

Wilson Prize article: From vacuum tubes to lasers and back again¹

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The first demonstration of an optical-wavelength laser by Theodore Maiman in 1960 had a transformational impact on the paths that would be blazed to advance the state of the art of short wavelength coherent electron beam-based radiation sources. Free electron lasers (FELs) emerged from these efforts as the electron beam-based realization of the pioneering model of atom-based “optical masers” by Schawlow and Townes, but with far greater potential for tunable operation at high power and very short wavelengths. Further opportunities for yet greater capabilities may be inherent in our still growing understanding of the underlying physics. This article focuses on the FEL efforts in which the author was directly and personally involved.

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I. INTRODUCTION

As a young amateur radio enthusiast in the 1950s [1] I was fascinated by the invisible but nonetheless very real mechanisms that operated in the vacuum tubes that my brother and I used in our transmitters: One could clearly observe their brightly glowing cathodes, but the electrons streaming from their cathodes were invisible to our eyes, as were their interactions with the fields and currents in the interconnected resonant circuits that converted their periodic pulsations to the sinusoidal currents that excited the free space waves launched by our carefully crafted Yagi antennas. And I was to have the unusual opportunity to closely examine the more sophisticated electron tubes that had been developed for operation at microwave and millimeter wavelengths when Howard Schrader, the brother of our next door neighbor, learned of my brother's and my interest in these devices. Howard had worked for many years as Princeton University's official photographer, and had—through his personal contacts with the Princeton faculty and their associates—come to amass one of the world's largest collection of vacuum tubes [2] including examples of everything from deForest's audion to the latest millimeter wave traveling wave tubes developed by Bell Labs and Hughes. I had also followed with interest Charles Townes's development of the first microwave maser [3].

By these means, I came to have an unusually complete knowledge of the status of vacuum tube technology by the time I graduated from high school in 1960, and had also

been introduced by Howard to the senior faculty members with whom he worked at Princeton including John Wheeler, who arranged to give my brother and I essentially unlimited access to the department's collection of surplus electronic chassis in the attic of the old Palmer Laboratories of Physics.

Another intangible aspect of life in northern New Jersey in those years was the heritage of that area's role as the predecessor to today's Silicon Valley with the evidence of the pioneering contributions of its inventors and entrepreneurs visible everywhere from the research complex that AT&T had developed at the nearby Bell Laboratories and the numerous pioneering microwave electronic companies in the area to the several still standing Edison-era laboratories and workshops scattered throughout the area including Edison's old workshop on Chestnut Street in Roselle Park that my dad drove by every morning on his way to work. For a young student interested in science and technology it was impossible to ignore the lesson that the ideas that had come to fruition in these laboratories and workshops had, when pursued with energy and determination, changed the world.

But the world of electron beam-based generators of coherent radiation which had seized my imagination appeared in 1960 to have been superseded by the new world of laser sources based on the principle of stimulated emission by inverted populations of excited atoms or molecules as embodied in Theodore Maiman's pioneering flashlamp pumped ruby laser [4]. In the years following Maiman's invention, I was to learn that the principles embodied in Maiman's laser could also be applied to develop short wavelength amplifiers and oscillators based on the bremsstrahlung radiation emitted by beams of relativistic free electrons moving through spatially periodic transverse magnetic fields. It is my purpose in preparing this article to provide an account of how my undergraduate friends and subsequent students and colleagues followed

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the trail blazed by the pioneering efforts of Einstein, Schawlow, Townes, Gould, and Maiman to devise the means—conceptual, analytic, and technical—needed to establish how these new completely tunable and highly coherent light sources could be developed.

In the interests of clarity, I have organized this account into five parts: (1) How were the first lasers different from the prior generations of electron tubes? (2) How did the FEL concept as it emerged in 1970 embody the principles of the new laser sources? (3) What was the significance of the early FEL amplifier and oscillator experiments? (4) What did it take to move from the first proof of principle experiments to today’s high brightness optical and x-ray sources? And (5) are there any further transformational developments of this nature still in prospect?

In the further interest of brevity, I have also organized this account around the efforts in which I was directly and personally involved. While this focus will inevitably omit mention of the many important contributions made by those other individuals who joined this effort elsewhere in the years after the first FEL experiments, I think this approach is nonetheless one of the more effective means available to convey a general understanding of the basic steps that were needed to transition the state of the art from the electron beam-based microwave and millimeter sources available for use in 1960 to the high brightness FEL light sources that now define the frontier of tunable coherent laser technology at wavelengths extending deep into the x-ray region.

II. A FIRST-HAND ACCOUNT

A. Electron tube technology in the 1960s and the impact of Ted Maiman’s new laser

The dramatic extension of operating wavelengths made manifest by the operation in 1961 of Ted Maiman’s flashlamp pumped ruby laser led many of those who were familiar with the older electron tube technology to wonder just what new physical principles made all of this possible. For despite the diverse approaches to amplification and extraordinary capabilities inherent in such devices as grid-modulated descendants of deForest’s audion, the Varian brother’s velocity-modulated klystron, Randall and Boot’s crossed field magnetron, and the coupled beam-wave traveling wave tubes of Kompfner and Pierce, none of these prior devices had the ability to operate at frequencies beyond 100 GHz, barely entering the THz spectral region. But here was the optical laser, operating at a frequency higher by a factor of 3000 and seemingly without any of the constraints previously held to limit the older electron devices to radio and microwave frequencies.

The prior theoretical analyses of Charles Townes and Arthur Schawlow [5] had identified four key elements of the new technology: (1) a new, more general description of the amplification process based on Einstein’s description

of the mechanism of stimulated emission applicable to the radiation process that operated on the quantum scale beyond the regime in which classical methods were applicable, (2) the application of this concept to characterize the amplification inherent in radiating systems in which the transition rate for stimulated emission exceeded the transition rate for absorption, (3) the need for radiating systems in which the spontaneous transition rate per mode of the electromagnetic field is high enough to achieve a net gain in excess of resonator losses, and (4) the use of highly overmoded open resonators to provide the radially confined Gaussian resonator modes needed to optimize the interaction volume while minimizing diffraction losses.

It now seems commonplace to minimize the significance and impact of these new principles. But a comparison of the physics and technology relied upon in the devices that anticipated one or another aspect of free electron lasers in the years before the demonstration of Maiman’s laser—Hans Motz’s exploration of the emission of optical radiation in undulator magnets in the mid-1950s [6] and R. M. Phillips subsequent Ubitron waveguide-based microwave amplifiers [7]—suffices to demonstrate the extent to which our present understanding of free electron lasers follows from the conceptual and technical advances that led to Maiman’s ruby laser.

Although Ginsburg appears to have been the first to point out that a relativistic beam of electrons moving through a periodic transverse magnetic field could emit a forward-directed beam of incoherent radiation at visible wavelengths [8], Hans Motz appears to have been the first to actually demonstrate the effect in a series of experiments using Stanford’s pioneering high energy *s*-band linac in the High Energy Physics Laboratory (HEPL) on the Stanford campus. Motz also investigated the theoretical possibilities for amplification of a copropagating electromagnetic wave using the theory developed by Pierce to describe the interaction responsible for amplification in traveling wave tubes [9]. Phillips actually designed, built, and operated a series of highly effective waveguide-based microwave and millimeter wave amplifiers based on the use of a lower energy electron beam but the same magnetic field configuration as employed by Motz at the nearby General Electric Research Laboratories in Palo Alto, and subsequently at SLAC. Through these seminal pioneering developments, Ginsburg, Motz, and Phillips’s research clearly anticipated several key aspects of the free electron lasers to be developed in subsequent years.

But the concept of the radially confined Gaussian resonator modes central to laser operation [10] was not to emerge until the need for interaction volumes large compared to the operating wavelength, but with small diffraction losses, emerged in the effort to design workable optical lasers. And that development was central to setting aside the limitation of the dimensions of the cavities or waveguides used in the prior generation of electron devices

to the scale of the operating wavelength. The analyses developed by Motz and Phillips were also purely classical, with no attention to the purely quantum effects which dominate startup and the approach to saturation at the shorter wavelengths at which FELs now operate, and incapable of identifying the opportunities now available for the further development of these sources.

