Narrow-band impedance of a round metallic pipe with a low conductive thin layer

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The new traveling wave structure with a single synchronous mode resonantly excited by the relativistic charge is presented. The structure is composed of a metallic tube with an internally coated low conductive thin layer. It is shown that the impedance of the internally coated metallic tube has a narrow-band single resonance at a high frequency. The analytical presentation of the narrow-band impedance, the wake function, and the frequency of the synchronous mode are obtained. The analytical solutions are compared with exact numerical simulations using the field matching technique.

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I. INTRODUCTION

The study of the new type of the slowly traveling wave structures is driven by the development of both novel coherent radiation sources [1–4] and advanced acceleration concepts [5–9]. The disc- or dielectric-loaded structures are the most known slowly traveling wave structures that are characterized by the infinite number of excited modes [10–13].

The structure with a single slow traveling wave has special interest, as the relativistic charge will excite the synchronous mode only. The radiated energy is then effectively stored in a single mode, and for bunch length $\sigma_z \leq \lambda$ (λ synchronous mode wavelength) the structure will serve as a source for a coherent monochromatic radiation.

The single slow wave structure is an attractive candidate also for particle acceleration due to the absence of high order propagating modes. In addition, such a structure may effectively support the high transformer ratio single-mode-based wake field accelerator schemes with linear ramped single- [14] and multibunch driver beams [15–17].

In this paper, the new traveling wave structure with single synchronous mode resonantly excited by the relativistic charge is presented. The structure is composed of a metallic tube with an internally coated low conductive thin layer.

We show that the longitudinal impedance of the internally coated metallic tube (ICMT) under certain conditions has a narrow-band single resonance at a high frequency observed numerically in Refs. [18,19] using the exact field matching technique [11]. We also show that the resonant mode is a single slow propagating TM mode synchronously excited by relativistic charge. The expression for the longitudinal impedance and the resonant frequency are obtained analytically. The point wake potential for a relativistic charge is given, and the numerical examples for the copper outer layer with low conducting thin internal coating are presented. Finally, the analytical results are compared with numerical simulations using the field matching technique.

II. DISPERSION RELATIONS AND TM MODES

Consider a round two-layer hollow pipe with a perfectly conducting outer wall and a thin inner metallic cover of thickness d and conductivity σ_1 (Fig. 1). The inner and outer radii of the internal cover are a and b, respectively, ($d = b - a$).

The eigenvalues for the axisymmetric TM modes (TM_{0n}, $n = 1, 2, \ldots$) of the structure may be obtained by matching the tangential components of the partial axisymmetric modes at the boundaries $r = a$ and $r = b$ [11]. In a high frequency range the eigenvalue equation can be presented as

$$\frac{\varepsilon_0}{\nu_{0n}} \frac{J_1(\xi_{0n})}{J_0(\xi_{0n})} = \frac{\varepsilon_1}{\nu'_{0n} tg(\nu'_{0n} d)}, \qquad (1)$$

where $\xi_{0n} = \nu_{0n} a$, ν_{0n}, and $\nu'_{0n} = \sqrt{\nu_{0n}^2 - \chi^2}$ ($n = 1, 2 \ldots$) are the transverse eigenvalues of the fundamental modes in a vacuum region ($0 \leq r \leq a$) and of the inner layer ($a \leq r \leq b$), respectively; ε_0 and $\varepsilon_1 = \varepsilon_0 + j\sigma_1/\omega$ are the

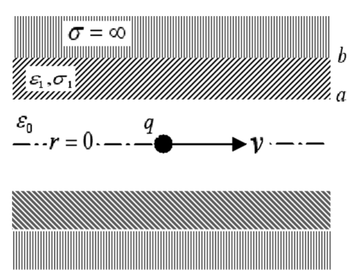

FIG. 1. Geometry of a two-layer pipe.

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dielectric constants of the vacuum and the inner layer, respectively; $\chi = \sqrt{-j\sigma_1\mu_0\omega}$ is the transverse propagation constant of the inner layer; μ_0 is the vacuum magnetic permeability; ω is the frequency; $k = \omega/c$ is the wave number; and c is the speed of light. In a frequency range $|\chi| \gg |\nu_{0n}|$ and $\omega \ll \sigma_1/\epsilon_0$ one gets $\nu'_{0n} \approx j\chi$, $\varepsilon_1 \approx j\sigma_1/\omega$ and the right-hand side of Eq. (1) is independent from the mode index n. For the thin inner layer $|\chi|d \ll 1$, the eigenvalue Eq. (1) is expressed as

$$\frac{1}{\xi_{0n}}\frac{J_1(\xi_{0n})}{J_0(\xi_{0n})} = \frac{1}{k^2ad}, \qquad (2)$$

and the solutions are independent from the cover metal conductivity σ_1 as well.

