Linear model formulation for superradiant and stimulated superradiant prebunched *e*-beam free-electron lasers

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We present a unified model and analysis of radiation modes excitation in free-electron lasers with a periodically premodulated electron beam. The formulation characterizes superradiant coherent radiation emission from electron beam prebunched at the synchronous frequency, including the case of stimulated superradiant where in addition to a prebunched electron beam an electromagnetic wave is injected into the undulator region. The derived formulation of radiation mode excitation by currents is applicable to general structures that can support orthogonal eigenmodes.

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I. INTRODUCTION

The subject of electromagnetic radiation emission by relativistic charged particles is a fundamental problem of classical electrodynamics [1,2]. In free-electron lasers (FELs) the electromagnetic radiation can be extracted in several different schemes from an electron beam, which passes through a magnetic undulator. In a conventional FEL, a uniform density electron beam passes through a magnetic undulator (wiggler), containing a spatially alternating transverse magnetic field. The electrons, therefore, traverse a path, oscillating transversely to the direction of propagation through the wiggler, and emit forward electromagnetic radiation [3]. In conventional synchrotron undulator radiation with a uniform, randomly distributed electron beam, the radiation field is only partially coherent and the total radiated power is proportional to the electron beam current. This spontaneous emission radiation is known as “undulator synchrotron radiation,” which was studied intensively [4–8].

Introduction of an electromagnetic field into the undulator region, in which a transversely oscillating uniform electron beam flows, results in a “beat wave” (the “ponderomotive wave”) between the undulator magnetic field and the transverse component of the electromagnetic field. As the electron beam propagates along the undulator, the ponderomotive field interacts with the electrons. This interaction results in energy modulation of the beam electrons, which transformed to spatial modulation (density modulation i.e. bunched electron beam). These bunches, if properly phased, and of proper velocity, enable energy extraction from the electrons and stimulate the electrons to emit longitudinally coherent radiation, leading to amplification of the electromagnetic wave [9–14].

Density modulation of the electron beam may occur as a consequence of interaction with the radiation field or by prebunching the electron beam before its entrance to the interaction region. Since the *e*-beam bunching is the driver of the radiation process, the concept of electron beam prebunching was derived as means for enhancing the radiative emission process and generation of harmonics [15–28]. In this case the longitudinally coherent radiation emission process does not require insertion of an external electromagnetic wave field. If the electron beam is periodically prebunched before it enters into the wiggler, all electrons radiate in phase with each other at the bunching frequency, resulting in longitudinally coherent superradiant emission at the bunching frequency [29–37]. In this paper we assume that the electron beam is already density modulated before its entrance to the interaction region (prebunched).

Besides superradiant or prebunched beam radiation, we employ our formulation also to the process of “stimulated superradiance” (SSR). This concept corresponds to the case where there is both prebunching of the *e*-beam and injection of radiation into the undulator at the same frequency and in phase [15,30,37,38]. It is therefore complementary to the case of spontaneous synchrotron undulator radiation (no prebunching, no seed radiation signal injection), FEL amplifier (no prebunching, injection of a radiation signal) and superradiance (*e*-beam prebunching, no radiation signal injection). In the SSR emission process “negative work” is performed by the wave on the electron bunches, stimulating them to radiate throughout the entire undulator length. The SSR regime in FELs provides the most efficient way to convert electron kinetic

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energy into optical energy. SSR emission was demonstrated experimentally in [37] using a scaled-down FEL experimental setup in the microwave regime (in the subsequent sections we will use the parameters of this setup in this paper for demonstrating the application of our formulation). Application of a SSR mechanism was recently suggested (and numerically simulated) as a tool for long wavelength spectral range extension in a waveguided FEL laser oscillator driven by a rf linear accelerator [39].

Beyond the proof of principle demonstrations of SR and SSR emission in the microwave regime, SR and SSR concepts have been considered in the short wavelength, usually in the tenuous beam limit. The radiation mechanisms are part and parcel of the well-known short wavelength FEL enhancement schemes of high gain harmonic generation (HGHG) [40–45] and optical klystron (OK) [46–50].

FELs research leads also to the development of devices designed to generate radiation at wavelengths up to the x-ray regime. Currently, no seed light source exists at x-ray wavelengths regime and thus, SSR is not part of short wavelengths schemes in the x-ray regime. One of the promising approaches for producing full-coherent radiation in short wavelengths region is high-order harmonic generation. An interesting research on seeding light, produced by high-order harmonic generation in a gas, covering wavelengths from the ultraviolet to soft x-rays, for self amplified spontaneous emission performance improvement, has been reported recently [51,52]. In this paper we present a theoretical model and thus, we can assume the existence of any desired wavelength of the injected electromagnetic wave and of the electron beam modulation. Recently, new schemes of beam prebunching and the superradiant harmonic radiation emission scheme, with orbital angular momentum modes, have been considered in the short wavelength high-gain regime [53–55].

The concept of periodically prebunched e-beam superradiant emission, formulated in this paper, is closely related to the concept of single bunch superradiance. The first case corresponds to a coasting beam, periodically prebunched at a frequency within the FEL radiation bandwidth, and the latter corresponds to single e-beam bunches shorter than the wavelength of the radiation emission. The first case is characterized by average radiation power proportional to beam current squared, and the latter by radiation energy proportional to the bunch charge squared [29]. The concept of superradiance is related to other free electron radiation mechanisms like coherent synchrotron radiation (CSR) [56–63].

In this paper we formulate the radiation emission process from a periodically modulated e-beam in terms of transverse modes excitation formulation and used here specifically for waveguide structures. However it can be employed with some approximations also to free space radiation modes and even e-beam guided radiation modes in a high-gain FEL. In this paper we consider the case when the ponderomotive field interaction with the electrons can be neglected and when the output signal is small. Contrary to single electron analysis [29], the e-beam modulation equations are derived from a linearized fluid plasma model, assuming a cold beam and a steady state continuous e-beam current. This formulation can thus be expanded to the collective regime. Moreover, a unique use of Lorentz reciprocity theorem was applied in this paper for development of the excitation equation of the waveguide modes.

The analytical model developed in [15] was done for free space radiation emission and under the assumption of having a single frequency radiation signal. In contrast, the present model describes the radiation process in a general structure of finite transverse dimensions and describes the excitation of radiation modes excited by currents in the structure (multimodes radiation emission).

