is equivalent to that of calculating the quadrature component of a narrow band signal from its in-phase component. In the terminology of transverse diagnostics, we are estimating the signal at a fictitious BPM that is 90° ahead of the original BPM in betatron phase.

When used alone, the discrete Hilbert transform produces significant errors at the edges of discrete data sets, since the corresponding filter has a long impulse response. This effect is minimized in practice by simultaneously subjecting the data to a smooth bandpass filter centered at the synchrotron or betatron sideband. The bandpass filter rejects noise outside the frequency band of interest. We also introduce appropriate delays in the bunch signals by means of a phase shift that is proportional to frequency. This compensates for the fact that the bunches are sampled at different instants and on different turns, due to the requirements of downsampling.

With the processing techniques described above, we have approximations to the action $a_n^k$ and normalized angle $\phi_n^k$ of each bunch $k$ at each sample instant $n$. If the data are digitized soon after feedback is switched off, we have enough information to test almost any theoretical prediction about the coupled-bunch instability. The most immediate application of this technique is in the diagnosis of fast transverse instabilities in short bunch trains. For example, one could distinguish between conventional instabilities and the FBII by matching the angle variation along a bunch train with the frequencies of various kinds of ions in the vacuum chamber. Tune shifts along the bunch train are also a strong indicator of the FBII. Bunch tunes can be tracked continuously by taking the derivative of smooth fits to the bunch angle.

Instabilities in beams with most of the buckets filled are well described by projections of the beam motion onto even-fill eigenmodes (EFEMs). These projections are calculated by taking the discrete Fourier transform of the sequence of analytic signals at each turn. The modal phase space coordinate $U_n^m$ of the $m$th EFEM at turn $n$ is thus given by

$$U_n^m = A_n^m e^{j(2\pi n D \nu + \Phi_n^m)} = \sum_{k=0}^{N-1} u_n^k e^{-j2\pi mk/N}, \qquad (6)$$

where $N$ is the ratio of the harmonic number to the bunch spacing, and $A_n^m$ and $\Phi_n^m$ are the magnitude and normalized angle, respectively, in modal phase space.

There is a subtle but important difference between the analytic signals $u_n^k$ used for studying bunch motion in the time domain and those used for calculating modal projections using the equation shown above. In the former case, data is processed to estimate the phase space coordinate of each bunch as it crosses the BPM (coincidence in space). This is how a localized impedance sees bunch motion. In the latter case, signals are processed to recreate instantaneous snapshots of the analytic signals of all bunches each time the first bunch crosses the BPM (coincidence in time). This is because bunch oscillations must be projected into the Fourier domain simultaneously for the discrete Fourier transform to correspond to a modal decomposition.

In the past, measurements of modal magnitude transients $A_n^m$ have been used to determine growth rates of coupled-bunch eigenmodes. Here we shall use normalized modal angle transients $\Phi_n^m$ to precisely measure coherent tunes in the linear regime. In addition to affording direct measurements of the imaginary part of the beam impedance, this approach yields new insights into uneven-fill instability dynamics.

## III. PHASE SPACE TRACKING OF BUNCH TRAINS

The PEP-II HER has exhibited vertical and horizontal instabilities at surprisingly low beam currents. The vertical instability was seen to grow and then saturate at amplitudes of around 100–300 $\mu$m at currents as low as 5 mA [3]. The low threshold and small saturation amplitudes triggered a search for a possible FBII.

The FBII is usually distinguished from conventional instabilities by studying the effect of variations in gas pressure, bunch spacing, train length, and bunch currents on the spectrum of betatron sidebands. Although such measurements have been used at the Advanced Light Source [8] and the Pohang Light Source [9], they are not always conclusive during commissioning, since conditions such as beam orbit, vacuum pressure, coupling, beam size, feedback, etc. are sometimes not well controlled.

To diagnose the HER vertical instability, we make use of the prediction that FBII growth in bunch trains is characterized by variation in the growth rate and bunch tune along the train. Conventional coupled-bunch instabilities caused by wake fields that persist over the length of the gap are not expected to exhibit such behavior. The approach of distinguishing between the two kinds of instability on the basis of a single growing transient has the advantage of being insensitive to artifacts such as parameter drift.

