Cherenkov-transition radiation in a waveguide with a dielectric-vacuum boundary

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We analyze the electromagnetic field of a charged particle moving uniformly in a circular waveguide and crossing the boundary between a dielectric and a vacuum. Our study focuses on the case when Cherenkov radiation is generated in the dielectric. Analytical and numerical investigation of the waveguide modes is performed. We show that a large radiation can be excited in the vacuum area. The mode amplitudes in the vacuum can be greater than those in the dielectric. The field from a Gaussian bunch is also studied. We note that the effect under consideration can be used to generate a large quasimonochromatic or multimode radiation.

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I. INTRODUCTION

The electromagnetic radiation produced by charge particles moving in the presence of material media has been investigated since the second quarter of the 20th century. Many experimental and theoretical works in this area have been published, and the most important results have been presented in monographs and reviews [1–5]. However, some essential questions have only been partially analyzed. This especially concerns the electromagnetic field structure in contrast to energetic characteristics studied in detail in many papers [5–7]. But the analysis of the field structure is important as well. For example, it allows clarifying the problem of generation of a large radiation in an area far from a material bulk. The radiation used in Cherenkov detectors is usually measured in a vacuum [1], but these devices do not generate powerful radiation. This is also true for detectors using transition radiation (TR).

A dramatically different situation occurs with the wakefield acceleration technique. One of the variants of this method consists of the application of the Cherenkov field (the so-called "wakefield") from a large bunch ("driver") moving in a vacuum channel in a circular waveguide loaded with a dielectric layer. Another bunch is accelerated in the driver field. The final experiments show an accelerating field >100 MV/m for frequencies 10–30 GHz [8,9] and >10 GV/m for the THz region [10]. However, this field exists in the vacuum channel behind the charge. Let us consider the following question: can we obtain a large field in the waveguide far from the area where the Cherenkov radiation (CR) is generated? Here we show that this is possible.

This paper is devoted first of all to the Cherenkov-transition radiation (CTR) effect which has not been practically described for problems with waveguides (we emphasize that CTR is not a usual TR). Note that this problem is of essential importance. CTR can be used for generation of large field in a vacuum zone and bunching of particles. As well, in this connection we can mention the idea of generation of radiation in a waveguide with periodic structure composed of dielectric plates and vacuum spaces [11]. For this perspective method, an analysis of penetration of CR through a dielectric-vacuum interface is a key problem.

II. A POINT CHARGE: ANALYTICAL INVESTIGATION

First, we consider the following problem. A point charge particle q moves in the metal circular waveguide of radius a along its axis (z axis) through the interface ($z = 0$) between homogeneous isotropic dielectrics with permittivity ε_1 for $z < 0$ and ε_2 for $z > 0$ (Fig. 1). The media are nonmagnetic and nondispersive. The charge moves uniformly with a velocity $\vec{V} = c\beta\vec{e}_z$ (where c is the light velocity in the vacuum) and intersects the boundary at the moment $t = 0$. Note that, in practice, the bunch moves in the vacuum channel. However, for simplicity, we assume that the channel radius is negligible. This assumption is warranted because a channel with a radius considerably less than the typical wavelength does not influence the generated radiation [3].

The analytical solution of this problem is traditionally found as expansions into a series of eigenfunctions of the transversal operator [6,7,12]. In each of two areas, the field has two summands:

$$\vec{E}_{1,2} = \vec{E}^q_{1,2} + \vec{E}^b_{1,2}. \tag{1}$$

The first summand ($\vec{E}^q_{1,2}$) is the field in a regular waveguide with homogeneous filling (a so-called "forced" field [2]). It contains CR if the charge velocity exceeds the Cherenkov threshold. The second summand ($\vec{E}^b_{1,2}$) (a so-called "free" field [2]) is connected with the influence of the boundary and includes TR.

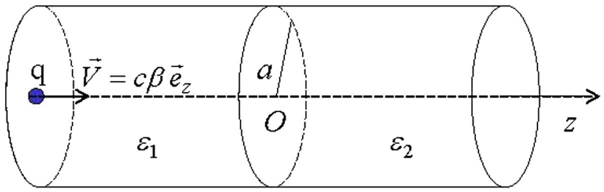

FIG. 1. Geometry of the problem.