In the analytical model developed in [17] the current density was expressed in a series of harmonics, where, for simplicity the spatial distribution of the charge density assumed to be symmetrical about the center of the bunch. In calculating the gain from a bunched e-beam (equivalent to the SSR term which we discuss in the following) the authors of [17] assume an external bunching, and assume that the beam dynamics is not affected by the interaction. In deriving the gain expression, they assumed a generic phase for the field mode and obtained a phase dependence of the radiation in a similar manner to that reported in this paper. However, in deriving the expression for the radiation obtained from a bunched e-beam, the main effort in their work was aimed to develop an expression for the saturation power in a resonator.

In the theoretical model developed in [26] the effect of finite boundaries has been investigated. In their paper the authors developed a nonlocal theory of a free-electron laser driven by a prebunched electron beam in a cylindrical waveguide, and they studied the three-wave nonlinear coupling involving a magnetostatic wiggler, a negative beam space charge wave, and an electromagnetic signal in the collective Raman and Compton regimes. The effect of SSR was not discussed in their work and the main effort was aimed to analyze the coupling between the e-beam and the waveguide boundaries.

A one-dimensional and nonlinear simulation of a free-electron laser with a prebunched electron beam moving in a planar wiggler and ion-channel guiding have been presented in [24]. Using Maxwell's equations and a full Lorentz force equation of motion for the electron beam, a set of coupled nonlinear differential equations is derived and solved numerically. The electron beam is assumed cold and propagates with a relativistic velocity. The main effort in their work was aimed at simulating the effect of a prebunched electron beam on saturation, and no analytical model for the effect of prebunched e-beam was presented.

The first part of this paper is devoted to an analytical model for modes excitation equation of radiation excited by currents in a general structure. Next we derive the transverse component of the e-beam current density of a prebunched e-beam. This is needed to calculate the excitation of the electromagnetic radiation modes. We use the fluid plasma model of the e-beam current propagating in the electromagnetic structure under the forces of the wiggler. At the last stage we apply the analytical model to the case of an FEL with a prebunched e-beam. In order to test the analytical model we restrict the multimodes radiation emission calculations to the case of a single mode, and compare the results with those obtained in previous models and simulations. This will allow the verification of the analytical formulation obtained in the previous stages.

II. AN ANALYTICAL MODEL FOR PREBUNCHED E-BEAM FEL

The equations, which describe the electromagnetic field radiation emission from a periodically modulated electron beam of rf current density J, propagating in the z direction of a waveguide, are presented in this chapter. The radiation emission process is described in terms of transverse mode excitation formulation. This formulation is derived from Maxwell equations and used here specifically for waveguide structures.

A. Mode excitation by currents in a general structure

We present here a derivation of the excitation equation of radiation modes excited by currents in a general structure that can support orthogonal eigenmodes (Fig. 1) [64]. A waveguide is certainly a particular structure that satisfies this requirement. For a waveguide structure the derivation of the mode excitation equation is a rigorous consequence of Maxwell equations. The derivation is also valid for any structure of finite transverse dimensions in which a complete orthogonal set of modes can be defined. This includes, in particular, the free-space modes in the paraxial approximation, such as Hermit-Gauss and Laguerre-Gauss modes.

Assuming a steady state (single frequency phasor) solution of the Maxwell equations

$$\underline{E}(\underline{r},t) = \mathrm{Re}\left\{\underline{\tilde{E}}(\underline{r})e^{-i\omega t}\right\},$$

$$\underline{H}(\underline{r},t) = \mathrm{Re}\left\{\underline{\tilde{H}}(\underline{r})e^{-i\omega t}\right\},$$

$$\nabla\times\underline{\tilde{E}} = i\omega\underline{\tilde{H}}, \qquad \nabla\times\underline{H} + i\omega\underline{E} = i\omega\mu_o\underline{J}.$$

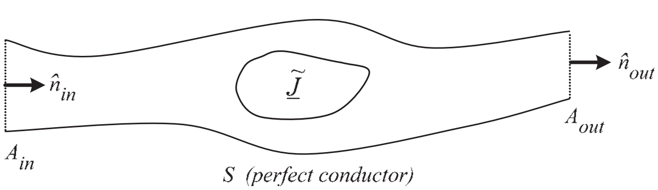

FIG. 1. Schematic of a general electromagnetic structure with excitation currents $\underline{\tilde{J}}(\underline{r})$ and perfectly conducting surfaces S (except for A_{in} and A_{out}).

It is assumed that the transverse components of the fields excited in the waveguide can be presented in terms of a series expansion of the eigenmodes of the structure

$$\underline{\tilde{E}}_{\perp}(\underline{r}) = \sum_q C_q\underline{\tilde{E}}_{\perp q}(\underline{r}), \tag{1}$$

$$\underline{\tilde{H}}_{\perp}(\underline{r}) = \sum_q C_q\underline{\tilde{H}}_{\perp q}(\underline{r}), \tag{2}$$

where C_q is the qth mode radiation field amplitude.

We assume that the modes of the structure are orthogonal and normalized to their mode power p_q

$$\frac{1}{4}\iint_{A_{\mathrm{in}}} dS\hat{n}\cdot(\underline{\tilde{E}}_{\perp q}\times\underline{\tilde{H}}_{\perp q'} + \underline{\tilde{E}}_{\perp q'}\times\underline{\tilde{H}}_{\perp q}) = \delta_{qq'}P_q, \tag{3}$$

where $\delta_{qq'}$ is the Kronecker delta function ($\delta_{qq'} = 1$ for $q = q'$ and $\delta_{qq'} = 0$ for $q \neq q'$).

We employ the Lorentz reciprocity theorem [65] that refers to two different solutions of the fields in the same structure of volume V and enclosing surface S $\{\underline{\tilde{E}}_1(\underline{r}), \underline{\tilde{H}}_1(\underline{r})\}$ and $\{\underline{\tilde{E}}_2(\underline{r}), \underline{\tilde{H}}_2(\underline{r})\}$ corresponding to two different current distributions in the structure $\underline{\tilde{J}}_1$ and $\underline{\tilde{J}}_2$:

$$\oint_S dS\hat{n}\cdot(\underline{\tilde{E}}_1\times\underline{\tilde{H}}_2 - \underline{\tilde{E}}_2\times\underline{\tilde{H}}_1) = \int_V dV(\underline{\tilde{E}}_2\cdot\underline{\tilde{J}}_1 - \underline{\tilde{E}}_1\cdot\underline{\tilde{J}}_2), \tag{4}$$

where $\hat{n}$ is an outward unit vector normal to the surface S.

