Monopole passband excitation by field emitters in 9-cell TESLA-type cavities

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We present an extension of the calculation of dipole-mode driven beam break-up instabilities, as calculated in [V. Volkov, Phys. Rev. ST Accel. Beams **12**, 011301 (2009); V. Volkov, J. Knobloch, and A. Matveenko, Phys. Rev. ST Accel. Beams (to be published)], to the monopole fundamental mode passband. The excitation of these modes has been observed in 9-cell TESLA cavities on test stands without beam [G. Kreps *et al.*, *Proceedings of SRF2009* (HZB, Berlin, Germany, 2009), pp. 289–291] and the same effect has been observed in klystrons with high DC currents.

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I. INTRODUCTION

During vertical cavity tests of TESLA 9-cell cavities, the spontaneous excitation of monopole modes with resonance frequencies close to the main mode is frequently observed [1]. When this occurs, generally the $7\pi/9$ mode with a frequency of 1297 MHz is excited and it grows exponentially with a time constant that depends on the quality factor. It has a high quality factor (Q of order 10^{10}) due to a weak external coupling. Interestingly, even though field emission occurs, little bremsstrahlung is observed outside the cryostat and the radiation energy is low, of order 100–200 keV. Measured bremsstrahlung on axis (shielded only by the cavity wall) yields energies up to 50 keV. One of the possible excitation mechanisms is due to field emitted electrons. The spontaneous excitation of the $7/9\pi$ passband mode suggests that the electrons are accelerated to a high energy and then are again decelerated to low impact energy, transferring their energy to the $7/9\pi$ mode. The power fed to the mode is relatively high, on the order of 10 W. Such power can only be transferred by the low emission currents of some microamperes if electrons reach the energy of the order of MeV and before being decelerated again. This transfer of energy is feasible due to a modulation of the beam trajectories and the emitted charge by the fields of the $7/9\pi$ mode.

In the article we analyze analytically and numerically this mechanism of $7/9\pi$ mode excitation. The actual trajectories of field emitted electrons and the threshold currents for mode excitation are calculated. Threshold currents of some microamperes are obtained.

II. THE PRINCIPLE OF MONOPOLE MODES EXCITATION

First, we consider the conventional theory. The monopole rf field inside a cavity forms an axially symmetric standing wave. The electric field $\bar{E} = \bar{E}(r, z)\cos(\omega t + \varphi)$ has $\bar{r}$ and $\bar{z}$ components and the magnetic field $\bar{B} = \bar{B}(r, z)\sin(\omega t + \varphi)$ has only an azimuthal component. Here ω is the angular frequency and φ is the initial phase of the mode. In an n-cell cavity there are n modes with resonance frequencies close to the main mode frequency which form the first monopole passband. We denote the frequency and phases of these modes by ω_i and φ_i, respectively. For clarity, we denote the π mode the accelerating mode and refer to the other modes as the passband modes.

The field emitters have a size much less than the cavity and are essentially pointlike. They are located at the cavity surface near the maximum of the electric rf field of the main mode. The emitted current cannot exceed some microamperes; otherwise the high current density will melt the emitter. The current density is described by the Fowler-Nordheim (FN) formula [2],

$$j(E) = \frac{A_{\rm FN}(\beta_{\rm FN}E)^2}{\phi}\exp\left(\frac{-B_{\rm FN}\phi^{3/2}}{\beta_{\rm FN}E}\right), \qquad (1)$$

where $A_{\rm FN} = 1.54 \times 10^6$, $B_{\rm FN} = 6.83 \times 10^3$, E is the surface electric field in MV/m, $\varphi = 4$ eV is the work function of niobium, $\beta_{\rm FN} = 50$–2000 is the field enhancement factor, and j is the current density in $\mathrm{A/m^2}$. The FN parameters, beta and emission area, should be used only as parameters to express the experimentally obtained dependence of the emitted current on the field in niobium cavities. For this reason we use the simplified FN expression of Eq. (1) [3] which ignores image charge effects.

