Determination of the particle energy in a waveguide with a thin dielectric layer

Andrey V. Tyukhtin
Physical Faculty of St. Petersburg State University, St. Petersburg, 198504, Russia
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An original method to determine the charged particle energy is developed. This method uses the dependency of waveguide mode frequency on the Lorentz factor of particles. It is central to this technique that the particle bunch generates Cherenkov radiation in a waveguide, and the mode frequencies depend essentially on the Lorentz factor. Here, we consider the case when radiation is excited in a circular waveguide with a dielectric layer. It is shown that structures with relatively thick layers are not convenient for the particle energy measurement because the dependence of the first mode frequency on the Lorentz factor is weak. In contrast, a structure with a thin layer is favorable for such a purpose because this dependency is more essential. Analytical and numerical investigations are performed. It is shown that the first mode amplitude is sufficient for measurements in the case of a pico-Coulomb bunch.

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I. INTRODUCTION

Cherenkov radiation (CR) is widely used for the detection of charged particles and the bunch diagnostics [1,2]. Usually, different open systems are applied for these purposes, but properties of CR in waveguide structures are not used. Meanwhile, CR in a waveguide is not an exotic effect. For example, it is actively used for wakefield acceleration of charged particles [3–5]. In this technique, a large bunch (driver) excites CR (so-called wakefield) in a waveguide loaded with a dielectric layer, and another (relatively small) bunch is accelerated in this field. Owing to this application, the techniques of CR generation in waveguides are well tested at present.

Here, we consider a method of bunch diagnostics that uses the dependence of the waveguide mode frequencies ω_m on the Lorentz factor of the bunch particles $\gamma = (1 - \beta^2)^{-1/2}$ ($\beta = V/c$, V is a bunch velocity, c is the velocity of light). The particle energy is proportional to the Lorentz factor ($\mathrm{E} = mc^2\gamma$); therefore, the determination of γ is equivalent to the determination of the particle energy (here, we assume that the bunch is practically monoenergetic, i.e., the energy spread of particles in the bunch is negligible).

For the method under consideration, it is critical that the particle bunch generates CR in a waveguide. This can be achieved in different ways, but it is obvious that the waveguide cannot be a simple regular structure with a perfectly conductive wall. Different variants of waveguide structures were offered for the indicated goals [6–13]. In Ref. [6], the authors have recommended the use of the simplest variant: a circular waveguide with a cylindrical layer of isotropic nondispersive dielectric. A disadvantage of this variant is that the dependency of the waveguide modes on the Lorentz factor is usually weak. Therefore, the accuracy of the Lorentz factor determination will be small, especially for the ultrarelativistic case $\gamma \gg 1$ (this peculiarity is typical for all detectors based on the Cherenkov phenomenon). However, it is possible to partially overcome this imperfection.

We have offered different ways to achieve an essential dependency $\omega_m(\gamma)$ both for some predetermined narrow range of γ values and for a sufficiently wide range of γ [5–11]. Here, we discuss the method, which is relatively simple for experimental realization and, at the same time, gives a sufficiently strong dependency of the mode frequency on the Lorentz factor. As in Ref. [6], we considered a waveguide with cylindrical dielectric layer. However, our suggestion is to use a thin dielectric layer instead of a traditional, relatively thick layer (this idea was mentioned for the first time in Ref. [11]). Further advantages of this approach will be shown.

II. ANALYTICAL INVESTIGATION

Consider a circular waveguide with a radius a that has a cylindrical layer of dielectric material. The thickness of the dielectric layer is $d = a - b$, where b is a vacuum channel radius (Fig. 1). A particle bunch possessing a charge q moves along the waveguide axis (the z axis) with a velocity $\vec{V} = c\beta\vec{e}_z$. The transverse dimension of the bunch is assumed to be negligible, and the longitudinal distribution of the charge is determined by the Gaussian function $\exp[-\zeta^2/(2\sigma^2)]$, where $\zeta = z - Vt$. Note that such bunches are typical, such as for the technique of a wakefield acceleration [3,5].