For a perfectly conducting surface S (for example a waveguide) the left-hand side of Eq. (4) vanishes since $\underline{\tilde{E}}_1\times\underline{\tilde{H}}_2\cdot\hat{n} = (\hat{n}\times\underline{\tilde{E}}_1)\cdot\underline{\tilde{H}}_2 = 0$. This is also true for any structure including free space, in which we may assume that the radiation fields vanish in the transverse dimensions. Considering a general boundary as shown in Fig. 1, the only surfaces contributing to the integral given by Eq. (4) are the input and output surfaces A_{in} and A_{out}, respectively.

In this case Eq. (4) can be written in the form

$$\begin{aligned}&\oint_{A_{\mathrm{out}}} dS\hat{n}\cdot(\underline{\tilde{E}}_1\times\underline{\tilde{H}}_2 - \underline{\tilde{E}}_2\times\underline{\tilde{H}}_1)\\ &\quad - \oint_{A_{\mathrm{in}}} dS\hat{n}\cdot(\underline{\tilde{E}}_1\times\underline{\tilde{H}}_2 - \underline{\tilde{E}}_2\times\underline{\tilde{H}}_1)\\ &\quad = \int_V dV(\underline{\tilde{E}}_2\cdot\underline{\tilde{J}}_1 - \underline{\tilde{E}}_1\cdot\underline{\tilde{J}}_2).\end{aligned} \tag{5}$$

In the particular, case of a cold structure solution with no currents in $-J_1 = J_2 = 0$, one may substitute in (5) an eigenmode solution $\{\underline{\tilde{E}}_1, \underline{\tilde{H}}_1\} = \{\underline{\tilde{E}}_q, \underline{\tilde{H}}_q\}$ and a conjugate (backward wave) mode solution $\{\underline{\tilde{E}}_2, \underline{\tilde{H}}_2\} = \{-\underline{\tilde{E}}_{q'}, \underline{\tilde{H}}_{q'}\}$. This confirms that the modes are orthogonal and keep conserving their power E_q at any cross section along the structure.

We now apply the Lorentz reciprocity theorem (4) to a general case, where inside the structure $\underline{\tilde{J}} \neq 0$ and the solution $\{\underline{\tilde{E}}, \underline{\tilde{H}}\}$ can be expanded in terms of the eigenmodes of the structure $\{\underline{\tilde{E}}_q, \underline{\tilde{H}}_q\}$. We substitute the expansion (1), (2), and $\underline{\tilde{J}}$ instead of $\{\underline{\tilde{E}}_1, \underline{\tilde{H}}_1, \underline{\tilde{J}}_1\}$ and $\{-\underline{\tilde{E}}^*{}_q, \underline{\tilde{H}}^*{}_q, 0\}$ instead of $\{\underline{\tilde{E}}_2, \underline{\tilde{H}}_2, \underline{\tilde{J}}_2\}$:

$$\oint_{A_{\text{out}}} dS\hat{n}_{\text{out}} \cdot \left[\left(\sum_{q'} C_{q'}\underline{\tilde{E}}_{\perp q'}\right) \times \underline{\tilde{H}}^*_{\perp q} + \underline{\tilde{E}}^*_{\perp q} \times \left(\sum_{q'} C_{q'}\underline{\tilde{H}}_{\perp q'}\right)\right]$$
$$- \oint_{A_{\text{in}}} dS\hat{n}_{\text{in}} \cdot \left[\left(\sum_{q'} C_{q'}\underline{\tilde{E}}_{\perp q'}\right) \times \underline{\tilde{H}}^*_{\perp q} + \underline{\tilde{E}}^*_{\perp q} \times \left(\sum_{q'} C_{q'}\underline{\tilde{H}}_{\perp q'}\right)\right]$$
$$= \int_V dV\,(-\underline{\tilde{E}}^*_{\perp q} \cdot \underline{\tilde{J}}). \tag{6}$$

Because of orthogonality of the eigenfunctions, all terms in the left-hand side of Eq. (6) vanish except for the terms $q = q'$. We, therefore, obtain from Eq. (6)

$$C_q^{\text{out}} \oint_{A_{\text{out}}} dS\hat{n}_{\text{out}} \cdot [\underline{\tilde{E}}_{\perp q} \times \underline{\tilde{H}}^*_{\perp q} + \underline{\tilde{E}}^*_{\perp q} \times \underline{\tilde{H}}_{\perp q}]$$
$$- C_q^{\text{in}} \oint_{A_{\text{in}}} dS\hat{n}_{\text{in}} \cdot [\underline{\tilde{E}}_{\perp q} \times \underline{\tilde{H}}^*_{\perp q} + \underline{\tilde{E}}^*_{\perp q} \times \underline{\tilde{H}}_{\perp q}]$$
$$= \int_V dV(-\underline{\tilde{E}}^*_{\perp q} \cdot \underline{\tilde{J}}). \tag{7}$$

According to Eq. (3), Eq. (7) results in

$$C_q^{\text{out}} - C_q^{\text{in}} = -\frac{1}{4p_q}\int_V dV\,\underline{\tilde{E}}^*_{\perp q} \cdot \underline{\tilde{J}}. \tag{8}$$

Inserting C_q^{in} in the right-hand side of Eq. (8), and taking the absolute square values of each side gives

$$|C_q^{\text{out}}|^2 = |C_q^{\text{in}}|^2 - \frac{1}{2\,p_q}\operatorname{Re}\left[C_q^{\text{in}^*}\int_V dV\,\underline{\tilde{E}}^*_{\perp q} \cdot \underline{\tilde{J}}\right] + \frac{1}{16p_q^2}\left|\int_V dV\,\underline{\tilde{E}}^*_{\perp q} \cdot \underline{\tilde{J}}\right|^2.$$

Multiplying both sides by the normalization power of the mode p_q gives the power carried by the qth mode:

$$P_q^{\text{out}} = P_q^{\text{in}} - \frac{1}{2}\operatorname{Re}\left[C_q^{\text{in}^*}\int_V dV\,\underline{\tilde{E}}^*_{\perp q} \cdot \underline{\tilde{J}}\right] + \frac{1}{16p_q}\left|\int_V dV\,\underline{\tilde{E}}^*_{\perp q} \cdot \underline{\tilde{J}}\right|^2. \tag{9}$$

The total electromagnetic power carried by all modes can be written as

$$\mathbf{P} = \sum_q P_q. \tag{10}$$

This means that the total radiated power is the linear sum of the power carried by each of the waveguide modes.