The formulas for the electromagnetic field components can be derived from the formulas for the general case of the boundary between arbitrary dispersive homogeneous media [6,7,12]. Here, we give only expressions for the longitudinal components of the electric field:

$$E^q_{z1,2} = \frac{-4q}{a^2\varepsilon_{1,2}} \sum_{n=1}^{\infty} \frac{J_0(\chi_{0n}r/a)}{J_1^2(\chi_{0n})} \times \begin{cases} \exp\left[-\frac{\beta\omega_n}{\sqrt{1-\beta^2\varepsilon_{1,2}}} \left|\frac{z}{c\beta} - t\right|\right], & \varepsilon_{1,2}\beta^2 < 1, \\ \cos\left[\Omega_{n1,2}\left(\frac{z}{c\beta} - t\right)\right]\theta(ct\beta - z), & \varepsilon_{1,2}\beta^2 > 1, \end{cases} \tag{2}$$

$$E^b_{z1,2} = \frac{2\mathrm{iqc}^2\beta}{\pi a^4 \varepsilon_{1,2}} \sum_{n=1}^{\infty} \frac{\chi_{0n}^2 J_0(\chi_{0n}r/a)}{J_1^2(\chi_{0n})} \int_{-\infty}^{\infty} B_{n1,2} \times \exp[-\mathrm{i}(\omega t - k_{z1,2}|z|)]d\omega, \tag{3}$$

where

$$B_{n1,2} = \frac{(\varepsilon_{1,2} - \varepsilon_{2,1})[\omega(\mp 1 \pm \varepsilon_{1,2}\beta^2) - \beta\sqrt{\omega^2\varepsilon_{2,1} - \omega_n^2}]}{g(\omega)[\omega^2(1-\varepsilon_{1,2}\beta^2) + \beta^2\omega_n^2](\omega \pm \beta\sqrt{\omega^2\varepsilon_{2,1} - \omega_n^2})},$$

$$k_{z1,2} = \sqrt{\omega^2\varepsilon_{1,2} - \omega_n^2}/c,$$

$$g(\omega) = \varepsilon_1\sqrt{\omega^2\varepsilon_2 - \omega_n^2} + \varepsilon_2\sqrt{\omega^2\varepsilon_1 - \omega_n^2}, \quad \omega_n = \chi_{0n}c/a,$$

$$\Omega_{n1,2} = \beta\omega_n(\beta^2\varepsilon_{1,2} - 1)^{-1/2},$$

χ_{0n} is the n^{th} zero of the Bessel function ($J_0(\chi_{0n}) = 0$), and $\theta(x)$ is the Heaviside step function. The radicals are defined on the real axis by the following rule: $\mathrm{sgn}\sqrt{\omega^2\varepsilon_{1,2} - \omega_n^2} = \mathrm{sgn}(\omega)$ if $\omega^2\varepsilon_{1,2} > \omega_n^2$, and $\mathrm{Im}\sqrt{\omega^2\varepsilon_{1,2} - \omega_n^2} > 0$ if $\omega^2\varepsilon_{1,2} < \omega_n^2$ (for the case of negligible losses when $\mathrm{Im}\varepsilon_{1,2} \to +0$). Note that the upper sign in $B_{n1,2}$ applies to B_{n1}, and the lower sign applies to B_{n2}.

Further, we analyze the field of a particle flying from the dielectric into the vacuum ($\varepsilon_2 = 1$). We investigate the free field (3) with two methods: analytical and numerical. They have been described in a prior paper [12] for the case of the boundary between a vacuum and plasma. We stress that, as there is no CR in such situation, it is dramatically different from the case considered here.

