Noninterceptive method to measure longitudinal Twiss parameters of a beam in a hadron linear accelerator using beam position monitors

A. Shishlo* and A. Aleksandrov

Oak Ridge National Laboratory, P.O. Box 2008, Oak Ridge, Tennessee 37831-6461, USA
(Received 27 February 2013; published 11 June 2013)

A new method of measuring of the rms longitudinal Twiss parameters of a beam in linear accelerators is presented. It is based on using sum signals from beam position monitors sensitive to the longitudinal charge distribution in the bunch. The applicability of the method is demonstrated on the superconducting section of the Oak Ridge Spallation Neutron Source linear accelerator. The results are compared to a direct measurement of the bunch longitudinal profiles using an interceptive bunch shape monitor in the linac warm section of the same accelerator. Limitations of the method are discussed. The method is fast and simple, and can be used to obtain the initial parameters for the longitudinal matching in linear accelerators where interceptive diagnostics are not desirable.

DOI: 10.1103/PhysRevSTAB.16.062801 PACS numbers: 29.20.Ej

I. INTRODUCTION

It is important to know the longitudinal Twiss parameters of the bunch for beam dynamics optimization and loss reduction in linear accelerators. This is especially true for accelerators with flexible longitudinal settings like superconducting linacs (SCL) having large numbers of independently powered accelerating cavities and uncorrelated amplitude and phase setpoints. Typically, the superconducting part of a linac has strong limitations on the use of interceptive diagnostics due to concerns regarding contamination of superconducting surfaces. This precludes the use of conventional longitudinal bunch profile diagnostics such as bunch shape monitors (BSM) [1] or similar devices. A method of measuring the longitudinal distribution, the energy spectra, and the emittance of the bunch in the superconducting linac without using interceptive diagnostics was suggested in Ref. [2]. That method is based on an amplitude scan of the first SCL cavity combined with a phase scan of the rest of the SCL linac, measurements of beam loss, and assumptions about sharpness of the edges of the longitudinal acceptance. The method was successfully used for the Spallation Neutron Source (SNS) superconducting linac. The weak points of the method are the difficulty of checking the assumption about the sharpness of the acceptance edges, the necessity for time consuming two-dimensional scans, and difficulties with determination of errors for the estimated parameters. In fact no error determination was done or discussed in [2].

In this paper we suggest a new method to measure the longitudinal Twiss parameters at the entrance of a superconducting linac based on the analysis of the beam position monitor (BPM) sum signals during a phase scan of the first accelerating cavity. These signals are proportional to the amplitude of the frequency spectrum of the longitudinal bunch distribution at the BPM's frequency. Assuming a Gaussian longitudinal distribution, information on the bunch length can be easily extracted. We will show that by combining information on the bunch length at multiple BPM locations and using a simple model of a cavity and a drift one can determine the longitudinal Twiss parameters at the linac entrance with good accuracy. The suggested method is fast, simple, and its accuracy can be easily estimated from the measured uncertainty of the BPM data. We describe the assumptions used in the method and derive all necessary equations in Sec. II. Application of the new method for the SNS superconducting linac is described in Sec. III. Verification of the result using direct BSM measurements in the preceding section of the warm linac is presented in Sec. IV. The error analysis is given in the Appendix.

II. METHOD DESCRIPTION AND ITS LIMITATIONS

The first step is to extract information on the bunch longitudinal size from the sum signal of the BPM. In the case of SNS, the BPM pickups are shorted striplines. For the beam near the center of the beam pipe, the amplitude of the BPM's sum signal is [3]

$$u_\omega = J_\omega z(\omega) \frac{1}{I_0(\frac{\omega R}{\beta\gamma c})}, \tag{1}$$

where J_ω is an amplitude of the beam current harmonic with the frequency ω, $z(\omega)$ is a factor describing transfer function of the BPM including the pickup geometry, amplifier gain, etc., R is the radius of the pickup aperture, c is the speed of light, β and γ are relativistic factors, and I_0 is

*shishlo@ornl.gov

the modified Bessel function. In the case of Gaussian longitudinal bunch shape,

$$J_\omega = Qe^{-(\omega\sigma)^2/2}, \tag{2}$$

where Q is the total charge of the bunch and σ is the rms bunch time length. Substituting (2) in (1) gives

$$u_\omega = Qz(\omega)\frac{1}{I_0(\frac{\omega R}{\beta\gamma c})}e^{-(\omega\sigma)^2/2}, \tag{3}$$

where the exponential factor contains information on the bunch longitudinal size. The expression for the bunch size is found by inverting (3):

$$\sigma = \frac{1}{\omega}\sqrt{-2\ln\left[\frac{u_\omega}{Qz(\omega)}I_0\left(\frac{\omega R}{\beta\gamma c}\right)\right]}. \tag{4}$$

