Radio frequency quadrupole for Landau damping in accelerators

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We propose using a radio frequency quadrupole (RFQ) to introduce both the longitudinal spread of betatron frequency and the transverse spread of synchrotron frequency for Landau damping of transverse beam instabilities in accelerators. The existing theory of stability diagrams for Landau damping is applied to the case of a RFQ. As an example, the required quadrupolar strength is calculated for stabilizing the Large Hadron Collider beams at 7 TeV. It is shown that this strength can be provided by a superconducting rf device only a few meters long.

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I. INTRODUCTION

In accelerators the effect of Landau damping [1] provides a natural stabilizing mechanism against collective instabilities if particles in the beam have a small spread in their natural (betatron or synchrotron) frequencies; see, for example, [2] and references therein. It is important to note that it is essential for Landau damping that the frequency spread depends on action, which means it is not averaged out over the oscillation period. This is the meaning of the *spread* which is used below if not stated otherwise.

The spread can come from several sources related to nonlinearities of different kinds. Nonlinear space charge forces cause the spread both in betatron and synchrotron frequencies. Nonlinearity in rf focusing voltage results in synchrotron frequency spread. Nonlinearities in a magnetic focusing system cause betatron frequency spread. Last but not least, in colliders, beam-beam effects at collision may also introduce strong frequency spreads. These nonlinearities are usually naturally present in the accelerator and have to be taken into account when analyzing the beam stability in accelerators. However, often in order to improve the beam stability, a dedicated nonlinear element is added to the system. For example, magnetic octupoles are used to introduce betatron frequency spread for Landau damping of dipole modes [3] of coherent beam oscillations caused by transverse beam coupling impedance of the accelerators. Each mode is characterized by the induced coherent shift of the betatron frequency that depends on the mode index, beam, and accelerator parameters and their coupling impedance. An approximate stability criterion is a coherent frequency shift that is smaller than the frequency spread. In the Large Hadron Collider (LHC) [4], a family of 84 focusing and 84 defocussing, 0.32 m long superconducting magnetic octupoles has been installed for Landau damping. In [5], the total integrated strength of all LHC octupoles in order to stabilize the strongest coupled bunch mode that has a coherent betatron frequency shift—$|\Delta\Omega^{\mathrm{coh}}| \sim 0.0002\omega_0$ at 7 TeV, where ω_0 is the revolution frequency—has been calculated using analytical theory developed for Landau damping in the presence of two-dimensional betatron frequency spread [6]. The LHC octupoles have been successfully used to stabilize the beams at top energy of 3.5–4 GeV/c [7,8]. The effect of the transverse spread, however, reduces as the transverse beam emittance goes down at higher energies due to adiabatic damping.

The purpose of this paper is to propose, for the first time, that a radio frequency quadrupole (RFQ) is used to introduce the longitudinal spread of betatron frequency for Landau damping of the transverse oscillations. The basic idea is to use the harmonic dependence of the quadrupolar focusing strength of the RFQ on the longitudinal position of the particles in the bunch. Moreover, since transverse momentum change of a particle (transverse kick) is related to the transverse gradient of longitudinal momentum change (acceleration) according to the Panofsky-Wenzel theorem [9], the RFQ also introduces the transverse two-dimensional spread of the synchrotron frequency. It will be shown that in high energy accelerators the longitudinal spread is more effective than the transverse one due to longitudinal emittance of the beam being much larger than the transverse one. The higher efficiency of the longitudinal spread for Landau damping allows for a compact, only a few meters long, rf device based on several 800 MHz superconducting cavities operating in a transverse magnetic (TM) quadrupolar mode to provide the same functionality as the LHC octupoles whose total length is about 56 m. This RFQ cavity is not to be confused with RFQ linac [10] which is a more complicated rf device focusing, bunching, and accelerating low energy ions at the same time from few tens of keV up to few MeV. Although, a simplified version of a RFQ linac cavity operating at

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transverse electric (TE) quadrupolar mode with rf focusing only can be used for our purpose at the expense of a more complicated shape and higher rf fields if smaller transverse dimensions of the cavity are desirable.