It is assumed that the charge velocity is more than the phase velocity of electromagnetic waves in the layer material: $V > c/\sqrt{\varepsilon}$, or $\varepsilon\beta^2 > 1$, where ε is the permittivity of the material (dissipation and dispersion are assumed to

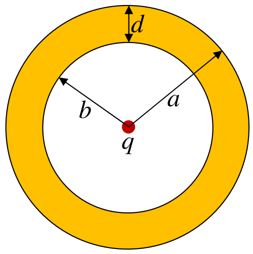

FIG. 1. Cross section of the structure.

be negligible, and permeability is $\mu = 1$). The expressions for the components of the electromagnetic wave field are well known [14]. They can be written in the following form (cylindrical coordinates ρ, ϕ, ζ are used):

$$E_z = \frac{4q(\tilde{\varepsilon}\beta^2 - 1)}{c^2\beta^2}\Theta(-\zeta)\mathrm{Re}\left\{\sum_{m=1}^{\infty}\omega_m \mathrm{Res}_m(\Phi)\right.$$
$$\left.\times \exp[-\omega_m^2\sigma^2(2V^2)^{-1} + \mathrm{i}\omega_m\zeta V^{-1}]\right\},$$
$$E_\rho = -\frac{4q}{c\beta}\Theta(-\zeta)\mathrm{Im}\left\{\sum_{m=1}^{\infty}\mathrm{Res}_m\left(\frac{\partial\Phi}{\partial\rho}\right)\right. \quad (1)$$
$$\left.\times \exp[-\omega_m^2\sigma^2(2V^2)^{-1} + \mathrm{i}\omega_m\zeta V^{-1}]\right\},$$
$$H_\phi = c\beta E_\rho,$$

where $\tilde{\varepsilon} = \varepsilon$ in the layer (at $b < \rho < a$) and $\tilde{\varepsilon} = 1$ in the vacuum channel (at $\rho < b$),

$$\Phi = \begin{cases} \alpha I_0(k\rho) & \text{for } \rho < b, \\ \varepsilon^{-1}\left[\frac{\pi \mathrm{i}}{2}\eta H_0^{(1)}(s\rho) + \delta J_0(s\rho)\right] & \text{for } \rho > b, \end{cases} \quad (2)$$

$$\alpha = \frac{sK_1(kb)\psi_1(s) + \varepsilon k K_0(kb)\psi_0(s)}{sI_1(kb)\psi_1(s) - \varepsilon k I_0(kb)\psi_0(s)},$$
$$\eta = -\frac{2\varepsilon k}{\pi b s}\frac{J_0(sa)}{sI_1(kb)\psi_1(s) - \varepsilon k I_0(kb)\psi_0(s)}, \quad (3)$$
$$\delta = -\frac{\pi \mathrm{i}}{2}\frac{H_0^{(1)}(sa)}{J_0(sa)}\eta,$$

$$\psi_0(s) = J_1(sb)N_0(sa) - J_0(sa)N_1(sb), \quad (4)$$
$$\psi_1(s) = J_0(sb)N_0(sa) - J_0(sa)N_0(sb),$$

$$k = \frac{\omega\sqrt{1-\beta^2}}{c\beta} = \frac{\omega}{c\sqrt{\gamma^2 - 1}},$$
$$s = \frac{\omega\sqrt{\varepsilon\beta^2 - 1}}{c\beta} = \frac{\omega}{c}\sqrt{\frac{\gamma^2(\varepsilon - 1) - \varepsilon}{\gamma^2 - 1}}, \quad (5)$$
$$\gamma = \frac{1}{\sqrt{1-\beta^2}}, \qquad \Theta(\xi) = \begin{cases} 0 & \text{for } \xi < 0, \\ 1 & \text{for } \xi > 0. \end{cases}$$