Had I known of the pioneering developments of Motz and Phillips at the time, I might have followed the directions that they had established as opposed to taking a fresh look at the opportunities for development of electron beam-based radiation sources operable at shorter wavelengths. But given my experience base in New Jersey and in the laser and high energy accelerator communities in California, I did not become aware of their work until many years later, leaving me free to find my own path to the development of these new sources.

B. Emergence of the FEL concept

I had just entered CalTech as a freshman the year that Maiman demonstrated the first visible-wavelength laser, and as I learned more about the physical principles responsible for its operation, my thoughts turned to the question of whether there were any similar principles that might be exploited to extend the operating range of the electron tubes with which I had become familiar through my prior interests in amateur radio and the history of electron tube development.

Though not starting with much in the way of insight, the pieces began to fall into place when I learned of the emission of high energy bremsstrahlung radiation by relativistic electrons as an undergraduate research assistant at the CalTech Synchrotron Laboratory where the process was used to create an external high energy photon beam using a thin internal target inserted into the synchrotron's circulating e -beam, and subsequently in further analytic detail in Professor Barnes's senior year modern physics course in which Professor Barnes also related the bremsstrahlung mechanism to the synchrotron radiation clearly visible through the glass ports in the synchrotron's vacuum chamber. I also subsequently had the benefit of Professor Yariv's description of the quantitative requirements for lasing in his subsequent graduate level quantum electronics course which I took as a master's degree candidate in electrical engineering and a further introduction to the analytic theory for the bremsstrahlung and synchrotron radiation mechanisms in Professor Walker's graduate electricity and magnetism course.

Two aspects of the bremsstrahlung mechanism appeared to be of special interest from the standpoint of its potential for use in an electron beam-based laser. The first of these aspects was the ability of the mechanism to produce radiation at energies deep into the x-ray and gamma ray spectrum, without the constraints to the microwave or millimeter wave region which had appeared to limit the

further prospects for development of conventional electron tube radiation sources. The second aspect of interest was that bremsstrahlung radiation was understood to be emitted as a consequence of the electrons' transverse acceleration, a mechanism very distinct from the velocity-dependent coupling of the electrons to the field in conventional microwave tubes. If this process was subject to enhancement through Einstein's stimulated emission mechanism, it thus seemed at least conceivable that the bremsstrahlung mechanism might serve as the basis for an entirely new type of electron beam-based coherent radiation source.

The general geometry of bremsstrahlung radiation sources was also attractive from the standpoint of laser operation given the possible use of near-filamentary electron beams aligned with the axis of an open optical resonator whose mirrors had been ground to optimize the e -beam's "filling factor" for the resonator's Gaussian modes, another important requirement for the attainment of lasing [11]. The use of such an open resonator would also simply and cleanly overturn the limits on the electron beam radius that had been imposed by the need for the low order resonators and/or waveguides with transverse dimensions on the scale of the operating wavelength that had been used in the prior microwave and millimeter wave sources including the devices previously considered by Motz and Phillips.

The increasingly effective collimation of the radiation emitted in the direction of the electrons motion, acting to further decrease the solid angle—and hence the number of modes—into which the radiation was emitted was also of evident interest. Indeed, it was this latter effect which in subsequent years was to overcome the limits to x-ray laser operation foreseen by Schawlow and Townes on the basis of the rapidly increasing number of modes into which the radiation emitted by the atomic systems they considered would be distributed at x-ray wavelengths [5].

These general aspects of the bremsstrahlung radiation emitted by high energy electron beams were sufficient by themselves to inspire the upperclassmen who occasionally gathered in 1963 for a physics-oriented bull session around the fireplace in the Blacker House student lounge at CalTech to speculate on the properties and uses of a hypothesized "stimulated bremsstrahlung" gamma ray laser, a possibility that intuitively seemed to those present to have at least a remote basis in the principles of the newly discovered atomic and molecular optical laser sources.

But could the bremsstrahlung mechanism generate the number of radiative transitions per mode of the electromagnetic field to provide a useful degree of amplification in competition with the inevitable absorption of these same photons by the incident electrons? And even more fundamentally, was there any experimental evidence that Einstein's model for simulated emission could be applied to simple scattering processes like bremsstrahlung when all the laser applications to date had relied on the spectrally

resolved bound-state transitions of inverted populations of atoms or molecules.

The further evolution of this concept was to consume another four years as part of my dissertation research as a member of Bill Fairbank's Low Temperature Physics Group at Stanford in which the primary subject of my research was the development of a low temperature thermalized positron source for Bill's electron/positron free fall experiment to determine the sign of the gravitational interaction between matter and antimatter. The positron thermalization project ended up relying on some of the newly discovered aspects of cavity electrodynamics to achieve the enhanced transition rates needed to thermalize the positrons' cyclotron motion in the time available for the process, an experience that further contributed to my familiarity with the principles of the quantum electrodynamics applicable to electrons' lower energy electromagnetic interactions [12].

I had originally applied to Stanford with the intent of learning more about the innovative microwave tubes that had been developed in the Applied Physics Department following the pioneering development of the klystron by the Varian brothers in the 1930s. In making this choice, I was also influenced by the longstanding history of the Bay Area's contributions to the development of radio and microwave technology [13]. But Stanford's microwave tube development effort had been shut down the year before I applied, with the redirection of the efforts in the department towards the further development of the new laser sources. Nonetheless, I was to have the benefit of the informal but immensely valuable advice and recollections of the pioneers of the old microwave tube effort—Marvin Chodorow and Rudolf Kompfner—as well as the seminal figures in the new field of quantum electronics—Felix Bloch, Arthur Schawlow, and Tony Siegman—during all the years I spent at Stanford.

It quickly became apparent in those years that the problem with the classic process of nuclear bremsstrahlung as a candidate mechanism for development of an electron beam-based laser is that the spectral distribution of the emitted photons is much too broad, resulting in very low transition rates per mode of the field. As I learned in Professor Oakes's class at Stanford on quantum field theory, Weizsacker and Williams [14] had demonstrated in their 1930's analysis of the spectrum and angular distribution of bremsstrahlung radiation, the process could—at sufficiently high electron energies—be treated as the Compton scattering of the real photons in a pulse of radiation with the amplitude and temporal duration of the Lorentz-contracted and Lorentz-boosted Coulomb field of the nuclei responsible for scattering. In the limit of ultra-relativistic motion, the amplitudes and vector directions of the rest frame electric and magnetic fields with which the electrons interacted were indistinguishable from the fields of a real traveling plane wave, or rather to a single half-cycle of the fields of such a plane wave.

Given the broad range of Fourier components that such a pulse would contain, the radiation scattered from the pulse via the Compton mechanism would contain an equivalently broad range of final state photon energies with very low transition rates per mode. But the Weizsacker-Williams demonstration of the equivalence of bremsstrahlung and Compton scattering, more precisely to inverse Compton scattering in which the scattered photons are upshifted from the electrons' rest frame by the electrons' relativistic motion, also suggested that the solution to this problem was to consider the bremsstrahlung radiation emitted by electrons moving through a periodic transverse field as assumed in the standard model for Compton scattering. The radiation emitted by an electron moving through such a periodic field would be restricted to a small frequency interval determined by the period of the field through which the electrons moved and the Doppler shifts defining the upshift of the field from the lab frame to the electron rest frame, and of the scattered radiation from the rest frame back to the lab frame as shown schematically in Fig. 1.

