Smith-Purcell radiation from rough surfaces

Zh. S. Gevorkian[1,2,*]
[1] *Institute of Radiophysics and Electronics, Ashtarak-2, 0410, Armenia*
[2] *Yerevan Physics Institute, Alikhanian Brothers St. 2, Yerevan 0036, Armenia*
(Received 9 February 2010; published 28 July 2010)

Radiation of a charged particle moving parallel to an inhomogeneous surface is considered. Within a single formalism periodic and random gratings are examined. For the periodically inhomogeneous surface we derive new expressions for the dispersion relation and the spectral-angular intensity. In particular, for a given observation direction two wavelengths are emitted instead of one wavelength of the standard Smith-Purcell effect. For a rough surface we show that the main contribution to the radiation intensity is given by surface polaritons induced on the interface between two media. These polaritons are multiply scattered on the roughness of the surface and convert into real photons. The spectral-angular intensity is calculated and its dependence on different parameters is revealed.

DOI: 10.1103/PhysRevSTAB.13.070705 PACS numbers: 41.60.−m, 41.75.Ht

I. INTRODUCTION

Smith-Purcell radiation (SP) [1] is originated when a charged particle travels parallel to a plane with diffraction grating. Recent renewed interest in this problem is caused by different applications. Among these applications are length determination for short electron bunches [2], creation of monochromatic light source in the far infrared region [3–8], etc. Various theoretical models were proposed for describing the SP; [9–13], for a brief review of recent theoretical works see [14,15]. Most of these models deal with the periodical grating in the strong scattering regime (see below). However, in many situations the interface over which the charge travels is rough. As an example, one can mention chamber walls in storage rings. Even the best treated surfaces contain roughness. Radiation appearing when a charged particle moves near a rough surface could be useful for beam diagnostics [16]. The influence of the surface roughness on the transition radiation (originating when particle crosses the interface between two media) was discussed in [17,18]. Roughness-induced radiation for a charged particle sliding over a surface was experimentally observed in [19]. In the present paper we study radiation emitted due to electromagnetic field scattering on inhomogeneities of dielectric constant. We will see below that in the weak scattering regime it is possible to develop a rigorous theory describing both periodical and random grating within a single formalism.

II. GENERAL RELATIONS

The geometry of the problem is shown in Fig. 1.

A charged particle moves uniformly in the vacuum at the distance d from the plane $z = 0$ separating vacuum and isotropic medium. We are interested in the radiation field far away from the charge and the interface. The Maxwell equation for the electric field reads

$$\nabla^2 \vec{E}(\vec{r}, \omega) - \text{graddiv}\vec{E}(\vec{r}, \omega) + \frac{\omega^2}{c^2}\varepsilon(\vec{r}, \omega)\vec{E}(\vec{r}, \omega) = \vec{j}(\vec{r}, \omega), \tag{1}$$

where $\vec{j}$ is the current density related to the charge

$$\vec{j}(\vec{r}, \omega) = -\frac{4\pi ie\omega\vec{v}}{vc^2}\delta(z-d)\delta(y)e^{i\omega x/v}. \tag{2}$$

Here $\vec{v}$ is the velocity of the particle moving on the $0x$ direction and $\varepsilon(\vec{r}, \omega)$ is the inhomogeneous dielectric permittivity of the system which for a rough surface can be chosen in the form

$$\varepsilon(\vec{r}, \omega) = \Theta[z - h(x, y)] + \varepsilon(\omega)\Theta[h(x, y) - z], \tag{3}$$

where $\Theta(z)$ is Heaviside's unit step function, and $h(x, y)$ is the amplitude of surface roughness. As it follows from

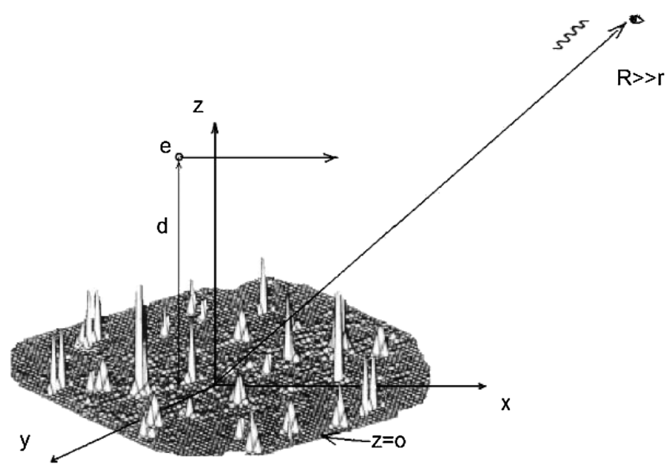

FIG. 1. Geometry of the problem. A charged particle moves parallel to the $0x$ axis. The observation point is far away from the system.

