Generating coherent THz radiation in electron storage rings using an ac sextupole magnet and a vertical kicker magnet

Yoshihiko Shoji

Laboratory of Advanced Science and Technology for Industry (LASTI), University of Hyogo, 1-1-2 Kouto, Kamigori-cho, Ako-gun, Hyogo 678-1205, Japan

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This paper proposes a new method for generating coherent radiation in the THz region from an electron storage ring. When the vertical chromaticity is modulated by an ac sextupole magnet, a vertical beam deflection caused by a kicker magnet produces a wavy spatial structure in the electron bunch after a number of revolutions. The vertically polarized synchrotron radiation from the wavy bunch becomes coherent at the wavelength of the spatial structure. This narrow bandwidth radiation is extremely strong, can be tuned by controlling ring parameters, and is easy to generate. By appropriate choice of ring parameters it is possible to generate radiation at 1 THz.

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I. INTRODUCTION

Applications and technologies of THz (terahertz) electromagnetic radiation is a rapidly progressing field of research. Until the end of the last century, the lack of suitable detectors and sources of radiation in this frequency region resulted in what became known as the "THz gap." However at present, several kinds of radiation sources and detectors have been developed in this spectral region and several applications are beginning to emerge. Of these new radiation sources, particle accelerators are expected to be of particular interest due to their ability to generate extremely strong and stable radiation. When the synchrotron radiation from many electrons is in phase, it becomes an extremely strong coherent synchrotron radiation (CSR). One recent achievement in electron storage rings was the quasi-isochronous operation at BESSY-II [1], which supplied stable sub-THz radiation. However in this case, the intensity was limited by a low threshold of electron beam instability and the generation of shorter wavelength radiation required extreme stability of a ring [2]. Some alternative ideas have been proposed to overcome these limitations, including laser bunch slicing [3–6] and temporal circulation of a short bunch [7,8]. This article proposes a new and easy method for generating CSR in the THz region using an electron storage ring.

To implement this new method two technical requirements must first be met. One is the production of a spatial structure in an electron bunch by means of a kicker magnet proposed by Guo *et al.* [9], intended for the emission of short-pulsed x rays. Another is the production of betatron tune spread using an ac sextupole magnet proposed by Nakamura [10,11] intended for the suppression of beam instabilities. The combination of these two elements enables the production of a spatial wave structure in the electron bunch. This spatial structure can be used as a source of narrow band CSR. This radiation is extremely strong, can be tuned by controlling ring parameters, is well reproducible, and easy to generate. The hardware required for this scheme consists of conventional magnets, which are known to operate with high controllability and good reproducibility. They are suitable to take full advantage of a high stability of electron storage ring.

II. PRODUCTION OF WAVY BUNCH

In the following analytical calculations we will ignore higher order and nonlinear contributions as well as the negligible effect of radiation damping. The vertical betatron motion $y(t)$ produced by a vertical kicker magnet at time $t = 0$ is given as

$$y(t) = y_0 \sin\left(2\pi \int_0^t \frac{\nu_y}{T_{\rm rev}} dt\right). \tag{1}$$

Here y_0 is the initial betatron oscillation amplitude, ν_y is the vertical betatron tune, and $T_{\rm rev}$ is the revolution time. In our case the vertical chromaticity ξ_y is modulated by the synchrotron oscillation frequency as

$$\xi_y = \xi_0 + \xi_1 \sin\omega_S t. \tag{2}$$

Here ξ_0 and ξ_1 are the amplitudes of the dc (off-set) and ac (modulation) components and ω_S is the angular frequency of the synchrotron oscillation. The chromaticity produces a betatron phase shift $\Delta\psi_y$ according to the energy displacement ε, which is given by

$$\Delta\psi_y = \frac{2\pi}{T_{\rm rev}} \int_0^t \xi_y \varepsilon dt. \tag{3}$$

The synchrotron oscillation of ε and the time displacement τ are expressed by the following equations:

$$\varepsilon = \varepsilon_0 \cos\omega_S t + (\omega_S/\alpha_P)\tau_0 \sin\omega_S t, \tag{4a}$$

$$\tau = \tau_0 \cos\omega_S t - (\alpha_P/\omega_S)\varepsilon_0 \sin\omega_S t. \tag{4b}$$

Here α_P is the momentum compaction factor, ε_0 and τ_0 are the initial parameters.