Equation (2) has an infinite countable set of solutions corresponding to TM_{0n} $(n = 1, 2, \ldots)$ modes. Each solution should be determined by an initial condition at a frequency close to zero $(k \to 0)$, and it should be a continuous function of the frequency. For zero frequency, Eq. (2) is converted into the equation for the roots j_{0n} of the zero-order Bessel function: $J_0(\xi_{0n}) = 0$ with solution $\xi_{0n} = j_{0n}$. For very high frequencies Eq. (2) is converted into the equation for the roots j_{1n} $(n = 2, 3, 4, \ldots)$ of the first-order Bessel function $J_1(\xi_{1n}) = 0$ with $\xi_{1n} = j_{1n}$.

Note that the left-hand side of Eq. (2) should be purely real, which is possible either for purely real or for purely imaginary values of $\xi_{0n} = \nu_{0n}a$. The frequency dependences of the transverse eigenvalues of TM_{0n} $(i = 1, 2, \ldots)$ modes, obtained by a numerical solution of Eq. (2), are shown explicitly in Fig. 2.

As presented in Fig. 2, the solutions for TM_{0n} $(n > 1)$ modes are purely real and decrease with a frequency increase.

The specific solution of Eq. (2) exists for the TM_{01} mode with zero transverse eigenvalue $(\nu_{01} = 0)$ at a frequency of $k_0 = \sqrt{2/ad}$. As the longitudinal propagation constant of the mode is $\beta = \sqrt{k^2 - \nu_{01}^2}$, this solution corresponds to a

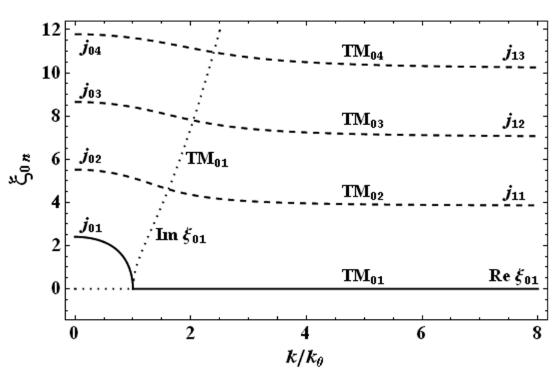

FIG. 2. Real (solid line) and imaginary (dotted line) parts of TM_{01} and purely real TM_{0n} $(n > 1)$ (dashed line) transverse eigenvalues versus frequency.

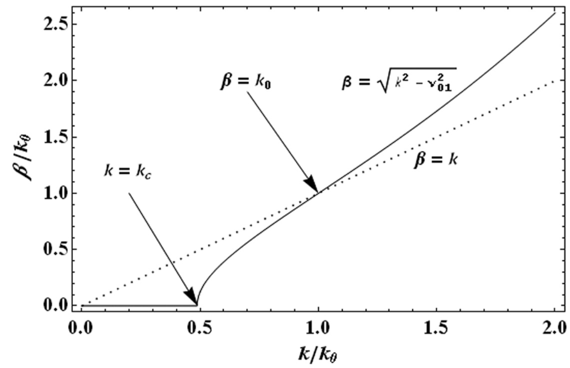

FIG. 3. Synchronous line (dashed line) and dispersion curve of TM_{01} mode (solid line); k_c is the cutoff frequency.

synchronous component of the mode with a phase velocity v_{ph} equal to the velocity of light c.

In a frequency range $k > k_0$ the TM_{01} mode transverse eigenvalue solution of (2) is purely imaginary and the phase velocity is $v_{ph} < c$. In a frequency range $0 < k < k_0$ the transverse eigenvalue is purely real and $v_{ph} > c$. At synchronous frequency $k = k_0$ the transverse eigenvalue vanishes and $v_{ph} = c$ (Fig. 2). Thus, the structure under consideration is characterized by a single slow TM_{01} mode.

