Measurement of coherent beam-beam tune shift under crabbing collision at KEKB

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Effective head-on collision and improvement in specific luminosity were established by the installation of crab cavities in KEKB. Tune spectra of a colliding bunch in a crabbing collision were observed by using a spectrum analyzer. The beam-beam spectrum showed strong nonlinear resonance. By taking into account this nonlinearity, the coherent beam-beam tune shift was measured as a function of the bunch current. The vertical beam-beam parameter estimated from the coherent beam-beam tune shift was found to agree with that observed by using a bunch-by-bunch luminosity monitor. The estimated vertical beam-beam parameter is saturated at approximately 0.05; this value is the beam-beam limit. Further, the bunch current corresponding to the beam-beam limit was found to be considerably lower than that used in usual operations. The horizontal beam-beam parameter was not saturated over 0.15. The horizontal beam-beam parameter estimated from the coherent beam-beam tune shift approximately agrees with a calculated beam-beam parameter considering the dynamic beam-beam effect.

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I. INTRODUCTION

KEKB [1] is a multibunch, high-current, electron/positron collider for producing *B* mesons. The collider consists of two storage rings: a low energy ring (LER) for a 3.5-GeV positron beam and a high energy ring (HER) for 8-GeV electrons, respectively. Both rings can store more than 1500 bunches, where the harmonic number is 5120 with an rf frequency of 509 MHz. The bunches are stored in two rings with a three-bucket (6 ns) or four-bucket (8 ns) spacing, forming a single bunch train followed by empty buckets that occupy approximately 5% of the circumference. Additional bunches, called pilot bunches, are placed just after the train at different locations in each ring so that they do not collide with each other.

The electron and positron beams collide at an interaction point (IP) with a horizontal crossing angle of 22 mrad. Crab cavities that were installed at KEKB in 2007 can provide horizontal deflections operating at the rf frequency to bunches without changing the central orbit [2]. These crab cavities enable the realization of an effective head-on collision, while maintaining the crossing orbit at the IP. The effect of the presence of the crab cavity becomes evident in the horizontal beam-beam kick. Figure 1 shows a comparison of two beam-beam kicks of particles colliding with and without a crossing angle for the same bunch intensity. The beam-beam kick increases when the collision mode is changed from the crossing collision to the head-on collision. As can be seen in Fig. 1, the slope of the linear part in the beam-beam kick in the case of a head-on collision is roughly twice the slope in the case of a crossing collision; the relative beam size is estimated to decrease by a factor of 1.4 in the case of crabbing collision (see the Appendix).

Crabbing collisions were successfully performed for the first time [3]. Since only one crab cavity is installed per ring, the effect of the crab kick can be observed in the entire ring. The crabbing collision resulted in an increase in the specific luminosity at low currents; however, the specific luminosity decreased with an increase in the bunch current. The maximum current was limited by the lifetimes of both beams. These trends observed at KEKB are of primary concern from the viewpoint of beam dynamics. However, the cause and the mechanism of these phenomena are unknown. KEKB currently operates at 0.5 to 0.6 mA^2, which is the product of bunch currents with a bunch spacing of 6 ns. This current product is almost half of that in the case of a crossing collision.

The coherent beam-beam tune shift can be used to evaluate the beam-beam parameter in colliders. However, measurement of the coherent beam-beam tune shift is not

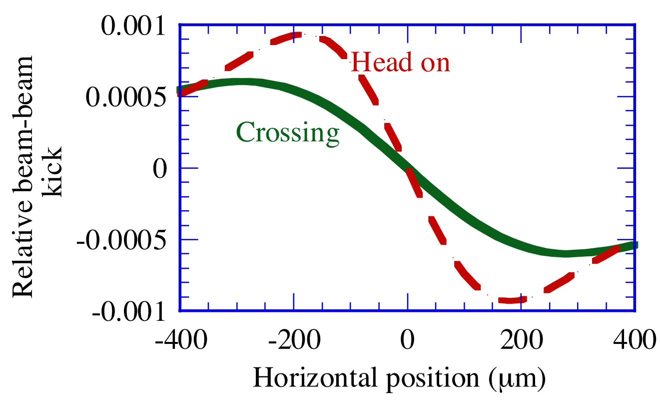

FIG. 1. (Color) Relative beam-beam kick as a function of the horizontal position shift at the IP; the dashed red line indicates the relative beam-beam kick in the case of a head-on collision, and the solid green line indicates the relative beam-beam kick in the case of a horizontal crossing angle of 22 mrad under identical beam conditions.

easy because of the difference in the beam parameters in double-ring colliders such as KEKB. Since KEKB is operated at a horizontal betatron tune near a half integer, dynamic beam-beam effects should be considered [4]. In this study, the horizontal and vertical tune shifts during a crabbing collision were measured, and special attention was paid to the nonlinearity in the tune spectrum. The sum of the coherent beam-beam parameters that represented an effective beam-beam force of a bunch with a small amplitude was estimated from the coherent beam-beam tune shifts. The next section describes the relationship between the tune shifts and the beam-beam parameters.