*gevork@yerphi.am

Eq. (3), the space $z > h(x, y)$ is vacuum while the space $z < h(x, y)$ is occupied by a medium with isotropic dielectric constant $\varepsilon(\omega)$. Assuming $h(x, y)$ small and expanding Eq. (3), one gets [20]

$$\varepsilon(\vec{r}, \omega) = \varepsilon_0(z, \omega) + \varepsilon_r(\vec{r}, \omega), \tag{4}$$

where

$$\varepsilon_0(z, \omega) = \begin{cases} 1, & z > 0 \\ \varepsilon(\omega), & z < 0 \end{cases} \tag{5}$$

and

$$\varepsilon_r(\vec{r}, \omega) = [\varepsilon(\omega) - 1]\delta(z)h(x, y). \tag{6}$$

Thus, the total ε is presented as a sum of a regular part ε_0 and an irregular part ε_r. To separate the radiation field, we decompose electric field $\vec{E} = \vec{E}_0 + \vec{E}_r$ analogous to Eq. (4). Here $\vec{E}_0$ and $\vec{E}_r$ are the background and radiation fields, respectively. They obey the following equations:

$$\nabla^2 \vec{E}_0(\vec{r}, \omega) - \mathrm{graddiv}\vec{E}_0(\vec{r}, \omega) + \frac{\omega^2}{c^2}\varepsilon_0(z, \omega)\vec{E}_0(\vec{r}, \omega) = \vec{j}(\vec{r}, \omega) \tag{7}$$

$$\nabla^2 \vec{E}_r(\vec{r}, \omega) - \mathrm{graddiv}\vec{E}_r(\vec{r}, \omega) + \frac{\omega^2}{c^2}\varepsilon_0(z, \omega)\vec{E}_r(\vec{r}, \omega) + \frac{\omega^2}{c^2}\varepsilon_r(\vec{r}, \omega)\vec{E}_r(\vec{r}, \omega) = -\frac{\omega^2}{c^2}\varepsilon_r(\vec{r}, \omega)\vec{E}_0(\vec{r}, \omega). \tag{8}$$

Note that, although the term $\varepsilon_r E_r$ in Eq. (8) is small, one should keep it because it causes multiple scattering of the electromagnetic field. We will see that multiple scattering effects are very important in radiation from the rough surface. Multiple scattering effects in SP radiation for a cluster of dielectric particles were discussed in [21]. At large distances from the system, the electromagnetic field can be treated as a plane wave in which electric and magnetic fields are equal to each other. Therefore the intensity of radiation at the frequencies $[\omega, \omega + d\omega]$ and at solid angles $[\Omega, \Omega + d\Omega]$ can be determined as follows:

$$dI(\omega, \vec{n}) = \frac{c}{2}|\vec{E}_r(\vec{R})|^2 R^2 d\Omega d\omega, \tag{9}$$

where $\vec{n}$ is unit vector on the direction of observation point $\vec{R}$, Ω is the corresponding solid angle; see Fig. 1 and also [14]. As usual at large distances $|\vec{E}_r(\vec{R})|^2$ behaves as $1/R^2$; therefore intensity does not depend on R. The expression Eq. (9) should be averaged over the realizations of random roughness $h(x, y)$. For this reason it is convenient to introduce the Green's functions of Eqs. (7) and (8):

$$\left[\varepsilon_0(z, \omega)\frac{\omega^2}{c^2}\delta_{\lambda\mu} - \frac{\partial^2}{\partial r_\lambda \partial r_\mu} + \delta_{\lambda\mu}\nabla^2\right] G^0_{\mu\nu}(\vec{r}, \vec{r}', \omega) = \delta_{\lambda\nu}\delta(\vec{r} - \vec{r}') \tag{10}$$

$$\left[\varepsilon_0(z, \omega)\frac{\omega^2}{c^2}\delta_{\lambda\mu} - \frac{\partial^2}{\partial r_\lambda \partial r_\mu} + \delta_{\lambda\mu}\nabla^2 + \varepsilon_r(\vec{r}, \omega)\frac{\omega^2}{c^2}\delta_{\lambda\mu}\right] \times G\mu\nu(\vec{r}, \vec{r}', \omega) = \delta_{\lambda\nu}\delta(\vec{r} - \vec{r}'). \tag{11}$$