Figure 3 shows the dependence of TM_{01} mode longitudinal propagation constant β on a wave number k (dispersion curve). The dispersion curve crosses the synchronous line $\beta = k$ (phase velocity equal to velocity of light) at a resonant frequency of $k = k_0 = \sqrt{2/ad}$.

III. NARROW-BAND RESONANCE

The longitudinal impedance for the arbitrary nonmagnetic layer in a high frequency range $|\chi|a \gg 1$ can be written as [18,19]

$$Z_{||}^0 = j\frac{Z_0}{\pi ka^2}\left(1 + \frac{2}{a}\frac{\varepsilon_1}{\varepsilon_0\chi}cth(\chi d)\right)^{-1}, \qquad (3)$$

where $Z_0 = 120\pi\ \Omega$ is the impedance of free space. For thin inner cover $|\chi|d \ll 1$ $(cth(\chi d) \approx 1/\chi d + \chi d/3)$ the impedance (3) is simplified to a form of the impedance of a parallel resonance circuit [11]:

$$Z_{||}^0(\omega) = \frac{R}{1 + jQ\left(\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0}\right)}, \qquad (4)$$

where $\omega_0 = ck_0$ is the resonant frequency, $R = Z_0c/\pi a_1^2A$ is the shunt impedance, $Q = \omega_0/A$ is the quality factor, and

$$A = \frac{2c}{\sqrt{3}a}(\varsigma + \varsigma^{-1}), \qquad \varsigma = d\sigma_1Z_0/\sqrt{3}. \qquad (5)$$

The frequency range of applicability for expression (4) is given as $a^{-1} \ll |\chi| \ll d^{-1}$. The real part of the impedance

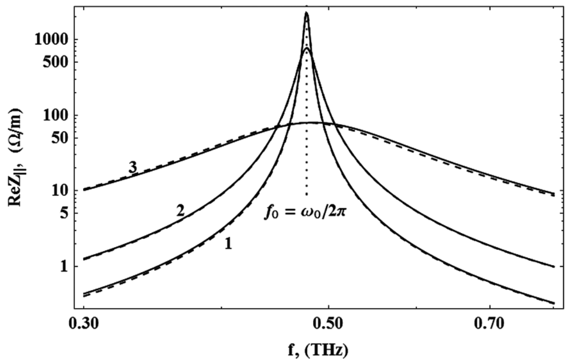

FIG. 4. The exact (solid line) and approximate (dashed line) solutions for the real part of longitudinal impedance for various conductivities of the inner layer: $\sigma_1 = 3\times10^3(1),\ 3\times10^4(2),\ 3\times10^5(3)\ \Omega^{-1}\,\mathrm{m}^{-1}$.

reaches its maximum value at a frequency of $\omega = \omega_0$, while the imaginary part vanishes at the same frequency. This corresponds to the necessary condition for the presence of a synchronous component in the eigenmode of the wave guide. Note, that the resonant frequency is independent of the cover metal conductivity σ_1. The real impedance of the resonant frequency is given by $\mathrm{Re}Z_{\|}^{0} = Z_0 c/(\pi a^2 A)$ and reaches its maximum value at $\varsigma = 1$ (maximal quality factor of the resonance circuit).

Figure 4 presents the comparison between the field-matching-based exact numerical calculations (solid line) and approximate solutions (4) (dashed line) of the ICMT structure. The dotted line indicates the resonant frequency given by $f_0 = \frac{c}{2\pi}\sqrt{2/ad}$. Curves 1, 2, 3 correspond to impedances for various conductivities σ_1 for the thin inner film: $3\times10^3,\ 3\times10^4,\ 3\times10^5\ (\Omega^{-1}\,\mathrm{m}^{-1})$. The inner radius and the inner layer thickness are $a = 2$ cm and $d = 1\ \mu$m, respectively. A good agreement is obtained in the validity region of expression (4). The resonant frequency is about $f_0 = 0.48$ THz independent from the inner layer conductivity.

IV. NUMERICAL EXAMPLES

The longitudinal impedance for a copper tube with a NEG (nonevaporable getter) thin film coating is presented in Fig. 5. The following material conductivities σ have been taken: copper: $58\times10^6\ \Omega^{-1}\,\mathrm{m}^{-1}$; NEG: $3\times10^4\ \Omega^{-1}\,\mathrm{m}^{-1}$. The inner and outer radii of the tube are 2 and 10 cm, respectively. The calculations have been performed using the exact field matching technique. The vertical lines show the resonant frequencies given by k_0.

