Scaling of the longitudinal electric field and transformer ratio in a nonlinear plasma wakefield accelerator

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The scaling of the two important figures of merit, the transformer ratio T and the longitudinal electric field E_z, with the peak drive-bunch current I_p, in a nonlinear plasma wakefield accelerator is presented for the first time. The longitudinal field scales as $I_P^{0.623\pm0.007}$, in good agreement with nonlinear wakefield theory ($\sim I_P^{0.5}$), while the unloaded transformer ratio is shown to be greater than unity and scales weakly with the bunch current. The effect of bunch head erosion on both parameters is also discussed.

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Plasma based acceleration is currently an exploding field of research, due to the large electric fields that plasmas are able to support. Recently, GeV scale particle energy gains were achieved in both the electron bunch driven [1] and the laser driven case [2], culminating in the demonstration accelerating gradients in excess of 50 GeV/m sustained in a meter scale plasma [3]. These gradients are 3 orders of magnitude larger than those achievable in conventional accelerators, offering an attractive future for plasma based accelerators in next generation collider designs.

This paper presents a measurement of the wakefield amplitude and unloaded transformer ratio with respect to incident peak bunch current in a nonlinear plasma wakefield accelerator (PWFA), parameters critical to a future collider design. The nonlinear PWFA, where a large plasma wave is driven by an ultrashort electron bunch whose density n_b is greater than that of the background plasma n_p, can be thought of as an energy transfer device. A drive electron bunch is injected into a plasma where it transfers energy by expelling the mobile plasma electrons, driving a decelerating field across itself, and therefore experiencing a longitudinally varying retarding potential. After the passage of the drive bunch, the expelled plasma electrons are attracted back by the space charge force of the immobile massive plasma ions setting up an accelerating field, meaning any witness electron bunch will experience an accelerating potential. As the drive and witness bunch are both typically ultrarelativistic, there is no phase slippage for any particles sampling the wake. The goal for a collider would be to maximize the energy gained by the witness bunch, while minimizing the structure length.

This leads to two figures of merit. First is the transformer ratio T, defined as the ratio of maximum voltage V_g, that can be gained by a trailing particle to the voltage lost by a particle in the drive bunch V_l. In this case

$$T = \frac{V_g}{V_l} = \frac{E_{a,\max}/L}{E_{d,\max}/L}, \tag{1}$$

where $E_{a,\max}$ is the maximum longitudinal accelerating field, $E_{d,\max}$ is the maximum longitudinal decelerating field, and L is the length of the plasma column. Note that in the experiment the transformer ratios are determined from the peak energy gain and loss by the beam particles. As the interaction of the bunch and the plasma can only be sustained as long as there is enough energy in the bunch to continue to drive the maximum decelerating field, the maximum energy that can be gained from a single plasma stage is TW_0, where W_0 is the energy of the incident drive bunch. However, T will only tell us the maximum possible energy gain. The minimum plasma length $L_{\min}$ necessary to reach this energy gain is determined by the maximum decelerating field as $L_{\min} = W_0/E_{d,\max}$, which can be rewritten as a function of T and $E_{a,\max}$ as $L_{\min} = TW_0/E_{a,\max}$. The second figure of merit is thus the maximum accelerating gradient itself, which will govern the length of the plasma structure at a given T.

The linear regime of a PWFA, where $n_b < n_p$, characterized by small density modulations in the plasma charge density and very small plasma electron velocities, is analytically tractable. Using a bunch with a constant longitudinal profile, the scaling of the wake amplitude in the linear regime is determined, yielding $E_z \sim 4\pi \frac{m_e c \omega_p}{e} \frac{I_p}{I_A}$, where m_e is the mass of the electron, c is the speed of light, e is the electron charge, ω_p is the plasma frequency, I_A is the Alfven current, and I_p is the peak current of the incident electron bunch. The linear wakefield response is therefore linear in I_p. In this regime, the transformer ratio is one for short Gaussian bunches [4], and is limited to two for any symmetric bunches [5].

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In the nonlinear regime $n_b > n_p$, the plasma electrons are all swept out of the bunch volume, leaving large modulations in the charge density at any given location. The excited radial currents source the wakefields [6], meaning the wakes are fully electromagnetic in nature. This requires a fundamentally different approach to determine the wake amplitude scale.