In the units of degrees at the bunch fundamental frequency the bunch length

$$\sigma_\varphi = \frac{180°}{\pi}\sqrt{-2\ln\left[\frac{u_\omega}{Qz(\omega)}I_0\left(\frac{\omega R}{\beta\gamma c}\right)\right]}. \tag{5}$$

If the energy of the particles does not change at the BPM location during the measurements, the Bessel function in (5) could be included into the calibration constant A:

$$\sigma_\varphi = \frac{180°}{\pi}\sqrt{-2\ln\left(\frac{u_\omega}{A}\right)}. \tag{6}$$

The bunch longitudinal size can be calculated using (6) if the calibration constant A is known. In practice, it is difficult to calculate A with a good accuracy, but it is easy to measure it if the bunch can be made so short that

$$\sigma \ll \frac{1}{\omega}. \tag{7}$$

In that case $A \approx u_\omega$. For example, the operating frequency of the BPMs in the SNS superconducting linac is 402.5 MHz, the rms bunch length is less than 5°, and the limits for the calibration constant are $u_\omega < A < u_\omega/0.9924$. Therefore, measuring the sum BPM signal under the nominal operation conditions provides an accurate estimate of the calibration constant to use in (5) or (6) for bunch length determination. Unfortunately, the condition (7) also defines the low sensitivity of the bunch length to the sum BPM signal

$$\frac{\delta\sigma}{\sigma} = \frac{1}{(\omega\sigma)^2}\frac{\delta u_\omega}{u_\omega}, \tag{8}$$

where $\delta\sigma$ and δu_ω are errors for the bunch length and the sum signal. When the condition (7) is met, $\omega\sigma$ becomes too small to allow the bunch length determination with an acceptable accuracy. This means that it is not possible to use the BPM sum signal to measure the bunch length in the nominal linac operation when the bunch is too short. In order to use formula (6) for the bunch length measurements, we need to have longer bunches. This can be achieved by switching off the accelerating field in the cavities and allowing the bunch to expand in the drift space. After some distance along the linac, the bunch becomes long enough to allow accurate measurements of its rms length using (6) (note that the calibration constants determined during the nominal linac operation are still valid).

In the absence of space charge, the dependence of the rms bunch length upon the distance along the linac will be determined by the initial longitudinal Twiss parameters and the known transformation matrix of the drift. Therefore the initial Twiss parameters and their uncertainties can be found from the measured rms bunch lengths by using the algorithm described in the Appendix. In most cases of interest, including the SNS linac, space charge plays an important role and cannot be neglected. In this case, a computer model which incorporates the space charge effect should be used for reconstruction of the initial Twiss parameters. As we will show later in case of the SNS linac, the error in the reconstructed Twiss parameters is too large due to noise in the measured data and model imperfections. In essence, the effect of the space charge on the bunch expansion is significantly larger than the influence of the initial Twiss parameters.

In order to increase the effect of the initial Twiss parameters on the measured signals we switched on the first accelerating cavity and performed a phase scan, while recording the BPM sum signal amplitude at each step. The cavity transforms the longitudinal beam phase-space distribution with the focusing strength determined by the rf phase [Ref. [4], page 19]. In some sense this procedure is similar to a "quad-scan" technique of measuring transverse Twiss parameters [5]. In the next section we will demonstrate that adding a controllable focusing element results in significant improvements in the accuracy of the longitudinal Twiss parameters determination.