The solid curves (Fig. 5) show the impedances of conventional homogeneous tubes with infinite thickness and an inner radius of $a = 2$ cm. In a frequency region where the skin depth is much larger than the cover thickness d, the impedance of the frame tube basically contributes to the two-layer tube impedance. At a frequency region where the skin depth is much smaller than the cover thickness, the cover layer of the tube defines the impedance. The resonance arises in intermediate frequencies when the skin depth is of the order of inner layer thickness.

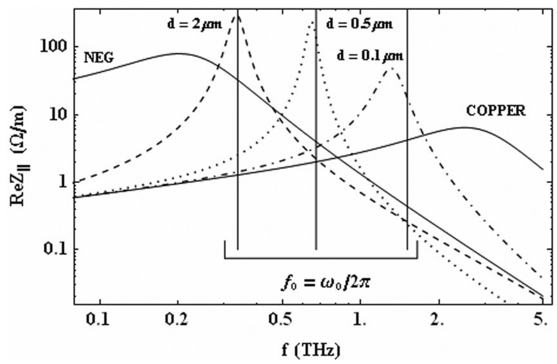

FIG. 5. Real part of copper-NEG tube impedances for various cover thicknesses.

V. WAKE FUNCTION

The longitudinal wake potential $W_{//}(s)$ produced by a relativistic charge moving along the structure axis is given by the inverse Fourier transformation of the impedance [11]:

$$W_{\|}(s) = -\frac{1}{2\pi}\int_{-\infty}^{\infty} Z_{\|}(\omega)e^{-j\frac{\omega}{c}s}d\omega. \tag{6}$$

The integration of the impedance in the complex plane is reduced to the calculation of residues of the impedance (4) in two simple poles, located in the lower half-plane:

$$\omega_{1,2} = \left(-jA \mp \sqrt{4\omega_0^2 - A^2}\right)/2. \tag{7}$$

The integrand has no cutoff in the complex plane and the explicit form of the wake function can be written as the sum of two residues:

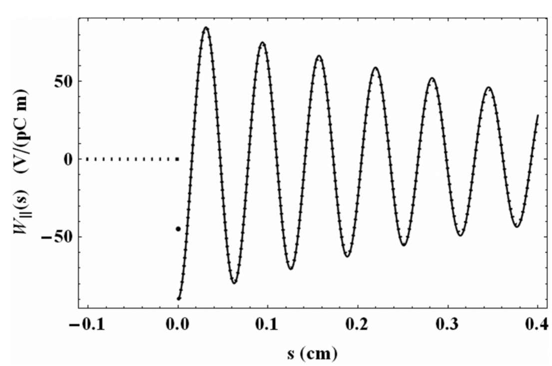

FIG. 6. Point wake function for a perfect conducting pipe with a NEG thin coating. $a = 2$ cm, $d = 1\ \mu$m, $\sigma_1 = 3\times10^4\ \Omega^{-1}\,\mathrm{m}^{-1}$.

$$W_{||}^{0}s = -\frac{Z_0 c}{\pi a^2} e^{-\alpha s}\left[\cos(k_\alpha s) - \frac{\alpha}{k_\alpha}\sin(k_\alpha s)\right], \quad (8)$$

where $k_\alpha = \sqrt{k_0^2 - \alpha^2}$ and $\alpha = A/2c$.

The wake function is given by the exponentially damping periodic curve. The minimum damping rate is $\alpha = 2/\sqrt{3}a$.

Figure 6 presents the point longitudinal wake potential given by formula (8) (dotted line) and numerically calculated using the field exact matching technique (solid line) for a perfect outer layer with a NEG thin film internal coating. A good agreement is obtained: the curves actually coincide. The loss factor is $W_s^0(s = 0) = -Z_0 c/\pi a^2$.

VI. SUMMARY

The resonant properties of a metallic tube with a low conductivity internal thin film coating are examined. It is shown that a structure of such a type has a single slowly propagating TM wave. The dispersion relation, impedance, and the synchronous frequency of the mode are obtained. The analytical presentation of the longitudinal point wake potential is given. The main results of the paper are verified by an exact numerical simulation based on the field matching technique. The experimental study of the presented structure at the AREAL test facility [20] is foreseen.

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