In principle, according to Weizsacker and Williams, the periodic acceleration needed to compress the spectrum of the radiation emitted as nuclear bremsstrahlung by a relativistic electron subject to this acceleration could be provided by either a real photon field or a stationary periodic transverse electric or magnetic field, the only difference being the period of the field perceived by the electrons in their rest frames. The rest frame spatial period of a periodic transverse field that was stationary in the lab frame is a factor of $1/\gamma$ shorter than the lab frame period. The rest frame spatial period of a counterpropagating

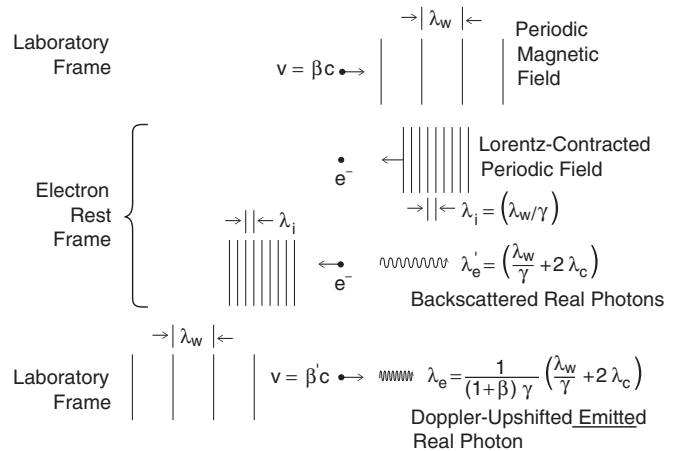


FIG. 1. Wavelength of magnetic bremsstrahlung emitted in a periodic magnetic field. In the electron rest frame, a stationary transverse magnetic field with period λ_w becomes indistinguishable from a traveling plane wave with a period λ_w/γ . Photons backscattered from this Lorentz-contracted field are shifted to a wavelength longer by $2\lambda_c$ by the Compton effect. The backscattered rest frame photons are Doppler upshifted by a factor of $(1 + \beta)\gamma$ to appear in the laboratory frame as real emitted photons with wavelength λ_e .

electromagnetic wave is $1/(1 + \beta)\gamma \sim 1/2\gamma$ shorter than the lab frame wavelength. In both cases, according to Weizsacker and Williams, the transition rates and angular distribution of the emitted bremsstrahlung are determined by the cross section for Compton scattering as evaluated in the electron rest frame and modified to include the factor for the density of states appropriate for scattering into a single mode of the field as determined by the direction and frequency of the scattered photons.

The transition rate per mode in a radiating system incorporating such periodic acceleration could thus be hundreds of times larger than for the radiation emitted in the course of passage through a single period or half-period of the field, with a net rate depending only on the available electron current. This was beginning to sound like a system that might actually be able to lase. But was there any experimental evidence to suggest that the process of stimulated emission would actually be operational for a radiation mechanism physically equivalent to the scattering of individual photons through the process of Compton scattering as opposed to the transitions between the well-defined atomic levels which served as the basis of Maiman’s laser?

To some extent, this concern reflected the unresolved, but longstanding questions regarding the underlying physical nature of the “photon” concept introduced by Albert Einstein. Are photons particles, as would appear to follow from the “ballistic model” of Compton scattering, or waves, as would appear to follow from the spatial and temporal coherence of the light emitted by Maiman’s new laser? And given this uncertainty, was the model of laser operation developed by Schawlow and Townes broad enough to accommodate both aspects of Einstein’s photons?

The 3-degree cosmic black body radiation had been discovered by Arno Penzias and Robert Wilson in 1965 [15], confirming the earlier speculations of Ralph Alpher and Robert Herman [16], and the literature of the day included several analyses of the detailed processes through which the radiation that had been emitted in the Universe’s prior epochs could have come to thermal equilibrium with the rapidly cooling cosmic ionized plasma just before its temperature fell to the level at which stable hydrogen atoms could be formed, effectively decoupling the radiation field from further interactions with the more strongly interacting but no longer available free electrons.

What seemed to be the most relevant conclusion of these analyses appeared in a paper by H. Dreicer at Los Alamos in which he concluded that it was only Einstein’s stimulated emission mechanism that made possible the strength of the coupling between the rapidly cooling cosmic plasma and the radiation field via stimulated Compton scattering needed to ensure the thermal equilibrium of matter and radiation at the time that matter and radiation became decoupled [17].

The Dreicer paper also suggested a name for the hypothesized new lasers: “free electron lasers” based on the characterization of the interactions responsible for the thermalization of the cosmic black body radiation as “free-free” transitions. With the specific intent of distinguishing these new lasers—if and when they were ever developed—from the prior atomic and molecular lasers, it occurred to me that these new lasers should be called “free electron lasers” or FELs.

With this “cosmic” demonstration of the role of Einstein’s stimulated emission mechanism in the enhancement of the transition rates for Compton and inverse-Compton scattering, I was confident that a bremsstrahlung laser was at least theoretically possible provided that the transition rate for stimulated emission exceeded the competing rate for absorption and that a sufficient electron current could be made available to support the operation of the device. What remained to be determined was the relative contributions of stimulated emission and absorption to the amplification available in a stimulated bremsstrahlung laser, and also the means available—if available—to achieve a useful net amplification factor.

The appearance of absorption as an intrinsic aspect of the bremsstrahlung process is best understood in the electron rest frame (Fig. 2). In the case of the emission of a photon in the direction of the electrons’ motion, opposite to the direction of the incoming periodic field perceived by the initially stationary electrons, the “backscattered” photon is shifted to a wavelength longer than the wavelength of the rest frame incident field by twice the Compton wavelength h/mc . This emission process will be enhanced by the presence of other previously emitted photons that are present in the same mode of the field as the newly backscattered photon.

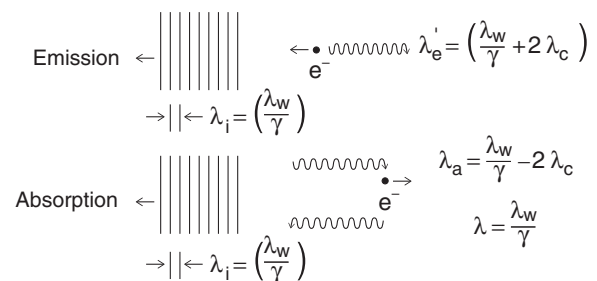


FIG. 2. Differing wavelengths for emission and absorption. Photons moving with the electrons through a periodic transverse magnetic field can also be absorbed if their wavelength λ_a is shorter by $2\lambda_c$ than the period of the Lorentz-contracted wave in the electron rest frame. Scattering of photons with this reduced initial wavelength in the direction of the Lorentz contracted periodic field is strongly enhanced by the high density of photons in their final state leading to a transition rate for absorption of these “blue shifted” photons equal in value to the rate for amplification of the “redshifted” emitted photons of wavelength λ_c .

But it is also possible for the previously emitted photons to scatter from one of the other electrons present in the rest frame thereby to reverse its direction and end up in the same mode of the field as populated by the strong incoming periodic field. The scattering of this photon out of the mode into which it was initially emitted, and back into the mode populated by the intense incoming field, will be strongly stimulated in proportion to the population of the mode occupied by the incoming periodic field. But such an absorption event will occur at a different wavelength from the wavelength for emission, shifted to shorter wavelengths by the same Compton shift $2 h/mc$ for backscattering as for the case of emission, but of the opposite sign.

For FELs designed to operate with short wavelength pumps, for example, the intense optical fields which may be possible in future years using specially designed near-confocal optical storage cavities, the line spectra for amplification and absorption may be at least partially resolvable if the linewidths in the electron rest frame are comparable to $4 * \lambda_c$. But for longer wavelength counterpropagating pump fields, at mm or microwave wavelengths, the small shift between the lineshapes for amplification and absorption is visible only as the appearance of a gain spectrum proportional to the derivative of the lineshape of the radiation attributable to the spontaneous Compton scattering process [18] absent any factors of Planck's constant, a result that I was also subsequently able to establish as the consequence of the purely classical aspects of the interaction.

There was thus a hint in these results, subsequently to be pursued at much greater length, that the lasing mechanism of interest might also be amenable to analysis using the methods of classical electrodynamics in those regimes in which the Compton wavelength was small compared to the rest frame linewidths for emission and absorption as determined by the operating wavelength, electron energy, and interaction times specific to the system of interest.

But it was perhaps of even greater fundamental interest to note that the condition corresponding to level inversion in these e -beam-based light sources would be satisfied whenever such a system was driven with a monoenergetic electron beam.