Here, $\mathrm{Res}_m(F)$ are residues of the function $F(\omega)$ in its poles $\omega = \omega_m$, $J_n(\xi)$ and $N_n(\xi)$ are Bessel and Neumann functions, respectively, and $I_n(\xi)$ and $K_n(\xi)$ are modified Bessel and Neumann functions, respectively. The frequencies ω_m included in (1) are positive solutions of the following dispersion equation:

$$f(\omega) \equiv s(\omega)I_1(k(\omega)b)\psi_1(s(\omega)) - \varepsilon k(\omega)I_0(k(\omega)b)\psi_0(s(\omega)) = 0. \quad (6)$$

Note that Eq. (1) is a wave field only (so-called wakefield, or field of CR). It is a part of the total field that also contains a quasi-Coulomb (quasistatic) field. However, a quasistatic field does not take the energy from the particle, and it is not important for our goal. Stress that the total field is continuous at $\zeta = 0$ [a discontinuity $\theta(-\zeta)$ in the formula (1) concerns the wave field only].

Properties of the solutions of Eq. (6) are the most important for our purposes. Let us consider some particular cases to analyze these properties. First, we assume that the condition

$$s(\omega)b \gg 1 \quad (7)$$

is fulfilled, i.e., that the channel radius is sufficiently large. Note that this inequality may be correct for only part of the frequency range; therefore, this assumption should be justified after determination of the dispersion equation solutions. The functions (4) can be replaced with their asymptotics:

$$\psi_0(s) \approx \frac{2\cos(sd)}{\pi s\sqrt{ab}}, \qquad \psi_1(s) \approx \frac{2\sin(sd)}{\pi s\sqrt{ab}}. \quad (8)$$

One can see that the dispersive equation (6) takes the form

$$\tan(s(\omega)d) = \frac{\varepsilon k(\omega)I_0(k(\omega)b)}{s(\omega)I_1(k(\omega)b)}. \quad (9)$$

If we use, in addition to (7), the condition

$$k(\omega)b \gg 1, \quad (10)$$

then Eq. (9) can be written in the form

$$\tan(s(\omega)d) = \varepsilon k(\omega)/s(\omega), \quad (11)$$

or

$$\tan\left(d\frac{\omega}{c}\sqrt{\frac{\gamma^2(\varepsilon-1)-\varepsilon}{\gamma^2-1}}\right) = \frac{\varepsilon}{\sqrt{\gamma^2(\varepsilon-1)-\varepsilon}}. \quad (12)$$

First, we are interested in the case of ultrarelativistic particles, when $\gamma \gg 1$. In this case, Eq. (12) can be written approximately in the form

$$\tan\left(d\frac{\omega}{c}\sqrt{\varepsilon - 1}\right) = \frac{\varepsilon}{\gamma\sqrt{\varepsilon - 1}}. \quad (13)$$

If we assume that the layer thickness is small enough, and therefore the condition

$$d\omega\sqrt{\varepsilon - 1}/c \ll 1 \tag{14}$$

is true, we obtain for the first waveguide mode the simple expression

$$\omega_1 \approx \frac{c\varepsilon}{d\gamma(\varepsilon - 1)}. \tag{15}$$

Thus, under the assumptions made above, the first mode frequency is inversely proportional to the Lorentz factor. This dependency is very favorable for determination of γ because the relative error of γ is close to the relative error of the measurement of frequency: $\Delta\gamma/\gamma \approx \Delta\omega_1/\omega_1$.

Substituting Eq. (15) into the inequalities (7), (10), and (14), one can obtain the following conditions on the validity of the obtained results:

$$\gamma \ll \frac{b\varepsilon}{d\sqrt{\varepsilon - 1}}, \qquad \gamma \ll \sqrt{\frac{b\varepsilon}{d(\varepsilon - 1)}}, \qquad \gamma \gg \frac{\varepsilon}{\sqrt{\varepsilon - 1}}. \tag{16}$$

If the first or second conditions (16) are disturbed, then the dependency $\omega_1(\gamma)$ will be weaker and the relative error of γ will be greater than the relative error of ω_1. The strongest limitation is determined from the second inequalities (16) [it follows from Eq. (10)]. If ε is not close to 1, then we have the condition $\gamma \ll \sqrt{b/d}$. Thus, to measure the large values of γ, we should decrease the relative thickness of the dielectric layer d/b.