Because of the nonlinearity of the process, studies in this regime have been carried out employing particle-in-cell (PIC) simulations. Using the results of these simulations the tendency of the plasma electrons to cohere into sheaths, as shown in Fig. 1, became apparent. This allows the motion of the plasma electron, and the wakes, to be parametrized by the trajectory taken by an electron on the inside of this plasma sheath, as shown by [7,8].

Using [8], it is possible to derive estimates for the expected fields and relationship of the wakes to the peak current of the driving electron bunch in the nonlinear regime. Because the plasma electrons are entirely swept out of the bunch volume, the radial size of the bunch couples very weakly to the wakefield response and can be ignored. The equation governing the trajectory of the plasma sheath R, after normalizing R and $\xi = z - ct$ to $1/k_p\sqrt{I_P/I_A}$, is given as

$$\ddot{R} = \frac{4F(\xi) - [2(R\dot{R})^2 + 1]}{R^3}, \tag{2}$$

where $\dot{R} = dR/d\xi$ and $F(\xi)$ are functions whose peaks are unity, representing the shape of the current profile. The longitudinal field is given from [8] as

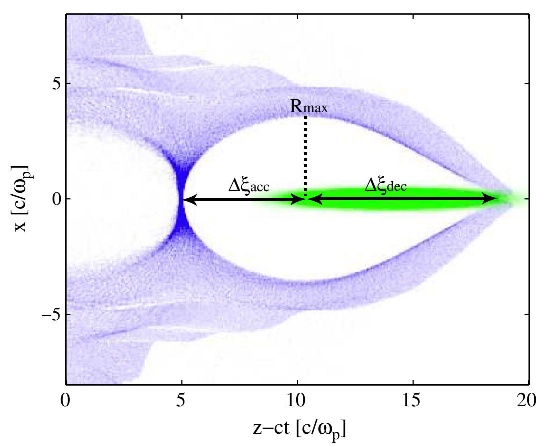

FIG. 1. An example of the plasma electron response, in blue, to a dense electron bunch, in green, propagating at an ultrarelativistic velocity to the right. Plasma electrons are fully evacuated from the bunch volume and beyond, and are traveling in a narrow sheath around the ions. The location of the peak radius of this bubble is where the wake changes sign, from decelerating to accelerating. The time scale of each is marked. The simulation code QUICKPIC [16] was used here.

$$E_z = \frac{m_e c\omega_p}{e}\sqrt{\frac{I_P}{I_A}}\frac{d(R^2)}{d\xi} = E_{gs}\frac{d(R^2)}{d\xi}. \tag{3}$$

The global scale for the electric fields in this system is therefore E_{gs}. Equation (2) describes the plasma sheath being driven off axis by the electron beam and returning due to the ion-column force while also subject to its own self-fields. It is symmetric in $\xi \to -\xi$ and $d/d\xi \to -d/d\xi$, meaning that knowing R and $\dot{R}$ at any point in the trajectory will define the entirety of the path. From Eq. (3), it is apparent that the longitudinal fields will vanish at the peak of the trajectory. This is where the phase flips from decelerating to accelerating. Also apparent in this equation, since $E_z \propto R\dot{R}$, the harder the sheath is driven off axis, the larger the decelerating wake. In the nonlinear regime the wakes can be expected to vary as $\sqrt{I_p}$, as opposed to I_p in the linear case.

The transformer ratio, from Eq. (3), is given as

$$T \sim \frac{R_{max}^2/\Delta\xi_{acc}}{R_{max}^2/\Delta\xi_{dec}} = \frac{\Delta\xi_{dec}}{\Delta\xi_{acc}}, \tag{4}$$

where $\Delta\xi$ is the time scale of each phase, shown in Fig. 1. The $\Delta\xi_{acc}$ is dependent only on the maximum excursion, R_{max}, not on how the sheath got there, as it occurs after $R = R_{max}$ and $\dot{R} = 0$. As R_{max} is only sensitive to I_p, and no significant bunch charge sits in this region, whether the bunch is long or short will have no real effect, meaning this parameter is set. However, $\Delta\xi_{dec}$ is strongly coupled to the bunch, both to the peak current and the shape. Long ramped bunches slowly drive the plasma sheath off axis and have a correspondingly long $\Delta\xi_{dec}$. Short bunches eject plasma electrons very fast, and have a short $\Delta\xi_{dec}$. Therefore T is larger for long bunches.