To obtain the highest possible spontaneous transition rates per mode the physics of the process appeared to favor operation with a long wavelength electromagnetic pump to optimize the photon density in the incident field, and the use of a high energy electron beam to take advantage of the reduced angular divergence of the backscattered radiation at the higher electron energies. But attention also had to be paid to the effects of pump wavelength on the linewidth of the backscattered photons. Since the available gain was limited by the competition between emission and absorption in these systems with the net gain determined by the difference between the lineshapes when shifted by $4 * \lambda_c$, optimization of gain also required the minimization of the

linewidths for these two processes. While it could reasonably be assumed that the line broadening due to the energy spread of the electrons available from the then-available accelerators could be reduced to a fraction of a percent, reduction of the transform limit of the backscattered photons to the same levels would require extended interaction times attainable only by extending the physical length of the interaction region.

In a parallel analysis of the amplification available due to the stimulated scattering of the real photons accumulated in a high Q superconducting cavity by a relativistic electron beam, Professor Richard Pantell and his associates G. Soncini and Hal Puthoff had established that the gain available due to this stimulated inverse-Compton scattering process was too low for operation at optical wavelengths given the peak electron currents then available from the linear accelerators of the day [19]. But much higher scattering rates and gain could be achieved through the use of a strong, spatially periodic transverse magnetic field along the lines of the stimulated bremsstrahlung model which I had worked to develop.

And so, when following extended discussions with fellow grad student Mark Levinson I computed the net amplification factor available for a stimulated bremsstrahlung laser using a kilogauss-level alternating magnetic field and the ampere-level peak electron currents available from the pulsed linear accelerators of the day, I obtained a result indicating that useful amplification should be available at wavelengths extending through the infrared. And if larger currents could be obtained at some point in the future, there was at least the possibility of achieving lasing at x-ray wavelengths, an objective that then as now seemed well beyond the capabilities of the then more well-established "conventional" atomic and molecular lasers [20].

As a further advantage of this new lasing mechanism, application of the Kramers-Kronig relations to the imaginary part of the gain medium's index of refraction (proportional to the gain per unit length) revealed that the real part of the gain medium's index would be greater than one suggesting the possibility of optical mode confinement and even higher gains than estimated the basis of the Gaussian resonator mode model.

A paper describing these results was submitted for publication in the *Journal of Applied Physics* on 20 February 1970 and appeared in the 21 August 1970 edition of that publication [21].

C. Significance of the first amplifier and oscillator experiments

At this point, it appeared that there was the basis for an experiment to test the predictions of the model which I had developed and investigate the operation of the stimulated bremsstrahlung mechanism in the strong signal regime which my small signal analysis did not address. If it could be established by these means that this new laser

mechanism was capable both of completely tunable operation and operation at high power, it seemed evident that there would be the resources needed to develop the higher peak current electron sources needed for the practical operation of these devices at optical and perhaps also at UV and x-ray wavelengths.

Subsequent events were to conspire to make possible precisely such a proof of principle experiment. Professors William Fairbank and Alan Schwettman had spent much of the past decade developing the technology needed for the world's first superconducting linear accelerator as the follow-on to the normal, *s*-band accelerators that had been developed by Hansen and Ginzton in the years following World War II in the High Energy Physics Laboratory on the Stanford campus. Most of HEPL's staff had migrated to Project M (for Monster) under the direction of Professor Panofsky to extend Hansen's pioneering work through the construction of a two-mile-long *s*-band linac with upgraded klystrons at a site northwest of the campus along Sand Hill Road, now known as the Stanford Linear Accelerator Laboratory or, simply, SLAC. But a small core of the engineers and technicians who had worked with Hansen and Ginzton stayed on at HEPL to develop the next generation of microwave accelerators, intended to exploit the higher gradients and reduced power consumption of superconducting niobium cavities cooled below the lambda point of liquid helium.

The accelerator structures developed by Professors Fairbank and Schwettman, their students and postdocs, though still subject to the multipactoring mechanism that had limited the gradient of the prior normal *s*-band cavities, nonetheless had proven to be capable of providing the highly stable, low energy spread electron beams [22] needed for an initial test of the new FEL concept. And by a stroke of good fortune, the Air Force Office of Scientific Research (AFOSR), then directed by Stanford engineering graduate Jack Gregory, was also looking for new laser candidates for its high power directed energy research program. Given the possibilities for a decisive test of the model for FEL operation which I had developed and with the support of Science Officer George Knausenberger, the AFOSR enthusiastically agreed to sponsor both an initial experimental test of the "small signal" theoretical model which I had developed and an experimental investigation of operation at the higher signal levels that would be encountered in practical high power laser systems. The AFOSR also agreed to fund a follow-on oscillator experiment if the amplifier experiment was successful.

With Air Force funding available in the fall of 1972, we assembled the team needed to develop the hardware for the amplifier experiment. I was responsible for the laser physics, electronic instrumentation, and cryogenic systems for the experiment; Luis Elias for the optics, optical instrumentation, and conventional laser sources; and Todd Smith assumed responsibility for the accelerator

systems and electron optics. Luis and I joined forces to design and wind the 5.2-meter-long 3.2 cm period helical superconducting magnet for the project including the development of a specialized precision winding lathe constructed for the project with the assistance of machinist Bill Richmond (Fig. 3).

A schematic of the initial "gain experiment" is shown in Fig. 4. A high peak power transversely excited CO₂ laser master oscillator was operated in a single longitudinal mode using a bleachable intracavity sulfur hexafluoride cell to suppress the unwanted longitudinal modes. The output of the laser was focused to a 3.3 mm waist at the entrance to the 10.2 mm inside diameter evacuated beam tube for the superconducting helix to selectively excite the beam tube's EH₁₁ waveguide mode. The incident on-axis power density could be varied from 100 to 1.4×10^5 watts/cm² using a set of optical attenuators. The polarization of the injected laser beam could be varied from right circular to linear to left circular through the use of a set of 10 μm birefringent quarter wave plates.

Signal detection was accomplished by measuring the 1.3 GHz modulation induced on the 10.6 μm CO₂ laser "carrier" by the gain or attenuation attributable to the 1.3 GHz repetition rate of the bunches in the cw electron beam provided by the new superconducting linac. With the CO₂ laser "master oscillator" operating at a fixed wavelength, tuning was accomplished by sweeping the energy of the electron beam in a small range around 24 MeV. Once gain was observed, the focusing and steering of the *e*-beam was adjusted to maximize the amplitude of the gain, with fluorescent screens installed at the input and output to the dewar in which the undulator was installed to document the dimensions and angular divergence of the input and output *e*-beams.

The instantaneous peak gain measured in the experiment reached 7% at an on-axis magnetic field of 2.4 kilogauss and a peak electron current of 70 mA. The measured gain was actually a bit higher than the gain predicted for these operating conditions, 5%. The measured gain varied

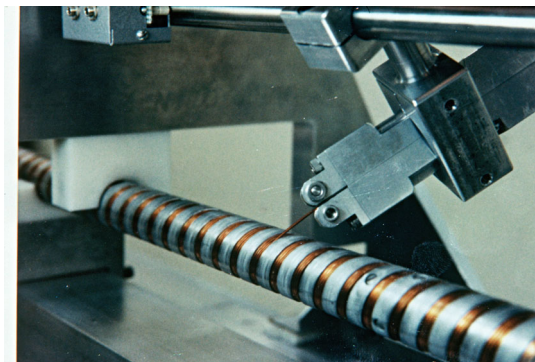


FIG. 3. Photograph of the five meter precision lathe developed to wind the superconducting helical magnet used in the first FEL amplifier and oscillator experiments.