The vertical instability was investigated using digitized records of the oscillations of each bunch immediately after switching off feedback. Figure 1 shows a typical growing transient in a 150-bunch train with a 4.2 ns spacing and a total beam current $I_0$ of 52 mA[1]. Feedback is switched off at approximately $t = 0$. The bunch oscillation amplitudes increase exponentially with time, with bunches at the tail of the train reaching higher amplitudes than those at the head. Although growth along the train is sometimes thought to be a symptom of the FBII, it is also a feature of conventional instabilities driven by an impedance resonance whose fill time is comparable to the length of the bunch train.

Bunch tune variation can be examined most easily by locating the peak in the Fourier spectrum of each bunch signal. Figure 2 zooms in on the bunch spectra in the region of the vertical tune peak. The trailing bunches show a more pronounced spectral peak than the leading bunches, since they oscillate at larger amplitudes. Since the data record is 20 ms long, these spectra have a frequency resolution of no more than 50 Hz, i.e., 0.0004 in tune units. At this resolution, we see no tune shift along the bunch train. In addition to the limited resolution of the discrete Fourier transform, such measurements are also often complicated by power supply ripple, which imposes a 60 Hz modulation on the betatron tune.

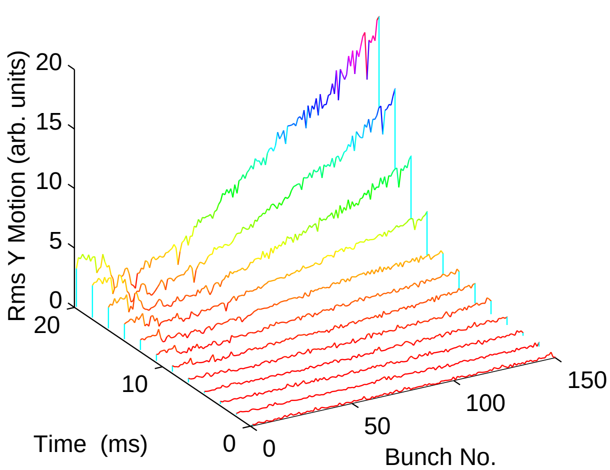


FIG. 1. (Color) Growing vertical instability transient in a 150-bunch train in the PEP-II HER at $I_0 = 52$ mA. Feedback is switched off at $t = 0$. The oscillation amplitude $a^k$ of each bunch $k$ grows exponentially over the 20 ms interval, with trailing bunches growing to larger amplitudes than leading bunches.

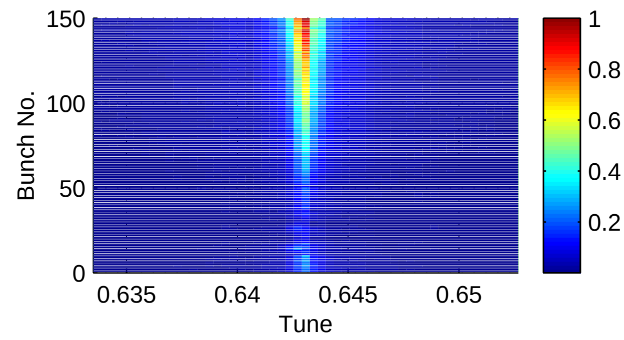


FIG. 2. (Color) Color-coded representation of magnitude spectra of the 150 bunches in Fig. 1. Peaks of Fourier transform of bunch transients lie at the same tune, indicating that tune spread across the train is ≤ the frequency resolution (50 Hz).

We can measure tune variations with greater sensitivity by tracking the normalized bunch phase space angles $\phi_n^k$. For example, we could subtract $\phi_n^{150}$ from all the other angles to get the phase space angle of each bunch relative to that of the last bunch. This automatically masks the tune variation due to power supply ripple. The slope of this angle differential directly yields the tune of the corresponding bunch relative to that of bunch 150. Figure 3 shows the phase space angle differentials $(\phi_n^k - \phi_n^{150})$ for all bunches. Only the last 7 ms of data are shown, since the signal-to-noise ratio is worse during the initial section of the growing transient. We see that the differential angles are almost constant over these 7 ms,