It is worthwhile mentioning that the use of a RFQ for introducing longitudinal linear variation of the betatron frequency along the bunch has been proposed in [11] to increase the transverse mode coupling instability (TMCI) threshold. It has been demonstrated analytically and numerically that the TMCI threshold can be significantly increased if the variation of the betatron frequency over the bunch length is of the order of the synchrotron frequency which is, for example, $\sim 0.002\omega_0$ in the LHC. This, however, may be limited in practice by the synchrobetatron resonances, which is difficult to avoid because of widening of the beam footprint in the betatron tune diagram [12]. The variation required in this case is relatively large because the effect of frequency mixing must take place within one synchrotron period in order to increase the TMCI threshold. The situation of Landau damping discussed below is different. It is related to the weak head-tail instability. Since the betatron frequency spread is linear in action, the frequency variation along the bunch is quadratic and a particle with large synchrotron amplitude always oscillates at the same betatron frequency independently whether it is in the tail or in the head of the bunch. As a result the frequency spread required for Landau damping must only be of the order of coherent frequency shift of the unstable mode. In principle, the same RFQ device can be used to cure both the TMCI and the head-tail instabilities depending on its rf phase with respect to the bunch which must be at zero crossing for the former and on crest for the latter.

II. STABILITY DIAGRAMS FOR LANDAU DAMPING

The generalized theory for stability of horizontal, vertical, and longitudinal oscillations in the presence of horizontal betatron, vertical betaron, and synchrotron frequency spreads has been developed in [13]. It will serve us as a basic for the following analysis. First, action-angle variables $[J_\alpha, \theta_\alpha]$ are defined so that the coordinates $[x, y, z]$ are $\alpha = [2J_\alpha \beta_\alpha(s)]^{1/2}\cos(\theta_\alpha)$, where β_α are the β functions or its longitudinal equivalent for which $\beta_\alpha = \sigma_\alpha^2/\varepsilon_\alpha$ holds with σ_α and ε_α being the RMS bunch sizes and emittances in all three directions. Second, the distribution function is defined in the form of

$$\Psi_0(\boldsymbol{J}) = \frac{(2\pi)^3}{\varepsilon_x \varepsilon_y \varepsilon_z} S(J_x/\varepsilon_x, J_y/\varepsilon_y)\lambda(J_z/\varepsilon_z),$$

where the integrals of the functions S and λ are 1 as are their first moments. Then the problem of stability of transverse oscillations is expressed by the following dispersion equations:

$$\frac{1}{\Delta\Omega_{x,y}^{\mathrm{coh}}} = -\int \frac{J_{x,y}\partial\Psi_0/\partial J_{x,y}}{\Omega_{x,y}^T - \omega_{x,y}(J_x, J_y) - m\omega_z(J_x, J_y)} d^3\boldsymbol{J}, \tag{1}$$

$$\frac{1}{\Delta\Omega_{x,y}^{\mathrm{coh}}} \int J_z^{|m|}\Psi_0 d^3\boldsymbol{J} = \int \frac{J_z^{|m|}\Psi_0}{\Omega_{x,y}^Z - \omega_{x,y}(J_z) - m\omega_z(J_z)} d^3\boldsymbol{J}, \tag{2}$$

where m is the longitudinal azimuthal mode number, $\Delta\Omega_{x,y}^{\mathrm{coh}}$ is the coherent betatron frequency shift induced by the machine impedance in absence of any frequency spread, $\Omega_{x,y}^T$ and $\Omega_{x,y}^Z$ are coherent frequencies of horizontal (x) and vertical (y) betatron oscillations in the presence of the transverse (T) and longitudinal (Z) spread of betatron frequencies, respectively, $\boldsymbol{\omega}^L$ is the vector of horizontal betatron, vertical betatron and synchrotron frequencies without any spread, i.e. given by linear lattice of the accelerators and $\boldsymbol{\omega} = \boldsymbol{\omega}^L + \boldsymbol{A}\cdot\boldsymbol{J}$ is the vector of the corresponding frequencies with the presence of the spread expressed in terms of vector of action variables $\boldsymbol{J}$ and a constant matrix $\boldsymbol{A}$, whose elements

$$A_{\alpha\beta} = a_{\alpha\beta}/\varepsilon_\beta. \tag{3}$$

The assumption of constant $\boldsymbol{A}$ is only valid if the spread is linear in terms of action, which is not always true. It can be linearized, however, in most of the cases as well as in ours. Equations (1) and (2) are decoupled and describe two different cases. Equation (1) describes the case of the transverse spread of the betatron and synchrotron frequencies whereas Eq. (2) treats the case of their longitudinal spread. Depending on which spread is present, Eq. (1) or Eq. (2) is solved for complex $\Omega_{x,y}^T$ or $\Omega_{x,y}^Z$, respectively, as an eigenvalue problem giving the stability region boundary in complex plane of coherent betatron frequency shift: $\Delta\Omega_{x,y}^{T,Z} = \Omega_{x,y}^{T,Z} - \omega_{x,y}^L - m\omega_z^L$. A mode is stable if its complex coherent betatron frequency shift is below the boundary. Recently, this problem was revised in [14], where Eq. (1) was re-derived, a better approximation for Eq. (2) was suggested, and the stability diagram for a general case of $\omega_{x,y}(J_x, J_y, J_z)$ was obtained.