The betatron phase shift of a particle after many revolutions is obtained by substituting Eqs. (4a) and (2) into Eq. (3). When we use ε and τ as the beam parameters instead of ε_0 and τ_0, the result is written as

$$\Delta\psi_y = \frac{2\pi}{T_{\text{rev}}}\left[\varepsilon\left(\frac{\xi_0}{\omega_S} + \frac{\xi_1}{2}t\right)\sin\omega_S t + \tau\left(\xi_0\frac{\cos\omega_S t - 1}{\alpha_P} + \xi_1\frac{\omega_S t\cos\omega_S t - \sin\omega_S t}{2\alpha_P}\right)\right]. \tag{5}$$

This shows that at $t = n\pi/\omega_S$, where n is an integer, the phase shift is independent of ε and is proportional to τ. The vertical position can then be expressed as a function of τ by

$$y = y_0 \sin(\omega_\psi \tau + \psi_{y0}), \tag{6}$$

where ψ_{y0} is the betatron phase at $\tau = 0$. The coefficient ω_ψ is given by

$$\omega_\psi = -\frac{2\pi}{T_{\text{rev}}}\frac{1}{\alpha_P}\left[(\pm 1 + 1)\xi_0 \pm \frac{1}{2}n\pi\xi_1\right] \quad \text{for } t = n\pi/\omega_S, \tag{7}$$

where plus and minus signs are for odd and even n, respectively. For any $t > 0$ the angular frequency of the wavy structure is given by

$$\omega_\psi = \frac{\partial\Delta\psi_y}{\partial\tau} = \left(\frac{2\pi}{T_{\text{rev}}}\right)\left(\xi_0\frac{\cos\omega_S t - 1}{\alpha_P} + \xi_1\frac{\omega_S t\cos\omega_S t - \sin\omega_S t}{2\alpha_P}\right). \tag{8}$$

As will be explained later, ω_ψ is the angular frequency of CSR. Using the natural energy spread σ_ε, the variance of the betatron phase shift produced by the energy spread is given by

$$\sigma_{\psi E}^2 = \left(\frac{\partial\Delta\psi_y}{\partial\varepsilon}\right)^2\sigma_\varepsilon^2 = \left[\frac{2\pi}{T_{\text{rev}}}\left(\frac{\xi_0}{\omega_S} + \frac{\xi_1}{2}t\right)\sin\omega_S t\right]^2\sigma_\varepsilon^2. \tag{9}$$

III. RADIATION FROM WAVY BUNCH

In the following calculations of synchrotron radiation we assume that the observation point is far enough away from the charge such that the vertical angle, θ, from the light source to the observer is small. The calculations are performed for free space and interference with the vacuum wall is not considered. The radiation intensity, I, per unit solid angle per angular frequency emitted by an electron in a bending magnet is calculated from the Liénard-Wiecheld potential [12] as

$$\frac{d^2I}{d\omega d\Omega} = \frac{e^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty}\mathbf{n}\times(\mathbf{n}\times\boldsymbol{\beta})e^{j\omega[t'-\mathbf{n}\cdot\mathbf{r}(t')/c]}dt'\right|^2. \tag{10}$$

Here Ω is the solid angle, ω is the angular frequency of radiation, e is the electron charge, $\boldsymbol{\beta}$ is the particle velocity vector normalized with the light velocity c, $\mathbf{x}$ and $\mathbf{r}$ are the position vectors of the observer and the particle, $\mathbf{n}$ is the unit vector in the direction of $\mathbf{x} - \mathbf{r}$. In the following calculations $\beta \equiv |\boldsymbol{\beta}|$ will be put equal to unity wherever possible. In the limit of a continuous charge distribution of $J(\mathbf{x}, t)$, the intensity distribution becomes

$$\frac{d^2I}{d\omega d\Omega} = \frac{e^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty}\mathbf{n}\times[\mathbf{n}\times J(\mathbf{x}, t')\boldsymbol{\beta}] \times e^{j\omega[t'-\mathbf{n}\cdot\mathbf{r}(t')/c]}dt'\right|^2. \tag{11}$$

Assuming a Gaussian distribution along the direction of the beam, the continuous charge distribution of the wavy bunch described by Eq. (6) can be expressed as

$$J(\mathbf{x}, t) = \frac{eN_e c}{\sqrt{2\pi}\sigma_\tau}\exp\left(-\frac{t^2}{2\sigma_\tau^2}\right)\delta[y - y_0\sin(-\omega_\psi t + \psi_{y0})]. \tag{12}$$

Here N_e is the number of electrons in the bunch and σ_τ is the natural bunch length.