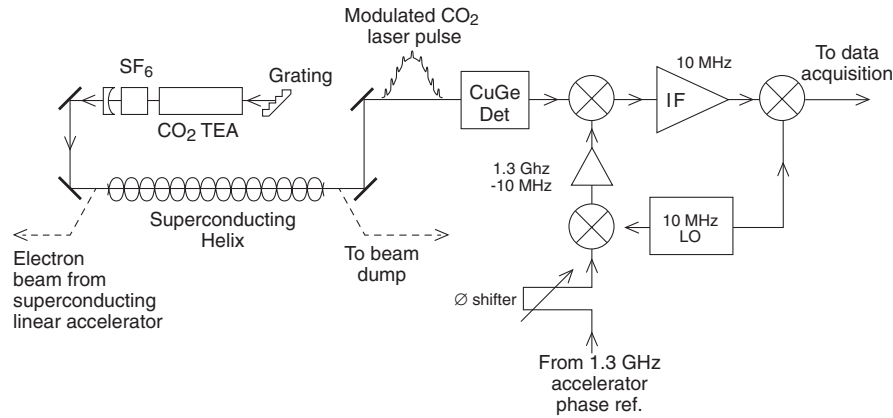


FIG. 4. Schematic drawing showing the setup of the helical undulator magnet, single longitudinal mode CO₂ laser “master oscillator,” high speed helium-cooled Cu:Ge detector, and 1.3 GHz phase sensitive detection system used to measure the modulation induced on the 10.6 μm pulses from the master oscillator by their interaction with the 1.3 GHz cw bunched beam from HEPL’s pioneering superconducting accelerator.

linearly in the electron current and was independent of the incident CO₂ laser power up to the largest values that could be achieved with the 10 μm TEA laser. At the highest laser powers used in the experiment, the electrons were estimated to have conveyed an average power of 4.2 kW to the power of the incident 10 μm laser beam.

The measured dependence of the system’s gain and attenuation on the kinetic energy of the incident electrons was an almost exact match to the derivative of the measured lineshape for the spontaneous radiation emitted by the electrons with the CO₂ laser turned off (Fig. 5)—an almost perfect verification of the relationship between amplification, attenuation, and spontaneous radiation predicted by my analysis.

The results of the amplifier experiment, which nicely verified the results of my small signal theoretical model and dramatically extended the applicability of those results into the strong signal regime, were published in the 29 March 1976 edition of *Physical Review Letters* [23]. The comprehensive quantitative success of the amplifier experiment also persuaded the Air Force to provide the continued funding needed to modify the system for operation as an oscillator.

Given the need to ensure the precise alignment of the resonator mirrors for the oscillator experiment and the small but finite losses that were incurred in the amplifier experiment by the conversion of the incident Gaussian beam from the CO₂ laser to the EH₁₁ waveguide mode in the helical superconducting magnet’s evacuated beam tube, we decided to attempt the oscillator experiment at a wavelength of 3.4 μm in the vicinity of the strong HeNe IR lasing line at 3.39 μm. By operating at this wavelength, we could propagate the desired fundamental Gaussian resonator mode through the magnet’s beam pipe with minimal diffraction losses, and use a custom intracavity HeNe plasma tube to align the resonator mirrors prior to operation of the system with an electron beam. Although the available gain at 3.4 μm would be lower than at the

10.6 μm used in the amplifier experiment, it was felt that the overall advantages of operation at this shorter wavelength outweighed the predicted loss of gain.

The upgrades of the system needed to achieve lasing at 3.4 μm also motivated an expansion of the project’s staff to include graduate student David Deacon and electrical engineer Gerry Ramian. David would be the first of the several truly extraordinary graduate students to join the effort during our years at Stanford, while Gerry (whom I had known at the old CalTech Synchrotron Laboratory in the years that I had worked there) was to devise the ingenious solutions needed to upgrade the superconducting accelerator’s injector as required to achieve the higher peak

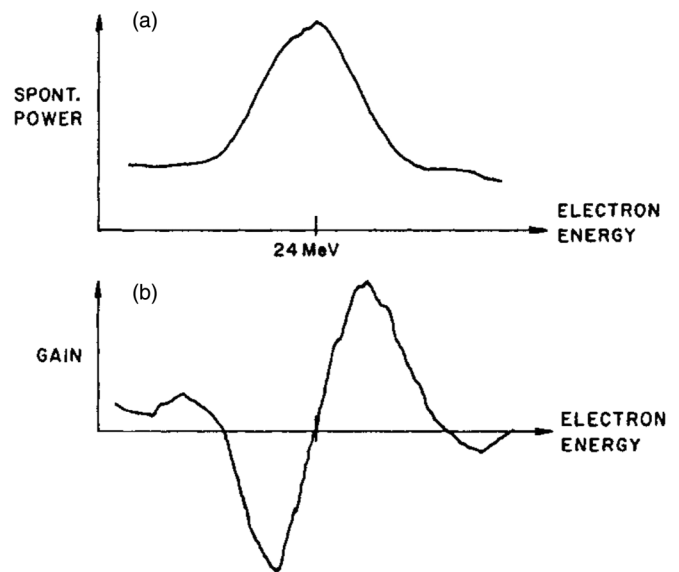


FIG. 5. Comparison of the measured spectrum of the 10.6 μm spontaneous radiation emitted by the 24 MeV electrons in the amplifier experiment (a) with the gain measured using the setup of Fig. 4(b).

currents needed for the oscillator experiment. Gerry also later served as the chief engineer for Luis Elias's highly successful and pioneering energy-recovered dc electrostatic accelerator THz FEL system at the University of California at Santa Barbara [24].

The major planned effort carried out in advance of the oscillator experiments was the development of a high peak current gridded electron gun that could provide the higher peak currents needed for laser operation, but at a reduced repetition rate consistent with the limited rf power available to accelerate e -beams in the superconducting linac. Excellent performance was obtained using a gridded dispenser cathode provided by Eimac and driven by a chain of GHz bandwidth grounded grid high transconductance microwave triode amplifiers [25] in a design that was subsequently adopted by SLAC. With the superconducting linac now capable of providing 4-psec-long electron bunches with estimated peak currents of 2.6 amps at a repetition rate of 11.8 MHz, the calculated gain available at $3\ \mu\text{m}$ approached 100% (3 dB)—more than enough to cross threshold given the estimated 3.5% resonator losses.

The major unplanned effort that had to be pursued in preparation for the oscillator experiment was the fabrication of a second superconducting helical undulator on short notice and with limited funds, the first undulator having been damaged by an unanticipated surge in the voltage provided by its high current power supply. But while a second undulator was successfully wound and prepared for operation in the short time available before the scheduled start of the long-planned oscillator experiments, the multi-strand NbTi superconducting wire provided by the then-bankrupt supplier had been rolled to its specified cross section after it had been coated leading to the failure of the wire's insulation along the edges of its rectangular cross section. Because of the ohmic currents flowing between strands (the "shorted strands" of concern are the wires shown in Fig. 3 in the process of being wound onto the helical coil form for the assembled magnet), the completed windings of the new superconducting helical magnet displaying an L/R charging time at helium temperatures of the order of a half hour, drastically limiting the rate at which the magnet could be ramped up or down during operation.

But even with these setbacks, the system was ready for operation in December 1976, and after some optimization of the steering and focusing of the electron beam lased above threshold in January 1977, at an average power output of 360 mW and an estimated peak power output of 7 kilowatts, nearly twice the power extracted from the electron beam in the earlier amplifier experiment. The circulating peak intracavity optical power at saturation in the experiment reached 500 kW.

The results of the oscillator experiment, though less significant as a detailed quantitative test of the theory for these devices than the results of the prior amplifier



FIG 6. Team photograph taken the evening the oscillator experiment first crossed threshold. From left to right: Todd Smith, John Madey, Luis Elias, and David Deacon.

experiment, proved significantly more persuasive as a compelling demonstration that new lasers of this type could actually cross threshold and lase at significant average and peak power levels. The results of the experiment were reported in the 18 April 1977 edition of *Physical Review Letters* [26]. A photo of the team responsible for these results is shown in Fig. 6.

At the conclusion of these experiments I also occasionally considered that we had succeeded in these experiments in demonstrating, for the first time in 13.6 billion years and in the first well controlled laboratory experiments, the operation of the mechanism—stimulated Compton scattering—that had played such a prominent role in the evolution of the Universe as we presently understand it.