If we assume $\varphi = 0$ and substitute $E = E_s\cos(\omega t) \approx E_s(1 - (\omega t)^2/2)$ near the surface field maximum in Eq. (1), we obtain the time dependence of the current close to a Gaussian:

$$j(t) \approx j(E_s)\exp(-t^2/2\tau^2), \qquad \omega\tau(E_s) = \left(\frac{\beta_{\rm FN}E_s}{B_{\rm FN}\phi^{3/2}}\right)^{1/2}. \qquad (2)$$

Here $\omega\tau$ is the rms width of the bunch in radians. According to (2) the field emitted current is a chain of short electron bunches launched at the phase $\varphi = 0$ of the main mode with a repetition frequency equal to the main mode frequency. Integration of (2) over an rf period and multiplication by the site area s [m^2] gives the bunch charge $q(E)$,

$$q(E_s) \approx j(E_s)\tau(E_s)s\sqrt{2\pi}. \tag{3}$$

Since the frequencies of passband modes are close to the main one $\omega \sim \omega_i$, we can express the electric surface field as a sum of the main mode and one of the passband modes at $t \sim 0$:

$$\begin{aligned} &E_s\cos(\omega t) + E_{si}\cos(\omega_i t + \varphi_i) \\ &= E_s\cos(\omega t) + E_{si}\cos(\omega_i t)\cos(\varphi_i) - E_{si}\sin(\omega_i t)\sin(\varphi_i) \\ &\approx [E_s + E_{si}\cos(\varphi_i)]\cos(\omega t). \end{aligned} \tag{4}$$

If we replace E_s in Eq. (3) by $E_s + E_{si}\cos(\varphi_i)$ from Eq. (4) and assume $E_{si} \ll E_s$, we obtain

$$\begin{aligned} q(E_s, \varphi_i) &= q(E_s) + \frac{\partial q(E_s)}{\partial E_s}E_{si}\cos(\varphi_i) \\ &= q(E_s)\left[1 + \frac{E_{si}}{E_s}\left(2.5 + \frac{B_{FN}\phi^{3/2}}{\beta_{FN}E_s}\right)\cos(\varphi_i)\right]. \end{aligned} \tag{5}$$

The particle motion in the rf field of a monopole mode is represented by the equation

$$\begin{aligned} &\frac{d}{dt}m\gamma\dot{\bar{r}} = e\bar{E}(\bar{r})\cos(\omega t + \varphi) + e\dot{\bar{r}} \times \bar{B}(\bar{r})\sin(\omega t + \varphi), \\ &\qquad \gamma = 1/\sqrt{1 - \dot{r}^2}, \end{aligned} \tag{6}$$

where the space charge forces are neglected since the emission currents are small. The solution of Eq. (6) is the trajectory $\bar{r} = \bar{r}(t, \varphi)$ that lies in the rz plane. We consider further the approximation of emitted bunch as a pointlike one having the particle trajectory with $\varphi = 0$ since this phase corresponds to the maximal charge density of the bunch [see Eq. (2)].

The energy gain is the integral along the electron trajectory. While the bunches are launched at the phase $\varphi = 0$ of the main mode, the phases of the other low-amplitude passband modes change from shot to shot by $\Delta\varphi_i = 2\pi(\omega_i - \omega)/\omega$, i.e., the passband mode has all launch phases (0–2π) with equal probability. Bunches will gain (on average) energy in the field of the passband mode if the trajectory or bunch charge is modified by the field of this mode. Because of the energy conservation, the energy gain by the passband mode is the same as the energy lost by the beam. If the current exceeds some threshold current, the power transferred to the passband mode will be larger than the power lost in the cavity wall, and the mode will grow exponentially (instability) until some nonlinearity limits the field. Further we reconsider these arguments analytically.