II. COHERENT BEAM-BEAM TUNE SHIFT

Assuming that both beams are Gaussian, the bunch-by-bunch luminosity L can be expressed in terms of the coherent vertical beam-beam parameter $\Xi_y^{\pm}$ [5] as follows:

$$L \approx \frac{f_0}{r_e} \frac{N^+\gamma^+ N^-\gamma^-}{N^+\gamma^+ + N^-\gamma^-} \frac{\Xi_y^+ + \Xi_y^-}{\beta_y^*} R_L, \qquad (1)$$

where f_0 is the revolution frequency; r_e, the classical electron radius; $N^{\pm}$, the number of particles in a positron/electron bunch; $\gamma^{\pm}$, the relativistic factor; β_y^*, the vertical beta function at the collision point (IP); and R_L, the luminosity reduction factor resulting from the hourglass effect. The superscripts + and − denote positron and electron bunches, respectively. Here, we assume that the conditions $\beta_y^{*+} = \beta_y^{*-} = \beta_y^*$ and a flat beam $\beta_x^* \gg \beta_y^*$ hold true. However, in reality, the condition for energy transparency $N^+\gamma^+ = N^-\gamma^-$ is not satisfied. The specific luminosity L_s, defined as $L_s \equiv L/(I_b^+ I_b^-)$, is inversely proportional to the cross-sectional area of the beams at the IP and is independent of the beam current. Here, $I_b^+ I_b^-$ is the product of bunch currents. The beam-beam interaction generates two oscillation modes: a higher (H) mode and a lower (L) mode; both modes are affected by the beam-beam force. Assuming the rigid Gaussian model, the resultant tunes in the horizontal or the vertical plane are expressed by [6]

$$\cos\mu_{qH} + \cos\mu_{qL} - (\cos\mu_{q0}^+ + \cos\mu_{q0}^-) = -2\pi(\Xi_q^+ \sin\mu_{q0}^+ + \Xi_q^- \sin\mu_{q0}^-). \qquad (2)$$

Here, $\mu_{q0}^{\pm}$ is the betatron phase advance without collision and the subscript 0 means noncollision. The tunes of the H mode and the L mode are calculated, when the unperturbed tunes and the coherent beam-beam parameters are given. The coherent beam-beam tune shift is defined as follows:

$$\Delta\nu_{bb} = \nu_H + \nu_L - \nu_0^+ - \nu_0^-, \qquad (3)$$

where ν_H and ν_L are the tunes of the H mode and L mode, respectively, and ν_0^+ and ν_0^- are unperturbed tunes. The sum of the coherent beam-beam parameter $\Xi_q^+ + \Xi_q^-$ was calculated as a function of the coherent beam-beam tune shift $\Delta\nu_{bb}$. It was confirmed that the coherent beam-beam tune shift did not depend on the ratio of the coherent beam-beam parameter Ξ_q^+/Ξ_q^-, when the sum of the coherent beam-beam parameter $\Xi_q^+ + \Xi_q^-$ was given. Thus, the coherent beam-beam tune shift defined in Eq. (3) is a simple and good index to evaluate the beam-beam parameter. Figure 2 shows the sum of the coherent beam-beam parameters as a function of the coherent beam-beam tune shift, assuming $\Xi_q^+ = \Xi_q^-$. Since the horizontal tune is close to a half integer, the relationship is nonlinear, as shown in Fig. 2(a). It is noted that the Yokoya factor Y [7] should be considered in the relation between the beam-beam parameter and the coherent tune shift. On the other hand, the sum of the coherent beam-beam parameters is approximately equal to the average incoherent beam-beam parameter $\bar{\xi}_q$; i.e., $\Xi_q^+ + \Xi_q^- \approx \bar{\xi}_q = (\xi_q^+ + \xi_q^-)/2$, under the condition that the sizes of both beams are not much different. When the tunes of both beams are equal, i.e. $\nu_0^+ = \nu_0^-$, the equations of $\nu_H = \nu_\pi$ and $\nu_L = \nu_0^{\pm}$ hold true. Then, the coherent beam-beam tune shift becomes

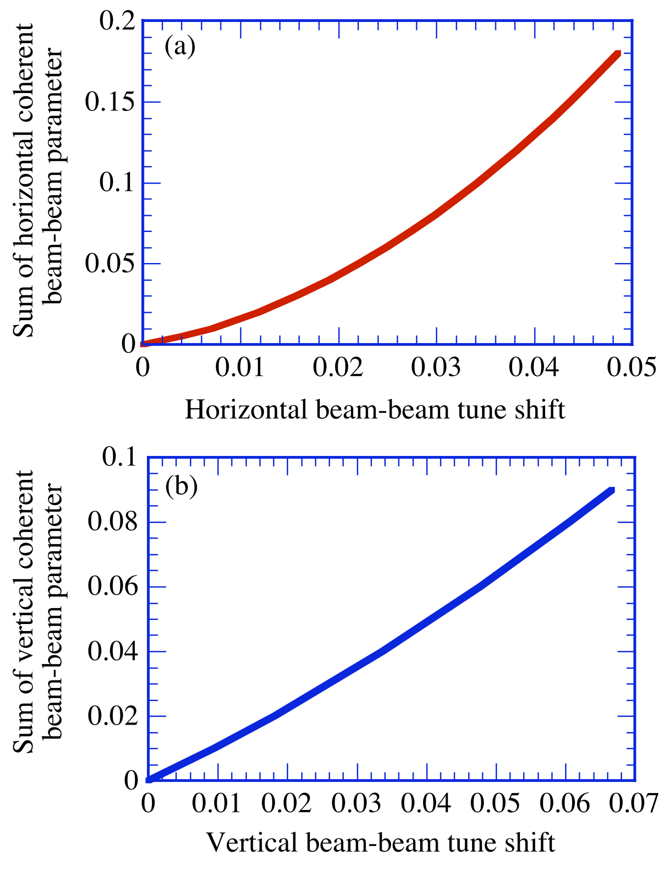

FIG. 2. (Color) Sum of coherent beam-beam parameters as a function of the coherent beam-beam tune shift. Part (a) shows the horizontal plane calculated with unperturbed fractional tunes $\nu_{x0}^+ = 0.506$ and $\nu_{x0}^- = 0.512$, and (b) shows the vertical plane calculated with unperturbed fractional tunes $\nu_{y0}^+ = 0.571$ and $\nu_{y0}^- = 0.589$, assuming $\Xi_q^+ = \Xi_q^-$.