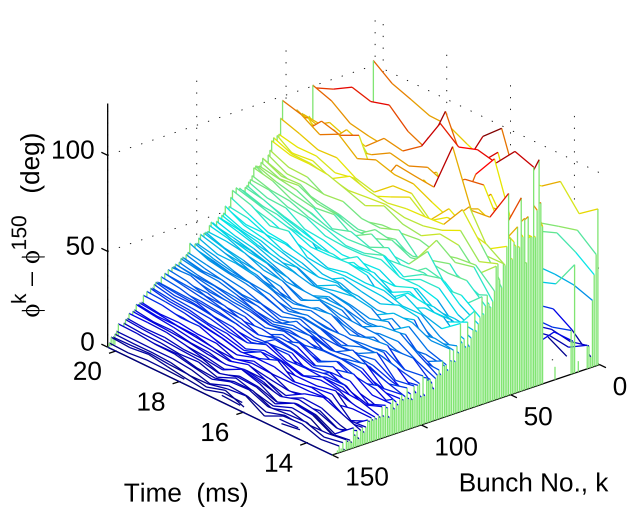


FIG. 3. (Color) Phase space angle differentials $(\phi_n^k - \phi_n^{150})$ for all 150 bunches. Differentials for the first few bunches are noisy due to smaller oscillation amplitudes. Differential angles are almost constant over 20 ms, indicating that the bunches oscillate coherently.

[1]The PEP-II HER design goal is 1 A in 1658 buckets. At a spacing of 4.2 ns, the entire ring can be filled with 1746 bunches.

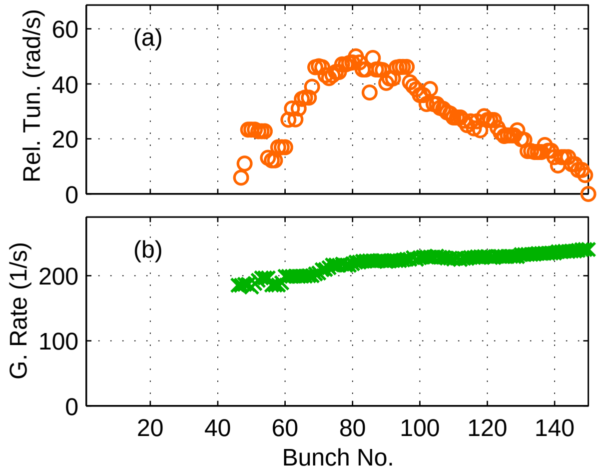


FIG. 4. (Color) (a) Relative tunes of bunches 46–150, calculated using linear fits to the phase space angle differentials in Fig. 3. (b) Growth rates of the same bunches, calculated using exponential fits to the magnitude transients $a^k$.

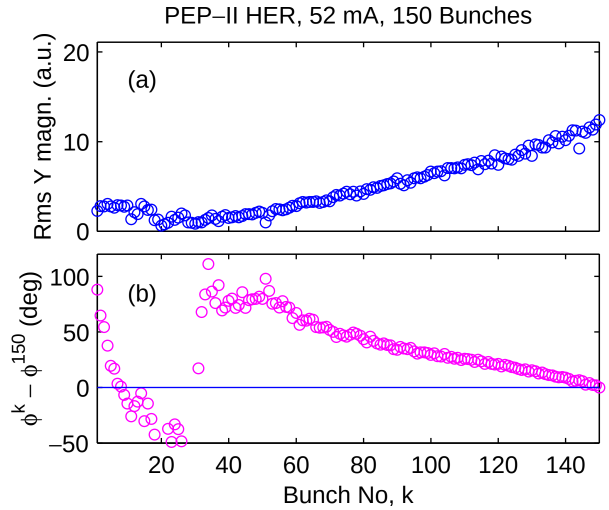


FIG. 5. (Video) A single frame from Video 1 depicts a vertical instability transient in a 150-bunch train in the PEP-II HER (same data as Figs. 1–4). (a) Oscillation magnitudes of the 150 bunches on a single turn and (b) relative phase space angles ($\phi^k - \phi^{150}$) of the bunches on the same turn.

with a small positive slope in the section from bunch 60 to bunch 140. This implies that the bunches oscillate coherently, with very little tune variation along the train. The exact tune variation can be extracted from linear fits to the relative angles ($\phi_n^k - \phi_n^{150}$). Instantaneous tunes are not calculated, since the relative angles vary linearly with time in this piece of data.