The next assumption is that the radiation wave period $1/\omega$ is much longer than the radiation pulse length from a single electron. If we use a time Δt much longer than the radiation pulse length, the integration in Eq. (10), which we will refer it as $\mathbf{A}$, can be approximated as

$$\mathbf{A}(t) \equiv \int_{-\infty}^{\infty}\mathbf{n}\times(\mathbf{n}\times\boldsymbol{\beta})e^{j\omega[t'-\mathbf{n}\cdot\mathbf{r}(t')/c]}dt' \approx \int_{t-\Delta t}^{t+\Delta t}\mathbf{n}\times(\mathbf{n}\times\boldsymbol{\beta})e^{j\omega[t'-\mathbf{n}\cdot\mathbf{r}(t')/c]}dt'. \tag{13}$$

In addition, $\omega\Delta t$ can be much smaller than unity, so that J and ωt are effectively constant in Δt. The integration in Eq. (11) can now be approximated as

$$\int dt\int dx^3\mathbf{n}\times[\mathbf{n}\times J(\mathbf{x}, t')\boldsymbol{\beta}]e^{j\omega[t'-(\mathbf{n}\cdot\mathbf{x})/c]} \approx \int J\mathbf{A}(t)e^{j\omega t}dt. \tag{14}$$

According to the well-known calculations [12], $\mathbf{A}$ is calculated to be

$$\mathbf{A} \approx \frac{1}{\sqrt{3}}\frac{\rho}{c}\left[-\mathbf{e}_{\text{II}}\left(\frac{1}{\gamma^2}+\theta^2\right)K_{2/3}(\xi) + \mathbf{e}_{\perp}\theta\left(\frac{1}{\gamma^2}+\theta^2\right)^{1/2}K_{1/3}(\xi)\right], \quad (15)$$

where $\mathbf{e}_{\text{II}}$ is the unit vector in the y direction, corresponding to horizontal polarization, and $\mathbf{e}_{\perp} = \mathbf{n} \times \mathbf{e}_{\text{II}}$ is the orthogonal polarization vector corresponding approximately to vertical polarization. $K_{2/3}$ and $K_{1/3}$ are modified Bessel functions and the parameter ξ is defined by

$$\xi = \frac{\omega\rho}{3c}\left(\frac{1}{\gamma^2}+\theta^2\right)^{3/2}. \quad (16)$$

Here γ is the Lorentz factor and ρ is the radius of curvature of the bending magnet. Assuming that the vertical displacements of electrons are small, θ can be approximated as

$$\theta \approx \theta_0 - y/R, \quad (17)$$

where R is the distance to the observer and θ_0 is the angle at $y = 0$. $\mathbf{A}$ can then be approximated as

$$\mathbf{A} \approx \mathbf{A}|_{\theta=\theta_0} + \frac{\partial\mathbf{A}}{\partial\theta}\bigg|_{\theta=\theta_0}(-y/R). \quad (18)$$

The differential in Eq. (18) is calculated to be

$$\frac{\partial\mathbf{A}}{\partial\theta} \approx \frac{1}{\sqrt{3}}\frac{\rho}{c}\left\{-\mathbf{e}_{\text{II}}3\theta\xi K_{1/3}(\xi) + \mathbf{e}_{\perp}\left[\left(\frac{1}{\gamma^2}+\theta^2\right)K_{1/3}(\xi) - 3\theta^2\xi K_{2/3}(\xi)\right] \times \left(\frac{1}{\gamma^2}+\theta^2\right)^{-1/2}\right\}. \quad (19)$$

Consequently Eq. (14) can be written as

$$\int dt \int dx^3\mathbf{n}\times[\mathbf{n}\times\mathbf{J}(\mathbf{x},t')]e^{j\omega[t'-(\mathbf{n}\cdot\mathbf{x})/c]} \approx \int dt\frac{eN_e c}{\sqrt{2\pi}\sigma_\tau}\exp\left(-\frac{t^2}{2\sigma_\tau^2}\right)\left\{\mathbf{A}|_{\theta=\theta_0} + \left[\frac{y_0\sin(-\omega_\psi t + \psi_{y0})}{R}\right]\frac{\partial\mathbf{A}}{\partial\theta}\bigg|_{\theta=\theta_0}\right\}e^{j\omega t}. \quad (20)$$