$\Delta\nu_{bb} = \nu_\pi - \nu_0$ that is widely used in single-ring colliders. The sum of the horizontal coherent beam-beam parameters is expressed as follows:

$$\Xi_x^+ + \Xi_x^- = \frac{r_e \beta_{x0}^*}{2\pi\Sigma_x^2}\left(\frac{N^-}{\gamma^+} + \frac{N^+}{\gamma^-}\right), \quad (4)$$

where Σ_x is the horizontal effective beam size defined as $\Sigma_x = \sqrt{(\sigma_x^+)^2 + (\sigma_x^-)^2}$ and $\beta_{x0}^+ = \beta_{x0}^- = \beta_{x0}^*$ are assumed. The sum of the horizontal coherent beam-beam parameters can be obtained from the effective beam size. Since the horizontal betatron tune is close to a half integer, the beam size would be affected by the dynamic beam-beam effect.

III. MEASUREMENT

A. Instruments

The betatron tune is measured by a swept-frequency method using a spectrum analyzer (Advantest R3132) and a sine-wave tracking generator. Such a measurement of the betatron tune is equivalent to the measurement of the frequency response function of the beams. Bunches are excited alternately in the horizontal and vertical planes, and the resultant oscillations are detected using independent devices. A gated tune monitor [8] is employed, as shown in Fig. 3, to excite a selected bunch and detect the oscillation of the excited bunch in a multibunched beam using fast gates. The excitation level can be controlled using a variable attenuator shown in Fig. 3. In order to eliminate a forced damping effect arising from beam feedback, the transverse feedback of the selected bunch is turned off. The spectrum analyzer has a resolution bandwidth of 100 Hz and a sweep time of 1.9 s. A measurement resolution of approximately 2×10^{-4} is achieved by using a fitted resonance curve. The noise level corresponds to an amplitude oscillation of 0.3 μm at the pickup, where $\beta_{x0} \approx 22$ m and $\beta_{y0} \approx 22$ m in the LER. The dynamic range is greater than 60 dB. A bunch-by-bunch luminosity monitor, termed a zero-degree luminosity monitor [9], is used to monitor the relative luminosity of a selected bunch with an accuracy of 10%. The absolute luminosity can be estimated by using conventional luminosity monitors [10].

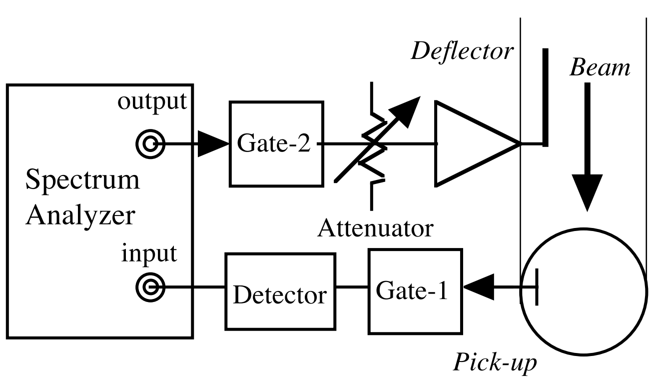

FIG. 3. Schematic gated tune monitor.

B. Observation of tune spectra

The frequency spectrum was observed by using the spectrum analyzer under the condition that the tunes of both the beams were almost identical. The frequency spectrum when expressed as a function of the betatron tune is called a tune spectrum. We observed two peaks in the vertical tune spectra of both the beams, as shown in the upper panel of Fig. 4: the peak in the lower tune side of the spectrum represents the L mode, and that on the higher tune side represents the H mode. For the L mode, a sym-

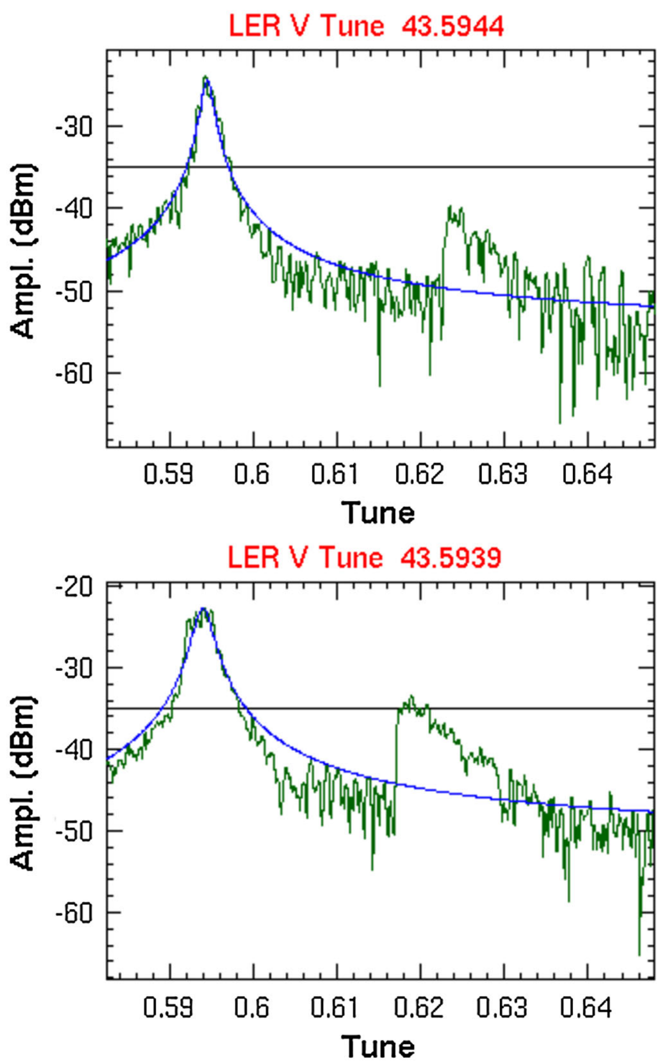

FIG. 4. (Color) Vertical tune spectra of a colliding positron bunch denoted by the green curve. The blue line is a resonance curve fitted to a single peak. The left peak represents the L mode and the right side spectrum represents the H mode. The upper and lower excitation levels are −28 and −25 dB, respectively. In the vertical axis, 0 dBm corresponds to an oscillation amplitude of 0.3 mm. An amplitude of −20 dBm at the detector corresponds to an amplitude of less than 1 μm at the IP.