We note some results of the analytical investigation for the area $z > 0$. Figure 2 shows the singularities of the function B_{n2} and the branch cuts (the positional relationship of the singularities was determined by taking into account the small losses). The poles of B_{n2} are located at the points

$$\pm\omega_{0n}^{(1)} = \pm\Omega_{n1} - i0,$$
$$\pm\omega_{0n}^{(2)} = \pm i\beta\omega_n(1-\beta^2)^{-1/2}.$$

The branch cuts are defined by the equations

$$\mathrm{Re}\sqrt{\omega^2 - \omega_n^2} = 0, \qquad \mathrm{Re}\sqrt{\omega^2\varepsilon_1 - \omega_n^2} = 0.$$

For obtaining asymptotic expressions, we use the steepest descent technique [13]. Preliminarily, it is convenient to make the following change of variables for the vacuum region: $\omega = \omega_n \cosh\chi$. This replacement removes the pair of branch points $\pm\omega_n$. The steepest descent path (SDP) consists of two branches, as described in [12]. The poles and the branch points can be crossed in the transformation of the initial integration contour into the SDP, and the contributions from the corresponding singularities should be included in asymptotic expressions.

Omitting all the transforms, we present the most important result of the analytical investigation concerning the contributions of the poles $\pm\omega_{0n}^{(1)}$. In a vacuum, these contributions give the transmitted wave of the CR, the so-called Cherenkov-transition radiation (CTR). This radiation exists only at

$$\beta_C < \beta < \beta_{CT},$$

where

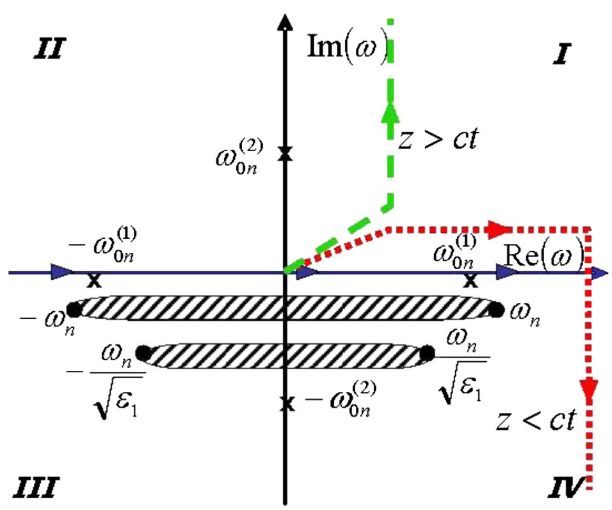

FIG. 2. Poles (crosses), branch points (circlets), branch cuts and integration paths (initial and transformed for computation) in a complex plane of ω for the field components in the vacuum region under the condition $\varepsilon_1\beta^2 > 1$.

$$\beta_C = \varepsilon_1^{-1/2}, \qquad \beta_{CT} = (\varepsilon_1 - 1)^{-1/2}.$$

The lower threshold β_C is the threshold for the CR, and the upper threshold β_{CT} is explained by total internal reflection of the CR from the boundary that occurs at $\beta > \beta_{CT}$. The longitudinal component of the CTR is written in the following form:

$$E_z^{\mathrm{CTR}} = \frac{-8q}{a^2} \sum_{n=1}^{\infty} \frac{J_0(\chi_{0n} r/a)\cos(\Omega_{n1} t - \varkappa_{n1} z)}{J_1^2(\chi_{0n})[1 + \varepsilon_1\sqrt{1-\beta^2(\varepsilon_1 - 1)}]}\theta(z_1 - z), \tag{4}$$

where

$$\varkappa_{n1} = \Omega_{n1}\sqrt{1-\beta^2(\varepsilon_1 - 1)}/c\beta,$$
$$z_1 = ct\sqrt{1-\beta^2(\varepsilon_1 - 1)}/\beta.$$

In the vacuum region, the CTR exists only in the area $0 < z < z_1$. Note that the value z_1 is determined by the group velocity of the waveguide waves V_g. Indeed, it is connected with the phase velocity V_p by the formula:

$$V_g = c^2/V_p = c^2\varkappa_{n1}/\Omega_{n1} = c\sqrt{1-\beta^2(\varepsilon_1 - 1)}/\beta.$$

Thus $z_1 = V_g t$, i.e., the area of the CTR increases in time with velocity V_g. A comparison of the analytical estimations and the computations (below) shows that the CTR is the main part of radiation in this region.