Applying expression (9) to the case of a FEL with a prebunched electron beam, the first term on the right-hand side of Eq. (9) is the power of the electromagnetic signal injected into the interaction region; the second term on the right-hand side of Eq. (9) is due to the transverse current $\underline{\tilde{J}}$ of a bunched e-beam oscillating in a wiggler,stimulated to radiate in the presence of an input signal power ($C_q^{\text{in}} \neq 0$); the third term on the right-hand side of Eq. (9) describes the radiation process of an oscillating current (prebunched beam) in the absence of an electromagnetic signal at the wiggler input.

B. The current density in a prebunched e-beam

Within the formulation of a fluid plasma model of a finite width e-beam current propagating uniformly in the electromagnetic structure under the forces of the wiggler we derive the transverse small signal component of the e-beam current density of a prebunched e-beam. Here we neglect longitudinal space charge force, but extension to the collective regime is possible. This is needed to calculate the excitation of the electromagnetic radiation modes [Eq. (9)].

We assume that a small-signal ac charge density wave $\tilde{\rho}_1(\underline{r})$ propagates along a radially confined electron beam modulating the dc charge density of the beam. The total charge density can be written as

$$\rho(\underline{r}, t) = \rho_o + \operatorname{Re}\{\tilde{\rho}_1(\underline{r})e^{-i\omega t}\}, \tag{11}$$

where ρ_o is the average dc charge density and the phasor $\tilde{\rho}_1$ is the first-order time dependent perturbation of the charge density. The ac component of the charge density is a result of prebunching the e-beam before its entrance to the interaction region. For a small diameter confined electron beam, the dc e-beam current density can be considered uniform at any cross section area (so that we can neglect the transverse variations of the dc and rf current density); longitudinal z-axis variation of the current density is, therefore, assumed to be the dominant term. This assumption may be justified as long as the transverse variation of the electromagnetic field, over the e-beam cross section area, is also negligible. This allows the

e-beam charge density, given in the form $\rho(\underline{r}, t)$, to be expressed as a function of the longitudinal (z) coordinate only $\rho(z, t)$. Thus, the continuity equation $\nabla \cdot \underline{J}(\underline{r}, t) = -\partial\rho(\underline{r}, t)/\partial t$ can be written in the form

$$\frac{\partial J_z(z,t)}{\partial z} = -\frac{\partial \rho(z,t)}{\partial t}. \tag{12}$$

Assuming a sinusoidally varying charge density $\rho(z, t)$, and using complex phasor notation we can write Eq. (12) as

$$\frac{d\tilde{J}_z(z)}{dz} = i\omega\tilde{\rho}_1(z). \tag{13}$$

Expressing the current density in the form

$$\tilde{J}_z(z) = \tilde{\rho}(z)V_z = (\rho_o + \tilde{\rho}_1(z))(V_{zo} + \tilde{V}_{1z}(z)) \cong \rho_o V_{zo} + \tilde{\rho}_1(z)V_{zo} + \rho_o\tilde{V}_{1z}(z) \equiv J_o + \tilde{J}_{1z}(z), \tag{14}$$

where $\tilde{J}_{1z} = \tilde{\rho}_1(z)V_{zo} + \rho_o\tilde{V}_{1z}(z)$ and we neglect the second-order term $\tilde{\rho}_1(z)\tilde{V}_{1z}(z)$.

At this stage we assume that all the electrons propagate inside the wiggler with a uniform axial velocity $V_z \cong V_{zo}$, and also that there is no velocity modulation on the e-beam, i.e. $\tilde{V}_{1z}(z) = 0$.

Thus, the longitudinal ac component of the current density can be written as

$$\tilde{J}_{1z}(z) = \tilde{\rho}_1(z)V_z.$$

Using this expression in Eq. (13) gives

$$V_z\frac{d}{dz}\tilde{\rho}_1(z) = i\omega\tilde{\rho}_1(z). \tag{15}$$

The solution of this first-order differential equation for the e-beam charge and current density is, respectively,

$$\tilde{\rho}_1(z) = \tilde{\rho}_1(0)\exp\left(i\frac{\omega}{V_z}z\right), \tag{16}$$

$$\tilde{J}_{1z} = \tilde{J}_{1z}(0)\exp\left(i\frac{\omega}{V_z}z\right), \tag{17}$$

where $\tilde{J}_{1z}(0) = \tilde{\rho}_1(0)\,V_z$. This describes a "frozen" traveling current density wave drifting with the e-beam at the beam velocity V_z. We define the current modulation index M_J as

$$\tilde{M}_J = \tilde{J}_{1z}(0)/J_o = M_J\exp(i\phi_J), \tag{18}$$

where $\tilde{J}_{1z}(0)$ is the ac complex current amplitude (due to prebunching) at the wiggler entrance ($z = 0$), and its phase ϕ_J is measured relative to the phase of the radiation field amplitude $C_q^{\text{in}}(0)$ at $z = 0$. The e-beam dc current density is $J_o = I_o/A_e$, where I_o is the dc current and A_e is the e-beam cross section area. Using these relations in Eq. (17) we find

$$\tilde{\rho}_1(0) = \frac{\tilde{J}_{1z}(0)}{V_z} = \frac{I_o M_J\exp(i\phi_J)}{A_e V_z}. \tag{19}$$

Interaction of the transverse component of the electromagnetic field with the ac current density requires a transverse component of current density. A transverse component of the e-beam current density is obtained due to the periodic magnetic field of the wiggler, which produces a transverse wiggling motion of the e-beam.