Figure 4(a) shows the fitted tunes of bunches 46 to 150, relative to the tune of bunch 150. The peak-to-peak variation is less than 50 rad/s. The first 45 bunches are excluded because they grow to smaller amplitudes and have smaller signal-to-noise ratios. Exponential fits to the magnitude transients $a^k$ yield the instability growth rates shown in Fig. 4(b). As can be expected of conventional instabilities, the growth rate variation across the train is small enough to be accounted for by measurement error and the presence of other eigenmodes at small amplitudes. The original theoretical studies [4,5] of the FBII predicted that oscillations should grow as $\exp(\sqrt{t/\tau})$, where $\tau$ is the growth time. However, the experimental data quite clearly show exponential growth. A more detailed theoretical analysis, which incorporates $\beta$-function variation around the ring, predicts exponential FBII growth and a linear variation in tune shift and growth rate along the train [10]. The linear variation of growth rates and tunes is not borne out by the data, as can be seen from Fig. 4. It is, of course, possible that some of the approximations made in [10] do not apply to the relative time scales of this experiment.

### Animation

It is convenient to use animation to represent the evolution of the vertical instability transient shown in Figs. 1–4. Figure 5 shows a single frame from Video 1. Each such frame shows the oscillation magnitudes $a^k$ and relative phase space angles ($\phi^k - \phi^{150}$) of the 150 bunches on a single turn. Successive frames are separated by 20 turns. Only the last 7 ms of data are animated. The relative phase pattern is largely fixed, with a small upward trend in the section from bunch 60 to bunch 140.

## IV. PHASE SPACE TRACKING OF COUPLED-BUNCH MODES

The previous section focused on applications of time domain phase space tracking, i.e., tracking of bunch trajectories $u_n^k$ in phase space. Coherent instabilities in rings with more filled buckets than empty buckets are better described by the projection of these trajectories onto the eigenmodes of an evenly filled ring [see Eq. (6)]. The magnitude $A^m$ of each projection $U^m$ corresponds to the magnitude of the sideband of the $m$th revolution harmonic in the bunch spectrum. If the coherent tune is a constant, the angle of $U^m$ evolves linearly with a constant slope $\nu + d\Phi^m/dt = \nu + \Delta\nu^m$. The coherent tune shift $\Delta\nu^m$ can thus be measured accurately by measuring the slope of the modal phase space angle as the instability grows linearly out of the noise floor. This is a direct measurement of the imaginary part of the beam impedance.

Of the 328 longitudinal coupled-bunch modes at the ALS, only modes 204 and 233 are unstable in most cases [11]. Figure 6 shows the measured linear evolution of $\Phi^{204}$ (dash-dotted line) and $\Phi^{233}$ (dotted line) as the two modes grow out of the noise floor. The ring is evenly filled at $I_0 = 157$ mA. The slopes give the coherent

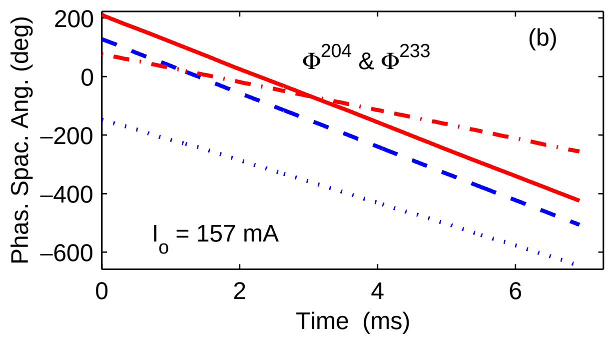


FIG. 6. (Color) Linear evolution of modal phase space angles $\Phi^{204}$ and $\Phi^{233}$ at the ALS (longitudinal instabilities, $I_0 = 157$ mA). Dash-dotted line: $\Phi^{204}$, even fill; dotted line: $\Phi^{233}$, even fill; solid line: $\Phi^{204}$, square-wave fill; dashed line: $\Phi^{233}$, square-wave fill. Square-wave fill couples the two frequencies and creates a mixed eigenmode, so that $U^{204}$ and $U^{233}$ are phase locked.

frequency shifts, which are $-132$ Hz and $-196$ Hz, respectively.