III. A POINT CHARGE: NUMERICAL INVESTIGATION

For the numerical calculations, the exact integral representations (1)–(3) are used. We develop an efficient algorithm based on a certain transformation of the integration path in the complex plane. Earlier, such an algorithm was developed for the computation of the electromagnetic field in different unbounded or semibounded media [12,14–17]. We demonstrate this method for the vacuum area. The initial integration path is transformed into a new contour in the upper half-plane (the dashed green line in Fig. 2) for $z > ct$ (before the "wave front" $z = ct$) and into another contour (the dotted red line) in the lower half-plane for $z < ct$ (behind the wave front). New contours bypass all of

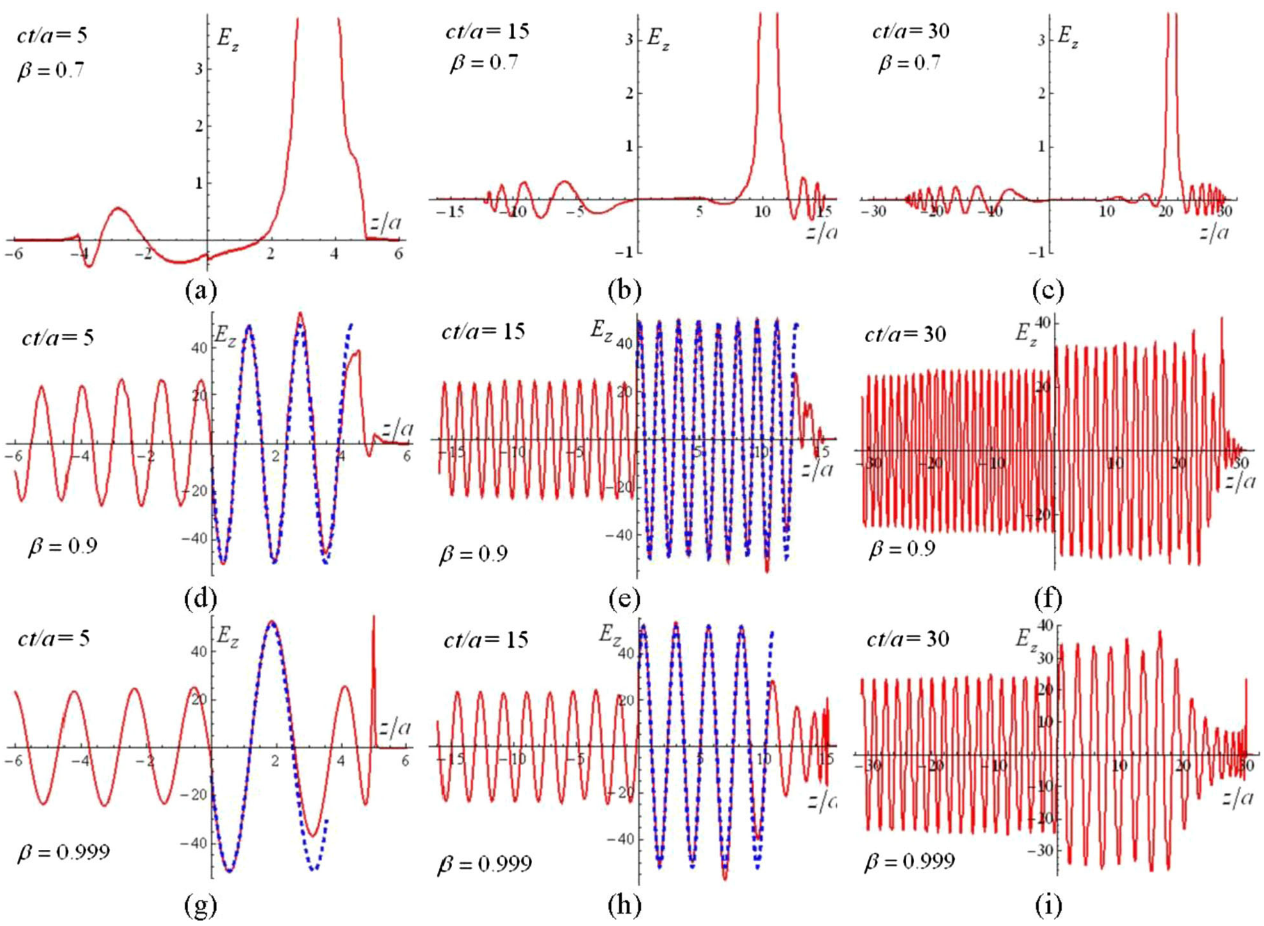

FIG. 3. The dependence of the longitudinal component E_z (kV/m) of the first mode of the total field (a continuous red line) on the distance z/a at $r = 0$ for different dimensionless times ct/a and velocities β; $\varepsilon_1 = 1.5$, $\varepsilon_2 = 1$, $a = 5$ cm, and $q = -1$ nC. A dashed blue line applies to the first mode of the CTR (4) in the vacuum region.

the singularities and travel parallel to the SDP for large values of ω (i.e., parallel to the imaginary axis). We stress that the computations are performed with the exact formulas and without any limitations of the problem parameters. The field in the dielectric region is computed analogously.