III. MEASUREMENT OF LONGITUDINAL TWISS PARAMETERS AT THE SNS SUPERCONDUCTING LINAC

The SNS superconducting linac is a part of the SNS 1 GeV linear accelerator, which includes six sections of a normal conducting drift-tube linac, four sections of a normal conducting coupled cavity linac (CCL), and 81 individually powered superconducting cavities (SCL). The input energy of the SCL is 185.6 MeV; the design peak current is 38 mA with a bunch frequency of 402.5 MHz. The SCL has 32 stripline BPMs installed along the linac between the cavities. A bunch shape monitor (BSM), an interceptive device for direct measurement of the longitudinal bunch profile, is installed close to the end of the CCL [6]. The schematic view of the experimental setup including the last section of the CCL, the BSM, and the SCL with

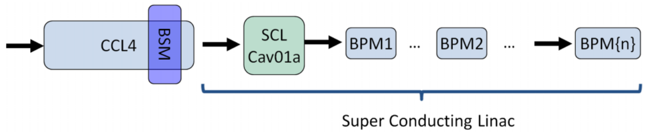

FIG. 1. The experimental setup layout.

the first accelerating cavity SCL:Cav01a and BPMs is shown in Fig. 1.

The first measurements were performed with the design settings in the warm linac and with all SCL cavities switched off including the first one. The bunch was allowed to expand freely in the SCL. The peak current was 35 mA, as measured by the beam current monitors in the beginning of the warm linac. To avoid beam loading in the SCL cavities, the bunch train was kept short, to a duration of 0.45 μs, equivalent to about 182 bunches. For the analysis with the algorithm described in the Appendix, we used 13 BPMs located in the SCL. This number of BPMs was chosen to be suitable for all cases in the study and will be discussed later. We used the XAL online model (OM) [7] as an accelerator model to generate the linear transport matrices necessary for the analysis algorithm. The OM is an envelope tracking accelerator code similar to TRACE3D [4] including the space charge effects. The initial SCL transverse Twiss parameters used in the model were found from the previous laser wire measurements for the warm linac design settings. The measured normalized BPM sum signal amplitudes and the model calculation are shown in Fig. 2. The longitudinal Twiss parameters alpha, beta, and emittance determined at the entrance of the SCL are $\alpha = -0.5 \pm 1.6$, $\beta = 33 \pm 86\,(\mathrm{deg}/\mathrm{MeV})$, and $\varepsilon = 0.7 \pm 4.2\,(\pi \cdot \mathrm{MeV} \cdot \mathrm{deg})$, respectively. The errors on these parameters are too large to make them useful. As discussed in the previous chapter, these large uncertainties are caused by strong space charge repulsion in the bunch.

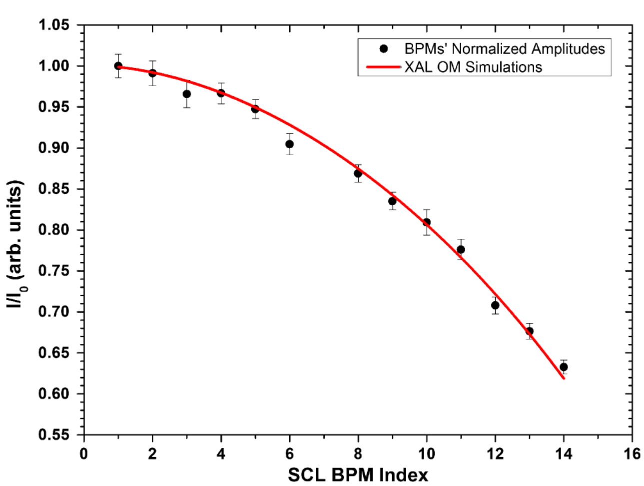

FIG. 2. Normalized BPM amplitudes in the SCL with all rf cavities switched off.

To reduce the errors we added one variable element in the lattice—the first superconducting cavity in the SCL. We performed a full phase scan of this cavity while collecting all BPMs sum signal amplitudes. The model parameters for this cavity (the amplitude of the field and the phase) were found from the BPM phases by using the time-of-flight method. The scan includes 72 cavity phase measurement points for each of the 13 BPMs. The acquisition time of the entire scan was about 2.5 minutes. Figure 3 shows the result, further details of the method are described in the Appendix. The longitudinal Twiss parameters found are $\alpha = 0.56 \pm 0.02$, $\beta = 19.1 \pm 0.5\,(\mathrm{deg}/\mathrm{MeV})$, $\varepsilon = 0.80 \pm 0.01\,(\pi \cdot \mathrm{MeV} \cdot \mathrm{deg})$. All errors are significantly reduced compared to the free expansion case. The error reduction is due to the SCL cavity's longitudinal phase-space manipulation during the phase scan. At each phase point a longitudinal kick transforms the longitudinal phase-space distribution before it starts debunching freely. Therefore, we create a set of unique and independent conditions which are linked to the initial Twiss parameters. All these measurements included into the matrix (A3) reduce the uncertainty of the found initial Twiss parameters. The cavity model is one of the important components of our analysis. The SCL cavities are short compared to the warm linac cavities (six rf gaps for the SCL cavity and 96 gaps for the CCL4), so a simple TRACE3D-like model of the SCL cavity is sufficiently accurate. The measured emittance is about 2 times higher than the design value $0.4\,(\pi \cdot \mathrm{MeV} \cdot \mathrm{deg})$ (see the table of the beam evolution parameters section in [8]). This result agrees with the measurements based on the SCL acceptance scan method [2]. The rms emittance value of $2.7\,(\pi \cdot \mathrm{MeV} \cdot \mathrm{deg})$ reported in [2] was mistakenly not divided by π, and the correct value of $0.86\,(\pi \cdot \mathrm{MeV} \cdot \mathrm{deg})$ is very close to our result.