However, it is also important to address whether the waveguide mode amplitude is sufficiently large for measurements in the case when the layer thickness is so small. Note that, in accordance with the boundary condition, $E_\rho|_{\rho=b-0} = \varepsilon E_\rho|_{\rho=b+0}$ and $E_z|_{\rho=b-0} = E_z|_{\rho=b+0}$. In the case of a thin dielectric layer, the longitudinal component E_z is small in the layer and at its boundary $\rho = b$. The component E_ρ is larger than E_z, and it has a maximum at $\rho = b - 0$. Thus, the value $E_\rho|_{\rho=b-0}$ is the most convenient for the method under consideration. Therefore, only this component will be analyzed further.

Omitting cumbersome transformations, we write out the approximate formula that follows from Eqs. (1)–(5) under condition (7):

$$E_\rho \approx -\frac{8q\Theta(-\zeta)}{\pi c\beta\gamma\sqrt{\varepsilon\beta^2 - 1}b\sqrt{a}} \times \sum_{m=1}^{\infty}\begin{Bmatrix} \frac{\varepsilon I_1(k\rho)}{\sqrt{b}I_1(kb)}\cos(sd) & \text{for } \rho < b \\ \rho^{-1/2}\cos[s(a-\rho)] & \text{for } \rho > b \end{Bmatrix} \times \frac{\exp[-\omega^2\sigma^2(2V^2)^{-1}]}{df/d\omega}\sin(\omega\zeta V^{-1})\Bigg|_{\omega=\omega_m}, \tag{17}$$

where

$$f(\omega) \approx 2\frac{sI_1(kb)\sin(sd) - \varepsilon kI_0(kb)\cos(sd)}{\pi s\sqrt{ab}}. \tag{18}$$

If both conditions (7) and (10) are fulfilled, then the following approximation takes place:

$$E_\rho \approx -\frac{4q\Theta(-\zeta)}{(\varepsilon-1)(\varepsilon\gamma^{-2}+1)} \times \sum_{m=1}^{\infty}\begin{Bmatrix} \frac{2\pi\varepsilon\omega}{c\beta\gamma^2 d}\mathrm{e}^{-2kb}I_1(k\rho) & \text{for } \rho < b \\ \frac{\sqrt{2\pi\omega}\cos[s(a-\rho)]}{d\sqrt{c\beta\gamma^3\rho\cos(sd)}}\mathrm{e}^{-kb} & \text{for } \rho > b \end{Bmatrix} \times \exp[-\omega^2\sigma^2(2V^2)^{-1}]\sin(\omega\zeta V^{-1})\Bigg|_{\omega=\omega_m}. \tag{19}$$

One can see that the amplitude of the m mode at the vacuum-dielectric boundary is

$$E_{\rho m}|_{\rho=b-0} \approx \frac{4q\varepsilon\sqrt{2\pi\omega}\exp[-kb - \omega^2\sigma^2/(2V^2)]}{(\varepsilon - 1)(\varepsilon\gamma^{-2} + 1)d\sqrt{c\beta\gamma^3 b}}\Bigg|_{\omega=\omega_m}. \tag{20}$$

Under the condition $\gamma \gg \varepsilon/\sqrt{\varepsilon - 1}$, we can use approximation (15), and the expression for the 1st mode amplitude has the form

$$E_{\rho 1}|_{\rho=b-0} \approx \frac{4q\sqrt{2\pi\varepsilon^3}}{(\varepsilon - 1)^{3/2}\gamma^2 b^2}\left(\frac{b}{d}\right)^{3/2}\exp\left[-\frac{\varepsilon b}{(\varepsilon - 1)\gamma^2 d}\right] \times \exp\left[-\frac{\varepsilon^2\sigma^2}{2(\varepsilon - 1)^2\gamma^2 d^2}\right]. \tag{21}$$