Since $\underline{J} = \rho\underline{\mathrm{v}}$ we can write the transverse component of the current density in the form

$$\begin{aligned}\underline{J}_\perp(z,t) &= \rho(z,t)\underline{\mathrm{v}}_{\perp w}(z) = (\rho_o + \rho_1(z,t))\underline{\mathrm{v}}_{\perp w}(z)\\ &= \rho_o\underline{\mathrm{v}}_{\perp w}(z) + \rho_1(z,t)\underline{\mathrm{v}}_{\perp w}(z)\\ &= \underline{J}_{o\perp}(z) + \rho_1(z,t)\underline{\mathrm{v}}_{\perp w}(z),\end{aligned} \tag{20}$$

where $\underline{\mathrm{v}}_{\perp w}(z)$ is the transverse velocity of the wiggling and calculated directly from the force equation in the wiggler:

$$\frac{d}{dt}[\gamma_o m\underline{\mathrm{V}}_\perp(z)] = -e\hat{e}_z \times \underline{B}_w(z),$$

where e, m are the electron's charge and mass, respectively, and γ_o is the relativistic factor.

Letting $d/dt = V_z(d/dz)$ and $B_w(z) = (\tilde{B}_w e^{-ik_w z} + \text{c.c.})/2$ results in the expression for $\underline{\mathrm{v}}_{\perp w}(z)$,

$$\underline{\mathrm{V}}_{\perp w}(z) = \frac{1}{2}\underline{\tilde{\mathrm{V}}}_{\perp w}e^{-ik_w z} + \text{c.c.}, \tag{21}$$

where c.c. denotes *complex conjugate* of the first term and

$$\underline{\tilde{\mathrm{V}}}_{\perp w} = \frac{i\tilde{a}_w c}{\gamma_o}\hat{e}_\perp, \tag{22}$$

where c is the speed of light in free space and $\hat{e}_\perp$ is a transverse unit vector in the wiggling direction. The wiggler parameter $\tilde{a}_w$ is defined as

$$\tilde{a}_w = \frac{e\tilde{B}_w}{mck_w}, \tag{23}$$

where $\tilde{B}_w$ is the wiggler magnetic field amplitude, $k_w = 2\pi/\lambda_w$ is the wiggler wave number, and λ_w is the length of one wiggler period.

Substituting the expressions for the e-beam charge density from Eq. (16), and the transverse velocity in the wiggler from Eq. (21), into Eq. (20) gives (in phasor representation)

$$\underline{J}_{\perp}(z,t) = \underline{J}_{o\perp}(z) + \left[\frac{1}{2}\tilde{\rho}_1(0)\exp(i\frac{\omega}{V_z}z - i\omega t) + \text{c.c.}\right]\left[\frac{1}{2}\tilde{\underline{\mathbf{V}}}_{\perp w}e^{-ik_w z} + \text{c.c.}\right]$$
$$= \underline{J}_{o\perp}(z) + \frac{1}{4}\left\{\tilde{\rho}_1(0)\tilde{\underline{\mathbf{V}}}_{\perp w}\exp\left[i(\frac{\omega}{V_z} - k_w)z - i\omega t\right] + \tilde{\rho}_1(0)\tilde{\underline{\mathbf{V}}}^*_{\perp w}\exp\left[i(\frac{\omega}{V_z} + k_w)z - i\omega t\right] + \text{c.c.}\right\}. \quad (24)$$

Not all the current density terms in Eq. (24) contribute to interaction between the e-beam and the mode electromagnetic field, since not all current density terms are in synchronism with the mode electromagnetic field. Only the phasors that includes the term $(\frac{\omega}{V_z} - k_w)$ can satisfy the synchronism condition with the interacting electromagnetic wave $\tilde{\underline{E}}_q$ and these must be taken into account. Under this approximation, which can be justified in the subsequent derivation, the transverse component of the current density may be written in the simple form

$$\underline{J}_{\perp}(z,t) = \frac{1}{4}\tilde{\rho}_1(0)\tilde{\underline{\mathbf{V}}}^*_{\perp w}\exp\left[i\,(\frac{\omega}{V_z} - k_w)z - i\omega t\right] + \text{c.c.} \quad (25)$$

If we define $\tilde{\underline{J}}_{\perp}(0) = \frac{1}{2}\tilde{\rho}_1(0)\tilde{\underline{\mathrm{v}}}^*_{\perp w}$, Eq. (25) can be written in the form

$$\underline{J}_{\perp}(z,\,t) = \frac{1}{2}\tilde{\underline{J}}_{\perp}(0)\exp\left[i(\frac{\omega}{V_z} - k_w)z - i\omega t\right] + \text{c.c.}$$
$$= \text{Re}[\tilde{\underline{J}}_{\perp}(z)e^{-i\omega t}],$$

where

$$\tilde{\underline{J}}_{\perp}(z) = \tilde{\underline{J}}_{\perp}(0)\exp\left[i(\frac{\omega}{V_z} - k_w)z\right] \quad (26)$$

is the transverse current rf phasor.

Substituting the transverse current rf phasor from Eq. (26) into the qth mode power emission expression of Eq. (9) enables calculation of the three radiation emission schemes depicted by Eq. (9).

III. SUPERRADIANCE AND STIMULATED SUPERRADIANCE FROM A PREBUNCHED E-BEAM

In the previous section, the analytical expression [Eq. (9)] for the power emission was derived. We now apply the analytical model to the case of a FEL with a prebunched e-beam. In order to verify the analytical model derived in this paper, by comparison to results obtained in previous models and experiments, we restrict the multimodes radiation emission calculation to the case of a single mode emission.

As will be shown in this chapter the calculations of superradiance and stimulated superradiance power emission based on the present model are in good agreement with previous models and experiments. These comparisons reinforce the validity of the analytical model developed in this paper, and allow the development of multimodes radiation sources, based on results derived in this analytical model, which may improve operation of existing and planned devices.