All BPMs that are far downstream from the beginning of the SCL have a two-peak shape in Fig. 3. The positions of

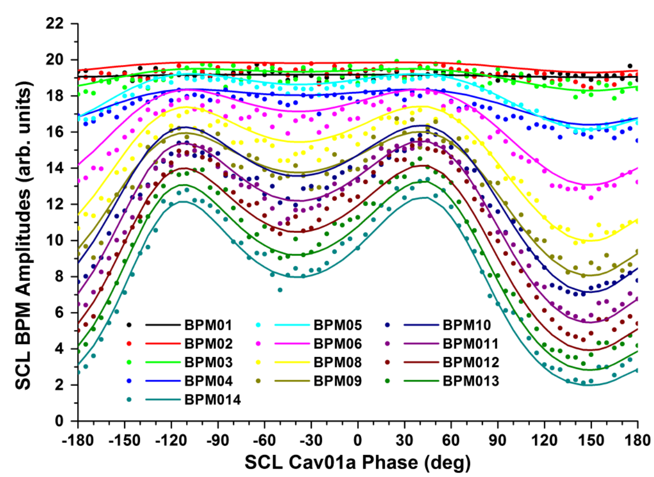

FIG. 3. The amplitudes of all 13 BPMs in the SCL as a function of the first SCL cavity phase. Points are measured values, and the curves are from the model.

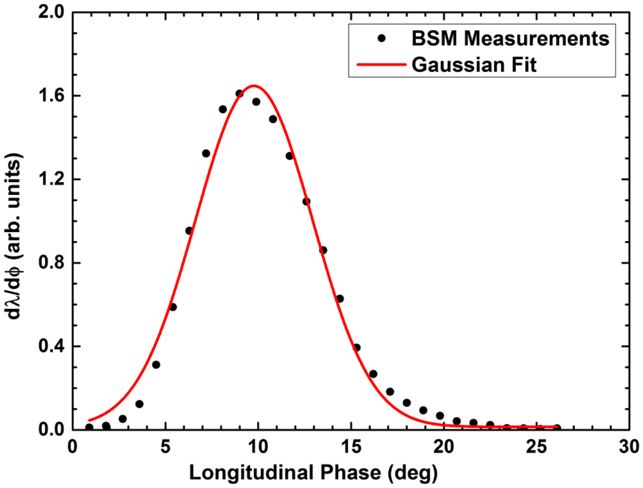

FIG. 4. The longitudinal beam profile measured by the BSM in the CCL. The red curve is a Gaussian fitting.

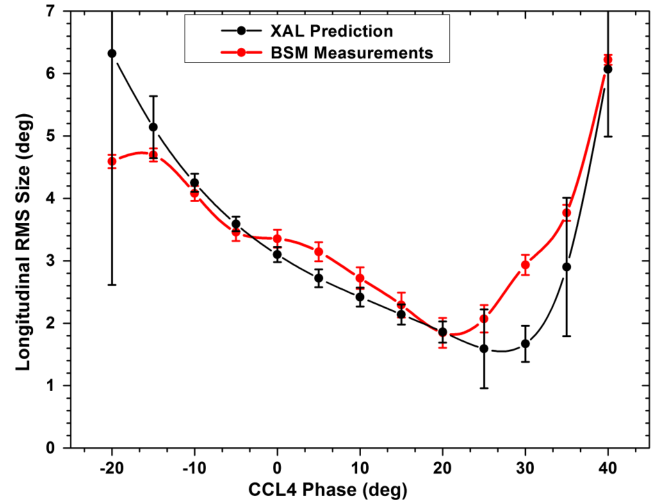

FIG. 5. The longitudinal bunch size at the CCL BSM410 as a function of the CCL4 cavity phase. The zero phase is the design phase of the cavity. Red: the direct BSM measurements. Black: SCL BPMs (new method) with XAL prediction at the CCL4 BSM410 location.