The experiment was carried out using the linear accelerator (linac) [9,10] at the SLAC National Accelerator Laboratory (SLAC) in the Final Focus Test Beam Facility located at the end of the SLAC linac. The electron bunches were delivered in a single bunch with 3 nC of charge or 1.8×10^{10} electrons, normalized emittances of $\epsilon_{N,x} \sim 60$ μm and $\epsilon_{N,y} \sim 6$ μm, and were focused down to a spot size of 10 μm. The bunch length σ_z was varied between 10 and 45 μm. The electron bunch and plasma source parameters used in this experiment are similar to those envisioned for a single stage of a multistage TeV class linear collider based on plasma wakefield acceleration [11]. The peak current of the incident bunch was measured using the longitudinal phase space matching method, whereby the incoming energy spectrum of the bunch was measured and matched to the corresponding longitudinal profile using the strong correlation between the two [1]. The bunch profiles can have long heads or tails, but are largely Gaussian and symmetric in nature. The bunch energy was set at either 28.5 or 42 GeV.

The plasma source was a neutral lithium vapor confined in a heat pipe oven by inert helium gas [3,12]. The neutral gas density was set at $n_0 = 2.7 \times 10^{17}/\mathrm{cm}^3$ and ramped up over a length of 5 cm yielding an estimated $n_b/n_p \sim 1$–5. Because of the effect of adiabatic damping [13], the actual n_b/n_p was likely larger. The heat pipe oven gave a flat longitudinal profile over 85 cm full width at half maximum (FWHM). As the beam head is known to erode over the 85 cm plasma length (42 GeV bunches) [3], the results were checked with a 10 cm FWHM plasma length and 28.5 GeV bunches. The gradient in this case is ambiguous, as the plasma density profile is Gaussian rather than flat, but the transformer ratio and its relationship to the peak current is unaffected.

Lithium, with a first ionization energy of 5.39 eV, was easily ionized by the electric fields of the head of the electron bunch to create the plasma. The experiment was designed to utilize a single bunch to sample the full wakefield response. The core of the bunch drove the wake while the tail of the bunch sampled the resulting accelerating fields. Using $n_0 = 2.7 \times 10^{17}/\mathrm{cm}^3$, along with the average bunch from the SLAC linac, with a length of 20 μm and $I_p = 17$ kA, gives $E_{\mathrm{gs}} \sim 50$ GV/m, in good agreement with the value measured in the experiment [3].

The wakefield amplitude is obtained by dividing the bunch particles energy change by the plasma length. The electron bunch length is on the same order as the plasma wavelength and it samples all phases of the wake. As the sampled fields are extremely large, the bunch exits the plasma with a large and continuous energy spread, depending on the propagation length. By locating the lowest and highest energy charge, and knowing their initial energy and propagation distance in the plasma, the peak decelerating and accelerating fields can be determined.

The post-plasma energy spectrum was measured using a dipole spectrometer, dispersing the bunch after the plasma exit. Figure 2 shows an example of the raw energy spectrometer image taken by a CCD camera and the derived energy profile, subdivided into four regions. In region I is the accelerated charge, region II contains the beam head, which had too low a current to ionize and therefore retains its initial energy, region III is the beam core which has driven the wake, and region IV is the lowest energy charge. Note that field ionization occurs early in the bunch (field of electrons in region II) and the trailing electrons in regions III, IV, and I propagate in the fully ionized plasma. Of most interest is identification of the highest and lowest energy charge. The lowest energy particles, affected only by the wake, are located in the peak in the spectrum in region IV. The lower energy roll-off is due to the particles radiating as they oscillate in the plasma, and not due to the wake itself [14]. Locating the highest energy charge is slightly more complicated. Because of the ~100% energy spread, imaging the plasma exit with magnetic optics was not possible. This necessitated capturing the bunch profile at two locations to distinguish energy gain from the possible transverse momentum that an off-axis beam tail can acquire from the large plasma focusing force [3]. The peak energy gain was then obtained from the highest energy particles in region I.