As we see, the first mode amplitude is a nonmonotonic function of the layer thickness. The amplitude increases with decrease in d/b [because of the factor $(b/d)^{3/2}$] if the following inequalities are fulfilled:

$$d \geq \frac{\varepsilon b}{(\varepsilon - 1)\gamma^2}, \qquad d \geq \frac{\varepsilon\sigma}{\sqrt{2}(\varepsilon - 1)\gamma}. \tag{22}$$

When these conditions are disturbed, the exponential factors in Eq. (22) play the main role, and the amplitude decreases with a decrease in d.

It is interesting that the amplitude maximum occurs for relatively small values of d. For example, if $\sigma = 0$, then this maximum is reached for

$$d/b = 2\varepsilon/[3(\varepsilon - 1)\gamma^2] \ll 1. \tag{23}$$

The formula (21) gives the following value for such a layer:

$$E_{\rho 1}|_{\rho=b-0} = 2\sqrt{27\pi e^{-3}}q\gamma b^{-2}. \tag{24}$$

It is important that the amplitude maximum takes place when $d \ll b$, i.e., in the range of d that is convenient for measurements. For bunches with a finite length, the amplitude has a maximum at larger values of d. However, as we will see, these values are sufficiently small in typical situations.

III. NUMERICAL RESULTS AND DISCUSSION

Here we give some results of computations obtained on the basis of the exact formulas for the first mode.

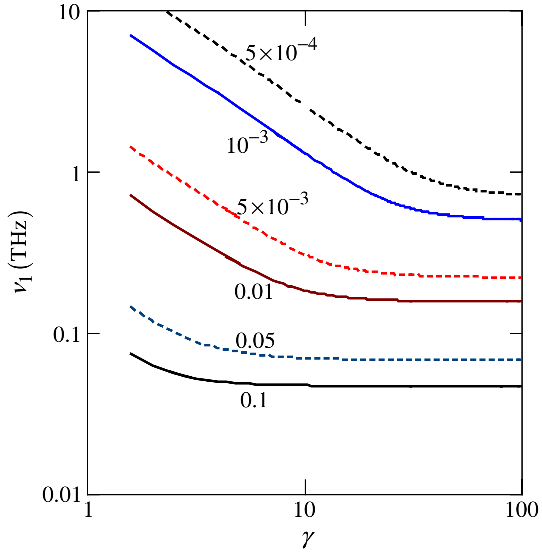

FIG. 2. The first mode frequency depending on the Lorentz factor for a waveguide with radius $a = 5$ mm and a dielectric layer with permittivity $\varepsilon = 4$; the magnitudes of d/a are given near the curves.

Figure 2 shows the dependencies of the first mode frequency $\nu_1 = \omega_1/(2\pi)$ on the Lorentz factor for different magnitudes of the dielectric layer thickness [computations have been performed on the basis of the exact dispersive equation (6)]. One can see that the essential dependency takes place for values of γ below some limit. This limit increases with a decrease in d/a. If $d/a = 0.1$, then we have a considerable dependency for $\gamma < 10$. However, for $d/a = 5 \times 10^{-4}$, the essential dependency takes place for $\gamma < 100$. It is interesting that the dependency of $\log\omega_1$ on $\log\gamma$ is almost linear for $\gamma < 70$ in the case of $d/a = 5 \times 10^{-4}$. In this range, the frequency ω_1 is