the peaks roughly coincide with the cavity phases for the maximum acceleration and deceleration of the bunch when we have the minimum longitudinal focusing and defocusing in the SCL cavity. The amplitudes of these peaks should be equal according to the envelope tracking model. The minima of the curves in Fig. 3 are created by focusing and defocusing longitudinal kicks from the cavity. The defocusing kick creates the more extreme minimum.

To check the assumption of a Gaussian shape for the longitudinal distribution we used the BSM in the CCL (see. Fig. 1). The measured distribution and a Gaussian fit are shown in Fig. 4. The measured distribution is very close to Gaussian. The fitted rms size is $\sigma_\varphi = 3.1 \pm 0.1$ degree, and the calculated rms value is 3.4 degrees, which justifies use of the Gaussian approximation in our analysis.

IV. COMPARISON WITH BSM MEASUREMENTS

To validate the results of the new method we compared it with a direct measurement of the bunch profile using an interceptive monitor (BSM) in the last section of the warm linac (CCL4). The position of the BSM relative to the SCL entrance is shown in Fig. 1. The distance between the BSM and the SCL entrance is 2.3 meters, in which there are only 16 rf accelerating gaps of the CCL4 and one quad. The Twiss parameters at the SCL entrance depend on the phase of the rf field in CCL4. Therefore the measurements were performed at several rf phase setpoints with 10° steps, ranging from −20° to +40° around the design phase, which allows comparison over a range of initial Twiss parameters. At each CCL4 phase point we measured the longitudinal profiles with the BSM and performed the SCL:Cav01a cavity phase scan as described in Sec. III. The longitudinal Twiss parameters at the SCL entrance were derived from the SCL first cavity phase scan data and used to calculate the longitudinal beam size at the BSM location. For both measurement methods we used the set of the transverse Twiss parameters found previously. The calculation of the longitudinal bunch size was done by backpropagating a bunch envelope with the measured longitudinal Twiss parameters using the same XAL online model as in the previous section. The results of the BSM-based direct bunch size measurements and the SCL BPMs data analysis are compared in Fig. 5. The error bar size in Fig. 5, calculated using (A6), combines contributions from all types of errors: BPM noise, the transformation matrix errors, and the calculated rms bunch size errors due to a non-Gausian shape. Therefore the error bar size is

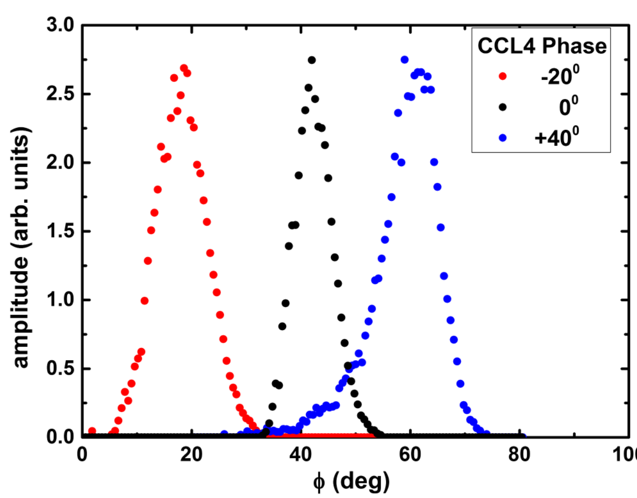

FIG. 6. The longitudinal distributions of the bunch at the CCL BSM410 measured for different phases of the CCL4 cavity. The profiles are scaled in amplitude and shifted in phase for better comparison.

a good criterion of validity of the Gaussian approximation, which is very useful when there are no tools for a direct bunch profile measurement.