metrical spectrum similar to usual tune spectra is obtained, since it is not significantly affected by the beam-beam force. On the other hand, the peak in the H mode spectrum is not at the center, but shifts to the lower tune side. This asymmetry in the spectrum is due to the nonlinearity of the beam-beam kick. When the excitation level is slightly increased by 3 dB in the gated tune monitor system, the peak in the H mode spectrum is shifted to the lower tune side by 0.005, although the peak in the L mode spectrum remains almost unaffected, as shown in the lower panel of Fig. 4. The tune spectra obtained in the horizontal plane also show an amplitude-dependent tune shift, as shown in Fig. 5. The peak corresponding to the H mode shifts by -0.006 when the excitation level is increased by 9 dB. The peak amplitudes showing the nonlinear resonance oscillations are less than the beam size. The nonlinearity in the beam-beam kick causes the amplitude dependence of the tune shift. Since the beam-beam parameter depends on the linear portion of the kick, the coherent tune shift should be determined from the amplitude response to an infinitely small excitation.

Next, the intensity dependence of the tune spectrum was studied. Figure 6 compares two spectra of a positron bunch that have almost the same bunch current under a constant excitation level. As shown in the upper panel of Fig. 6, at a low electron bunch current, two peaks appear in the spectrum; the peak toward the left represents the L mode, and the broad peak with a tail toward the higher tune side represents the H mode. The edge of the H mode spectrum

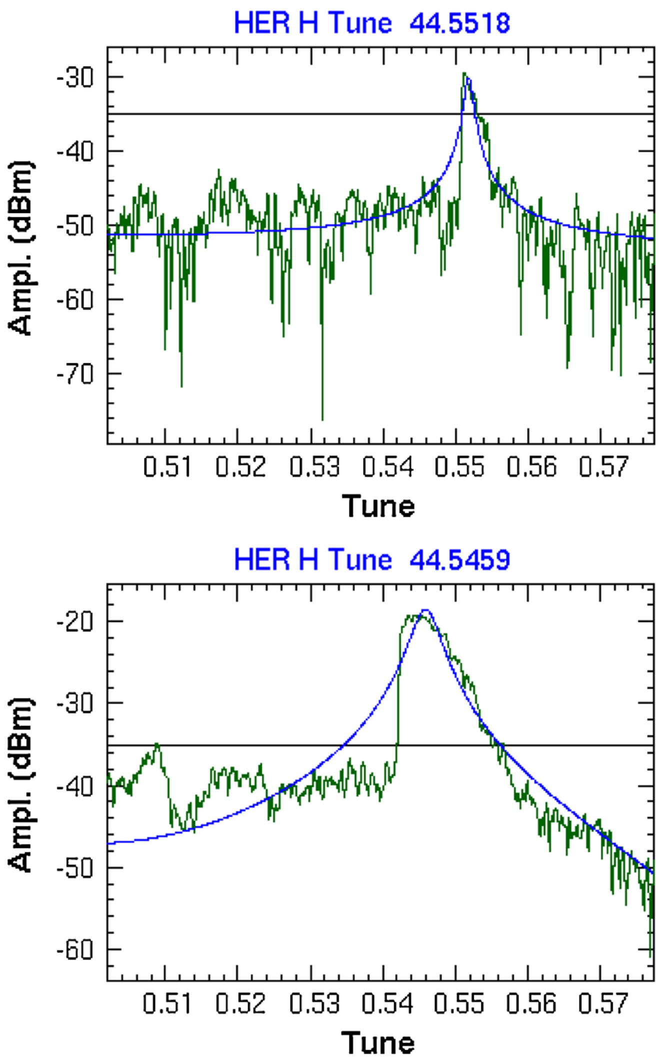

FIG. 5. (Color) Horizontal tune spectra of an electron bunch denoted by the green curve. The blue line is a resonant curve fitted to a single peak. The peak corresponds to the H mode. The excitation level changes from -39 dB (upper) to -30 dB (lower). The L mode appears as a second peak that is located around a tune of 0.509 in the lower panel. An amplitude of -20 dBm at the detector corresponds to an amplitude of 6 μm at the IP.