D. The search for workable approaches to practical operation

As satisfying as it was to have completed two key proof of principal experiments, it was also clear that the development of useful devices based on this new gain mechanism would require both further theoretical and technical efforts. Although the experiments had established the capability of the new mechanism to operate at respectable signal levels, some significant questions remained as to the physical basis of these results. Higher electron currents and lower e -beam emittances would also clearly be required for operation of shorter wavelength and more compact systems. And finally, assuming the availability of higher electron currents and higher gains, an improved understanding would be needed of operation under conditions in which the amplification attributable to the new mechanism would substantially increase the signal level during the course of the electrons' passage through the interaction region. The bulk of the efforts we have pursued in connection with these issues took place in the 10 odd years between the completion of the oscillator experiments and our move to the new laboratory at Duke in 1989, with some further efforts along these lines continuing through the present date.

With respect to the technical aspects of these efforts, the most important developments to emerge from this period were the invention, demonstration, and development of the “microwave electron gun”—which provided the means needed to increase the electron current by an additional order of magnitude while achieving even lower e -beam emittances than had been available from the Stanford superconducting accelerator—and the concept of energy-recovered FELs using the low energy beams from electrostatic accelerators.

By placing either a thermionic or photoemissive cathode within a high field pulsed TM_{010} microwave cavity, the longitudinal electric field gradients available to accelerate the emitted electrons would exceed by more than an order of magnitude the accelerating gradient in the upgraded but still conventional dc electron gun used in the superconducting accelerator experiments. The increased field gradient in the microwave gun cavities we built to test this concept increased the current density available from the gun’s lanthanum hexaboride cathode in both the thermionic and photoemissive mode while reducing the aberrations attributable to space charge by decreasing the time interval that the emitted electrons moved at nonrelativistic velocities.

And while the energy spectrum of the electrons emerging from such a cavity now included a much larger range of energies due to the accelerating gradient’s oscillations in time, Glen Westenskow’s analysis of the electrons’ time-dependent trajectories revealed that this time-varying accelerating field could also be used to prebunch the electrons in advance of their acceleration to higher energies by adjusting the length of the gun cavity [27]. The electrons not included in this bunch could be removed from the beam by including a momentum filter between the gun cavity and the accelerator that brought them to their final energy. By exploiting the dispersion intrinsic to the momentum filter, the prebunched beam emerging from the gun cavity could be further compressed to obtain picosecond bunch lengths with peak currents in excess of 30 amps in the prototype pulsed s -band gun we developed during this period [28]. These were—and continue to be—remarkable specifications, particularly for a system of the simplicity, compactness, and low cost of the prototype Stanford thermionic gun.

A photograph of the microwave thermionic gun used in our present research at the University of Hawai’i appears in Fig. 7.

The current available from this new gun was an order of magnitude higher than used in the 1977 oscillator experiment, and made possible the development of the relatively compact normal linac-based MkIII FEL oscillator [29]. Developed with the able assistance of engineer Marcel Marc, the new MkIII FEL was tunable throughout the infrared with a small signal gain of the order of 3 dB, adequate to support further experimental research on the

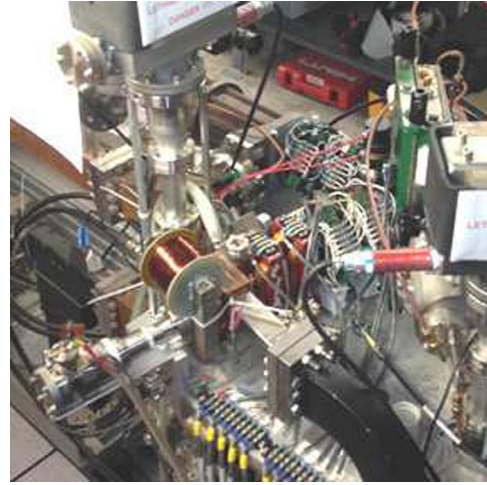


FIG. 7. Photograph showing the updated version of the microwave thermionic electron gun now operated as the injector for the 40 MeV linac presently used for our FEL and electron beam research at the University of Hawai’i.

physics of the FEL interaction as well as serving as a laboratory light source to explore the research applications of these new devices.

A duplicate of the microwave gun we developed in those years was also included as the injector for the clone of the MkIII FEL that my company Sierra Laser Systems developed for Vanderbilt University in the early 1990s [30] in an effort led by Marcel Marc. Vanderbilt’s FEL was the first to be used in a human surgical procedure and held the average power record for FELs (at 10 watts) for more than 5 years until superseded by the higher average current FELs developed at the Jefferson Laboratory [31]. An upgraded version of the Stanford microwave gun developed by Stanford graduate student Michael Borland was subsequently used at Argonne as the electron source for the first demonstration of high gain SASE FEL operation at ultraviolet wavelengths [32]. And yet more specialized versions of this gun, now specifically designed for operation in the photoemissive mode, have served as the source of the low emittance, kiloamp peak current electron bunches used in the new SASE x-ray FELs [33].

In our own continuing research efforts, an extended pulse length version of the original Stanford microwave gun is being developed by Jeremy Kowalczyk for use in the compact, high brightness cavity-enhanced inverse-Compton x-ray and gamma ray light sources we are developing as workable and affordable laboratory light sources at these wavelengths [34].

The concept of highly efficient energy recovered dc electrostatic accelerator systems using the depressed collectors developed by Litton Industries for use in their high efficiency microwave power tubes was also developed by Luis Elias during these years [35]. The concept proved the key to the attainment of the high average currents needed for the long pulse and cw operation of these systems as

demonstrated in the groundbreaking energy recovered Thz FEL that Luis subsequently constructed at UCSB [36]. Versions of Luis's energy recovered electrostatic accelerator system have also served very effectively in other applications requiring high average currents at MeV energies including the electron cooling system incorporated in the antiproton accumulator ring at Fermilab [37].

A collaborative experimental effort to explore the opportunities for the development of storage ring-based FEL oscillators was also conducted following the first linac-based amplifier and oscillator experiments. In cooperation with Yves Farge's LURE Laboratory at the University of Paris at Orsay in a series of experiments led by Dave Deacon and Kem Robinson from Stanford and Claude Bazin, Michel Bergher, Michel Billardon, Pascal Elleause, Jean-Michel Ortega, Ann-Marie Couprie, and Yves Petroff from Orsay, the collaboration led not only to the demonstration of the first visible-wavelength FEL but also to the observation and explanation of several unanticipated aspects of storage ring FEL operation including the suppression of the microwave instability responsible for anomalous bunch lengthening in low energy storage rings and the giant pulse mode of operation of these interesting systems [38].

The development of first-principles extensions of the descriptions of these devices adequate to identify the key physical features of operation in the strong signal and high gain regimes was also of high interest, both with respect to the results of those efforts and the light that they shed on the challenges to the development of models of device operation which were consistent with the practical or theoretical limits to the approaches available to us.

The analytic model that I had developed was perfectly acceptable for use at signal levels in which the number of photons emitted as stimulated radiation by each electron during its interaction with the signal and the periodic magnetic field through which it passed was small compared to one. But in the strong signal regime, each electron could emit many thousands of stimulated photons during each pass, implying the need to extend the small signal analysis to include very large numbers of sequential stimulated emission and absorption events, an unprecedented challenge to the application of the perturbation theory that serves as the basis of contemporary QED.

Physical processes that take place in the limit of large quantum numbers as would typically be the case in the limit of strong signals have also traditionally been considered to be describable in terms of classical electrodynamics, a possibility further highlighted by the cancellation of the factors of Planck's constant that occurs when taking the difference between the transition rates for stimulated emission and absorption for FELs designed to produce visible and longer wavelength radiation with highly relativistic electron beams to yield a net transition rate independent of Planck's constant.

But the cancellation of Planck's constant intrinsic to the calculation of the net transition rate for stimulated emission was limited to the longer radiated wavelengths and the use of higher energy electron beams, and hence not always an enabling factor in applying classical techniques to the analysis of these systems. And even when Planck's constant did not explicitly appear in the final expression for the net transition rate for stimulated emission in these systems, the finite value of the commutator of the operators for the electrons' spatial coordinates and momentum, and also the commutator for the amplitude and phase of the electromagnetic fields with which the electrons interacted, set independent limits to the levels of signal power that could be addressed using classical methods.