The two types of measurement in Fig. 5 are in good agreement, and results overlap within errors. The SCL data analysis results in increasing errors as the CCL4 phase is set far from the nominal design value, and that raises the question about applicability of the new method for these CCL4 phases. The problem can be explained by the very long non-Gaussian tails in the bunch longitudinal distributions which were seen in the BSM measurements. The measured distributions are shown in Fig. 6. The plots show, for large CCL4 phase values, e.g. $+40^\circ$, that the longitudinal bunch shape becomes asymmetric with long tails. For non-Gaussian distributions our formula (6) does not work correctly, and we have poor agreement between the measured and simulated BPMs' amplitudes, yielding large errors for the initial parameters and the predicted sizes in CCL BSM410. Further analysis of the errors should be done with PIC simulations which are beyond the scope of the present paper. At this time, it should be noted, even though this error increase seems to be a limitation of the new method, it is not very significant for the practical use. In practice, the Twiss parameters have to be measured in the vicinity of the design value. The phase variations of ± 20 degrees are already extremely large, and cannot be used for the nominal CCL operation.

V. CONCLUSIONS

A new method of measuring the longitudinal Twiss parameters in the superconducting rf hadron linacs is presented. It is based on the combination of controllable transformations of the bunch in longitudinal phase space by changing the phase settings of one superconducting cavity and analyzing the sum signals of the BPMs. The method allows the estimation of the accuracy of the resulting parameters based on an envelope tracking accelerator code. At this moment the method can be used only for bunches which have approximately Gaussian longitudinal distributions. The existence of non-Gaussian components will reduce the accuracy of the method. The method was successfully applied for the SNS superconducting linac and benchmarked with the BSM measurements in the last section of the warm linac.

ACKNOWLEDGMENTS

The authors are grateful to J. Brian and Dr. Craig Deibele for help with the SNS beam instrumentation during the measurements. The work was performed at Spallation Neutron Source accelerator at Oak Ridge National Laboratory (ORNL). This manuscript has been authored by UT-Battelle, LLC, under Contract No. DE-AC05-00OR22725 with the U.S. Department of Energy.

APPENDIX: CALCULATION OF THE LONGITUDINAL TWISS PARAMETERS USING BPMS' AMPLITUDES

Let us consider a lattice of a linear accelerator with N beam position monitors. A bunch of charged particles is transported through this lattice, and it generates a sum signal at each BPM. We want to calculate the initial longitudinal Twiss parameters of the bunch by using these signals. We use a simple linear transport model, as it is implemented in many accelerator codes. In this model the longitudinal coordinates of particles at the BPM locations are defined by their initial values and the transport matrix $m^{(i)}$, where $i = 1, \ldots, N$,

$$\begin{bmatrix} \varphi^{(i)} \\ E^{(i)} \end{bmatrix} = \begin{bmatrix} m_{1,1}^{(i)} & m_{1,2}^{(i)} \\ m_{2,1}^{(i)} & m_{2,2}^{(i)} \end{bmatrix} \begin{bmatrix} \varphi^{(0)} \\ E^{(0)} \end{bmatrix}, \tag{A1}$$

where φ and E are the longitudinal position and the kinetic energy deviations from the synchronous particle of the bunch. By calculating the square of both sides of the first equation of the (A1) system and averaging over the whole ensemble of particles in the bunch, we have the expression for the second order moments of the longitudinal distribution:

$$\begin{aligned} \langle(\varphi^{(i)})^2\rangle = (m_{1,1}^{(i)})^2\langle(\varphi^{(0)})^2\rangle + 2m_{1,1}^{(i)}m_{1,1}^{(i)}\langle\varphi^{(0)}E^{(0)}\rangle \\ + (m_{1,1}^{(i)})^2\langle(E^{(0)})^2\rangle. \end{aligned} \tag{A2}$$

We have N BPMs, so our system of equations will be written as

$$\begin{bmatrix} \langle(\varphi^{(1)})^2\rangle \\ \cdots \\ \langle(\varphi^{(N)})^2\rangle \end{bmatrix} = M_{N\times 3} \begin{bmatrix} \langle(\varphi^{(0)})^2\rangle \\ \langle\varphi^{(0)}E^{(0)}\rangle \\ \langle(E^{(0)})^2\rangle \end{bmatrix},$$

$$\text{where } M_{N\times 3} = \begin{bmatrix} (m_{1,1}^{(1)})^2 & 2m_{1,1}^{(1)}m_{1,1}^{(1)} & (m_{1,2}^{(1)})^2 \\ \cdots & \cdots & \cdots \\ (m_{1,1}^{(N)})^2 & 2m_{1,1}^{(N)}m_{1,1}^{(N)} & (m_{1,2}^{(N)})^2 \end{bmatrix}. \tag{A3}$$