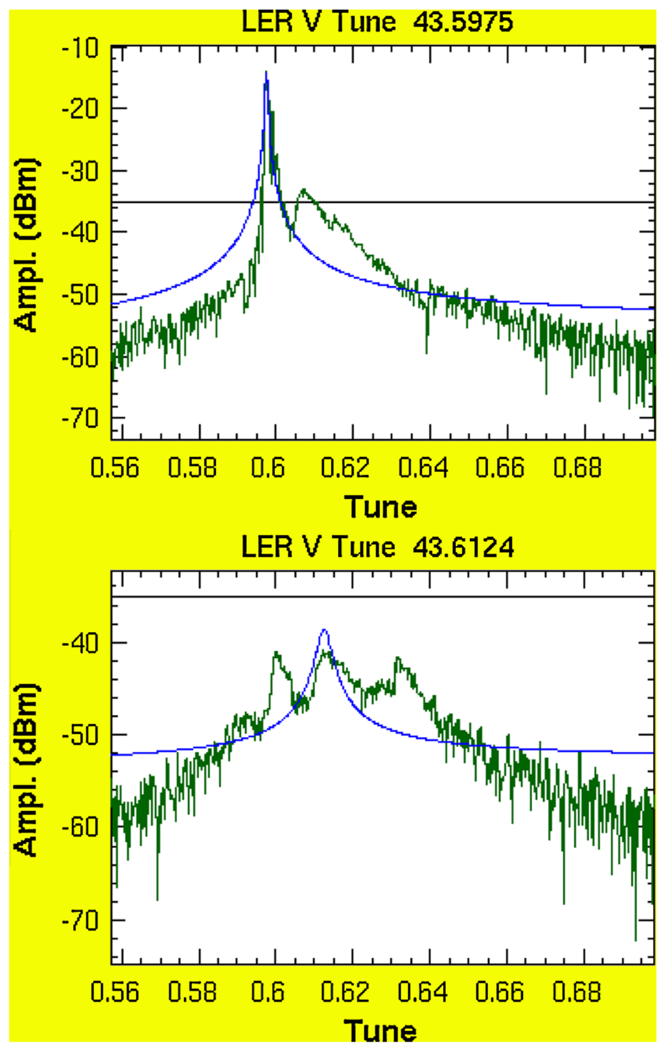

FIG. 6. (Color) Vertical tune spectra of a positron bunch while increasing the electron bunch current denoted by the green curve. The positron bunch current is 0.82 mA, while the electron bunch current is changed from 0.38 mA (upper) to 0.58 mA (lower). The blue line is a resonance curve fitted to a single peak.

is regarded as the tune of the *H* mode. However, at a higher electron current, the *H* mode spectrum splits into several peaks. The major two peaks are separated by approximately 0.02. Since the separation is close to the synchrotron tune, these spectra might be related to synchrobetatron beam-beam modes [11]. However, the phenomena should be confirmed by other methods. At the same time, we observed the broad spectrum or the large tune spread together with the decrease in the amplitude of the *L* mode. Besides the changes in the spectrum, the lifetime of the positron bunch became short. Widening of the spectrum was also observed in the horizontal plane at high bunch currents.

The tune spectrum along a bunch train was observed. Since the positron bunches are affected by the presence of an electron cloud, the vertical tune shifts along the bunch train depending on the cloud density. The vertical tune was observed to shift by 0.01 at a beam current of 1600 mA in the absence of collisions [8]. The vertical tune spectrum of a leading bunch in a bunch train during collisions is compared to that of a trailing bunch. Figure 7 shows that the *L* mode tunes of the two spectra differ by approximately 0.015. The effect of the presence of the electron cloud is superimposed on the beam-beam force acting on the trailing bunch. The broad spectrum of the trailing bunch is indicative of this complex effect. An additional peak is observed in the tune spectrum of the trailing bunch, as shown in the lower panel of Fig. 7. This second peak represents the *H* mode, because the same tune was observed in the partner electron beam. The peak corresponding to the *H* mode did not appear in the tune spectrum of the leading bunch, as shown in the upper panel of Fig. 7. The difference in the spectra of the leading and trailing bunches suggests a difference in the collision states. However, the bunch-by-bunch luminosity monitor did not show any characteristic changes in the specific luminosity along a bunch train.

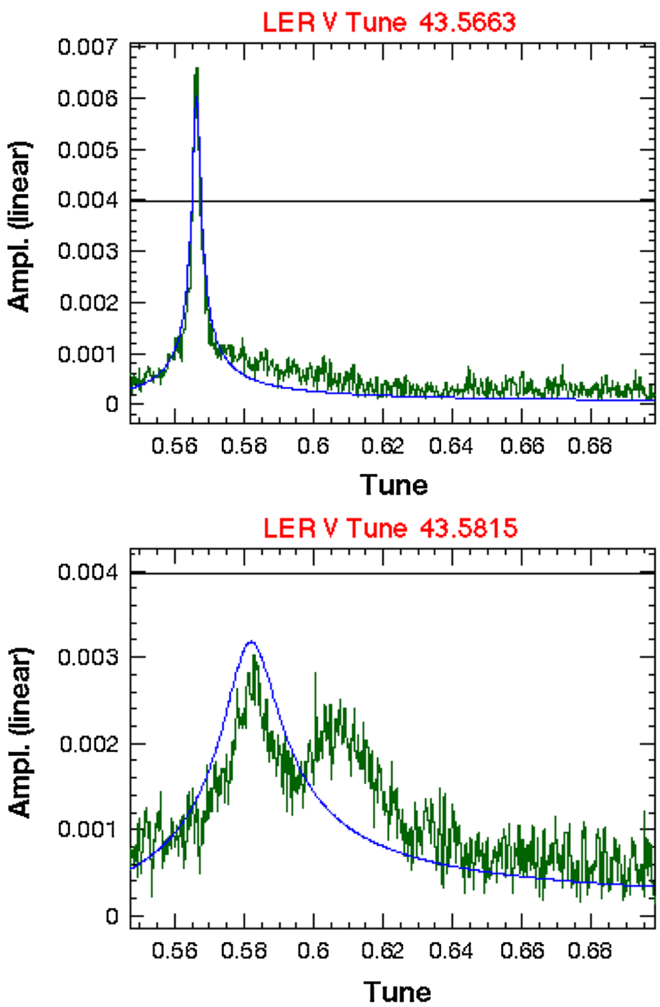

FIG. 7. (Color) Vertical tune spectra of a positron bunch at bucket number 0 (upper) and 4753 (lower). The bunch currents of the positron and electron are 0.93 and 0.45 mA, respectively. The total currents of the positron and electron are 1600 and 850 mA, respectively. An amplitude of 1.0 in the vertical scale corresponds to an oscillation amplitude of 0.3 mm.