In Eqs. (10) and (11) a summation over the repeated indices is supposed. Solutions of inhomogeneous Eqs. (7) and (8) can be expressed through the Green's functions Eqs. (10) and (11). Using Eqs. (7), (8), (10), and (11), one can represent the averaged radiation intensity tensor $\langle I_{ij}(\vec{R})\rangle = \langle E_{ri}(\vec{R})E^*_{rj}(\vec{R})\rangle$ in the form

$$\langle I_{ij}(\vec{R})\rangle = \frac{\omega^4}{c^4}\int d\vec{r}d\vec{r}'\langle G_{i\mu}(\vec{R}, \vec{r})\varepsilon_r(\vec{r})G^*_{\nu j}(\vec{r}', \vec{R})\varepsilon_r(\vec{r}')\rangle \times E_{0\mu}(\vec{r})E^*_{0\nu}(\vec{r}'), \tag{12}$$

where the background electric field $E_{0\mu}(\vec{r})$ is expressed through the bare Green's function

$$E_{0\mu}(\vec{r}) = \int d\vec{r}_1 G^0_{\mu\lambda}(\vec{r}, \vec{r}_1)j_\lambda(\vec{r}_1). \tag{13}$$

Here $\langle \cdots \rangle$ means averaging over the surface random profile $h(x, y)$. Note that in the original Smith-Purcell experiment [1], as well as in subsequent works on SP, a periodical grating in one direction is used. In this case $h(x, y) \equiv h(x)$ is some periodical function of one coordinate. In the present paper within a single approach we consider both periodical and random gratings. In the random case we suppose that h is a Gaussian stochastic process characterized by two parameters,

$$\langle h(\vec{\rho})\rangle = 0 \qquad \langle h(\vec{\rho}_1)h(\vec{\rho}_2)\rangle = \delta^2 W(|\vec{\rho}_1 - \vec{\rho}_2|), \tag{14}$$

where $\vec{\rho}$ is the two-dimensional vector in the xy plane, and $\delta^2 = \langle h^2(\vec{\rho})\rangle$ is the average deviation of surface from the plane $z = 0$. Correlation function W is characterized by a correlation length σ at which it is essentially decreased.

The Maxwell equations for electric fields, Eqs. (7) and (8), and Green's functions, Eqs. (10) and (11), should be amended by boundary conditions. As usual, it is required that tangential components of electric field be continuous across the plane $z = 0$. The exact field, of course, will satisfy the boundary conditions across the surface $z = h(x, y)$ rather than the plane. However, this approximation seems reasonable for small roughness $\lambda \gg \delta$ and is widely used in the literature. The Green's function $G_{\mu\nu}(\vec{r}, \vec{r}', \omega)$, when considered a function of z for fixed z', satisfies the same boundary condition as the μth Cartesian component of the electric field.

III. GREEN'S FUNCTIONS

The equation for the bare Green's function, Eq. (10), with correct boundary conditions for arbitrary $\varepsilon(\omega)$ was solved in [20]. To obtain radiation intensity in vacuum, we will need Green's functions in the half space $z > 0$. In order to simplify the problem, we will consider the case when an isotropic medium is a metal with very large negative dielectric constant $|\varepsilon(\omega)| \gg 1$. Using expressions for Green's functions from [20], we find the following basic components:

$$G^0_{zz}(\vec{p}|0, z) = G^0_{zz}(\vec{p}|z, 0) = \frac{ip^2}{k^2}\frac{\varepsilon(\omega)e^{iqz}}{k_1 - \varepsilon(\omega)q}$$
$$G^0_{xz}(\vec{p}|z, 0) = -G^0_{zx}(\vec{p}|0, z) = -\frac{ip_x}{k^2}\frac{\varepsilon(\omega)qe^{iqz}}{k_1 - \varepsilon(\omega)q}, \tag{15}$$

where $G^0_{ij}(\vec{p}|z, z')$ is the two-dimensional Fourier transform of $G^0_{ij}(\vec{r}, \vec{r}')$ and $z > 0$. In the coordinate representation

$$G^0_{ij}(\vec{r}, \vec{r}') = \int \frac{d\vec{p}}{(2\pi)^2} e^{i\vec{p}(\vec{\rho}-\vec{\rho}')} G^0_{ij}(\vec{p}|z, z'), \tag{16}$$