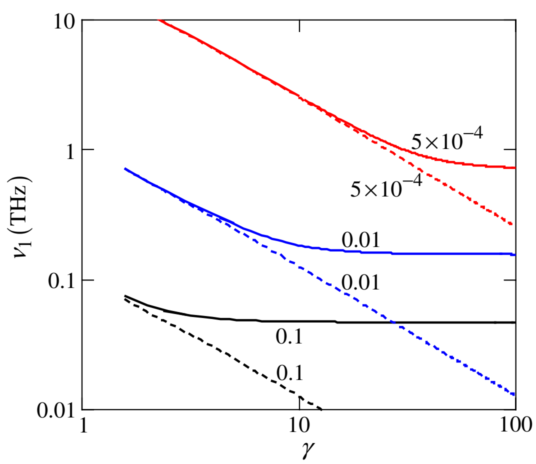

FIG. 3. The exact (solid) and approximate (dashed) magnitudes of the first mode frequency depending on the Lorentz factor for $a = 5$ mm and $\varepsilon = 4$; the values of d/a are given near the curves.

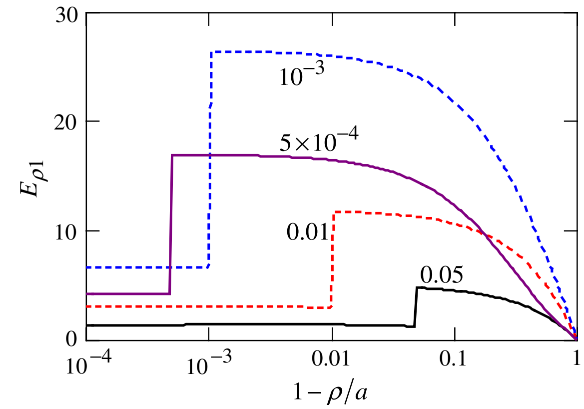

FIG. 4. Dependency of the radial electric field amplitude $E_{\rho 1}$ (KV/m) of the first mode on the distance from the waveguide boundary for $a = 5$ mm, $\varepsilon = 4$, $|q| = 1$ pC, $\sigma = 0$, and $\gamma = 30$; the magnitudes of d/a are given near the curves.

proportional to $1/\gamma$, i.e., the approximation (15) is true. This result is corroborated by Fig. 3, where the comparison between the solution of the strong dispersive equation (6) and the solution of the approximate equation (12) is given [note that Eq. (12) practically results in Eq. (15) for $\gamma > 3$].

It should be underscored that the linear parts of the curves in Fig. 2 are the most convenient for the determination of the Lorentz factor because the relative error of γ is equal to the relative error of the frequency. For the nonlinear parts of curves, obtaining the Lorentz factor is possible as well, but the accuracy will be lower. For example, in the case of $d/a = 5 \times 10^{-4}$, one can obtain that $\Delta\gamma/\gamma \sim 10 \times \Delta\omega_1/\omega_1$ for $\gamma = 100$. This result is favorable because the mode frequency can be measured with a very small error (10^{-4} and less).

Thus, it is important to use a thin dielectric layer for the determination of particle energy. However, the mode amplitude should be sufficiently large for measurement. Figure 4 shows the dependency of the radial electric field

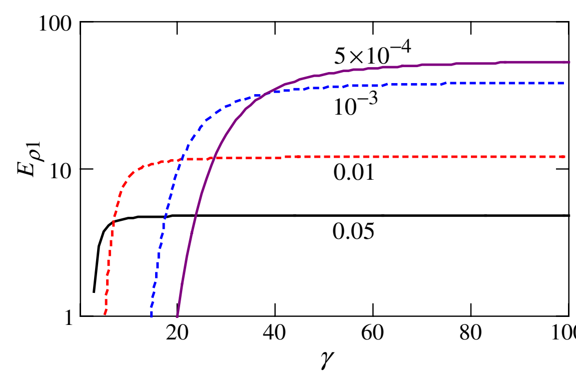

FIG. 5. Dependency of the radial electric field amplitude $E_{\rho 1}$ (KV/m) at $\rho = b - 0$ on the Lorentz factor for $a = 5$ mm, $\varepsilon = 4$, $|q| = 1$ pC, and $\sigma = 0$; the magnitudes of d/a are given near the curves.