C. Beam-beam tune shift

After tuning KEKB at a large bunch spacing of 192 ns, an additional bunch was injected into an empty bucket. The beam-beam tune shift and the luminosity were measured while increasing the injected bunch charge step by step. The major optical parameters are listed in Table I.

The vertical beam-beam parameter is estimated from the coherent tune shift as well as the luminosity. Figure 8 shows the vertical average beam-beam parameter estimated from the measured luminosity and from the coherent beam-beam tune shift as a function of the product of bunch currents. Coefficients $Y = 1.23$ and $R_L = 0.83$ are used in the estimation [12]. The beam-beam parameter increases at low currents; however, the vertical beam-beam parameters estimated from the tune shift and from the luminosity tend to be saturated at a level of approximately 0.05, which is termed the beam-beam limit. The corresponding bunch current product is 0.14 mA^2.

The horizontal coherent beam-beam tune shift is expressed as a function of the bunch current product. The tune shift increases with the current product, as shown in Fig. 9. In order to obtain the sum of the coherent beam-beam parameter from the measured tune shift, a second-

TABLE I. Beam and machine parameters

Parameter	LER	HER
Horizontal emittance (nmrad): ϵ_x	24	24
Beta's at IP (cm): β_x^*/β_y^*	80/0.59	80/0.59
Horizontal betatron tune: ν_x	45.507	44.510
Vertical betatron tune: ν_y	43.595	43.595
Synchrotron tune: ν_s	−0.0249	−0.0216
Crab voltage (MV)	0.94	1.43

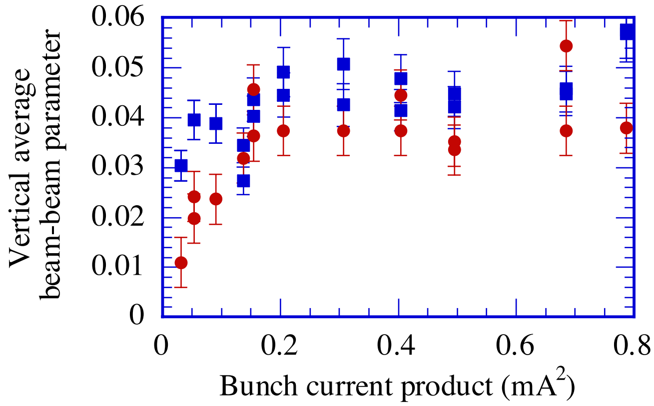

FIG. 8. (Color) Vertical average beam-beam parameter as a function of the product of bunch currents. The red dots represent the average coherent beam-beam parameter estimated from the tune shift, and the blue squares represent the beam-beam parameter estimated from the luminosity measurement.

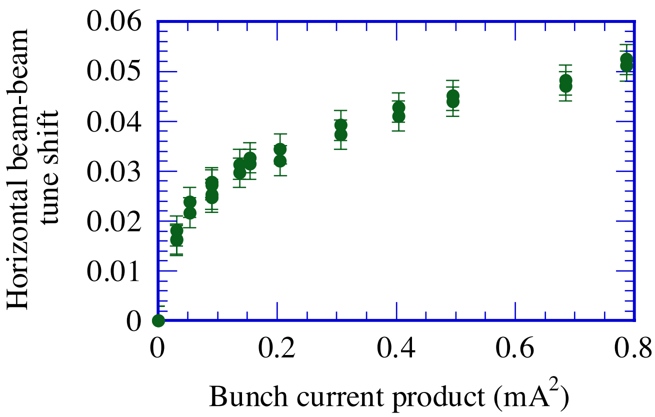

FIG. 9. (Color) Horizontal coherent beam-beam tune shift as a function of the product of bunch currents.

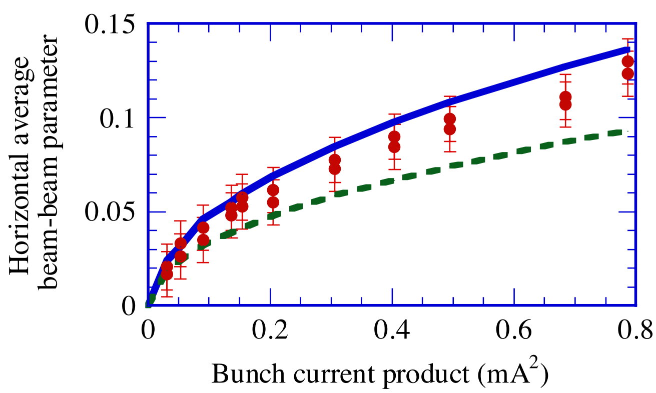

FIG. 10. (Color) The red dots represent horizontal average beam-beam parameter as a function of the product of the bunch currents. The blue solid line represents the horizontal average beam-beam parameter calculated using the beam size considering the dynamic beam-beam effect. The green dashed line is the beam-beam parameter calculated using the nominal beam size. The tune of the *H* mode is used in the calculation of the dynamic beam size.

order fitting curve is approximately used in the relation between the beam-beam tune shift and the beam-beam parameter as shown in Fig. 2. A Yokoya factor Y of 1.31 is used, assuming that the aspect ratio is 5%. The estimated horizontal average beam-beam parameter is not saturated above $\bar{\xi}_x = 0.1$ as shown by dots in Fig. 10. Calculated beam-beam parameters are also shown by the solid line and the dashed line in Fig. 10. The average beam-beam parameter estimated from the tune shift is close to the solid line that means a calculated average beam-beam parameter using the effective beam size considering the dynamic beam-beam effect. It is noted that tunes of the *H* mode are used in the calculation.