Here $\vec{p}$ and $\vec{\rho}$ are two-dimensional vectors with Cartesian components $p_x, p_y, 0$ and $x, y, 0$. Also $k = \omega/c$, k_1 and q are determined as follows:

$$q = \begin{cases} \sqrt{k^2 - p^2}, & k^2 > p^2 \\ i\sqrt{p^2 - k^2}, & k^2 < p^2 \end{cases} \tag{17}$$

$$k_1 = -[\varepsilon(\omega)k^2 - p^2]^{1/2}. \tag{18}$$

In Eq. (18) a branch cut for the square root along the negative real axis is assumed [20]. Other components of Green's function are small over the parameter $1/|\varepsilon|$. To determine radiation intensity we will need asymptotics of Green's functions at large distances. Substituting Eq. (15) into Eq. (16), one finds

$$G^0_{zz}(\vec{R}, \vec{\rho}, 0) \approx \frac{1}{2\pi\sqrt{2}R}\left[n_z\sqrt{n_\rho}\cos\left(k(R - \vec{n}_\rho\vec{\rho}) - \frac{\pi}{4}\right) + \frac{n_z}{\sqrt{n_\rho}}\cos\left(k(R - \vec{n}_\rho\vec{\rho}) + \frac{\pi}{4}\right)\right]$$
$$+ \frac{i}{2\pi\sqrt{2}R}\left[\sqrt{n_\rho}\cos\left(k(R - \vec{n}_\rho\vec{\rho}) + \frac{\pi}{4}\right) - \frac{1}{\sqrt{n_\rho}}\cos\left(k(R - \vec{n}_\rho\vec{\rho}) - \frac{\pi}{4}\right)\right]$$
$$G^0_{xz}(\vec{R}, \vec{\rho}, 0) = -G^0_{zx}(\vec{\rho}, 0, \vec{R}) \approx \frac{1}{2\pi\sqrt{2}R}\left[n_x\sqrt{n_\rho}\sin\left(k(R - \vec{n}_\rho\vec{\rho}) + \frac{\pi}{4}\right) + \frac{n_x}{\sqrt{n_\rho}}\sin\left(k(R - \vec{n}_\rho\vec{\rho}) - \frac{\pi}{4}\right)\right] \tag{19}$$
$$+ \frac{i}{2\pi\sqrt{2}R}\left[\sqrt{n_\rho}n_x n_z\sin\left(k(R - \vec{n}_\rho\vec{\rho}) - \frac{\pi}{4}\right) - \frac{n_z n_x}{\sqrt{n_\rho}}\sin\left(k(R - \vec{n}_\rho\vec{\rho}) + \frac{\pi}{4}\right)\right],$$

where $\vec{n}$ is the unit vector on the direction of the observation point $\vec{R} = \vec{n}R$, $n_{x,z}$ and $n_\rho = \sqrt{n_x^2 + n_y^2}$ are its corresponding components. When obtaining Eq. (19) we use asymptotics of Bessel functions for large argument [22]. Equation (19) is correct provided that $kR \gg 1$, $R_\rho \gg \rho$ and we use approximate equation $|\vec{R} - \vec{r}| \approx R - \vec{n}\,\vec{r}$.

IV. RADIATION INTENSITY

Spectral-angular radiation intensity, Eq. (12), can be represented as a sum of two contributions, $I(\vec{R}, \omega) = I^0(\vec{R}, \omega) + I^D(\vec{R}, \omega)$, where I_0 and I_D are single scattering and diffusive contributions, respectively [23]. First consider the single scattering contribution to the radiation intensity. Substituting the Green's functions in Eq. (12) by the bare ones, we obtain

$$I^0_{ij}(\vec{R}) = (\varepsilon - 1)^2\delta^2 k^4 \int d\vec{\rho}d\vec{\rho}' G^0_{iz}(\vec{R}, \vec{\rho}, 0)$$
$$G^{*0}_{zj}(\vec{\rho}', 0, \vec{R}) W(|\vec{\rho} - \vec{\rho}'|) E_{0z}(\vec{\rho}, 0) E^*_{0z}(\vec{\rho}', 0), \tag{20}$$

where $(ij) \equiv (xz)$. The background electric field in the limit $|\varepsilon(\omega)| \gg 1$ can be found from Eqs. (2), (13), and (15),

$$E_{0z}(\vec{\rho}, 0) = -\frac{4ee^{ik_0x}}{v}\frac{dk_0}{\gamma\sqrt{y^2 + d^2}} K_1\left(\frac{k_0\sqrt{y^2 + d^2}}{\gamma}\right), \tag{21}$$

where $k_0 = \omega/v$, $\gamma = (1 - v^2/c^2)^{-1/2}$ is the Lorentz factor of the particle, and K_1 is the first order Macdonald function. As it follows from Eq. (21) the background electric field and correspondingly radiation intensity is exponentially small when $\omega d/v\gamma \gg 1$, see also [24]. One can expect essential intensity provided that $\omega d/v\gamma \ll 1$. Far away from the system at the observation point, one can use asymptotic expressions for Green's functions, Eq. (19). Substituting Eqs. (19) and (21) into Eq. (20), for the spectral-angular radiation intensity $I(\omega, \Omega) = cR^2 I_{ii}(\vec{R})/2$, one obtains