A. Superradiance from a prebunched e-beam

We first derive the expression for the superradiance power (the power radiated from a prebunched e-beam for the case where no electromagnetic signal is launched into the interaction region ($C_q^{\text{in}} = 0$)). In this case the expression for the qth mode radiated power [Eq. (9)] reduces to the form

$$P_q^{\text{out}} = |C_q^{\text{out}}|^2 p_q = \frac{1}{16 p_q}\left|\int_V dV\,\tilde{\underline{E}}^*_{\perp q}\cdot\tilde{\underline{J}}\right|^2. \quad (27)$$

We substitute into Eq. (27) the transverse component of the current density from Eq. (26) and assume that the mode electric field can be expressed in the form $\tilde{\underline{E}}_q(\underline{r}) = \tilde{\underline{E}}_{\perp q}(x,y)\exp(ik_{zq}z)$, where k_{zq} is the mode wave number. In a waveguide structure $k_{zq} = (\omega^2 - \omega^2_{\text{c.o.q.}})^{1/2}/c$ where $\omega_{\text{c.o.q.}}$ is the cutoff frequency of mode q. This is approximately valid also for free space modes along their waist $k_{zq} \approx \omega/c$,

$$P_q^{\text{out}} = \frac{1}{16 p_q}\left|\int_V dV\,\tilde{\underline{E}}^*_{\perp q}(x,y)e^{-ik_{zq}z}\cdot\frac{1}{2}\tilde{\rho}_1(0)\tilde{\underline{\mathrm{v}}}^*_{\perp w}\exp\left[i(\frac{\omega}{V_z} - k_w)z\right]\right|^2. \quad (28)$$

Assume a transversely confined e-beam and negligible transverse variations of the mode electric field $\tilde{\underline{E}}_{\perp q}(x,y)$ across the e-beam cross section. The narrow electron beam transverses through the wiggler at coordinates $x = x_e$, $y = y_e$ and one may approximate

$$\left|\int_V dV\,\tilde{\underline{J}}_{\perp}(x_e,y_e,z)\cdot\tilde{\underline{E}}_{\perp q}(x_e,y_e)\right| = A_e|\tilde{\underline{E}}_{\perp q}|\left|\int_z dz\,\tilde{J}_{\perp}(z)\right|,$$

where A_e is the e-beam cross section area. Consequently:

$$\mathbf{P}_q^{\rm out} = \frac{A_e^2}{64 p_q} |\tilde{\rho}_1(0) \underline{\tilde{\rm v}}_{\perp w}^* \cdot \underline{\tilde{E}}_{\perp q}^*|^2 \left| \int_z dz\, e^{i\,(\frac{\omega}{{\rm v}_z} - k_{zq} - k_w)\,z} \right|^2. \tag{29}$$

For a TE waveguide mode (and in particular for the TE_{10} mode of the rectangular waveguide used in the Tel-Aviv University Free Electron Maser experiments [37]) the normalization power of the qth mode can be expressed in the form [66]

$$p_q = \frac{A_{\rm em}}{2 Z_q} |\underline{\tilde{E}}_{\perp q}|^2, \tag{30}$$

where $A_{\rm em}$ is the effective cross section area of the mode, and Z_q is the mode impedance defined by

$$Z_q = \frac{k_o}{k_{zq}} Z_o, \tag{31}$$

where $Z_o = \sqrt{\mu_o/\varepsilon_o}$ is the free space impedance.

The detuning parameter θ is defined as

$$\theta = \left(\frac{\omega}{{\rm V}_z} - k_z(\omega) - k_w \right). \tag{32}$$

Substituting Eq. (32) into Eq. (29), and integrating over the interaction length L_w, results in

$$P_q^{\rm out} = \frac{A_e^2}{64 p_q} \left| \frac{I_o M_J \exp(i\varphi_J)}{A_e {\rm v}_z} \ \frac{a_w c}{\gamma_o} \underline{\tilde{E}}_{\perp q} \right|^2 \left| \int_{z=0}^{L_w} dz\, e^{i\theta z} \right|^2.$$

This equation can be written in the form

$$\mathrm{P}_q^{\rm out} = M_J^2 P_B \sin c^2(\bar{\theta}/2), \tag{33}$$

where P_B is the prebunching power parameter defined as

$$p_B = \frac{1}{32} I_o^2 L_w^2 \left(\frac{a_w}{\gamma_o \beta_{oz}} \right)^2 \frac{Z_q}{A_{\rm em}}. \tag{34}$$

$\bar{\theta} \equiv \theta L_w$ is the normalized detuning parameter and $\sin c(x) \equiv \sin x / x$.

The expression obtained in Eq. (33) is the same as that obtained in the analytical model described in [15]. These were proved experimentally in [37,38]. This result confirms the analytical model derived in this paper.

The superradiant radiation emission power, given by Eq. (33), is maximal when $\bar{\theta} \cong 0$, which corresponds to a synchronism (or phase matching) condition: $k_{zq}(\omega) = \omega/V_z - k_w$.

In the TAU FEM experimental system [37] (with parameters as per Table I), a rectangular waveguide was used operating at the fundamental TE_{10} mode at the emission frequency range. The synchronous frequencies can be found from the intersection between the shifted electron beam line $k_z(\omega) = \omega/V_z - k_w$ and the dispersion curve of the TE_{10} mode $k_z(\omega) = \sqrt{\omega^2 - \omega_{\rm co}^2}/c$.

Figure 2 shows the shifted electron dispersion line $k_z = \omega/V_z - k_w$ (dashed line) and the dispersion curve of the TE_{10} mode (solid line) for the experimental parameters of Table I, and a 70 keV e-beam energy. The two curves intersections indicate two synchronous interaction frequencies that are well separated. The upper synchronous frequency is near 4.95 GHz, and the lower synchronous frequency is near 3.153 GHz. For these parameters the lower synchronous frequency is just slightly above the waveguide cutoff frequency ($f_{\rm co} = 3.152$ GHz). The

TABLE I. The parameters of the prebunched beam FEM experimental setup.

Electron accelerator	Pierce electron gun + acceleration gap
E-beam energy	55–70 keV
E-beam current	0.05–1.2 Amp.
E-beam prebuncher	Traveling wave tube
Prebuncher frequency band	3 GHz $\le f_m \le$ 12 GHz
Prebuncher input power	$0 \le P_{\rm bunch} \le 3$ W
Prebuncher current modulation index	$0 \le M_J \le 1$
Wiggler type	HALLBACH magnetostatic
Magnetic induction	300 Gauss
Wiggler period length	4.444 cm
Number of periods	$N_w = 17$
Waveguide type	WR − 187
Rectangular waveguide dimension	2.215×4.755 cm
Mode of interaction	TE_{10}
Interaction length	$L_w = 74.8$ cm
Cutoff frequency (TE_{10})	$f_{\rm co} = 3.152$ GHz

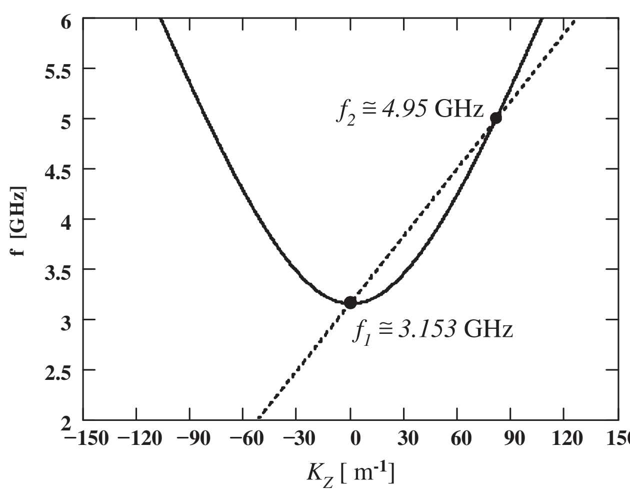

FIG. 2. The electron beam line (dashed) and the TE_{10} mode dispersion curve (solid) (for parameters as per Table I).

radiation process in the lower synchronous frequency has special features that are discussed in [67,68].