Magnetic octupoles provide linear (in action) variation of the betatron frequencies in the transverse plane $\omega_{x,y}(J_x, J_y)$ [5], so that matrix elements a_{xx}, a_{yy}, a_{xy}, and a_{yx} are constant and nonzero and Eq. (1) applies. $\omega_z(J_x, J_y) = \omega_z^L$, since there is no variation of the synchrotron frequency in the transverse plane and the matrix elements: a_{zx}, a_{zy} are zero. Furthermore, in [13], Eq. (2) has been applied to the case of the longitudinal spread of the synchrotron frequency $\omega_z(J_z)$, due to nonlinearity of the longitudinal rf focusing voltage, when a_{zz} is nonzero. Since in this case there is no effect on the betatron frequencies, $\omega_{x,y}(J_z) = \omega_{x,y}^L$ and a_{xz}, a_{yz} are zero. In a RFQ, however, the situation is different. Both the longitudinal spread of the betatron frequencies $\omega_{x,y}(J_z)$ and the transverse spread of synchrotron frequency $\omega_z(J_x, J_y)$ are present and matrix coefficients: a_{zx}, a_{zy}, a_{xz}, a_{yz} are constant and nonzero. Thus, both Eq. (1) and Eq. (2) apply to the case of RFQ. Although as it will be shown in Sec. III, the effect described by Eq. (1) is much smaller and can be neglected in comparison with the one described by Eq. (2).

The exact shape of the stability boundary depends very much on the assumed distribution function [2,4,5,6,13,14,15] and is not discussed in this paper. However, in most cases a rough estimate holds: $|\Delta\Omega^{T,Z}_{x,y}| > a_{\alpha\beta}$, depending on which matrix elements are nonzero. It will be used later on to estimate the spread $a_{\alpha\beta}$ necessary for Landau damping of a mode with coherent betatron frequency shift $\Delta\Omega^{\mathrm{coh}}_{x,y}$ according to the following condition:

$$a_{\alpha\beta} > |\Delta\Omega^{\mathrm{coh}}_{x,y}|. \tag{4}$$

III. RF QUADRUPOLE

For an ultra-relativistic particle of charge q and momentum p traversing a RFQ along the z-axis at the time moment t, the transverse kick in the thin-lens approximation is given by

$$\Delta \boldsymbol{p}_\perp = pk_2(x\boldsymbol{u}_x - y\boldsymbol{u}_y)\cos\omega t, \tag{5}$$

where ω is the RFQ frequency, $\boldsymbol{u}_a$ is the unity vector along the α coordinate, and k_2 is the amplitude of the normalized integrated quadrupolar strength. This can be calculated in a similar way as for magnets by taking the quadrupolar term in the azimuthal Fourier decomposition of the magnitude of the integrated transverse Lorenz force due to complex electric $\boldsymbol{E}$ and magnetic $\boldsymbol{B}$ fields of the RFQ. Taking, for instance, the horizontal component of the Lorenz force yields:

$$k_2 = \frac{q}{pc}\frac{1}{\pi r}\int_0^{2\pi}\left\|\int_0^L \left(E_x - cB_y\right)e^{j\omega z/c}dz\right\|\cos\varphi d\varphi, \tag{6}$$

where c is the speed of light, L is RFQ length, and $[r, \phi, z]$ are cylindrical coordinates. Assuming that the bunch center ($z = 0$) passes the thin-lens RFQ at $t = 0$, substitution $t = z/c$ gives the dependence of the quadrupolar strength along the bunch, $\sim\cos\omega z/c$, which can be approximated as $\sim 1 - (\omega z/c)^2/2$, for a small argument. Substituting this dependence in the expression for the betatron frequency shift due to quadrupolar focusing, and taking into account that $z^2 = J_z\beta_z = \sigma_z^2 J_z/\varepsilon_z$ after averaging over the synchrotron period, results in the expression for the variation of the betatron frequency in terms of the synchrotron action:

$$\Delta\omega_{x,y} = \pm\beta_{x,y}\frac{\omega_0}{4\pi}k_2\left[1 - \frac{1}{2}\left(\frac{\omega\sigma_z}{c}\right)^2\frac{J_z}{\varepsilon_z}\right]$$

from which corresponding matrix coefficients are expressed as

$$a_{xz,yz} = \mp\beta_{x,y}\sigma_z^2 k_2\left(\frac{\omega}{c}\right)^2\frac{\omega_0}{8\pi}. \tag{7}$$

From Eq. (7) and condition (4), the integrated strength of the RFQ necessary for Landau damping is calculated. Before doing this and discussing at the end of the paper the implementation of the RFQ, it is interesting to investigate whether or not matrix $\boldsymbol{A}$is symmetric as it is in the case of magnets.