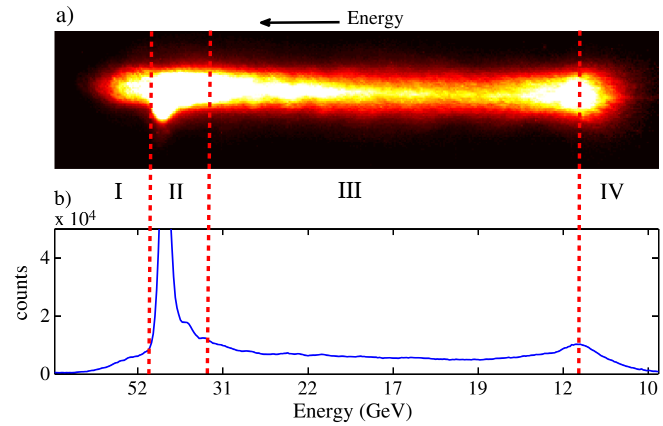

FIG. 2. An example of (a) the raw image, and (b) derived energy profile, from the energy spectrometer diagnostic (incoming bunch energy: 42 GeV). Four distinct regions can be seen. Region I contains the accelerated charge, region II the beam head which has too low a current to ionize, region III the beam core driving the wake, and region IV the lowest energy charge. The picture color table is chosen saturated over most of the energy range to make the trailing particles that gain energy (region I) visible on the same picture as the large number of bunch core electrons that lose energy.

Figure 3 shows the measured accelerating and decelerating fields plotted against the incident peak current. The measured maximum decelerating wake amplitude, shown in red, is nicely correlated to the peak current. The wake amplitudes vary from 10–30 GV/m. A power law fit is shown in black, where

$$E_{z,\mathrm{max\ decel}} \propto I_p^{0.623\pm0.007}. \tag{5}$$

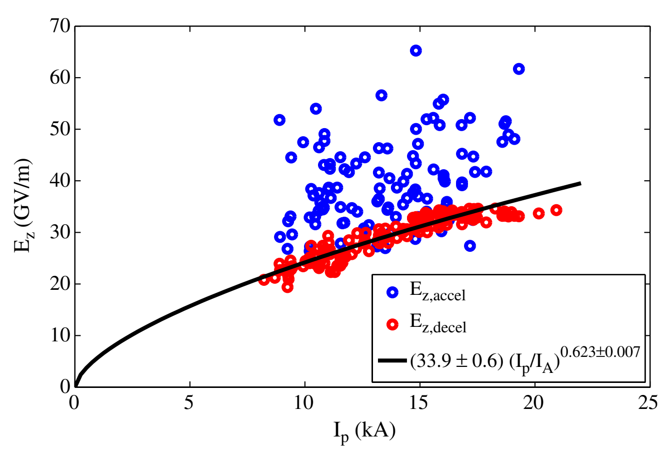

FIG. 3. The maximum accelerating and decelerating wake vs peak current for $L = 85$ cm. The accelerating wake can be seen to follow the decelerating one, but with a larger spread. A fit to a power law is shown in black. Incoming bunch energy: 42 GeV.

This result exhibits good agreement with the exponent predicted by Eq. (3): $1/2$. This value is in contrast to what was calculated in the linear case, namely an exponent of 1, showing that the nonlinear regime is indeed reached.

The accelerating wake amplitude (blue points on Fig. 3) can be seen to generally follow the decelerating one, but with a much larger spread. The measurement relies on there being significant charge in the tail of the single bunch to sample the high fields. The detection threshold of 3×10^6 electrons/GeV [3] is a small fraction of the total bunch charge of 1.8×10^{10} electrons, and its position can shift by as much as 10 μm for a given peak current due to accelerator jitter. Noting that the (unloaded) accelerating field can be approximated as linear, with $dE_z/d\xi = \frac{1}{2} \times \frac{m_e c \omega_p}{e} k_p$ [8], at the densities used here this translates into a 25 GV/m difference in the measured field, accounting for the large spread in the measured values. Accelerating wake amplitudes of 30–50 GV/m were measured, again agreeing with the order predicted.