We assume the passband-mode amplitudes $\bar{E}(\bar{r})_i$ and $\bar{B}(\bar{r})_i$ are initially very low. The particle motion is determined by the combination of the accelerating mode and the passband mode and can be represented by trajectory $\bar{r} + \bar{r}_i$. If we replace in (6) $\dot{\bar{r}}$ with $\dot{\bar{r}} + \dot{\bar{r}}_i$, $\bar{E}\cos(\omega t + \varphi)$ with $\bar{E}\cos(\omega t) + E_{mi}\bar{E}_i\cos(\omega_i t + \varphi_i)$, and $\bar{B}\sin(\omega t + \varphi)$ with $\bar{B}\sin(\omega t) + E_{mi}\bar{B}_i\sin(\omega_i t + \varphi_i)$, where the passband-mode fields E_i and B_i are normalized to their maximal on-axis electric field value E_{mi}, we obtain

$$\begin{aligned} \frac{d}{dt}m\gamma(\dot{\bar{r}}_i/E_{mi}) \approx{}& e\bar{E}_i(\bar{r})\cos(\omega_i t + \varphi_i) + e(\dot{\bar{r}}_i/E_{mi}) \\ &\times \bar{B}(\bar{r})\sin(\omega t) + e\dot{\bar{r}} \\ &\times \bar{B}_i(\bar{r})\sin(\omega_i t + \varphi_i), \end{aligned} \tag{7}$$

i.e., the velocity variation $\dot{\bar{r}}_i$ is small and is proportional to the passband-mode field amplitude. Since it is a periodical function of phase φ_i, it can be described approximately by first order terms of Fourier series,

$$\gamma\dot{\bar{r}}_i/E_{mi} \approx \bar{a}(t)\cos(\varphi_i) + \bar{b}(t)\sin(\varphi_i) + \bar{g}(t), \tag{8}$$

where $\bar{a}(t)$, $\bar{b}(t)$, and $\bar{g}(t)$ are the time dependent functions.

The average energy (qV_i) gained by the bunches due to interaction with the ith passband mode is found by integrating the field along trajectories and averaging over all phases φ_i:

$$\begin{aligned} qV_i &= \frac{1}{2\pi}\int_0^{2\pi} q(E_s, \varphi_i)\int_s E_{mi}\bar{E}_i \cdot d(\bar{r} + \bar{r}_i)d\varphi_i \\ &= \frac{E_{mi}}{2\pi}\int_0^{2\pi} q(E_s, \varphi_i)\int_s (\bar{E}_i\cos(\omega_i t + \varphi_i)) \\ &\quad \cdot (\dot{\bar{r}} + \dot{\bar{r}}_i)dtd\varphi_i. \end{aligned} \tag{9}$$

Then we use the formulas $\cos(\omega_i t + \varphi_i) = \cos(\omega_i t) \times \cos(\varphi_i) - \sin(\omega_i t)\sin(\varphi_i)$, $\cos(\varphi_i)^2 = 1/2 + 1/2\cos(2\varphi_i)$, and $\sin(\varphi_i)^2 = 1/2 - 1/2\cos(2\varphi_i)$. In Eq. (5) we must use a normalized surface field similar to other ones for the passband, i.e., replace E_{si} by $E_{mi}E_{si}$. Inserting $q(E_s, \varphi_i)$ from (5) and $\dot{\bar{r}}_i$ from (8) into (9) yields, after the averaging and neglecting terms with ${E_{mi}}^3$,

$$\begin{aligned} qV_i ={}& \frac{1}{2\gamma}q(E_s)E_{mi}^2\int_s[\bar{E}_i \cdot \bar{a}(t)\cos(\omega_i t) \\ &- \bar{E}_i \cdot \bar{b}(t)\sin(\omega_i t)]dt \\ &+ \frac{1}{2}\frac{\partial q(E_s)}{\partial E_s}E_{si}E_{mi}^2\int_s \bar{E}_i \cdot \dot{\bar{r}}\cos(\omega_i t)dt. \end{aligned} \tag{10}$$

The first term is the energy gained due to the changing of the bunch trajectory by the passband field, and the second term is the energy gained due to modulation of the current by the passband field. Note that both terms are proportional to the square of passband field strength, similar to the power dissipation of the mode in the cavity walls. If the energy gain given by (10) is larger than the mode's energy loss in the cavity wall, the field will grow exponentially until it is limited by other mechanisms. Note that a negative energy means that bunch is losing energy due to the bunch-passband interaction.