$$I^0(\omega, \Omega) = \frac{e^2}{c\beta^2}\frac{gL_x(1 - n_x^2)(1 + n_z^2)(1 + n_\rho^2)}{16\pi n_\rho d}, \tag{22}$$

where L_x is the system size in the x direction, $g = (\varepsilon - 1)^2\delta^2\sigma^2 k^4$ and $\beta = v/c$. When obtaining Eq. (22) we neglect strongly oscillating terms in the limit $kR \gg 1$ and suppose that $W(\vec{\rho} - \vec{\rho}') \equiv \sigma^2\delta(\vec{\rho} - \vec{\rho}')$. Beside that we substitute the Macdonald function by its asymptotics for small argument assuming that $k_0 d/\gamma \ll 1$. In the opposite limit, as was mentioned above, radiation intensity is negligible. The components of unit vector $\vec{n}$ are determined through the polar θ and azimuthal ϕ angles of observation direction: $n_z = \cos\theta$, $n_\rho = \sin\theta$, $n_x = \sin\theta\sin\phi$. We consider radiation into the half space $z > 0$ (vacuum) which means $\theta < \pi/2$. Note that the coupling constant $g = k^4(\varepsilon - 1)^2\delta^2\sigma^2$ in Eq. (22) is a dimensionless parameter. From the condition $R_\rho \gg \rho$, one obtains a restriction on angles $\sin\theta \gg L/R$, where L is a characteristic size of the system. To avoid misunderstanding note that $1/\beta^2$ dependence of radiation intensity, Eq. (22), is correct in an intermediate regime for not very low velocities $\omega d/v\gamma \ll 1$. When $\beta \to 0$, as was mentioned above, radiation disappears.

Note that the background field $\vec{E}_0$ can originate radiation without any roughness provided that Cherenkov condition $v^2\varepsilon > c^2$ is fulfilled. Cherenkov radiation is possible for dielectric surfaces with positive large ε. For metallic surfaces in the optical region we are interested in, the present paper dielectric constant is negative and Cherenkov radiation is absent.

V. PERIODICAL CASE

Analogously, one can consider the case when surface grating is a periodical function. For simplicity we will assume that $h(\vec{\rho}) \equiv \delta\sin 2\pi x/b$, where b is the period of grating. Substituting $W(|\vec{\rho} - \vec{\rho}'|)$ by $\sin 2\pi x/b \sin 2\pi x'/b$ in Eq. (20) and using Eq. (19), after integration, for spectral-angular radiation intensity, one has

$$I_{SP}(\vec{n}, \omega) = \frac{e^2}{c\beta^2}\frac{g_1(1 + n_z^2)(1 - n_x^2)(1 + n_\rho^2)L_x}{8\pi n_\rho} \times \left[\delta\left(kn_x + k_0 - \frac{2\pi}{b}\right) + \delta\left(k_0 - kn_x - \frac{2\pi}{b}\right)\right] \times F(kn_y), \tag{23}$$

where $g_1 = k^4(\varepsilon - 1)^2\delta^2$ and F is determined as follows:

$$F(kn_y) = \left|\frac{dk_0}{\gamma}\int_0^\infty dy \frac{K_1\left(\frac{k_0\sqrt{y^2+d^2}}{\gamma}\right)}{\sqrt{y^2 + d^2}} e^{ikn_y}\right|^2. \tag{24}$$

When obtaining Eq. (23) we keep only the terms proportional to L_x. In the most interesting case $n_y \sim 0$ and $dk_0/\gamma \ll 1$, substituting K_1 by its asymptotic expression, one finds from Eq. (24), $F(0) \sim \pi^2/4$. As follows from Eq. (23), because of the δ functions, for a given observation direction, only two discrete wavelengths are emitted:

$$\lambda_\pm = b\left(\frac{1}{\beta} \pm n_x\right). \tag{25}$$

This is a generalization of well-known Smith-Purcell dispersion relation [1] to the weak scattering (see below) case. For an arbitrary periodical grating, one can expand the surface profile $h(x)$ into Fourier series and for each term one can obtain analogous dispersion relation with b substituted by b/m, where m is the diffraction order. Note that the dispersion relation, Eq. (25), and the spectral-angular radiation intensity, Eq. (23), differ from reported earlier. The reason of those differences are following. First, we are considering the weak scattering regime instead of the strong one considered in the above-mentioned papers. Our theory is applicable provided that $[\varepsilon(\omega) - 1]^2\delta^2/\lambda^2 \ll 1$ although $|\varepsilon(\omega)| \gg 1$. Probably this regime was realized in the experiment on SP radiation in the optical region for shallow gratings [25]. As mentioned in [25], the traditional formula of SP radiation failed to explain the results of the experiment in the shallow grating case. The second reason is the boundary conditions. As follows from Eqs. (7) and (8), Maxwell equations for $\vec{E}_0$ and $\vec{E}_r$ contain the same disruptive function $\varepsilon_0(z, \omega)$. Therefore both of them should satisfy the same boundary conditions at $z = 0$. In our consideration this goal is achieved automatically because the Green's functions, Eqs. (15) and (19), satisfy the correct boundary conditions [20]. To the contrary, in traditional consideration [11], only the total field $\vec{E}_0 + \vec{E}_r$ satisfies the boundary conditions at $z = 0$. Probably this difference leads to a different dispersion relation, Eq. (25).

VI. DIFFUSIVE CONTRIBUTION, SURFACE POLARITONS

Using Eq. (12), one finds diffusive contribution to the radiation intensity in the form

$$I_{ij}^D(\vec{R}) = g\int d\vec{\rho}_1 d\vec{\rho}_2 d\vec{\rho}\, G_{im}(\vec{R}, \vec{\rho}_1, 0)G_{hz}(\vec{\rho}_2, \vec{\rho}) \times P_{mnhs}(\vec{\rho}_1 - \vec{\rho}_2)G_{zs}^*(\vec{\rho}, \vec{\rho}_2)G_{nj}^*(0, \vec{\rho}_1, \vec{R}) \times |E_{0z}(\vec{\rho}, 0)|^2, \tag{26}$$

where $G_{ij}(\vec{\rho}_2, \vec{\rho}_1) \equiv G_{ij}(\vec{\rho}_2, 0, \vec{\rho}_1, 0)$, and where diffusive propagator P is determined by the sum of ladder diagrams; see Fig. 2 and [26].

All integrations over z coordinates make them equal to 0 because of $\delta(z)$ in the fluctuation part of the dielectric constant, Eq. (6). Averaged two-dimensional surface polariton Green's function [27] satisfies the Dyson equation

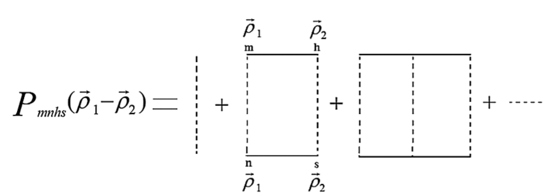

FIG. 2. The dashed line is the correlation function of roughness $g\delta(\vec{\rho}_1 - \vec{\rho}_2)$ and the solid line is the averaged over the randomness two-dimensional Green's function of the surface polariton.

$$G_{\mu\nu}(\vec{p}) = G^0_{\mu\nu}(\vec{p}) + gG^0_{\mu m}(\vec{p}) \int \frac{d\vec{p}_1}{(2\pi)^2} G^0_{mn}(\vec{p}_1) G_{n\nu}(\vec{p}). \tag{27}$$

Remember that $G\mu\nu(\vec{p}) \equiv G_{\mu\nu}(\vec{p}|0,0)$, see Eq. (15). Bare Green's functions are determined by Eq. (15). Further, we will be interested in the behavior of Green's function close to the pole. These values play a main role in the limit $g \to 0$. As it follows from Eq. (15), the two-dimensional Green's functions of surface polariton have a pole at $p^2 = k^2\varepsilon/(\varepsilon + 1)$, see [27]. The corresponding velocity of a surface polariton is equal to $c\sqrt{(\varepsilon+1)/\varepsilon} < c$. Remember that we consider the case when $\varepsilon \ll -1$. When electron velocity becomes equal to this velocity, a super-radiant emission is possible provided that the grating is periodical [6,8]. Close to the pole and for large negative $|\varepsilon(\omega)| \gg 1$, the Green's functions of the surface polariton can be represented in the form

$$\begin{aligned} G^0_{zz}(p) &\simeq \frac{-k}{\sqrt{-\varepsilon_1(\omega)}} \frac{1}{k^2 - p^2 - i\alpha} \\ G^0_{zx}(\vec{p}) &\simeq \frac{ip_x}{\sqrt{-\varepsilon_1(\omega)}} \frac{1}{k^2 - p^2 - i\alpha}, \end{aligned} \tag{28}$$

where $\alpha = k^2\varepsilon_2/\varepsilon_1^2$, $\varepsilon = \varepsilon_1 + i\varepsilon_2$, and $\varepsilon_2 \ll |\varepsilon_1|$. In Eq. (28) α describes the damping of the surface polariton on the flat surface due to the inelastic processes in the medium, i.e. $\varepsilon_2(\omega)$. It follows from Eq. (28) that $\int d\vec{p}\,\vec{p}\,G^0_{zx}(\vec{p}) \equiv 0$. Therefore only G^0_{zz} gives a contribution to the integral in Eq. (27). Solving the Dyson equation, Eq. (27), one can represent the averaged Green's functions in the form