While it might seem that the highly relativistic electrons typically needed to support the operation of free electron lasers more or less automatically fulfilled the test for operation in the limit of large quantum numbers, the microscopic scale of the evolution of the electrons' position and momentum associated with the transition from the classically random distribution of the electrons in initial phase to their increasingly localized distributions as saturation at optical wavelength is approached yield products of their changes in position and momentum in the electron rest frame which are inevitably small compared to Planck's constant at startup, and only grow to exceed Planck's constant as saturation is approached for operation at optical, UV, and x-ray wavelengths.

It follows that the evolution of the electrons' position and momenta during startup are not describable in terms of a classical model of their interactions with the signal wave and transverse periodic dc field through which they move in the lab frame, even in the strong signal regime.

Independently, the nonzero commutator for the amplitude and phase of the signal field limit the precision with which the amplitude and phase of the field can be specified at low signal levels. The limit of our ability to specify these key variables is most often specified in terms of the resultant zero-point fluctuations of the field. To justify the use of a classical description of the signal field, the power density of the field, defined in terms of the square of the amplitudes of the field's electric and magnetic fields times the velocity of light, has to be substantially larger than the power density attributable to the quantum fluctuations. We have shown that this sets a lower limit to the power densities that can be treated classically in these systems equal to $\sim \hbar c^3 / [(1 + \beta)\lambda_w \lambda_f^3]$, a rapidly increasing function of the signal frequency that reaches hundreds of watts per cm^2 at ultraviolet wavelengths [39].

So if a classical model of FEL operation is to be developed, it can only be held to be physically realistic when the product of the electrons change in position and change in momentum during the interaction is large compared to Planck's constant, and when the power density

of the signal field is large compared with the power density attributable to the field's quantum fluctuations.

And even when these limits are satisfied, the failure of classical electrodynamics to identify the forces responsible for conservation of energy in the case of the coherent emission into "free space" responsible for operation of SASE FELs [40], and to identify the effects of the electron beam's interactions with the field's quantum fluctuations as saturation is approached, limit the confidence with which classical methods can be applied to the analysis [41] of these systems as discussed further in the next section. But when no exact fully quantized model was available to treat the dynamics of these new FELs at the high signal levels, longer wavelengths, and high electron energies needed to surmount the limits set by Heisenberg's uncertainty principle, the classical approach was all that we had to work with.

Given this critique of the limits of applicability of the classical limits to electrodynamics, our first effort at extending the range of applicability of our quantum analysis of FEL operation was the development of an approximate approach to the evaluation of the Feynman path integrals for electrons of specified initial energy moving through a periodic transverse field of specified period and amplitude and a copropagating signal field of specified frequency and amplitude included as part of Dave Deacon's Ph.D. dissertation [42]. The amplitude for an electron to emit N final state photons in this approach could be identified as the sum over all K of the amplitudes for emission of $(N + K)$ photons and the absorption of (K) photons. Given the assumption of an undefined and unknowable signal field phase, and the ordered, sequential occurrence of the hypothesized events of emission and absorption, the integral for each of the $\{N + K\}$ interactions of an electron propagating through the interaction region could be integrated numerically, yielding the probability for the stimulated emission of the specified N photons and a series of contour plots showing the dependence of this probability on the specified initial conditions.

But while we were pleased that this approach indicated no hint of a mechanism that might result in the early saturation of the stimulated emission gain mechanism, the obvious restriction of the method to the signal powers at which it could be assumed that the local phase of the field was unknowable left us searching for a model that could be used with confidence at the higher signal levels in which both the phase and amplitude of the local signal field could be assumed to be well defined.

This objective represented the starting point of our and many others' attempts to develop the range of classical models of the interaction that could provide an indication of the nature of the interaction between the electrons and signal field that was operational at high signal levels. Given the diversity and world-wide extent of these efforts I will not attempt to present a comprehensive summary of their

objectives or results, focusing instead on the specific individual efforts that were pursued at Stanford in those years. And because of the diversity of those efforts, I will simply list those efforts by topic without attempting to provide a fuller account of their methods or results.

Ordering these efforts in approximate chronological order, they included the following:

1. My development of the coupled equations of motion for the energy and phase (relative to the phase of the signal field) of the electrons moving through the interaction region in a low gain FEL, including the demonstration that a derivative relationship would exist for the energy extracted from the electrons in the strong signal limit and the radiation emitted by the electrons at zero signal field amplitude matching the relationship that I had observed between these quantities in the earlier quantum analysis [43].

Indicative of the distinctions between my earlier small signal quantum analysis and this new strong signal classical analysis, the radiation emitted by the electrons in the new classical model occurred as a consequence of the work done by the electrons on the field as measured by the time integral of $\mathbf{j} * \mathbf{E}$, not the integral of the power radiated as a consequence of the electron's acceleration according to Larmor's theorem as in the classical models for bremsstrahlung and Compton scattering.

2. The theoretical observation that this derivative relationship could be exploited to increase the gain available from an undulator magnet of given length by modifying the positions and amplitudes of the undulator's individual magnet poles [44], a result subsequently reduced to practice in the "optical klystrons" designed and operated by Nikolai Vinokurov and his team at the Budker Institute in Novosibirsk [45].

3. The further characterization of the classical aspects of the strong signal FEL interaction by Bill Colson who reduced the coupled equations for electron energy and phase in the strong signal, low gain limit to a single second order equation identical in form to the equation for a simple gravitational pendulum [46].

4. The further development by Bill Colson and Sally Ride of a self-consistent classical model for FEL operation in the strong signal regime based on the integration of the coupled Lorentz force and one-dimensional inhomogeneous wave equations, making possible for the first time the numerical modeling of the coupled evolution of the electrons distributions in phase space and the time-dependent amplitude and phase of the copropagating signal field [47].

5. Dave Deacon's use of the pendulum model to explore the prospects for development of an isochronous storage ring FEL in which the phase of the circulating electrons could be preserved from pass to pass thereby suppressing the energy spread attributable to FEL operation that was

observed to limit the average power of nonisochronous storage ring FELs [42].

6. Steve Benson's exploration of the effects of introduction of shot noise and quantum fluctuations in these classical models [48].

7. The modification and application by John LaSala [49] of the 3D FEL model developed by Ted Scharleman at Lawrence Livermore National Laboratory to model the operation of high gain FEL oscillators with emphasis on the effects of the "guiding" observed to occur at high gain in these systems and the resultant need to modify the focal lengths of the resonator mirrors for high gain FELs to optimize their performance, results that were subsequently explored experimentally using the new MkIII [50] and high gain Rocketdyne undulators as related at further length below [51].

8. The development of a 3D analytic model for high gain FELs by Ming Xie as part of his dissertation research in which Ming identified for the first time the eigenmodes of the strongly interacting electron beam and copropagating signal field in these systems [52] anticipating the means and capabilities of the SASE FELs to be developed in future years.

9. The detailed analytic and numerical analysis by Eric Szarmes of FEL oscillators driven by the periodically pulsed e -beams characteristic of rf linac e -beam sources and including either a Michelson or Fox-Smith intracavity interferometer [53], leading to the demonstration that the prior mode of operation of these devices, in which each optical pulse circulating in the resonators of these devices had a statistically independent phase could be altered to achieve near-perfect phase coherence between the circulating pulses [54]. This discovery was to dramatically enhance the phase coherence and spectral brightness of these sources, making possible extensive new applications in both spectroscopy and high field QED.