The effective longitudinal impedance $Z^{\rm eff}$ for a beam with $N$ evenly spaced bunches is defined as

$$Z^{\rm eff}(\omega) = \frac{1}{\omega_{rf}} \sum_{p=-\infty}^{\infty} (pN\omega_0 + \omega) Z(pN\omega_0 + \omega), \quad (7)$$

where $Z(\omega)$ is the total longitudinal beam impedance, $\omega_{rf}$ is the frequency of the klystron drive, and $\omega_0 = 2\pi f_0$ is the revolution frequency. $Z^{\rm eff}$ is related to the even-fill coherent tune shift by [12]

$$\mathrm{Im}\{Z^{\rm eff}(m\omega_0 + \omega_s)\} = -\frac{4\pi E \nu_s}{\alpha e h I_0} \Delta\nu^m, \quad (8)$$

where $\alpha$ is the momentum compaction factor, $h$ is the harmonic number, $E/e$ is the nominal beam energy in volts, and $\nu_s = \omega_s/\omega_0$ is the nominal synchrotron tune. By scaling the measured tune shifts according to the above equation, we get $\mathrm{Im}\{Z^{\rm eff}(204\omega_0 + \omega_s)\} = -157\ \mathrm{k}\Omega$ and $\mathrm{Im}\{Z^{\rm eff}(233\omega_0 + \omega_s)\} = -232\ \mathrm{k}\Omega$. Together with the measured growth rates, these numbers have been used to estimate the shunt impedance of the cavity resonances that drive the instabilities [13].

In addition to aiding in impedance measurement, graphs of modal phase space angles provide information about the shape and nature of the eigenvectors of an unevenly filled ring. Recent theoretical studies of uneven-fill dynamics suggested that two even-fill eigenmodes could be coupled to each other by means of uneven fills that contain Fourier components at their spatial beat frequency [13]. For example, the ALS even-fill modes at $204f_0$ and $233f_0$ can be coupled together using a square-wave fill with a periodicity of roughly $1/29f_0$. The coupling creates a new pair of eigenmodes which are linear combinations of the two even-fill eigenmodes. If we measure the growth of one of the “mixed” eigenmodes, we should naturally see that projections of the motion onto even-fill modes 204 and 233 show exactly the same growth rate and coherent tune shift. Such a measurement was performed at the ALS on the same day and at the same beam current (157 mA) as the above-mentioned even-fill measurement. The normalized phase space angles $\Phi^{204}$ (solid line) and $\Phi^{233}$ (dashed line) of the projections $U^{204}$ and $U^{233}$ are shown in Fig. 6. The existence of a mixed eigenmode is confirmed by the fact that $\Phi^{204}$ and $\Phi^{233}$ have identical slopes (the slopes were different by 64 Hz in the even-fill case). Mixtures of unstable even-fill eigenmodes are generally to be avoided, since the mixed mode is more unstable than either of the even-fill modes [13].

Longitudinal coupled-bunch instabilities in PEP-II exhibit more complicated behavior, since they are driven by damped cavity resonances which span tens of revolution harmonics [14]. Other complications include irregular fill shapes during commissioning and gap-induced interbunch tune spreads [2,15], which tend to couple neighboring even-fill eigenmodes to each other. Conventional measurements of instability growth rates are difficult to interpret under such circumstances.

The uneven-fill eigenmodes can be thought of as linear combinations of even-fill eigenmodes. Thus an uneven-fill eigenmode could show up at more than one sideband in the beam spectrum, and a single sideband could be a superposition of many eigenmodes. Since different modes, in general, have different coherent frequencies, we should see beating of sideband amplitudes on a spectrum analyzer in zero span mode, and beating of the $A^m$’s in the reconstructed phase space trajectories.

Figure 7 shows the magnitude growth of two sets of projections of a single longitudinal instability transient in the PEP-II low energy ring (LER). The data were taken at

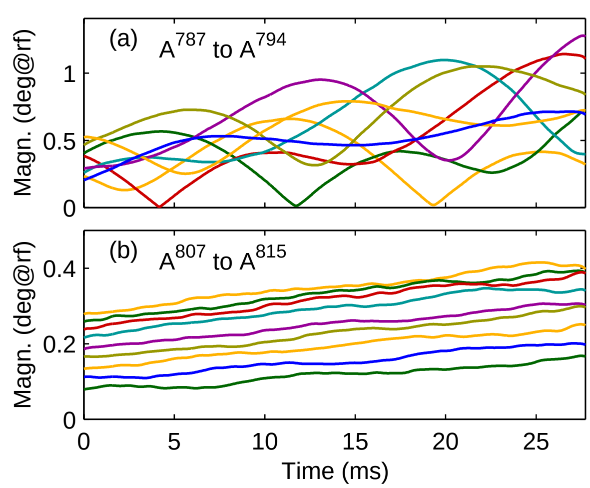


FIG. 7. (Color) Growth in magnitude of two sets of projections of a longitudinal instability transient in PEP-II LER. Uneven fill, $I_0 = 703$ mA. (a) $A^{787}$ to $A^{794}$, beating is evidence of at least two uneven-fill eigenmodes in this frequency range. (b) $A^{807}$ to $A^{815}$, quasiexponential growth indicates that this set of sidebands oscillates coherently as a single eigenmode.