IV. DISCUSSION

As shown in Figs. 4 and 5, the position of the peak in the *H* mode depends on the excitation amplitude. This dependence is caused by the nonlinearity of the beam-beam kick. The nonlinearity is the cause behind the hysteresis phenomena [13], where a peak in the spectrum shifts according to the sweep direction of the spectrum analyzer.

As shown in Fig. 8, the vertical average beam-beam parameter tends to be saturated at bunch current products greater than 0.14 mA^2. During actual operations, the bunch current product is 0.5 to 0.6 mA^2. According to our findings, the decrease in the specific luminosity with an increase in the bunch current is caused by the saturation in the vertical beam-beam parameter. In our experiment, the vertical beam-beam parameter is less than 0.06. On the other hand, the peak ξ_y is achieved to be greater than 0.08 in actual operations, as estimated from the average luminosity monitor [3]. There are two reasons for the discrepancy. The first reason is that the measured bunch and its partner bunch oscillate coherently because of the excitation applied for measuring the tune spectrum. The vibration reduces the luminosity by a factor of approximately 10%. The second reason is that a correction factor that accounts for the hourglass effect in the vertical beam-beam parameter was used in Ref. [3]: however, this factor is not used in our experiment. The introduction of this factor in the equation changes the vertical beam-beam parameter by 15%. Considering these two factors, the vertical beam-beam parameter obtained by our study is not very different from that in Ref. [3]. Further, the measured coherent tune shift is found to agree with the results of the luminosity measurement.

According to the dynamic beam-beam effect, the horizontal emittance increases with the beam-beam parameter. On the contrary, the betatron function at the IP decreases with the beam-beam parameter. Since the decreasing rate in the betatron function is higher than the increasing rate in the emittance, the beam size at the IP results in a decrease with an increase in the beam-beam parameter. The reduction in the beam size raises the horizontal beam-beam parameter as expected from Eq. (4). The estimated beam-

beam parameter from the tune-shift measurement is higher than the calculated beam-beam parameter using the nominal beam size, as shown in Fig. 10. Another measurement for the horizontal beam-beam parameter was carried out with low nominal emittances of $\epsilon_{x0}^{+} = 15$ nmrad and $\epsilon_{x0}^{-} = 18$ nmrad, as shown in Fig. 11. The average beam-beam parameter increases up to a value of 0.2 in these parameters. The estimated horizontal average beam-beam parameter agrees with a calculated beam-beam parameter including the dynamic beam-beam effect. It was found that the luminosity was affected by changing the horizontal beam-beam parameter or the beam size. The reduction rate of the specific luminosity, as a function of the bunch current product, tends to be high when the collision occurs at $\epsilon_{x0}^{+} + \epsilon_{x0}^{-} = 15 + 18$ nmrad when compared to that observed when the collision occurs at $\epsilon_{x0}^{+} + \epsilon_{x0}^{-} = 18 + 24$ nmrad. The reduction rate of the specific luminosity depended on the horizontal emittance or the beam size, as shown in Fig. 12. We suspect that the horizontal beam size and the magnitude of the horizontal beam-beam parameter might be related to the reduction in the luminosity by a nonlinear *x-y* coupling. Further, we observed that the tune spectra in the *L* mode broadened with an increase in the bunch current. Moreover, the horizontal beam profile estimated from the beam-beam kick indicates that the central part of the beam profile shrinks due to the dynamic beam-beam effect; however, the density in the peripheral regions is lower than that in a Gaussian profile, as shown in Fig. 13. The measured beam-beam kick suggests that the horizontal profile would expand in the peripheral regions with a low density. These observations might be related to the facts that the maximum bunch current is limited by the lifetime of the beams. We need to investigate the relationship between the tune spread caused by the collision and the momentum aperture with a help of simulation.

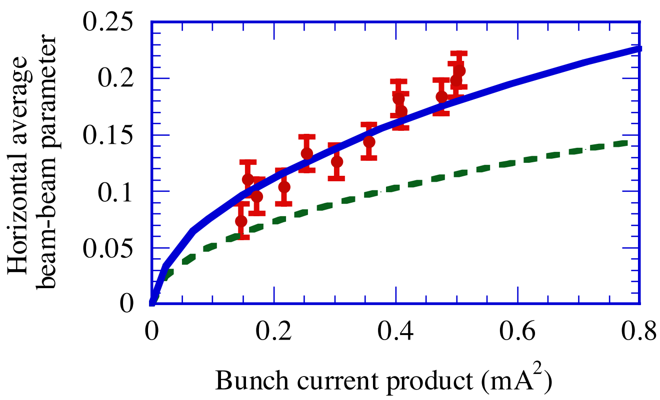

FIG. 11. (Color) The red dots represent horizontal average beam-beam parameter as a function of the product of bunch currents with low emittances of $\epsilon_{x0}^{+} = 15$ nmrad and $\epsilon_{x0}^{-} = 18$ nmrad. The blue line represents the horizontal beam-beam parameter calculated using the beam size considering the dynamic beam-beam effect. The green dashed line is the beam-beam parameter calculated using the nominal beam size. The tune of the *H* mode is used in the calculation of the dynamic beam size.