Figure 3 shows the longitudinal component E_z of the first mode of the total electric field in the dielectric and the vacuum for different velocities β and at different time moments. If $\beta < \beta_C$, then the field both in the dielectric and in the vacuum consists of TR and a quasi-Coulomb field that is essential only near the charge [Figs. 3(a)–3(c)]. If $\beta > \beta_C$, then CR is excited in the dielectric [Figs. 3(d)–3(i)]. It reflects off the boundary and can penetrate through one (i.e., the CTR effect can take place in the vacuum region). Note that the upper threshold for the CTR in vacuum $\beta_{CT} = (\varepsilon_1 - 1)^{-1/2}$ is of importance only for $\varepsilon_1 > 2$. We use in Fig. 3 the value $\varepsilon_1 = 1.5$; therefore, CTR is generated in the vacuum at $\beta_C < \beta < 1$. One can see that the field of the mode is quasimonochromatic in the dielectric and in some parts of the vacuum area [Figs. 3(d)–3(i)].

As the total field is approximately equal to the CTR field (4) in the area $0 < z < z_1$ [Figs. 3(d), 3(e), 3(g), and 3(h)], we can use the formula (4) for the analysis of the wave field from a bunch of finite size.

IV. CTR FROM A GAUSSIAN BUNCH

A typical bunch used as a driver for the wakefield acceleration technique is characterized by a Gaussian distribution along the z axis and a negligible thickness, i.e., the charge density is

$$\rho = q\delta(x)\delta(y)\exp[-\zeta^2/(2\sigma^2)]/(\sqrt{2\pi}\sigma),$$

where $\zeta = z - Vt$. The amplitudes of the waveguide modes of this bunch are equal to the ones of the point charge multiplied by $\exp[-\Omega_{m1}^2\sigma^2(2V^2)^{-1}]$. This exponential factor results in the reduction of the significance of the modes with large numbers.

The frequencies and amplitudes of the CTR in the vacuum are shown in Fig. 4. The frequencies of the CTR decrease with the velocity β, but the amplitudes increase. For the