The first term corresponds to CSR produced by the longitudinal structure of the entire electron bunch with the factor

$$F_0 = \int dt\frac{1}{\sqrt{2\pi}\sigma_\tau}\exp\left(-\frac{t^2}{2\sigma_\tau^2}\right)e^{j\omega t} = \exp\left(-\frac{\omega^2\sigma_\tau^2}{2}\right), \quad (21)$$

whose square is known as the CSR form factor. The second term is the CSR produced by the spatial structure within the bunch, which is proportional to the spatial form factor

$$F_1 = \int dt\frac{1}{\sqrt{2\pi}\sigma_\tau}\exp\left(-\frac{t^2}{2\sigma_\tau^2}\right)\sin(-\omega_\psi t + \psi_{y0})e^{j\omega t} = \exp\left[-\frac{(\omega_\psi^2+\omega^2)\sigma_\tau^2}{2}\right]\frac{y_0}{R} \times [\sin\psi_{y0}\cosh(\omega_\psi\omega\sigma_\tau^2) - j\cos\psi_{y0}\sinh(\omega_\psi\omega\sigma_\tau^2)]. \quad (22)$$

For $\omega_\psi\omega\sigma_\tau^2 \gg 1$, F_1 can be approximated as

$$F_1 \approx \exp\left[-\frac{(\omega_\psi-\omega)^2\sigma_\tau^2}{2}\right]\frac{y_0}{2R}. \quad (23)$$

Finally the radiation intensity is given by

$$\frac{d^2I}{d\omega d\Omega} = \frac{e^2\omega^2N_e^2}{4\pi^2c}\left|(\mathbf{A}|_{\theta=\theta_0})F_0 + \left(\frac{\partial\mathbf{A}}{\partial\theta}\bigg|_{\theta=\theta_0}\right)F_1\right|^2. \quad (24)$$

IV. REDUCTION BY RADIATION EXCITATION

The spatial structure can be greatly reduced as a result of the radiation excitation which is inherent in the system. The displacement of the phase shift by the energy displacement ε_N at $t = t_N$ is given by substituting $\varepsilon = -\varepsilon_N$ and $\tau = 0$ into Eq. (5) as

$$\Delta\psi_{y-N} = -\frac{2\pi}{T_{\text{rev}}}\varepsilon_N\left(\frac{\xi_0}{\omega_S}+\frac{\xi_1}{2}t_N\right)\sin\omega_S t_N. \quad (25)$$

The phase spread variance at $t > t_N$ is obtained by the integration of $\Delta\psi_{y-N}{}^2$ from the deflection. When ε_N is a random radiation loss, the expected phase shift variance $\sigma_{\psi N}{}^2$ is given by

$$\sigma_{\psi N}^2 = \langle\varepsilon_N^2\rangle\int_0^t\left[-\frac{2\pi}{T_{\text{rev}}}\left(\frac{\xi_0}{\omega_S}+\frac{\xi_1}{2}t_N\right)\sin\omega_S t_N\right]^2dt_N. \quad (26)$$

Here $\langle\varepsilon_N^2\rangle$ is the expected variance of ε_N given by the longitudinal damping time τ_L and the natural energy spread σ_ε as

$$\langle\varepsilon_N^2\rangle = 4\sigma_\varepsilon^2/\tau_L. \quad (27)$$

Equation (26) then becomes

$$\sigma_{\psi N}^2 = \left(\frac{2\pi}{T_{\text{rev}}}\right)^2\frac{\sigma_\varepsilon^2}{\tau_L\omega_S^3}\left\{\xi_0^2(2\omega_S t - \sin2\omega_S t)\cos^2\omega_S t + \xi_0\xi_1\left(\omega_S^2t^2\cos^2\omega_S t - \frac{1}{4}\omega_S t\sin4\omega_S t\right) + \left(\frac{\xi_1}{2}\right)^2\left[\frac{2}{3}\omega_S^3t^3 - \omega_S t + \sin2\omega_S t\left(\frac{1}{2}+\omega_S^2t^2\sin^2\omega_S t\right)\right]\right\}. \quad (28)$$