We have to find the initial second moments $\langle(\varphi^{(0)})^2\rangle$, $\langle(\varphi^{(0)}E^{(0)}\rangle$, and $\langle(E^{(0)})^2\rangle$ that will minimize the sum of the squared deviation of the calculated second moments at each BPM from the longitudinal rms sizes measured by the BPMs according to formula (6):

$$S = \sum_{i=1}^{N} [\langle(\varphi^{(i)})^2\rangle - (\sigma_\varphi^{(i)})^2]^2. \tag{A4}$$

This is a typical linear least square method problem, and the solution is

$$\begin{bmatrix} \langle(\varphi^{(0)})^2\rangle \\ \langle\varphi^{(0)}E^{(0)}\rangle \\ \langle(E^{(0)})^2\rangle \end{bmatrix} = (M_{N\times 3}^T M_{N\times 3})^{-1} M_{N\times 3}^T \begin{bmatrix} (\sigma_\varphi^{(1)})^2 \\ \cdots \\ (\sigma_\varphi^{(N)})^2 \end{bmatrix}. \tag{A5}$$

The variances of the initial parameters can be estimated by the following formula:

$$\begin{bmatrix} \mathrm{var}(\langle(\varphi^{(0)})^2\rangle) \\ \mathrm{var}(\langle\varphi^{(0)}E^{(0)}\rangle) \\ \mathrm{var}(\langle(E^{(0)})^2\rangle) \end{bmatrix} \cong \frac{S}{N-3}\{(M_{N\times3}^T M_{N\times3})^{-1}\}_{\mathrm{diagonal}}. \tag{A6}$$

The rms Twiss parameters emittance, alpha, and beta are calculated from the second moments (A5):

$$\begin{aligned} \epsilon &= \sqrt{\langle(\varphi^{(0)})^2\rangle\langle(E^{(0)})^2\rangle - (\langle\varphi^{(0)}E^{(0)}\rangle)^2}, \\ \alpha &= -\langle\varphi^{(0)}E^{(0)}\rangle/\varepsilon, \qquad \beta = \langle(\varphi^{(0)})^2\rangle/\epsilon. \end{aligned} \tag{A7}$$

The errors of the Twiss parameters can be estimated by using the variances of the second order moments (A6).

In the presence of space charge forces the transport matrices will be dependent on the initial Twiss parameters for the longitudinal and transverse directions. The transverse parameters can be found by analyzing wire or laser wire scanner data. As for the longitudinal Twiss parameters, there are several possible ways to find the initial values. First, we can use the design Twiss values to generate transport matrices to solve (A5) and iterate the calculations. There is no guarantee that the iterations will converge. In practice we saw divergence in many cases. Second, we can use a nonlinear minimizing method like the simplex minimization to find the initial conditions for the minimal S value (A4) and then use these parameters to generate transport matrices and solve (A5) and (A6) to get the initial parameters and their uncertainties. In this paper we use the second approach.

[1] A. V. Feschenko, AIP Conf. Proc. **281**, 185 (1992).
[2] Y. Zhang, J. Galambos, and A. Shishlo, Phys. Rev. ST Accel. Beams **11**, 104001 (2008).
[3] J. H. Cupérus, Nucl. Instrum. Methods **145**, 219 (1977).
[4] K. R. Crandall and D. P. Rusthoi, Los Alamos National Laboratory Report No. LA-UR-97-886, 1997 [http://laacg1.lanl.gov/laacg/services/traceman.pdf].
[5] J. T. Seeman, A. W. Chao, and M. Tigner, in *Handbook of Accelerator Physics and Engineering* (World Scientific, Singapore, 1999), pp. 564–566.
[6] A. V. Feschenko *et al.*, in *Proceedings of the 2007 Particle Accelerator Conference, Albuquerque, New Mexico* (IEEE, New York, 2007), pp. 2608–2610.
[7] C. K. Allen, C. A. McChesney, C. P. Chu, J. D. Galambos, W.-D. Klotz, T. A. Pelaia, and A. Shislo, ICALEPCS 2003 Conference Proceedings, Kyongju, Korea, 2003 (Pohang Accelerator Laboratory, Pohang, Korea, 2004), ISBN: 89-954175-2-8 98420.
[8] SNS Parameters List, Oak Ridge National Laboratory Report No. SNS-100000000-PL0001-R13, 2005 [http://neutrons.ornl.gov/media/pubs/pdf/sns_parameters_list_june05.pdf].