The energy dissipation by the passband mode over one rf period in the cavity wall is $2\pi U/Q$, where U is the stored field energy. U is proportional to E_{mi}^2 and was calculated with the CLANS code [4]. Since (10) can be rewritten as $qV_i = AU$, with A being a proportionality constant, energy conservation yields

$$qV_i - 2\pi U/Q = AU - 2\pi U/Q = \Delta U, \tag{11}$$

where ΔU is the change in stored energy over an rf period $T_i = 2\pi/\omega_i$. After dividing (11) by T_i and approximating $\Delta U/T_i \sim dU/dt$, we obtain the equation $dU/U = (A/2\pi - 1/Q)\omega_i dt$. Integration yields $U = U_0 \exp(t/\tau)$, where $\tau = 1/(A/2\pi - 1/Q)\omega_i$ is the time constant of the instability. We rewrite it by replacing A as follows:

$$\tau = 1/(IV_i/U - \omega_i/Q), \tag{12}$$

where $I = q/T_i = q\omega_i/2\pi$ is the beam current. The threshold current from (12) thus is

$$I_{\mathrm{th}} = \omega_i U/QV_i. \tag{13}$$

The quality Q must be high enough to provoke the spontaneous excitation with appreciable strength. If the beam current is twice the threshold current, the instability time constant of (12) equals the cavity decay time constant $\tau_i = Q/\omega_i$. This fact is observed in experiments [1] where τ varies when Q is modified by changing the external loading of the cavity via the power coupler even though the emission current remains constant.

The power is the product of the emission current and the energy gain V_i in (10)

$$P_i = IV_i \approx 30 \text{ W}.$$

Below we will show that indeed the energy gain V_i is large enough to excite 30 W of power in the $7/9\pi$ mode.

III. NUMERICAL MODELING OF MONOPOLE MODE EXCITATION

We consider the excitation of the $7\pi/9$ mode in a 9-cell TESLA cavity by field emitted electrons by simulating the field emission trajectories at increasing electric field levels.

The electric field patterns and surface electric fields of the π and $7\pi/9$ modes are shown in Figs. 1 and 2, respectively. The field distributions are nearly identical except in the 3rd and the 7th cell. For potential field emission sites we considered emission spot pairs located on the surface of every cell aperture near the electric field maximum (altogether 18).

The field intensity in a cavity is usually increased gradually during tests. Since the electric field is not high at the beginning of tests, the field enhancement factor (β_{FN}) in

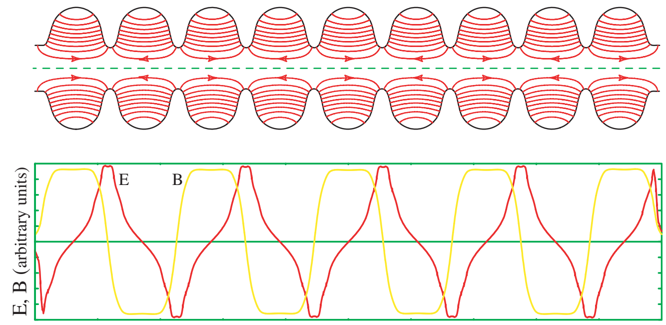

FIG. 1. (Color) Electric field pattern and surface field distribution of the 1300 MHz accelerating mode.

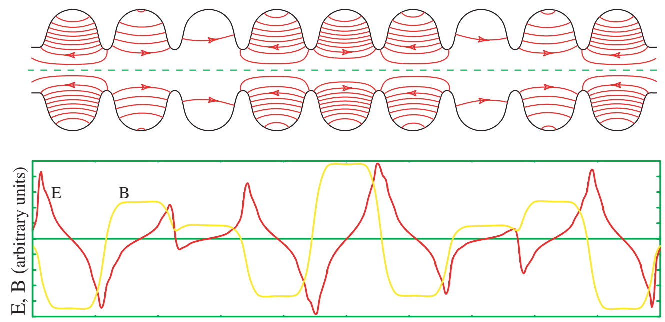

FIG. 2. (Color) Electric field pattern and surface field distribution of the 1297 MHz $7\pi/9$ mode.