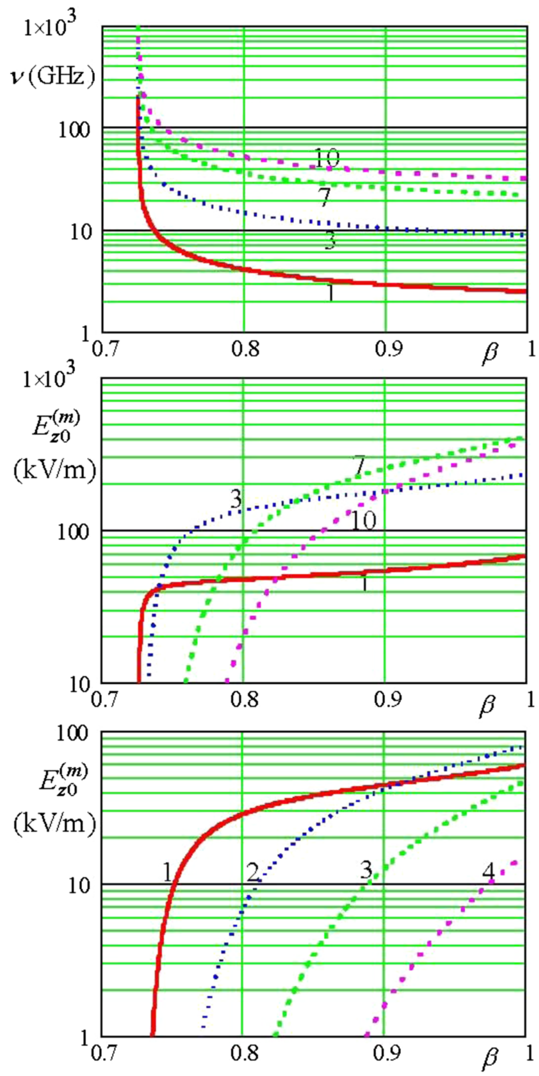

FIG. 4. The frequencies ν_m (top) and amplitudes E_{z0}^m of several CTR modes in the vacuum part of the waveguide for $\varepsilon_1 = 1.9$, $a = 5$ cm, $q = -1$ nC, $\sigma = 2$ mm (middle), and $\sigma = 10$ mm (bottom); the mode numbers are shown close to the curves.

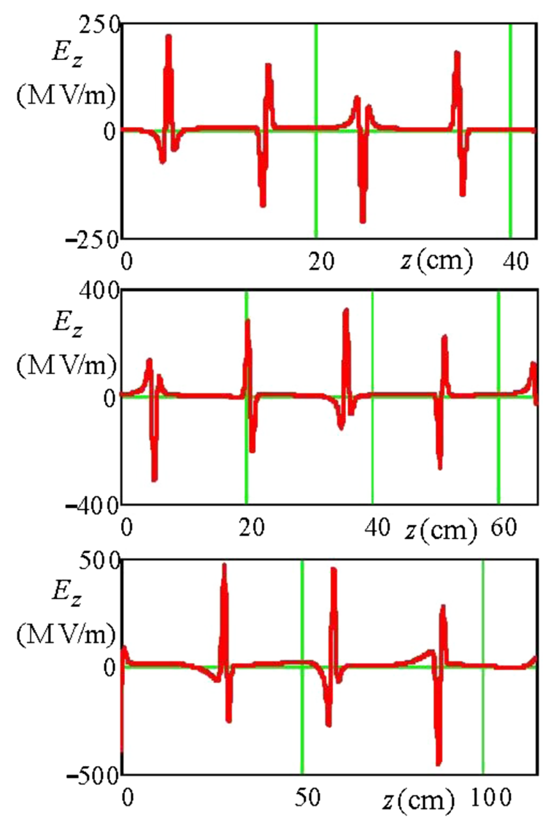

FIG. 5. The dependence of the longitudinal component of the electric field of the CTR on the distance from the boundary for $r = 0$, $a = 5$ cm, $q = -100$ nC, $\sigma = 2$ mm, $\beta = 0.999$, $\varepsilon_1 = 1.5$ (top), $\varepsilon_1 = 1.7$ (middle), and $\varepsilon_1 = 1.9$ (bottom).

relatively short bunch ($\sigma = 2$ mm), many modes have significant amplitudes (CTR is multimode). For the longer bunch ($\sigma = 10$ mm), only a few of modes are of importance.

Some examples of multimode CTRs in the vacuum area are presented in Fig. 5 for the case of electron bunch with $\sigma = 2$ mm and $q = -100$ nC. Such bunches can be formed nowadays [18]. Note that the field has maximum magnitude up to 500 MV/m. The structure of CTR is similar to the multimode wakefield of a bunch in an infinite regular waveguide with a dielectric [19].

V. CONCLUSION

In summary, we have shown that Cherenkov-transition radiation can be the main part of the field in the vacuum area of the waveguide. The modes of the CTR are monochromatic, similar to the modes of CR in an infinite regular waveguide. The amplitudes of the CTR modes in the vacuum can be greater than the amplitudes of the CR modes in the dielectric. This radiation is generated at a certain bunch speed range, and the area of the CTR increases in time with the group velocity of the waveguide waves. The radiation from a bunch with finite length can be both quasimonochromatic and multimode. The CTR effect can be used to generate large electromagnetic fields in the vacuum area of the waveguide.

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