an above-threshold beam current of 703 mA. There is a clear qualitative difference between the upper traces, which show beating at a frequency of 50–80 Hz, and the lower traces, which show slow quasiexponential growth. The obvious conclusion is that the "modes" in Fig. 7(a) are actually superpositions of two or more uneven-fill eigenmodes with slightly different coherent frequencies. The modes in Fig. 7(b) look like projections of a single uneven-fill eigenmode, since they all show about the same growth rate.

The LER instability transient described above is localized to the frequency range between $775 f_0$ and $815 f_0$, which coincides with the aliased frequencies of the two largest cavity resonances [14]. Figures 8(a) and 8(b) show the average magnitudes and tunes of the modal phase space trajectories $\{U^{775}, U^{776}, \ldots, U^{815}\}$ in the same piece of data. We see a clear transition at mode 798, which is marked with a dotted line. The coherent tune spectrum to the right of the dotted line shows no tune variation, confirming our earlier conclusion that this band of projections onto even-fill modes contains just a single uneven-fill eigenmode. The other possibility, which is much less likely, is that this band contains multiple eigenmodes whose growth rates are very close and whose coherent tunes are identical to within 4 Hz. The modes to the left of the dotted line seem to have a relatively large coherent tune variation. This frequency band contains two or more eigenmodes that beat against each other over time scales comparable to the length of the data set. The modal phase space angles $\Phi^m$ do not evolve linearly in this band, and therefore these calculated tunes have errors of the order of the beat frequency.

The phase space trajectories of some of the modes which comprise the single uneven-fill eigenmode above $798 f_0$ are shown in Fig. 9(a). Here we see the simple exponentially growing single-frequency spirals that we usually expect. Although the actual modal phase space trajectories complete roughly 95 revolutions around the origin in the duration of this piece of data, the figure shows less than a single revolution for each mode. This is because the phase space angle of a reference mode has been subtracted from the angles of each of the displayed trajectories, to reduce clutter in the graphical representation. In other words, we plot $U^m(t)\exp(-j\omega_{\mathrm{ref}} t)$ in the complex plane rather than $U^m(t)$.

The phase space trajectories of a few beating modes $U^m$ are shown in Fig. 9(b). Most of these trajectories look approximately like circles with a stationary or slowly rotating center. This indicates that the complicated beating in Fig. 7 is largely explained by the superposition

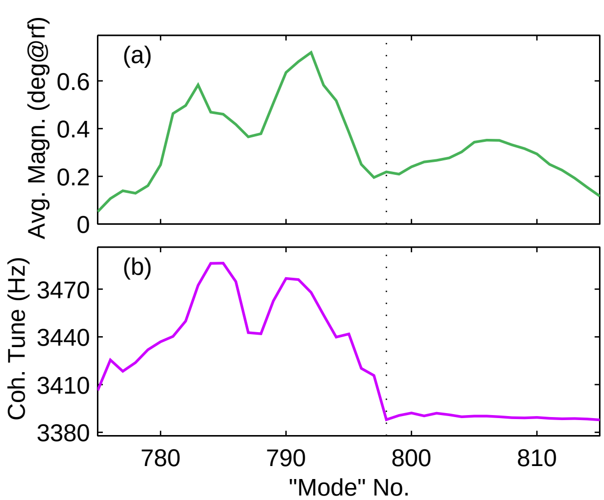


FIG. 8. (Color) (a) Average of modal phase space magnitudes $A^m$; $m = 775, 776, \ldots, 815$ (same data as Fig. 7). (b) Average coherent tunes, calculated using linear fits to $\Phi^m$; $m = 775, 776, \ldots, 815$. Fitted tunes show negligible variation above $m = 798$, implying that the band of projections on the right-hand side of the dotted line contains only one eigenmode.