The Panofsky-Wenzel theorem relates transverse kick and accelerating voltage by

$$\Delta\mathbf{p}_\perp e^{j\omega t} = \frac{jq}{\omega}\nabla_\perp V e^{j\omega t}. \tag{8}$$

This means that there is a 90° phase difference between the transverse kick and the acceleration, i.e. if the kick is on crest [time dependence in Eq. (5)] of the rf wave the acceleration is at a zero crossing. Taking the real part of the right-hand side of Eq. (8) gives the time dependence of accelerating voltage. Then the RFQ accelerating voltage is expressed as

$$V = -V_2(x^2 - y^2)\sin\omega t, \tag{9}$$

where V_2 is the quadrupolar expansion coefficient of the accelerating voltage given by the azimuthal Fourier transformation of the magnitude of the accelerating voltage integrated over the RFQ length L as

$$V_2 = \frac{1}{\pi r^2}\int_0^{2\pi}\left\|\int_0^L E_z e^{j\omega z/c}dz\right\|\cos 2\varphi d\varphi. \tag{10}$$

Substituting Eq. (9) into Eq. (8) gives the following relation between k_2 and V_2:

$$k_2 = \frac{2q}{p\omega}V_2. \tag{11}$$

Equation (11) is used to calculate the quadrupolar strength from the longitudinal electric field only, which is often more convenient. Moreover, comparing the results of Eq. (6) and Eqs. (10) and (11) gives a good estimate of numerical accuracy of the field maps used in the calculations.

Now, the main rf voltage, defined as $V_0\sin(h\omega_0 t + \phi_s)$, where h is the main rf harmonic number and ϕ_s is the synchronous phase of the main rf voltage with respect to the bunch, is considered. Then the expression for the square of the unperturbed synchrotron frequency is given by

$$\omega_s^2 = \omega_0^2 \frac{|\eta| q h V_0 \cos\phi_s}{2\pi p c}, \tag{12}$$

where η is the phase slip factor of the accelerator. Since the slopes of the main rf voltage and the RFQ voltage simply add up, taking into account the rf phases and harmonic numbers the following expression for the synchrotron frequency in the presence of RFQ is derived using Eq. (9):

$$\omega_z = \omega_s \sqrt{1 + \frac{h_2 V_2 (y^2 - x^2)}{h V_0 \cos\phi_s}}, \tag{13}$$

where h_2 is the RFQ harmonic number. Since the main rf voltage is much larger than the RFQ accelerating voltage the second term of Eq. (13) is much smaller than 1. Taking this into account and substituting expression for $hV_0\cos\phi_s$ derived from Eq. (12) into Eq. (13) yields

$$\omega_z = \omega_s \left[1 + \frac{1}{2} \frac{\omega_0^2 |\eta| q h_2 V_2 (y^2 - x^2)}{\omega_s^2 2\pi p c}\right]. \tag{14}$$

Then, taking the second term of Eq. (14), substituting with Eq. (11) and replacing the coordinate variables with action variables, the final expression for the synchrotron frequency variation in terms of betatron actions is derived:

$$\Delta\omega_z = \frac{\omega_0 |\eta| \omega^2 k_2}{\omega_s 8\pi c} (J_y \beta_y - J_x \beta_x) \tag{15}$$

from which the corresponding matrix coefficients are expressed as

$$a_{zx,zy} = \mp \beta_{x,y} \varepsilon_{x,y} \frac{|\eta| c}{\omega_s} k_2 \left(\frac{\omega}{c}\right)^2 \frac{\omega_0}{8\pi}. \tag{16}$$

In order to evaluate the symmetry of matrix $\boldsymbol{A}$, the ratio of matrix coefficients is found by dividing Eq. (16) by Eq. (7):

$$\frac{a_{zx,zy}}{a_{xz,yz}} = \frac{\varepsilon_{x,y}}{\sigma_z^2} \frac{|\eta| c}{\omega_s}. \tag{17}$$