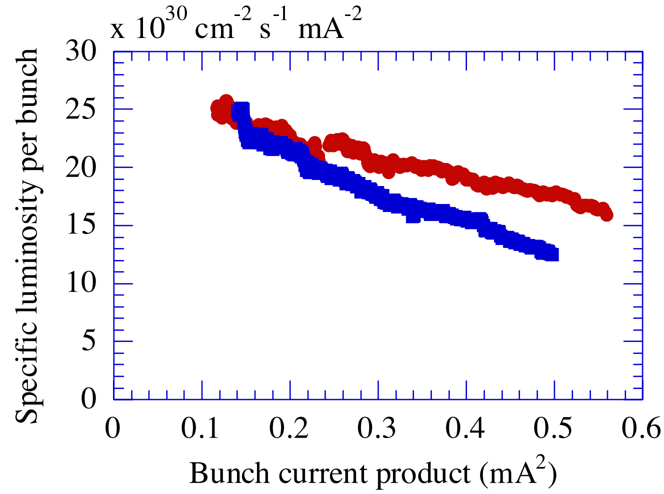

FIG. 12. (Color) Specific luminosity per bunch as a function of the product of bunch currents. The red dots and blue dots represent the specific luminosity at $\epsilon_{x0}^{+} + \epsilon_{x0}^{-} = 18 + 24$ nmrad and $\epsilon_{x0}^{+} + \epsilon_{x0}^{-} = 15 + 18$ nmrad, respectively, measured with a fixed betatron function at the IP.

We measured the coherent beam-beam tune shift during a crabbing collision and obtained the following experimental results: (i) The estimated value of $\bar{\xi}_y$ is saturated at approximately 0.05 at a low bunch current product of 0.14 mA^2; this value of $\bar{\xi}_y$ is termed a beam-beam limit. This value suggests that the specific luminosity decreases as the bunch current increases. (ii) The $\bar{\xi}_x$ is not saturated, and increases up to a value of 0.2. The $\bar{\xi}_x$ estimated from the coherent beam-beam tune shift approximately agrees

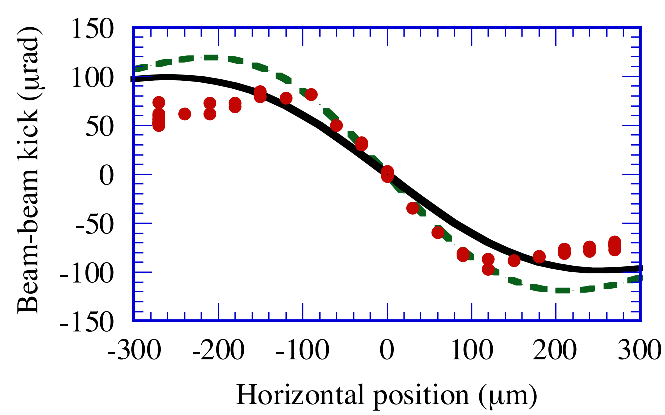

FIG. 13. (Color) The red dots represent a beam-beam kick as a function of the horizontal position, measured at a bunch current product of 0.4 mA^2. The dashed green line represents a beam-beam kick with an effective size of 162 μm considering the dynamic beam-beam effect. The solid black line represents a beam-beam kick with an effective nominal size of 196 μm. The kicks were calculated, assuming a Gaussian profile.

with a calculated beam-beam parameter considering the dynamic beam-beam effect. (iii) Although the central part of the horizontal beam profile shrinks due to the dynamic beam-beam effect, the horizontal profile deviates from the Gaussian profile with a low density in the peripheral region. The tune spread is also observed in the horizontal and vertical planes at high bunch currents. These phenomena might be related to the fact that the maximum bunch current is limited by the lifetime of the beams.

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APPENDIX

When two beams collide with an orbit offset Δ_x^* at the IP, bunches are kicked by an electromagnetic force generated by the opposite beam, and the orbits of both the beams are distorted around the ring. A position monitor located at a phase advance of $|\Delta\phi_d|$ from the IP detects the position shift caused by the collision. A position shift recorded by the detector is represented as follows:

$$\Delta X_{\rm det} = \frac{\sqrt{\beta_{\rm det}\beta^*}}{2\sin(\pi\nu)}\theta_{bb}\cos(\pi\nu - |\Delta\phi_d|). \tag{A1}$$

Here, $\beta_{\rm det}$ and β^* are the betatron functions at the detector and the IP, respectively; ν is the betatron tune; θ_{bb} is the beam-beam kick angle. Assuming that the vertical offset is zero, the horizontal beam-beam kick angle θ_{bbx} is expressed by using the following rigid Gaussian model [14]:

$$\theta_{bbx}(\Delta_x^*) = \frac{-2r_eN_b}{\gamma}\Delta_x^*\int_0^\infty \frac{\exp(-\frac{(\Delta_x^*)^2}{(t+2\Sigma_x^2)})}{(t+2\Sigma_x^2)^{3/2}(t+2\Sigma_y^2)^{1/2}}dt, \tag{A2}$$

where Σ_x and Σ_y are the effective horizontal and vertical beam sizes, respectively. The effective beam size is defined as $\Sigma_{x/y} = \sqrt{(\sigma_{x/y}^+)^2 + (\sigma_{x/y}^-)^2}$. The superscript $\pm$ denotes positron or electron bunches. The betatron function changes dynamically with the beam-beam parameter, depending on the betatron tune. Calculation using the optics parameters shows that the product of the betatron functions is constant, i.e., $\sqrt{\beta_{\rm det}\beta^*} \approx 4.0$ m. Thus, the position shift at the detector is proportional to the beam-beam kick. When the horizontal offset is smaller than the beam size, Eq. (A2) can be approximated as follows:

$$\theta_{bbx}^\pm \approx \frac{-1.94r_eN^\mp}{\gamma^\pm}\frac{\Delta_x^*}{\Sigma_x^2}. \tag{A3}$$

By using Eq. (A3), we can estimate the effective horizontal beam size at the IP from the slope $\theta_{bbx}^\pm/\Delta_x^*$.

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