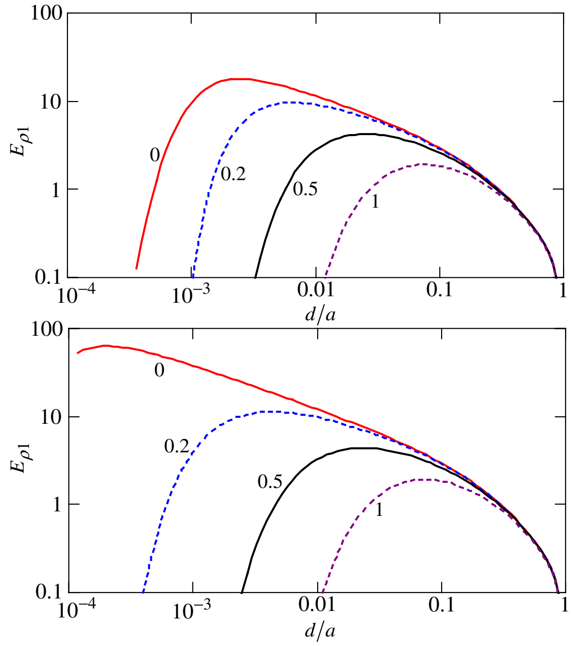

FIG. 6. Dependency of the radial electric field amplitude $E_{\rho 1}$ (KV/m) at $\rho = b - 0$ on the relative layer thickness d/a for $a = 5$ mm, $\varepsilon = 4$, $|q| = 1$ pC, $\gamma = 20$ (top) and $\gamma = 70$ (bottom); the magnitudes of σ (mm) are given near the curves.

amplitude on the distance from the waveguide boundary. As one would expect, the mode field has a maximum at $\rho = b - 0$, that is on the dielectric-vacuum boundary.

Note that the first mode amplitude increases with the Lorentz factor and approaches some constant for large γ (Fig. 5). As mentioned above, the dependency of the mode amplitude on the layer thickness is not monotonic (Fig. 6); instead, it increases with an increase in d/a up to some value, and then it decreases with further increase in d/a. The component $E_{\rho 1}$ has a maximum, as a rule, at small magnitudes of d/a ($\sim 10^{-4}$–10^{-1}). This fact is favorable for the method under consideration. Figure 6 shows $E_{\rho 1}$ for different values of parameter σ characterizing the bunch length (from $\sigma = 0$ to $\sigma = 1$ mm). One can see that the amplitude of the radial component is sufficiently large for measurements if the bunch charge is 1 pC.

It should be stressed that measurements are possible for smaller values of charges as well. The formulas (21) and (24) show that the radial field increases with decrease in the channel radius. Therefore we can hope that the method under consideration can be used for diagnostics of very small bunches and even for detection of single particles (if a very thin waveguide with radius <1 mm will be used).

The analysis performed here concerns the case of infinite waveguide. It is clear that a real waveguide must be long enough for implementation of the method under consideration. Estimation shows that the waveguide length L must be more than $a\gamma$. One can see that for $\gamma < 100$ and $a < 1$ cm we have $L > 1$ m that is quite realistic.

IV. CONCLUSION

We considered the nondestructive method of particle energy determination based on the measurement of the waveguide mode frequency. Note that Cherenkov radiation can be applied as well for other methods of measurement of the bunch energy. For example, an interesting technique using both deflecting magnetic field and Cherenkov effect has been developed in the paper [2]. This technique gives high time resolution but it is destructive for bunches.