The reduction of F_1 by the phase spread $\sigma_\psi^2 = \sigma_{\psi E}^2 + \sigma_{\psi N}^2$ is obtained by replacing $e^{j\omega x}$ by the following convolution:

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_\psi} \exp\left[-\frac{\omega^2(x-t)^2}{2\sigma_\psi^2}\right] e^{j\omega t} dt$$

$$= \exp\left(-\frac{\sigma_\psi^2}{2}\right) e^{j\omega x} dt. \quad (29)$$

This means that the large intensity enhancement due to coherence is obtained while $\sigma_\psi^2 < 1$. The values of time and ω_ψ which give $\sigma_{\psi N}^2 = 1$, referred to as t_C and $\omega_{\psi C}$, are important parameters. For large values of t_C these parameters are given by

$$t_C \approx \left(\frac{3}{2}\tau_L\right)^{1/3} \left(\frac{T_{\rm rev}}{\pi \xi_1 \sigma_\varepsilon}\right)^{2/3}, \quad (30)$$

$$\omega_{\psi C} \approx \frac{1}{\sigma_\tau} \left(\frac{3\pi}{2T_{\rm rev}} \xi_1 \sigma_\varepsilon \tau_L\right)^{1/3}. \quad (31)$$

The FWHM spectral bandwidth is roughly given by

$$\Delta\omega/\omega_\psi \approx 2\sqrt{\ln 2}/(\sigma_\tau \omega_\psi). \quad (32)$$

V. CALCULATIONS USING PARAMETERS OF NEWSUBARU

We will estimate the CSR power using the parameters of NewSUBARU [13,14] listed in Table I. At an electron energy of 1.0 GeV, $\sigma_\tau = 16$ ps, which results in CSR emission up to 0.01 THz (normal CSR). After the deflection by the kicker magnet, the phase fluctuation σ_ψ^2 reaches unity at $t_C \approx 13\pi/\omega_S$ and $\omega_{\psi C} \approx 2\pi \times 0.04$ THz. This means that coherent radiation with a wavelength 4 times shorter is emitted. After another 6.5 synchrotron oscillation periods ($t = 26\pi/\omega_S$), the center radiation frequency becomes 0.092 THz with a power reduction of $\exp(-\sigma_{\psi N}^2) = 4 \times 10^{-4}$ and spectral width of $\Delta\omega_\psi/\omega_\psi \approx 0.12$. The radiation power in this case is still 2 orders of magnitude larger than in the case of horizontally polarized incoherent radiation.

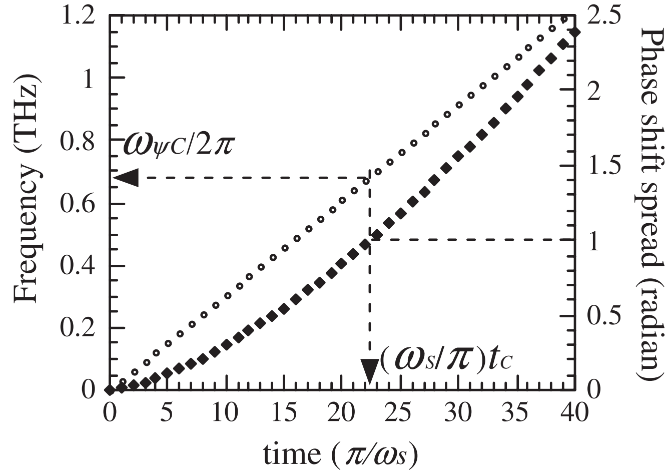

FIG. 1. Evolution of the coherent radiation frequency (open circle) and the phase shift spread (shaded diamond).