On the other hand, the dimensionless ratio of transverse to longitudinal emittances is given by:

$$\frac{\varepsilon_{x,y}}{\varepsilon_z} = \frac{\varepsilon_{x,y}}{\pi \sigma_z \sigma_E / E} = \frac{\varepsilon_{x,y}}{\pi \sigma_z^2} \frac{\lambda}{\Delta\hat{E}/E} = \frac{\varepsilon_{x,y}}{\sigma_z^2} \frac{|\eta| c}{\omega_s}. \tag{18}$$

where $E = pc$ and σ_E are the bunch energy and the RMS energy spread, respectively, and $\lambda = 2\pi c/\omega$ and $\Delta\hat{E} = 2E\omega_s/|\eta|\omega$ are the length the height of a stationary bucket ($\varphi_s = 0$) of the RFQ. It is obvious that the right-hand sides of Eqs. (17) and (18) are equal and matrix $\boldsymbol{A}$ is symmetric also in the RFQ case taking into account Eq. (3). This means that the longitudinal and transverse spreads induced by a RFQ are the same in terms of synchrotron and betatron actions, respectively. This is a very interesting result by itself. Furthermore, it leads to a very important consequence that since the dimensionless ratio of the transverse and longitudinal emittances is typically a very small number in high energy, high brightness accelerators, the transverse spread in the bunch is smaller than the longitudinal one by that ratio as follows from Eq. (17). For example, in the LHC at 7 TeV [4], the transverse and longitudinal emittances at 1σ is about 0.5 nm and 7 μm, respectively, resulting in a factor of at least 10^4 larger longitudinal spread in the bunch. This difference is the reason why the longitudinal spread is much more effective and will be the dominant mechanism of Landau damping using a RFQ. Furthermore, this also explains why a RFQ for Landau damping in LHC discussed below is a more compact device than the LHC octupoles even so that the typical field strength in superconducting magnets is higher than in superconducting rf cavities.

IV. RFQ DEVICE

For illustration, a design of a RFQ device with the same functionality as the LHC octupoles is proposed below. Combining Eqs. (4) and (7) the required quadrupolar strength is expressed as

$$k_2 = \frac{2}{\pi} \frac{|\Delta\Omega_{x,y}^{\mathrm{coh}}|}{\omega_0 \beta_{x,y}} \left(\frac{\lambda}{\sigma_z}\right)^2.$$

Its value $k_2 = 1.4 \times 10^{-5}\ \mathrm{m}^{-1}$, required for Landau damping of a coupled bunch mode with the coherent betatron frequency shift of $\sim 0.0002\omega_0$, is calculated for nominal parameters of the LHC at 7 TeV [4]: $\sigma_z = 0.08$ m; β function of 200 m at a potential location in IR4 near the main rf system; $\lambda = 0.375$ m for 800 MHz RFQ frequency, which is the second harmonic of the main rf frequency and for which the bunch still fits in the RFQ bucket: $4\sigma_z = 0.32\ \mathrm{m} < \lambda$. On the other hand, the normalized quadrupolar strength of a cylindrical 800 MHz 0.15 m long pillbox cavity operating in a TM quadrupolar mode is calculated from a complex electromagnetic field map obtained numerically using the code HFSS [16]. In Fig. 1, the distribution of the magnetic field in the transverse plane of the cavity is shown for illustration. The strength value per cavity is $k_2 = 6 \times 10^{-6}\ \mathrm{m}^{-1}$ for the maximum values of electric and magnetic fields on the cavity surface of 46 MV/m and 120 mT, respectively. Taking this value as a maximum that can be achieved in one cavity due to limitations on the surface fields coming from a rf superconductivity quench or an electrical discharge in vacuum [17], the total number of cavities needed can be determined to be three. Adding the same factor 2 margin as