Fowler-Nordheim current dependence in (1) must be high for the current to reach some microamperes (of the order of 1000). Therefore, the current pulse widths will be about $\pm$(20–25) degrees of 1.3 GHz [see $\omega\tau$ in Eq. (2)]. Further, we approximate all field emitted electron trajectories by the trajectory of the electron emitted at the phase of 0° of main mode.

It should be noted that β_{FN} usually decreases during the cavity tests to about 50–100 as emission sites high power rf process. Therefore, when a processed cavity is retested the excitation of passband modes in such a cavity may be absent. In other words, the excitation depends on the cavity's surface condition and preparation history. This is consistent with the fact that the excitation is not observed in all cavity tests [1].

In the calculations, we increased gradually the intensity of the accelerating mode similar to a test while the intensity of the $7/9\pi$ mode was as small as possible. For each emitter spot we calculated the electron trajectories with the ASTRA [5] code for different phases of the passband mode φ_i and $\varphi = 0$ for the main mode. These were used to calculate the energy gain of the passband mode. After averaging this energy over all trajectories, the average energy gain of the passband mode of (10) was found. The charge modulation was taken into account by multiplying the integrated voltage with its charge found from (5) normalized to the charge at $\varphi_i = 90$ degrees.

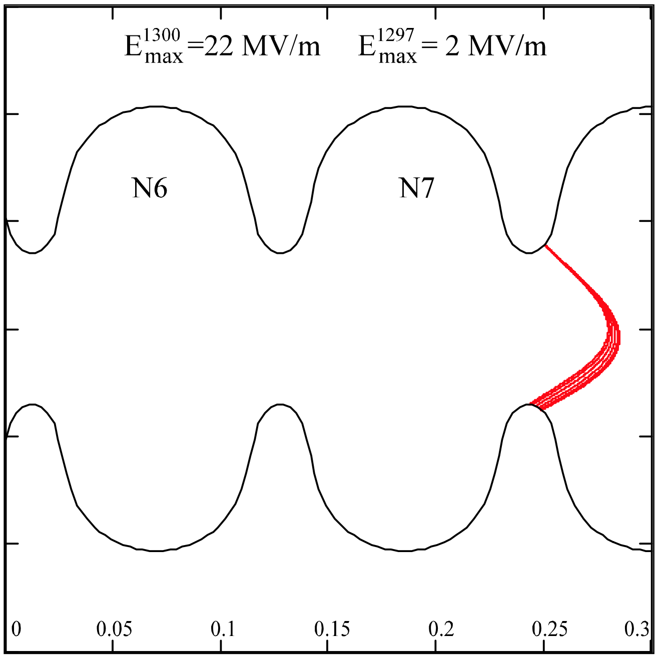

FIG. 3. (Color) Trajectories terminated in the emission cell ($E_{max} > 20$ MV/m). Impact energies >1 MeV. No passband-mode excitation occurs.

Finally, we used (13) to find the threshold current (for the $7\pi/9$ mode $U = 10$ J at $E_{mi} = 23.7$ MV/m). The quality factor of the mode is assumed to be $Q = 10^{10}$.

The trajectories of field emitted electrons with different amplitudes in both modes (1297 MHz mode field is small) are shown in Figs. 3–9. Configurations that led to the excitation of the passband mode are also listed in Table I. Four types of trajectories dependent on the field amplitude of the main mode E_{max} are found: (i) Trajectories that terminate in the same cell where electrons were launched ($E_{max} > 20$ MV/m, Fig. 3); (ii) trajectories that terminate in the next cell ($15 < E_{max} < 20$ MV/m, Fig. 4); (iii) trajectories that propagate through the cavity ($7 < E_{max} < 15$ MV/m, Fig. 5); (iv) trajectories that terminate in the third cell ($E_{max} \sim 7$ MV/m, Figs. 6 and 7).