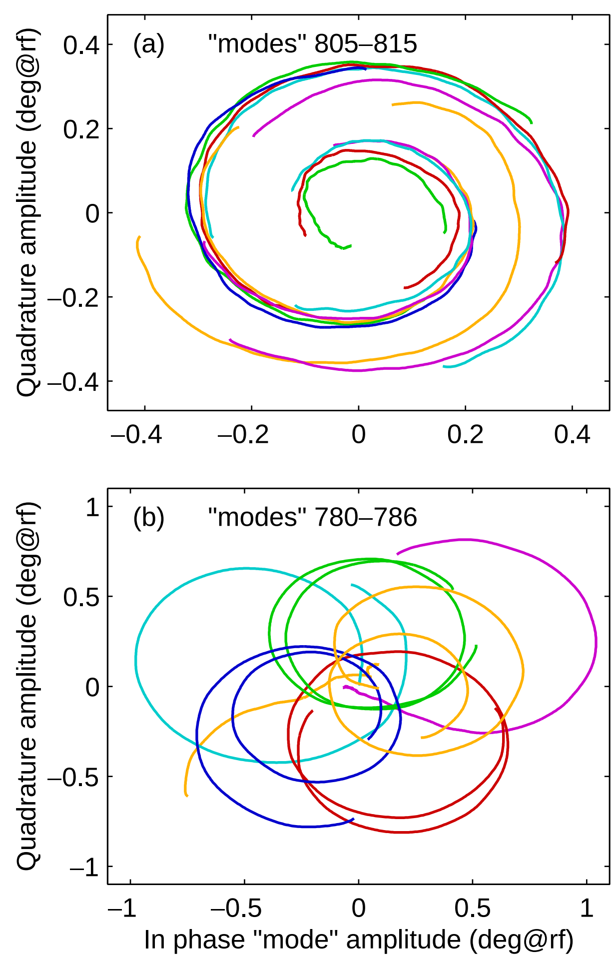


FIG. 9. (Color) Modal phase space trajectories of growing PEP-II LER longitudinal instability (same data as Fig. 8). (a) Representative selection of modes above $m = 798$. Expanding spirals about the origin indicate a single uneven-fill eigenmode. (b) Trajectories of a few modes below $m = 798$, whose magnitude transients show beating.

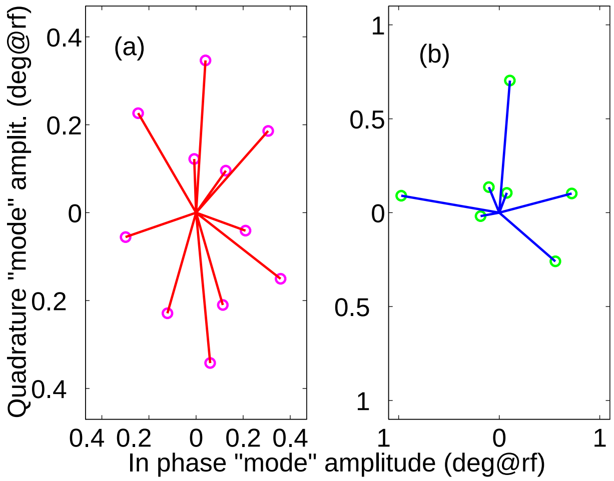


FIG. 10. (Video) A single frame from Video 2 depicts the instantaneous modal phase space coordinates of two sets of longitudinally unstable modes in the unevenly filled PEP-II LER (same data as Fig. 9). (a) Phasors representing modes 805–815: These modes are merely projections of a single eigenmode. (b) Modes 780–786: These are superpositions of two or more eigenmodes.

of just two uneven-fill eigenmodes. The slowly rotating (and diverging) centers of the circles are the tips of phasors that represent an eigenmode whose coherent frequency is very close to $\omega_{\mathrm{ref}}$ ($2\pi \times 3416$ rad/s, in this case). The circular orbits are formed when another eigenmode with a slightly larger coherent frequency is superimposed on the original mode. We foresee the use of such plots as visual aids in precisely measuring the growth rates and coherent tunes of unstable uneven-fill modes that beat against each other.