$$\begin{aligned} G_{zz}(p) &\simeq \frac{-k}{\sqrt{-\varepsilon_1(\omega)}} \frac{1}{k^2 - p^2 - i\Lambda} \\ G_{zx}(\vec{p}) &\simeq \frac{ip_x}{\sqrt{-\varepsilon_1(\omega)}} \frac{1}{k^2 - p^2 - i\Lambda}, \end{aligned} \tag{29}$$

where $\Lambda = \int \frac{d\vec{p}}{(2\pi)^2} \mathrm{Im} G^0_{zz}(p) = gk^2/4\varepsilon_1(\omega)$. The real part of the integral leads to renormalization of the parameters and does not play any role. The integral is calculated in the limit $\varepsilon_2 \to 0$. Λ describes the damping of the surface polariton by its roughness-induced conversion into radiative modes [27]. It is convenient also to introduce the polariton mean-free path on the rough surface $l = k/\Lambda$. Note that the neglected terms in diagram expansion are small on parameter $\lambda/l \ll 1$ [26].

Using Eqs. (29), one sees that the main contribution to the diffusive radiation intensity, Eq. (26), gives the term proportional to P_{zzzz} which contains a diffusive pole at small momentums. Summing the ladder diagrams in Fig. 2, one finds a Bethe-Salpeter equation for diffusive propagator $P(K) \equiv P_{zzzz}(K)$:

$$P(K) = f(K) + gf(K)P(K), \tag{30}$$

where

$$f(K) = \int \frac{d\vec{p}}{(2\pi)^2} G(p) G^*(|\vec{p} - \vec{K}|). \tag{31}$$

Here $G(p) \equiv G_{zz}(p)$. Using Eq. (29) and calculating the integral in Eq. (31) in the limit $g \to 0$, one finds $P(K)$ at small $Kl \ll 1$:

$$P(K) = \frac{2}{K^2 l^2}. \tag{32}$$

Substituting Eq. (19) into Eq. (26), for the diffusive contribution to the radiation intensity, one has

$$I^D(\omega, \Omega) = \frac{c(n_z^2 + 1)(1 - n_x^2)(1 + n_\rho^2)}{64\pi^2 n_\rho} \times P(K \to 0) \int d\vec{\rho} |E_{0z}(\vec{\rho}, 0)|^2. \tag{33}$$

As follows from Eqs. (32) and (33) radiation intensity diverges. This divergence is caused by the infinite size of the system, see also [23,28]. If one takes into account the finite size of the system, the minimal momentum will be of order $\sim 1/L$. As was mentioned above, the radiation intensity is exponentially small provided that $d \gg \gamma/k_0$. In the opposite limit $d \ll \gamma/k_0$ substituting K_1 by its asymptotic expression and integrating Eq. (33), we finally obtain

$$I^D(\omega, \Omega) = \frac{e^2}{c\beta^2} \frac{(1 + n_z^2)(1 - n_x^2)(1 + n_\rho^2)}{8\pi n_\rho} \frac{L_x L^2}{dl^2}. \tag{34}$$

In this consideration weak $l \ll l_{\mathrm{in}}$, where $l_{\mathrm{in}} = k/\alpha = \varepsilon_1^2/k\varepsilon_2$ is the inelastic mean-free path of surface polariton, absorption can be taken into account as follows [29]. When $L > (ll_{\mathrm{in}})^{1/2}$, L in Eq. (34) should be substituted by $(ll_{\mathrm{in}})^{1/2}$. Comparing single scattering, Eq. (22), and diffusive, Eq. (34), contributions, one has $I^D/I^0 \sim L^2/gl^2 \gg 1$. Therefore the diffusion of surface polaritons is the main mechanism of radiation. Let us make some numerical estimates for the optical region. For Ag at $\lambda \sim 4500$ Å, $\varepsilon_1 \sim -7.5$ and $\varepsilon_2 \sim 0.24$. Taking $\delta \sim 50$ Å and $\sigma \sim 1000$ Å [27], one has $g \sim 0.68$, $l \sim 7.7\lambda$, and $l_{\mathrm{in}} \sim 47.94\lambda$. Thus the conditions for diffusive mechanism $\lambda \ll l \ll l_{\mathrm{in}}$, L are fulfilled. Evidently, depending on grating parameters δ, σ emission in other wavelength regions is possible too.