Only cells 3 and 7 have unconventional behavior: they do not have propagating trajectories, and the trajectories terminated in the end cells with low impact energies of order 130 keV ($E_{max} \sim 9$ MV/m, Figs. 8 and 9 and Table I). In this case, these trajectories excite the $7\pi/9$ mode with a threshold current of some microamperes. This is consistent with the experiments [1], where the radiation

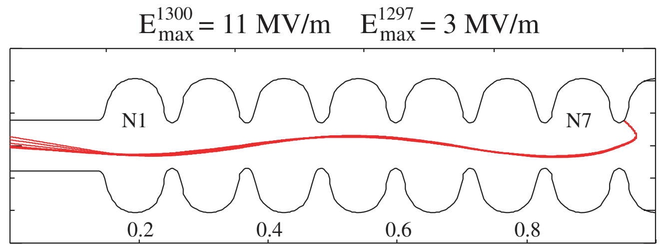

FIG. 5. (Color) Trajectories propagated through the cavity ($7 < E_{max} < 15$ MV/m) gaining an energy up to 830 keV. $7/9\pi$ mode spontaneously excites due to the emitted current modulation effect (see Table I).

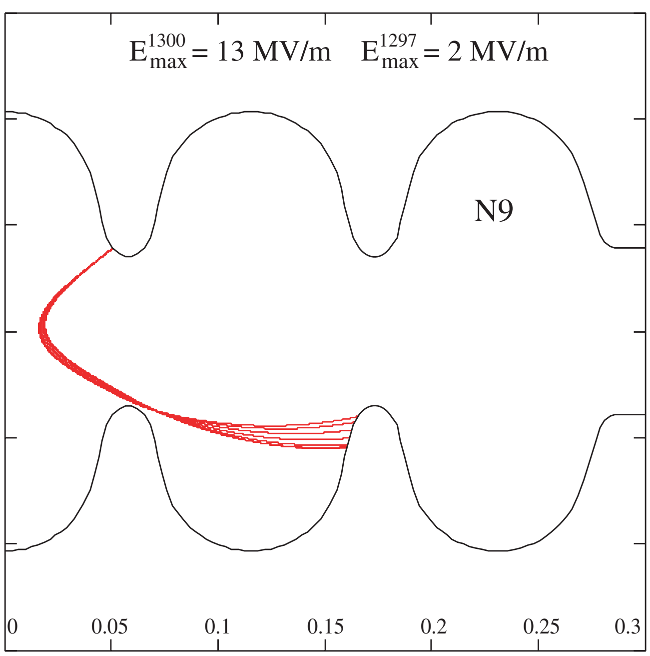

FIG. 4. (Color) Trajectories terminated in the neighboring cell ($15 < E_{max} < 20$ MV/m, for emission in the 3rd and 7th cell $E_{max} \sim 13$ MV/m). Passband-mode excitation occurs.

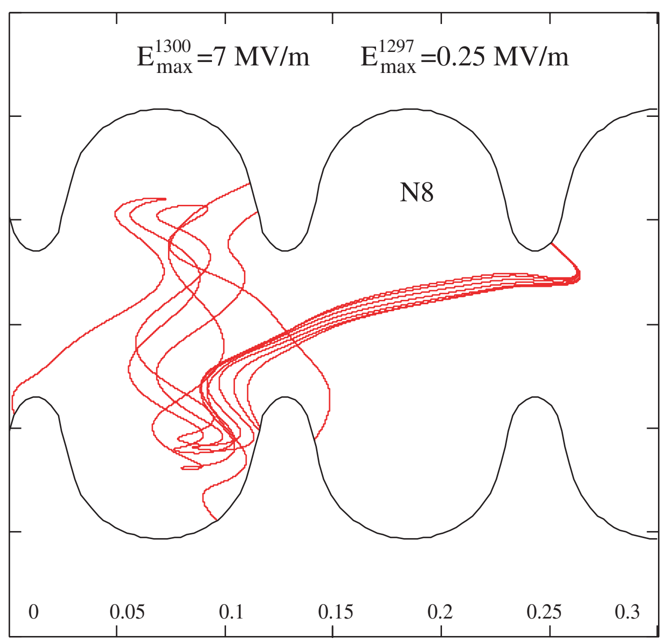

FIG. 6. (Color) Trajectories terminated in the 3rd cell ($E_{max} \sim 7$ MV/m). Impact energy is 80 keV. Stable (no spontaneous excitation).