The method under consideration can be realized in different ways. Here, we have analyzed the variant that is connected with the use of the dielectric layer. It has been shown that a decrease of the layer thickness results in an increase in the dependency of the mode frequency on the Lorentz factor γ. Therefore, this method is effective if the layer thickness is much less than the waveguide radius. In such a situation, there is a wide range of γ where the accuracy of the determination of γ is close to the accuracy of the mode frequency measurement. Computations show that, as a rule, the first mode amplitude is sufficient for measurement in the case of a typical pico-Coulomb bunch. The mode frequency can be measured with very small error ($\leq 10^{-4}$); therefore the method under consideration can be the base for real-time particle energy monitoring with high accuracy. It is essential as well that this method is nondestructive, i.e., measurements practically have no effect on the bunch.

Note that there are also other variants of the method under consideration [7–13]. For example, we can use a waveguide containing a certain "wire metamaterial" (a system of parallel wires having a dielectric coating) [12]. This method allows us to determine the Lorentz factor with high accuracy, but only in some narrow range. The other version consists of the use of a circular waveguide with a grid wall with small rectangular cells [13]. Such a structure gives the essential dependency of the mode frequency on the Lorentz factor in a wide range of values. However, it should be noted that the method analyzed in the present paper is the simplest for experimental realization.

It should be mentioned that recently the waveguide with a dielectric layer was offered as well for measurement of the electron bunch length [15]. Thus, we can believe that this structure is suitable for versatile nondestructive diagnostics of bunches.

ACKNOWLEDGMENTS

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[1] V. P. Zrelov, *Vavilov-Cherenkov Radiation in High-Energy Physics* (Israel Program for Scientific Translations, Jerusalem, 1970).

[2] L. N. Elberson, Yu. Ping, R. L. Shepherd, P. K. Patel, A. J. Mackinnon, and W. T. Hill III, Rev. Sci. Instrum. **80**, 023302 (2009).
[3] J. G. Power, M. E. Conde, W. Gai, R. Konecny, P. Schoessow, and A. D. Kanareykin, Phys. Rev. ST Accel. Beams **3**, 101302 (2000).
[4] A. M. Cook, R. Tikhoplav, S. Y. Tochitsky, G. Travish, O. B. Williams, and J. B. Rosenzweig, Phys. Rev. Lett. **103**, 095003 (2009).
[5] A. Kanareykin, J. Phys. Conf. Ser. **236**, 012032 (2010).
[6] V. V. Poliektov, A. A. Vetrov, K. A. Trukhanov, and V. I. Shvedunov, Instrum. Exp. Tech. **51**, 191 (2008).
[7] A. V. Tyukhtin, S. P. Antipov, A. Kanareykin, and P. Schoessow, in *Proceedings of the 2007 Particle Accelerator Conference, Albuquerque, New Mexico* (IEEE, New York, 2007), p. 4156.
[8] A. V. Tyukhtin, Tech. Phys. Lett. **34**, 884 (2008).
[9] A. V. Tyukhtin, Tech. Phys. Lett. **35**, 263 (2009).
[10] A. V. Tyukhtin, P. Schoessow, A. Kanareykin, and S. Antipov, AIP Conf. Proc. **1086**, 604 (2009).
[11] A. V. Tyukhtin, S. P. Antipov, A. Kanareykin, and P. Schoessow, in *Proceedings of the 2009 Particle Accelerator Conference, Vancouver, Canada* (IEEE, Piscataway, NJ, 2009), p. 4033.
[12] A. V. Tyukhtin, E. G. Doilnitsina, and A. Kanareykin, in *Proceedings of the IPAC'10 Conference, Kyoto, Japan* (ICR, Kyoto, 2010), p. 1071.
[13] A. V. Tyukhtin, in *Proceedings of IPAC'11, San Sebastian, Spain* (EPS-AG, Spain, 2011), p. 1302.
[14] B. M. Bolotovskii, Sov. Phys. Usp. **4**, 781 (1962).
[15] S. V. Shchelkunov, T. C. Marshall, J. L. Hirshfield, and M. A. LaPointe, Phys. Rev. ST Accel. Beams **8**, 062801 (2005).