We have considered multiple scattering effects in radiation for uncorrelated roughness. However, they are very important for the periodical as well as correlated grating cases too. These cases are more complicated and will be discussed elsewhere later. Our result, Eq. (23), for SP radiation intensity with only single scattering contribution is correct in the cases when the multiple scattering contribution is negligible. Such a situation can occur for the metals with relatively large absorption when the condition of multiple scattering of polaritons $l_{\mathrm{in}} \gg l$ is not fulfilled.

ACKNOWLEDGMENTS

I am grateful to A. Allahverdyan for helpful discussions. This work is supported by the ISTC Grant No. A-1602.

[1] S. J. Smith and E. M. Purcell, Phys. Rev. **92**, 1069 (1953).
[2] A. Doria *et al.*, Nucl. Instrum. Methods Phys. Res., Sect. A **483**, 263 (2002).
[3] L. Schachter and A. Ron, Phys. Rev. A **40**, 876 (1989).
[4] Yukio Shibata *et al.*, Phys. Rev. E **57**, 1061 (1998).
[5] Kwang-Je Kim and Su-Bin Song, Nucl. Instrum. Methods Phys. Res., Sect. A **475**, 158 (2001).
[6] H.L. Andrews and C.A. Brau, Phys. Rev. ST Accel. Beams **7**, 070701 (2004).
[7] S.E. Korbly, A.S. Kesar, J.R. Sirigiri, and R.J. Temkin, Phys. Rev. Lett. **94**, 054803 (2005).
[8] J. Urata, M. Goldshtein, M.F. Kimmit, A. Naumov, C. Platt, and J.E. Walsh, Phys. Rev. Lett. **80**, 516 (1998).
[9] G. Toraldo di Francia, Nuovo Cimento **16**, 61 (1960).
[10] Casper W. Barnes and K.G. Dedrick, J. Appl. Phys. **37**, 411 (1966).
[11] P.M. van den Berg, J. Opt. Soc. Am. **63**, 689 (1973); P.M. van den Berg and T.H. Tan, J. Opt. Soc. Am. **64**, 325 (1974).
[12] G. Kube, Nucl. Instrum. Methods Phys. Res., Sect. B **227**, 180 (2005).
[13] A.S. Kesar, Phys. Rev. ST Accel. Beams **8**, 072801 (2005).
[14] H.L. Andrews, C.H. Boulware, C.A. Brau, and J.D. Jarvis, Phys. Rev. ST Accel. Beams **8**, 110702 (2005).
[15] D. V. Karlovets and A. P. Potylitsyn, Phys. Rev. ST Accel. Beams **9**, 080701 (2006).
[16] Zh. S. Gevorkian, S. G. Arutunian, and K. B. Oganesyan, in *Proceedings of the 23rd Particle Accelerator Conference, Vancouver, Canada, 2009* (IEEE, Piscataway, NJ, 2009).
[17] R. A. Baghiyan, Phys. Rev. E **64**, 026610 (2001).
[18] S. Reiche and J.B. Rosenzweig, in *Proceedings of the Particle Accelerator Conference, Chicago, IL, 2001* (IEEE, New York, 2001).
[19] F. R. Harutunian, A. Kh. Mkhitarian, R. A. Hovhanissian, B. O. Rostomian, and M. G. Sarinyan, Zh. Eksp. Teor. Fiz. **50**, 895 (1979), and references therein.
[20] A. A. Maradudin and D. L. Mills, Phys. Rev. B **11**, 1392 (1975).
[21] F. J. de Abajo, Phys. Rev. Lett. **82**, 2776 (1999); Phys. Rev. E **61**, 5743 (2000).
[22] I.S. Gradshteyn and I.M. Ryzhik, *Table of Integrals, Series and Products* (Academic Press, New York, 1965).
[23] Zh. S. Gevorkian, Phys. Rev. E **57**, 2338 (1998).
[24] K.J. Woods, J.E. Walsh, R.E. Stoner, H.G. Kirk, and R.C. Fernow, Phys. Rev. Lett. **74**, 3808 (1995).
[25] G. Kube *et al.*, Phys. Rev. E **65**, 056501 (2002).
[26] A.A. Abrikosov, L.P. Gorkov, and I.E. Dzyaloshinski, *Methods of Quantum Field Theory in Statistical Physics* (Englewood, New York, 1963).
[27] A.R. McGurn, A.A. Maradudin, and V. Celli, Phys. Rev. B **31**, 4866 (1985).
[28] Zh.S. Gevorkian *et al.*, Phys. Rev. Lett. **97**, 044801 (2006).
[29] P.W. Anderson, Philos. Mag. **52**, 505 (1985).