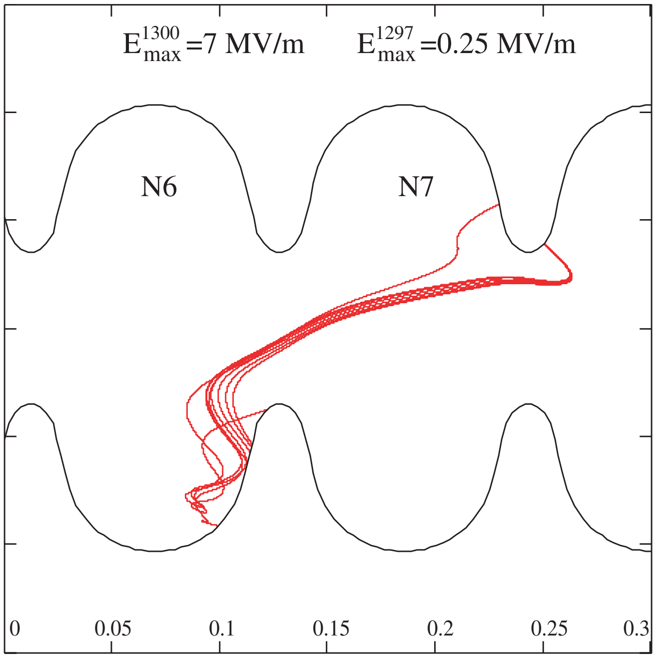

FIG. 7. (Color) Trajectories terminated in the 3rd cell ($E_{max} \sim$ 7 MV/m). Impact energy is 80 keV. Stable (no spontaneous excitation).

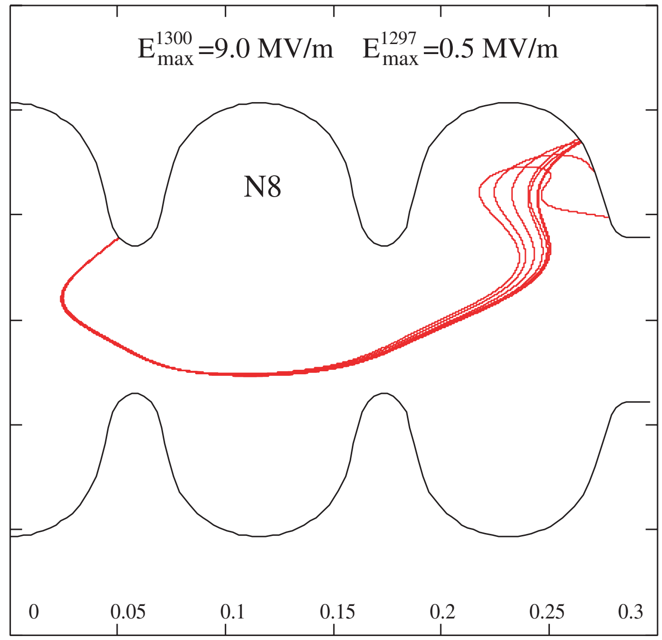

FIG. 9. (Color) Trajectories originate in the 7th cell and terminate in the end cell with impact energy of 120 keV. Passband-mode excitation occurs (see Table I).

source was found to be primarily in the end cells with maximum photon energy around 50 keV (measured outside of the cavity). It should be noted that the emission current is not modulated by the $7\pi/9$ mode since its surface field at the emission spot is very small (see Fig. 2).