As the bunch propagates, its head expands, thus eroding the position where the ionization occurs [3]. Data from the 10-cm case showed agreement with Eq. (5), indicating that the impact of erosion of the bunch head occurs quickly. As the erosion rate is proportional to $1/I^{3/2}$, the bunch current at the ionization location [15], a low current bunch head will erode away quickly, while the bunch core can propagate more stably over meter scale distances. As the head of the bunch can be expected to disappear within a few centimeters of propagation, the shape of the current profile ceases to matter. The wakes, both accelerating and decelerating, become dominated by the peak current of the incident electron bunch.

This process, however, does have strong implications for the transformer ratio. With the plasma electrons born further and further back from the bunch head, the bunch shortens with respect to the plasma. The erosion during propagation thus causes larger decelerating wakes, while not affecting the accelerating phase. The transformer ratio will then be reduced due to the propagation of the bunch, a result confirmed in simulation.

Using Fig. 1 as a reference, a lower limit on T can be derived, for the case where the incident bunch length is not known. Because of the symmetric nature of Eq. (2), a particle sitting at the trajectory peak, $R_{\max}$ can be examined and propagated both forward and backward in time, to estimate the ratio of time scales $\Delta\xi$. Propagating to the left, this particle is in the accelerating phase, sees a bare ion-column and travels to the axis with the time scale $\Delta\xi_{\rm acc}$. Propagating to the right, this particle is in the decelerating phase and will be affected by a screened ion column due to the presence of the negatively charged drive bunch. It will take longer to reach the axis, giving $\Delta\xi_{\rm dec} > \Delta\xi_{\rm acc}$. Thus even the eroding case should yield

$$T \gtrsim 1 \tag{6}$$

in the nonlinear regime. Mathematically this is evident in Eq. (2), where the bunch term reduces the magnitude of the second derivative $\ddot{R}$, meaning the trajectory is less curved and takes longer to approach the axis. This important result demonstrates that reasonable transformer ratios are possible even under heavy erosion.

Identifying events where both accelerating and decelerating wakes could be measured and dividing one by the other, the unloaded transformer ratio in the nonlinear regime is measured. The result, plotted in Fig. 4, shows that the transformer ratio is greater than one, as expected, and varies rather slowly with peak current for both plasma lengths, 85 and 10 cm FHWM, respectively. The spread evident in the accelerating wake measurement is naturally imprinted on the transformer ratio measurement. The results were binned in peak current, with bin sizes of 0.5 kA, on the order of the error in the peak current measurement. The data for each plasma length were taken two years apart, with the events in each taken within a 15 min time frame. The agreement in the overlapping region gives confidence in the result. The requirement of having charge to measure in the high field region is more difficult to satisfy as I_p rises and may result in a higher actual transformer ratio for those events.

In summary, the relationship of the longitudinal field to the peak current of the incident electron bunch in the nonlinear regime was derived and shown to be consistent with experimental results. The measurement of the unloaded transformer ratio in the nonlinear regime was shown to be greater than unity and weakly varying with I_p. This is a major result relevant to a collider stage design, demonstrating that in field-ionized plasmas the aim is for the highest current bunch possible, since the transformer ratio remains nearly constant, but the wake amplitude rises as the bunch current is increased. The next phase calls for wake loading experiments, with a proper drive-witness bunch configuration, to demonstrate high energy gains with narrow energy spreads while extracting significant energy from the wake.

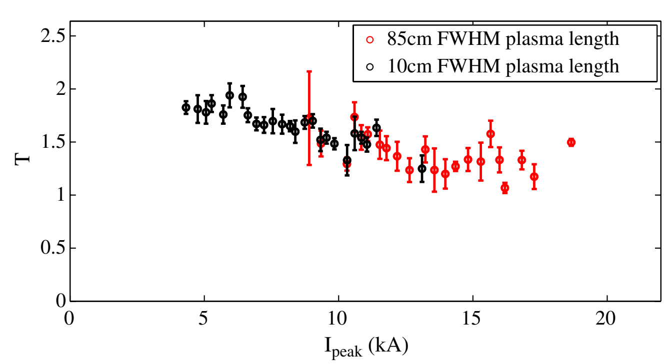

FIG. 4. Transformer ratio T vs bunch peak current I_p for $L = 85$ cm and $L = 10$ cm, and incoming bunch energies of 42 and 28.5 GeV, respectively.

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