In the following we shall compare the superradiance power calculations, based on the developed analytical model, to simulations with FEL3D code [66]. The FEL3D code calculates the electromagnetic field excited by an e-beam, which passes through a waveguiding structure located inside a periodic magnetic field (wiggler). The FEL3D is a single frequency code. It solves the Lorentz force equation together with the excitation equations of the electromagnetic wave. The code includes space-charge effects and, in particular, calculation of the 3D plasma frequency reduction factor of an e-beam in arbitrary waveguide geometry.

Figure 3 shows the superradiance power emission, in the vicinity of the upper frequency synchronism point ($f \approx 4.95$ GHz), based on the parameters given in Table I (for an e-beam energy of 70 keV, a 0.2 A e-beam current, prebunched power parameter of $P_B \approx 35$ W, and a current modulation index of $M_J = 0.2$). The solid curve is plotted based on the analytical model developed in this paper [Eq. (33)] and the dashed curve is plotted based on simulations with FEL3D code.

One can note that both the analytical and simulated curves obtained single maxima at the same frequency, which corresponds to the upper frequency synchronism point ($f \approx 4.95$ GHz, $\theta \approx 0$). One can also note that radiated power can be extracted from the prebunched e-beam, even in the absence of an electromagnetic wave injected into the interaction region. Also, one can note that the radiated power is positive for all frequencies. This is in contrast to the case of stimulated emission (FEL gain), where the electromagnetic wave is amplified in part of the frequency range and attenuated for another part. The difference between the two curves spectral widths is a result of the e-beam discretization performed in the simulation.

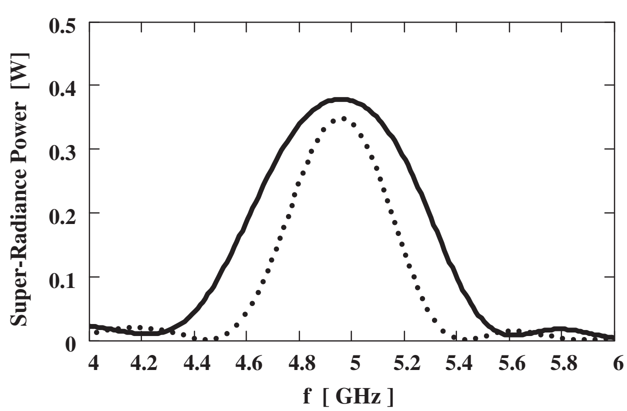

FIG. 3. Superradiance power vs frequency from a prebunched e-beam (for an e-beam current of 0.2 A, 70 keV e-beam energy, and a current modulation index of $M_J = 0.2$). The solid curve is plotted based on the analytical model developed in this paper [Eq. (33)] and the dashed curve is plotted based on simulations with FEL3D code.

For a best fit between the analytical results and simulations we assumed a plasma frequency reduction factor of 0.35, in calculations based on the analytical model, and we assumed a plasma frequency reduction factor of 0.3 in simulations. In most devices, a finite transverse cross section electron beam moved close to a metallic interaction structure. As a result, the space-charge field ceases to be purely radial and, therefore, the axial electric field intensity, and hence the restoring force on the electrons, reduced, as compared to their values in the case of an infinite transverse cross section of the beam. Plasma frequency reduction factors are used to describe the influence of the electron beam geometry, and the proximity of the metallic tunnel of rf interaction structure, on the space-charge field in finite cross section electron beams.

B. Stimulated superradiance from a prebunched *e*-beam

In the preceding section it was found that even without launching an external electromagnetic signal into the interaction region ($C_q^{\mathrm{in}} = 0$) a prebunched electron beam emits longitudinally coherent radiation, if the bunch duration is considerably shorter than the wavelength of the radiation field. An additional contribution to the radiated power can be obtained in the case that both an e-beam current density modulation and an electromagnetic signal are introduced at the input of the interaction region. This additional radiation power is given by the second term on the right-hand side of Eq. (9), namely,

$$\mathrm{P}_q^{\mathrm{out}} = \frac{1}{2}\,\mathrm{Re}\left[C_q^{\mathrm{in}^*}\int_V dv\,\underline{\tilde{E}}_{\perp q}^{*}\cdot\underline{\tilde{J}}\right]. \tag{35}$$

Again we substitute into Eq. (35) the transverse component of the mode electric field $\underline{\tilde{E}}_q(\underline{r}) = \underline{\tilde{E}}_q(x,y)\exp(ik_{zq}z)$ and the transverse component of the current density from Eq. (26):

$$\mathrm{P}_q^{\mathrm{out}} = \frac{1}{2}\,\mathrm{Re}\left\{C_q^{\mathrm{in}^*}\,\underline{\tilde{E}}_{\perp q}^{*}\,\frac{I_o M_J}{A_e \mathrm{v}_z}\frac{i a_w c}{\gamma_o}\,A_e e^{i\varphi_J}\int_{z=0}^{L_w} dz\, e^{i\theta z}\right\}. \tag{36}$$

Multiplying Eq. (30) by $|C_q^{\mathrm{in}}|^2$ and applying the relation $P_q = |C_q|^2 p_q$ we find that

$$|C_q^{\mathrm{in}}||\underline{\tilde{E}}_{\perp q}| = \sqrt{\frac{2Z_q}{A_{\mathrm{em}}}\,p_q^{\mathrm{in}}}\,. \tag{37}$$

Substituting Eq. (37) and using the definition of the prebunching power parameter from Eq. (34) in Eq. (36) gives

$$P_q^{\text{out}} = 4M_J\sqrt{P_B p_q^{\text{in}}}\text{Re}\left\{e^{i\varphi_J}\frac{e^{i\theta L_w}-1}{\theta}\right\}. \quad (38)$$

Therefore, the calculated output power, due to the input signal P_q^{in} and the prebunching of the e-beam, can be written in the form

$$P_q^{\text{out}} = 2M_J\sqrt{P_B P_q^{\text{in}}}\sin c(\bar{\theta}/2)\,\cos(\bar{\theta}/2+\varphi_J). \quad (39)$$

The expression obtained in Eq. (39) is the same expression as the one obtained in the analytical model described in [15] and some aspects of it were proved experimentally in [37]. Again, as in the superradiance emission case, this result confirms the analytical model derived in this paper.