If we reduce the stored electron energy to 0.5 GeV and increase the power of both the rf acceleration and the sextupole as listed in Table I, the CSR frequency would become higher. The shift of $\omega_\psi/2\pi$ and σ_ψ at $t = n\pi/\omega_S$ is shown in Fig. 1. The natural bunch length is $\sigma_\tau = 3.2$ ps, which emits CSR up to 0.05 THz. At $t_C \approx 22\pi/\omega_S$ the CSR frequency is $\omega_{\psi C} \approx 2\pi \times 0.68$ THz. Figure 2 shows the simulated bunch profile in the τ-y plane. The power spectrum and the spatial intensity distribution at the observer calculated using the analytical formulas are shown in Figs. 3 and 4, respectively. Unlike other methods, the spectral line does not have higher harmonic components. After more revolutions at $t_C \approx 33\pi/\omega_S$ (1.1 ms), the CSR frequency reaches 1 THz with a power reduction of $\exp(-\sigma_{\psi N}^2) = 0.04$. However, in this situation, we need to consider some additional effects. One effect is the transient bunch lengthening due to transverse and longitudinal coupling [15], which can occur at 0.8 THz in the worst case. The other effect which needs to be considered is the phase variation caused by the energy

TABLE I. Parameter of NEWSUBARU[12].

Stored electron energy	1 GeV	0.5 GeV
Momentum compaction factor (α_P)	0.0013	
Revolution frequency ($1/T_{\rm rev}$)	2525 kHz	
Curvature of radius of bending magnet (ρ)	3.221 m	
Natural energy spread (σ_ε)	0.047%	0.024%
Longitudinal damping time (τ_L)	12 ms	96 ms
Synchrotron oscillation frequency ($\omega_S/2\pi$)	6 kHz	15 kHz
dc chromaticity (ξ_0)	0	0
ac chromaticity amplitude (ξ_1)	1.0	10
Number of stored electrons in a bunch (N_e)	10^{10}	
Vertical oscillation amplitude (y_0)	2 mm	