### Animation

The distinction between projections of a single eigenmode and superpositions of two or more eigenmodes is quite clear in Video 2, which animates the data of Fig. 9. Figure 10 shows a single frame from the video. As before, we plot $U^m(t)\exp(-j\omega_{\mathrm{ref}}t)$ in the complex plane rather than $U^m(t)$. Successive frames, separated by 60 turns, show snapshots of the instantaneous modal phase space coordinates of the two sets of modes. Subframe (a) shows modes 805–815. These modes rotate at identical speeds as they grow, since they are merely projections of a single uneven-fill eigenmode. Subframe (b) shows the phasors corresponding to modes 780–786. The large variation in relative amplitude and relative phase is due to the fact that each of these phasors is a superposition of two or more uneven-fill eigenmodes, with two or more coherent frequencies and growth rates. The paths that the tips of these phasors trace are the trajectories in Fig. 9.

## V. SUMMARY

We have demonstrated the use of phase space tracking as a new diagnostic for coupled-bunch instabilities. Phase space tracking in the frequency (modal) domain has been shown to be useful in accurately measuring the imaginary part of the effective beam impedance at the ALS. Measurements of modal phase space trajectories at the ALS and PEP-II confirm qualitative predictions about uneven-fill coupled-bunch eigenmodes. These trajectories can conceivably be used to measure coherent tunes and growth rates in cases where conventional methods are frustrated by beating between multiple uneven-fill eigenmodes.

Tracking in the time domain has been used to study a low-threshold vertical instability in the PEP-II HER, and to compare features of the bunch phase space trajectories to characteristics of conventional instabilities and the FBII. The trajectories fail to match qualitative features described in the existing literature on FBII theory. The method shows promise as a tool for analyzing data from future FBII experiments, and for revealing aspects of instability growth that have until now remained unexamined.

## ACKNOWLEDGMENTS

The authors thank G. Stupakov of SLAC for elaborating on aspects of the fast beam-ion instability and B. Podobedov and C. X. Wang of SLAC for discussions of existing literature on phase space tracking of single-particle effects. We also thank W. Barry and G. Stover of LBNL and M. Minty of SLAC for help with the measurements. This work was supported by DOE Contract No. DE-AC03-76SF00515.

[1] J. D. Fox *et al.*, in *Proceedings of the Sixth European Particle Accelerator Conference, Stockholm, Sweden, 1998*, edited by S. Myers *et al.* (Institute of Physics, Bristol, UK, 1998), p. 296.
[2] D. Teytelman *et al.*, in *Beam Instrumentation Workshop*, edited by R. Hettel, S. Smith, and J. Masek, AIP Conference Proceedings No. 451 (AIP, New York, 1999), p. 222.
[3] S. Prabhakar *et al.*, SLAC Report No. SLAC-PEP-II-AP-NOTE-99-04.
[4] T. Raubenheimer and F. Zimmermann, Phys. Rev. E **52**, 5487 (1995).
[5] G. Stupakov, T. Raubenheimer, and F. Zimmermann, Phys. Rev. E **52**, 5499 (1995).
[6] A. N. Dubrovin *et al.*, in *Proceedings of the Fifth European Particle Accelerator Conference, Sitges, Spain, 1996*, edited by S. Myers *et al.* (Institute of Physics, Bristol, UK, 1996), p. 1585.
[7] G. Morpurgo, in Proceedings of the 1999 Particle Accelerator Conference, New York (to be published).
[8] J. Byrd *et al.*, Phys. Rev. Lett. **79**, 79 (1997).
[9] J. Y. Huang *et al.*, Phys. Rev. Lett. **81**, 4388 (1998).

[10] G. Stupakov, in *Proceedings of the 1997 Particle Accelerator Conference, Vancouver, Canada* (IEEE, Piscataway, NJ, 1998), p. 1632.
[11] S. Prabhakar *et al.*, Part. Accel. **57**, 175 (1997); SLAC Report No. SLAC-PUB-7717.
[12] A. Chao, *Physics of Collective Instabilities in High Energy Accelerators* (Wiley, New York, 1993), p. 210.
[13] S. Prabhakar, Ph.D. thesis, Stanford University, 1999.
[14] R. Rimmer *et al.*, in *Proceedings of the Fifth European Particle Accelerator Conference, Sitges, Spain, 1996* (Ref. [6]), p. 2035.
[15] S. Prabhakar *et al.*, in *Beam Instrumentation Workshop* (Ref. [2]), p. 529.