As can be seen from Eq. (39) the output power is proportional to the square root of the rf input power $\sqrt{P_q^{\text{in}}}$. The power P_q^{in} is the power injected at the entrance to the interaction region. The radiation power calculated for the case of presence of both a modulated e-beam and a rf input signal to the interaction region is significantly different from the radiation power obtained if e-beam modulation only exists. The maximum radiated output power is obtained in this case only if in addition to the synchronism condition ($\bar{\theta} \cong 0$) the signal wave and the electron beam (plasma) wave are in phase ($\phi_J = 0$). In this case the radiated power has its maximum at $\bar{\theta} = 0$, as was found also for the superradiance case described in the previous section. However, if $\phi_J \neq 0$ the radiated power will not reach a maximum for $\bar{\theta} = 0$. Numerically it was found that the maximum power in that case is obtained approximately at $\bar{\theta}$ which corresponds to the intersection between the term $\sin c(\bar{\theta}/2)$ and the term $\cos(\bar{\theta}/2+\phi_J)$ given in Eq. (39). The maximum radiated power is also less than the maximum power for the case where $\phi_J = 0$.

We calculated the stimulated superradiant power obtained from a prebunched e-beam in the vicinity of the upper synchronism frequency (4.95 GHz). Figure 4 shows the calculated stimulated superradiance power for two different values of the phase difference ϕ_J. The rf input power P_q^{in} injected at the entrance to the interaction region was taken to be 2 W. The other parameters were taken to be the same as those used for calculating the superradiance power as per Fig. 3.

One can notice that for $\phi_J = 0$ (solid curve) the maximum stimulated superradiance power is indeed obtained at exact synchronism $\bar{\theta} = 0$ (corresponding to $f \approx 4.95$ GHz). This is the same $\bar{\theta}$, which gives maximum radiated power for the case of pure prebunched beam radiation (superradiance power in Fig. 3). On the other hand, for $\phi_J = \pi/2$ (dashed curve) the maximum stimulated superradiance power is obtained at a lower frequency

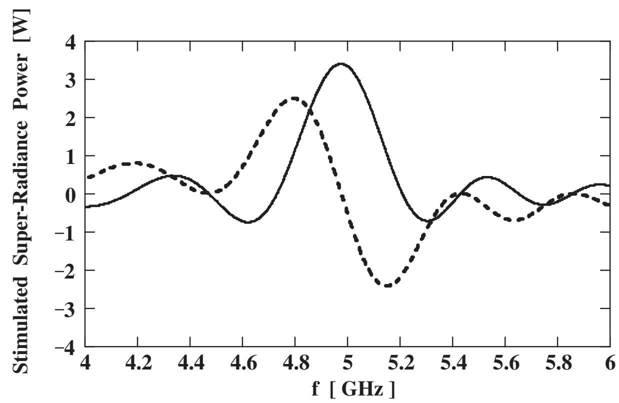

FIG. 4. Calculated stimulated superradiance power vs frequency from a prebunched e-beam. The e-beam current and energy are 0.2 A and 70 keV, respectively. The current modulation index is $M_J = 0.2$ and the input signal power is $P_q^{\text{in}} = 2$ W. The solid curve corresponds to $\phi_J = 0$ and the dashed curve corresponds to $\phi_J = \pi/2$.

(corresponding to $\bar{\theta}_{\text{max}} < 0$), and in addition the maximum power is lower as compared to the case where $\phi_J = 0$.

Note that the stimulated superradiance power shown in Fig. 4 can be positive or negative depending on $\bar{\theta}$. For some values of $\bar{\theta}$ there is a phase mismatch between the e-beam current modulation and the introduced signal P_q^{in} resulting in a "negative power" (absorption of radiation power and e-beam acceleration). This differs from superradiance power, which is a positive symmetrical function of the normalized detuning parameter $\bar{\theta}$ (Fig. 3). The stimulated superradiance power variation with $\bar{\theta}$ has symmetry properties depending on the value of ϕ_J. For a given value of $\bar{\theta}$ [i.e. a fixed value of e-beam energy and radiation frequency, as given by the synchronism condition in Eq. (32)] the stimulated superradiance power varies sinusoidally with ϕ_J [see Eq. (39)].

IV. SUMMARY AND CONCLUSION

A unified formulation for the characterization of superradiant and stimulated superradiant longitudinally coherent radiation emission from a periodically prebunched electron beam was presented in this paper. We showed that the excitation equation of radiation modes excited by currents in a waveguide structure can be obtained using the Lorentz reciprocity theorem. The small signal transverse component of the prebunched e-beam current density was derived within the formulation of a fluid plasma model, and further extension to the collective regime is possible. The e-beam prebunching schemes can result in enhanced longitudinally coherent radiation emission and are more intense than the emission processes in the case of unbunched e-beams.

The analytical calculations based on the present model are in good agreement with previous models, simulations, and experiments. These comparisons reinforce the validity of the analytical model developed in this paper, and allow the development of multimodes radiation sources, based on results derived in this analytical model, which may improve operation of existing and planned devices.

The unified formulation for the enhanced radiation emission processes can be exhibited in a wide variety of e-beam radiation schemes and devices, when the *e*-beam is bunched or density modulated. Specifically, the presented model may be useful for the analysis of seeding-based FEL schemes in the tenuous e-beam regime and may be employed also to coherent synchrotron radiation devices.

Although the subject of subharmonics in the prebunched *e*-beam was not part of this paper, a further work will include such an effect, which enables the development of longitudinally coherent radiation sources in the high-frequency spectrum.

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