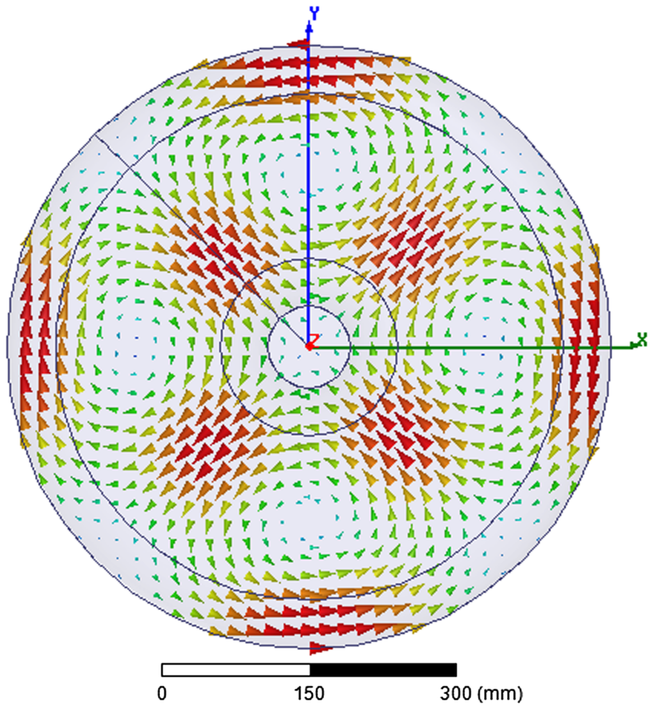

FIG. 1 (color online). Magnetic field distribution in the transverse plane of the TM quadrupolar mode cavity of the RFQ.

for the LHC octupoles, we conclude that six cavities whose total active length is less than a meter can provide the same Landau damping as the 56 m of LHC magnetic octupoles with a nominal current of 500 A. The whole device including the rf power couplers and the coupler for lower, same, and higher order parasitic modes suppression could be integrated in a single few–meters-long cryostat.

V. CONCLUDING REMARKS

It has been shown that a rf quadrupole introduces longitudinal spread of the betatron frequencies that can be used for Landau damping of the transverse coupled bunch instabilities. Moreover since typically the longitudinal emittance of a bunch is much larger than the transverse one, the longitudinal spread is much larger and is more efficient in Landau damping of the instabilities than the transverse spread of the synchrotron frequency that is also present in the case of a RFQ. As an example, the required strength of the RFQ providing the same functionality as the LHC octupoles has been calculated applying the same analytical theory of stability diagrams for Landau damping. Although this theory has been proven by operating the LHC octupoles and stabilizing the LHC beams at top energy, its validity in the case of a RFQ is still to be benchmarked with simulations, which is a subject for future work. This is even more true for the case of operating both octupoles and a RFQ at the same time, which is not described by the analytical theory. Furthermore, a possible implementation of the RFQ using a set of superconducting cavities in one few-meters-long cryostat has been shown.

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[1] L. D. Landau, *J. Phys. USSR* **10**, 25 (1946).
[2] A. W. Chao, *Physics of Collective Beam Instabilities in High Energy Accelerators* (John Wiley & Sons, New York, 1993).
[3] D. Moehl, *Part. Accel.* **50**, 177 (1995).
[4] LHC Design Report No. CERN-2004-003, 2004.
[5] J. Gareyte, J. P. Koutchouk, and F. Ruggiero, LHC project Report No. 91, 1997.
[6] J. S. Berg and F. Ruggiero, Report No. CERN SL-96-71 AP, 1996.
[7] E. Metral, B. Salvant,, and N. Mounet, in *Proceedings of the 2nd International Particle Accelerator Conference, San Sebastián, Spain* (EPS-AG, Spain, 2011), p. 775; Report No. CERN-ATS-2011-102, 2011.
[8] N. Mounet *et al.*, Report No. CERN-ATS-Note-2012-073 MD, 2012.
[9] W. K. H. Panofsky and W. A. Wenzel, Rev. Sci. Instrum. **27**, 967 (1956).
[10] I. M. Kapchinskiy and V. A. Tepliakov, *Prib. Tekh. Eksp.* **2**, 19 (1970).
[11] V. V. Danilov, Phys. Rev. ST Accel. Beams **1**, 041301 (1998).
[12] E. A. Perevedentsev and A. A. Valishev, in *Proceedings of the 8th European Particle Accelerator Conference, Paris, 2002* (EPS, Geneva, 2002), p. 1574.
[13] J. S. Berg and F. Ruggiero, in *Proceedings of the Particle Accelerator Conference, Vancouver, BC, Canada, 1997* (IEEE, New York, 1997), p. 1712; LHC Project Report No. 121, 1997.
[14] A. Burov, http://arxiv.org/abs/1309.0044 (2013).
[15] E. Metral and A. Verdier, Report No. CERN-AB-2004-019 ABP, 2004.
[16] ANSYS HFSS, http://www.ansys.com/.
[17] H. Padamsee, J. Knobloch, and T. Hays, *RF Superconductivity for Accelerators* (John Wiley & Sons, New York, 1998).