10. Theory for operation of "gas loaded" FELs, using a low pressure hydrogen gas in the interaction region to extend the tuning range of the prior "vacuum" FELs, carried out by Professor Pantell, his associates, and students [55]

The availability of the new high performance microwave electron gun also made possible a number of key experimental investigations of the effects which we were attempting to study by analytic or numerical means, including the following:

1. The use of the new MkIII FEL to investigate the transition of FEL operation—particularly the appearance of the optical guiding phenomenon predicted by theory—from the low gain regime at low electron currents to the high gain regime at the largest currents available from the new gun in an effort pursued by John LaSala and Dave Deacon [50],

2. The use of the new gun in the photoemissive mode to investigate the underlying physics of an integrated master oscillator-power amplifier system using the MkIII

undulator and resonator as the master oscillator and the new high gain Rocketdyne undulator as the power amplifier [56]. The experiment was the first to demonstrate operation of an integrated FEL system pumped by a microwave electron gun with a photoemissive cathode [57], and the first to test the theories for optimization of the match from a FEL master oscillator to a high gain power amplifier [58]. Participants in the project included Steve Benson, Anup Bhowmik, Mark Curtin, Wayne McMullin, Bruce Richman, and Louis Vintro,

3. An experimental test of the gas loaded FEL concept using the MkIII FEL undulator modified to include a thin boron nitride window in the input beamline to permit pressurization of the interaction region carried out by Professor Pantell, Alan Fisher, Joe Feinstein, A. H. Ho, M. Ozcan, H. D. Dulman, and Mike Reid [58].

This period, extending approximately through the 10-odd years from 1977 through 1989 was characterized by growing worldwide interest in FELs and their prospects for operation at high power and at ultraviolet and x-ray wavelengths. The projects pursued and names of the pioneers who pursued these efforts are simply too numerous to attempt to identify in this paper. But the fruits of these efforts were to be realized in the years to come in the development of the High power FEL facility at Jefferson Laboratory exploiting the high cw currents available from the Jefferson Laboratory's superconducting linac [59], the very productive High Intensity Gamma Ray Source (HIGS) at Duke developed through the collaboration of Duke and the Budker Institute [60], Argonne's key demonstration of an ultraviolet SASE FEL using an upgraded version of the Stanford microwave thermionic gun [61], the development of a highly capable X-ray SASE FEL at SLAC using an optimized high peak current microwave gun in the photoemissive mode with key contributions from Argonne, Brookhaven, and UCLA [62], and the very impressive new SASE FEL projects at the Elettra Laboratory in Trieste [63], Spring 8 in Japan [64] and at DESY in Germany [65].

E. Are there any further transformational developments still in prospect?

As evident from the brief account above of the last three decades' developments in FEL science and technology, responsibility for the development of these large new systems has now largely shifted to the national laboratories with the capabilities needed to design, integrate, and commission these elaborate facilities in the U.S., Europe, and Asia. When I consider the possibilities for further transformational developments similar to those that followed from the introduction of the stimulated bremsstrahlung and microwave electron gun concepts, I am therefore led to consider those aspects of the physics and technology of the field whose roots lie deeper in the underlying physics and technology, and could benefit from the imagination and efforts of the inspired individuals and smaller university

groups that most frequently play a role in such basic new developments.

Three such subject areas now appear ripe for investigation:

1. *The inability of classical macroscopic electrodynamics to fully describe the key radiation mechanism of coherent emission.*—It has long been recognized that the present formulation of the macroscopic classical theory of electrodynamics does not provide an explanation or expression for the forces needed to conserve energy for the mechanism of coherent radiation emitted into free space [40]. Pardis Niknejadi and co-authors have suggested that the problem may lie with the longstanding but erroneous application of Sommerfeld's theorem [66] to rule out the inclusion of the advanced solutions to the inhomogeneous wave equation, also pointing out that the 1945 analysis of Wheeler and Feynman [67] that specifically includes these advanced terms correctly accounts for energy conservation in the case of coherent emission.

From the standpoint of cosmology, the demonstration by Wheeler and Feynman that the nonlocal component of the radiation reaction force required to satisfy energy conservation in the case of coherent radiation can be seen as a quantitative “reduction to practice” of Mach's principle according to which the local properties of matter and radiation are determined at least in part by their interactions with all of the rest of the matter and radiation in the Universe. The experimental verification of this relationship would constitute a fundamental development in physics on a par with the recent experimental demonstrations of quantum entanglement.

Towards this end, Ms. Niknejadi and her colleagues have proposed a direct experimental test of the Wheeler Feynman model [68]. If verified, the model would have a profound impact both on our understanding of the role of advanced forces in the Universe in which we live and on the means by which the performance of the present generation of SASE x-ray FELs might be optimized, for example, by including an energy selective Mossbauer absorber as part of the targets in which the radiation emitted from these sources is absorbed.

2. *The roles of the acceleration-dependent coherent radiation and velocity-dependent amplification of the harmonic radiation emitted by FELs approaching saturation.*—In a 1998 experiment by Teng Chen (Fig. 8), the fluctuations in intensity of the coherent spontaneous harmonic radiation emitted by FEL oscillators as they approach saturation were found experimentally and theoretically to be smaller than that expected on the basis of shot noise and the Poisson statistics normally caused by the zero point fluctuations in the field [69]. At present, it appears that the suppression of the fluctuations normally caused by the zero point fluctuations may be due to the velocity-dependent amplification of the component of those fluctuations in quadrature with the

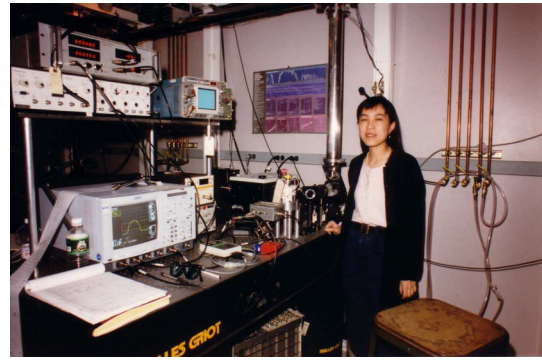


FIG. 8. Photograph showing Teng Chen and the instrumentation she used to measure the fluctuations in the 9th coherent spontaneous harmonic of the MkIII FEL as it approached saturation in 1998.

acceleration-dependent radiation emitted by those tightly bunched electrons [70].

If this mechanism can be confirmed by further research, FELs may also prove to have a role as a source of the intense, amplitude squeezed light needed for research in quantum optics, particularly, in the high repetition-rate generation of the pure single photon states needed for all-optical quantum computing.

3. *The development of optical storage cavities capable of integrating the high peak power phase coherent pulsed output of phase-locked FEL oscillators.*—Eric Szarmes's demonstration of the ability to generate microsecond pulse trains of tunable, phase coherent picosecond pulses with GHz repetition rates and energies in excess of 10 millijoules has raised the possibility of integrating the output of these powerful phase coherent sources by coherently stacking the pulses in a near-concentric optical storage cavity in which the pulses are stably brought to a focus with a μm -scale waist within the cavity [71]. While higher peak field strengths are now routinely generated at low repetition rates by pulse-compressed Ti-sapphire lasers, the circulating optical pulses in such an optical cavity would persist for at least the $1/e$ decay time of the storage cavity, increasing the duty cycle at which these pulses would be present by orders of magnitude.

The development and integration of such storage cavities could provide a significant new experimental capability for research in high field QED and high order multiphoton spectroscopy, as well as dramatically enhancing the capabilities of the laboratory-scale cavity-enhanced inverse-Compton x-ray and Gamma ray sources currently under development at the University of Hawai'i [72].

While the development of such an optical storage cavity would constitute a transformational *technical* advance, not a conceptual or theoretical advance as in the case of the first two opportunities described above, there are times in which technical breakthroughs of this kind enable their respective fields of research to evolve into important new directions as

with the development of the microwave electron gun at Stanford in the early 1980s.

III. CONCLUSIONS

The emergence of the stimulated bremsstrahlung concept in the 1960s and its experimental validation in the first amplifier and oscillator experiments in the mid-1970s has led both to important new technical capabilities and to a significantly improved understanding of both the quantum and classical theoretical models available to analyze the operation of these devices thereby extending the physics and technology of the electron devices developed in the first half of the 20th century to operation at wavelengths and power levels that were undreamed of in the heyday of vacuum tube technology in 1960. Further attention to the opportunities which have emerged as a consequence of the results secured to date hold the promise of at least several further transformational developments to come including potentially radical improvements in our understanding of the underlying physics.

ACKNOWLEDGMENTS

Special thanks are due to my wife Susan, my brother Julius, and my parents Julius and Helen Madey for their decades-long encouragement and support, without which none of this would have been possible.

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