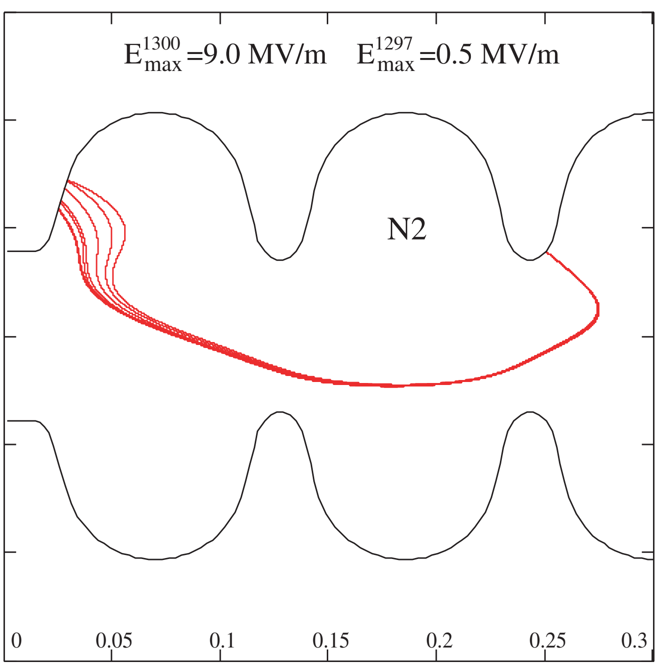

FIG. 8. (Color) Trajectories originate in the 3rd cell and terminate in the end cell with impact energy of 135 keV. Passband-mode excitation occurs (see Table I).

Now we compare the calculated power of the $7\pi/9$ mode with the measured ~30 W in experiments [1], where the passband-mode field was in order of the main field $E_{7\pi/9} \sim E_{max} = 9$ MV/m (see first two columns of Table I). In this case, according to (10), the energy gain $V_{7\pi/9}$ is more than $V_i \sim 2$ keV presented in Table I for $E_{mi} = 0.5$ MV/m: $V_{7\pi/9} = V_i(E_{7\pi/9}/E_{mi})^2 \sim 650$ keV. This energy is sufficient to transfer 30 W of power to the $7/9\pi$ mode for an emission current of 46 μA. A number of emission spots can be active simultaneously (radiation was observed at both ends of the cavity) and the 46 μA value should be divided between all spots.

Only the four cases listed in Table I were found to spontaneously excite the passband mode. Excitation only occurred in narrow intervals of the main mode intensity ($\Delta E_{max} \sim 1$ MV/m). The effect of the current modulation is only significant in the case of Fig. 5, but in this case the impact energy is high (830 keV) and all emitted electrons leave the cavity through the beam pipes. This situation may occur in multicell photocathode rf guns with field emission occurring at the photocathode surface exposed to the maximum electric field of the accelerating mode. This case requires careful investigation in rf gun practice.

The same effect of high order mode (HOM) spontaneous excitation may take place in devices operating with a DC beam: in high current DC guns of klystrons or induction accelerators [6,7]. In this case the main mode frequency is zero but (10) still is valid. If we take the value of the threshold current estimated in this calculation as $I_{th} \cdot Q \sim 2 \times 10^4$ A, the HOM quality factors in such devices must

TABLE I. Configurations that led to spontaneous excitation of the passband mode.

Figure numbers	8	9	4	5
Maximum on-axis electric field 1300 MHz, MV/m	9	9	13	11
Maximum on-axis electric field E_{mi}, MV/m	0.5	0.5	2	3
Energy gain without current modulation, keV	-1.53	-2.15	-2.26	-0.32
Energy gain with current modulation (V_i), keV	-1.55	-2.1	-3.83	-1.96
Impact energy, keV	135	120	824	830
Impact power, W	0.32	0.2	12.5	0.83
Threshold current, μA	2.33	1.7	15.1	1.0

be $Q < 2000$ for a 10 A beam and $Q < 10$ for 2 kA to avoid mode excitations.

IV. SUMMARY

The numerical estimations confirm the experimentally observed features of spontaneous excitation of the monopole modes in the first passband in 9-cell TESLA-type cavities. Electron field emission from high-field regions in the cavity is a likely excitation mechanism. Features such as the low threshold currents and small impact energies, the maxima of measured bremsstrahlung close to the end cells of the cavity, and the necessary high quality factor of the cavity needed for excitation are explained. Both the modification of the electron trajectory as well as the emission current modulation plays an important role in determining the level of mode excitation. The latter mechanism is new when compared to the excitation of dipole modes during beam break-up instabilities [8]. Still, spontaneous excitation of the monopole mode has the same instability-like process with the threshold current being dependent on the mode quality factor.

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