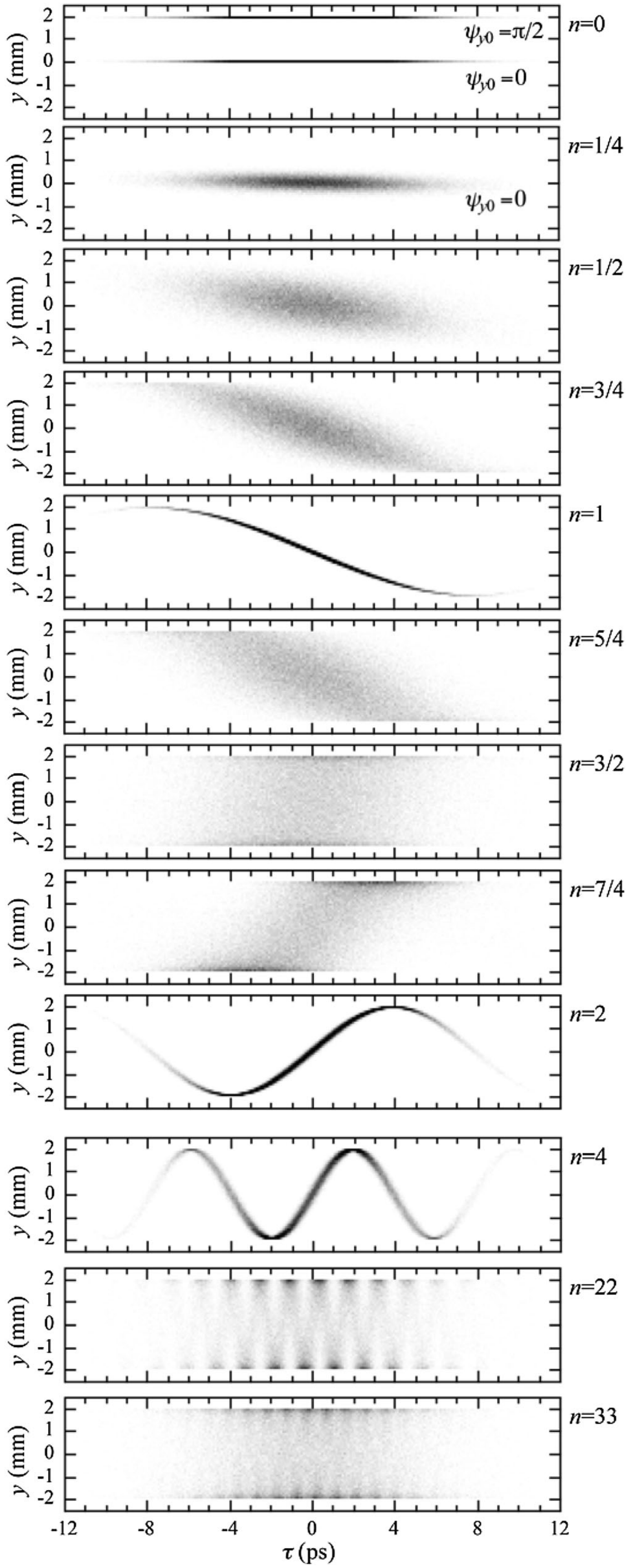

FIG. 2. Typical spatial bunch structures in the τ-y plane. Two images at $n = 0$ show structures at two betatron phases. Both the longitudinal radiation excitation as well as damping were considered. The contributions of the horizontal movement and the nonlinear effect were not considered.

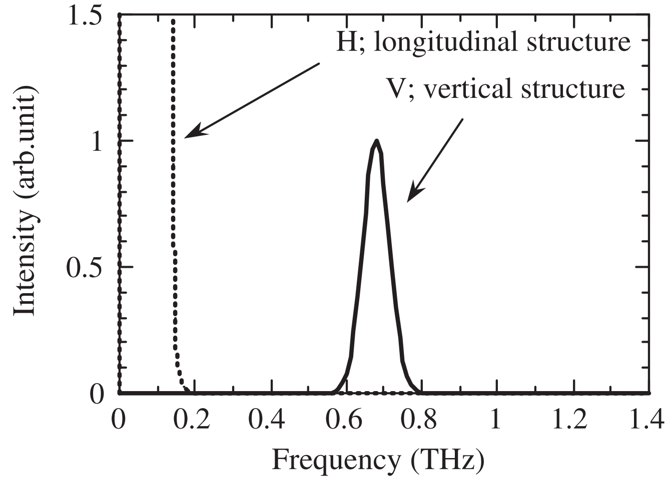

FIG. 3. Power spectrum of CSR at $n = 22$ ($t = 22\pi/\omega_S$) and $\theta_0 = 0$. The "H" and "V" indicate the direction of the polarization. The vertical axis is normalized with the peak for the vertically polarized CSR produced by the vertical structure.

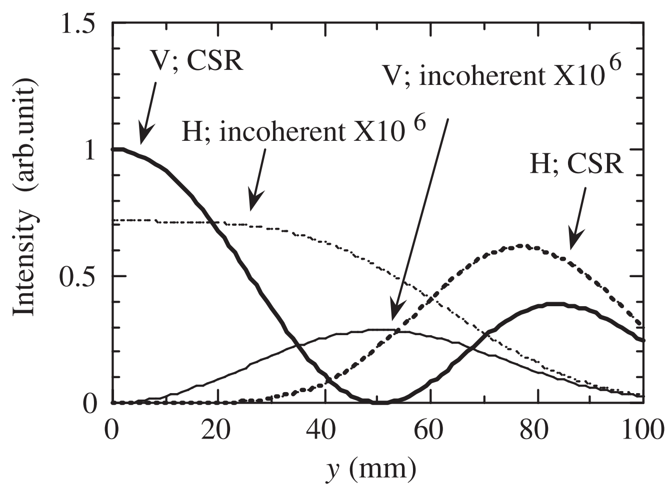

FIG. 4. Spatial intensity distribution along the y direction at $n = 22$ ($t = 22\pi/\omega_S$) for $\omega_\psi \approx 2\pi \times 0.68$ THz. The same normalization for the vertical axis as that for Fig. 3 is applied.

spread given in Eq. (9). At $t_C \approx 33\pi/\omega_S$ the condition for small spread, $\sigma^2_{\psi N} < 1$, is satisfied only for 2.6 revolutions. We ignored the nonlinear effect of the rf bucket, the amplitude dependence of a synchrotron tune, which might be harmful. However, it is adjustable by means of multipole magnets set at dispersive sections. Apparently the longer damping time means reduction of the deflection rate. However, it would be less serious than Guo's method because the CSR intensity does not depend on the vertical emittance.

VI. CONCLUSION

We have shown that a low energy storage ring with a longer damping time can be used to generate strong radiation at 1 THz. The parameters listed in Table I are not optimized which means that there is much scope for improving this technique. For example, according to Eq. (31) a reduction in the linear momentum compaction factor α_P will result in an increase in $\omega_{\psi C}$.

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