Small signal gain of a Smith-Purcell free electron laser

G. F. Mkrtchian*

Department of Quantum Electronics, Yerevan State University, 1 A. Manukian, Yerevan 375025, Armenia
(Received 18 June 2007; published 7 August 2007)

In this paper the operability of a Smith-Purcell free-electron laser for an arbitrary grating is considered. An arbitrary grating is described in terms of impedance matrix. The consideration is based on the self-consistent set of Maxwell-fluid equations. The dispersion relation is established for the TM mode of electromagnetic waves. The latter is studied considering amplification of the evanescent as well as radiation modes. The small signal gain in various limits is calculated and the dependence of the gain on the electron beam current is analyzed.

DOI: 10.1103/PhysRevSTAB.10.080701 PACS numbers: 41.60.Cr, 42.25.Fx, 07.57.Hm

I. INTRODUCTION

A traveling charged particle induces radiation when it passes over the metallic grating. This radiation, called Smith-Purcell (SP) radiation [1], has several remarkable properties. Since its first observation, SP radiation has been studied both experimentally and theoretically by many authors [2–4] (for more recent discussions and other references on the subject see [5]). Owing to the unique properties of SP radiation, it has long been recognized that SP radiation can be a basic mechanism for a compact free-electron laser (FEL) [6,7].

Recently, there has been a renewed interest in the SP system related to both a possibility of using it to generate intense radiation from the millimeter to optical region [8] and to implement it in nondestructive beam diagnostics [9]. The experiment at Dartmouth College [10,11], where superradiance in the SP system was observed, has stimulated new investigations concerning the SP FEL as an open slow wave structure [12–16].

In the paper [7], the authors considered the initial incident propagating wave, which is amplified during the multiple reflection upon grating. They predicted a small signal gain of SP FEL to be proportional to $I_b^{1/3}$, where I_b denotes the electron beam current. The authors of [7] considered magnetized electron beam and described grating in terms of reflection matrix. It an additional requirement was invoked that the reflection matrix is singular at the frequency of the incident propagating wave. But this is actually not the case. As will be shown below, the singularities of the reflection matrix correspond to eigenmodes of grating which are the evanescent waves.

In [12], the authors considered an infinitely thin electron beam passing close to a grating. They assumed that both an evanescent and a propagating mode have the same frequency. The electron beam amplifies the evanescent mode, which scatters into the propagating radiation mode. The gain predicted by this model is $\propto I_b^{1/2}$.

A different result has been obtained in Ref. [13]. In this paper it is considered a rectangular grating, assuming that the entire space above the grating is filled by a uniform electron beam. The authors have established the dispersion relation and found the dispersion law $\omega(k)$. It turns out that the dispersion equation allows only evanescent solutions and the operating point of a SP FEL is fixed by the intersection of the dispersion curve with the beam line. The corresponding small signal gain follows the $I_b^{1/3}$ law. However, the coefficient differs from that of Ref. [7]. In Refs. [14,15] it has also been shown that for a low-energy electron beam the group velocity of the resonant evanescent wave may be negative. This means that the device operates on an absolute instability regime in contrast to the conventional FEL, where a convective instability takes place. At the absolute instability, the SP FEL operates like a backward wave oscillator [17] and without external feedback if the current is above a threshold value called the start current.

In [16], the authors have revised the work [12] with proper evaluation of singularities of reflection matrix for rectangular grating. The authors considered an infinitely thin electron beam extending analysis also to the nonlinear regime by performing one-dimensional numerical simulations. Particularly, they found that the growth rate equation becomes cubic near the singularity of the reflection matrix as obtained in Ref. [7]. Otherwise the growth rate equation is quadratic as obtained in Ref. [12]. The latter regime corresponds to the spectral region, where some of the spectral orders are radiative.

Recently, SP FEL dynamics has also been investigated using the 2D particle-in-cell (PIC) code [18,19]. The numerical results of [18] in general are in agreement with the analysis of [13,14], but the gain calculated in [18] is within a factor of 2 of that predicted in the work [13].

Hence, there is no general consent on the mechanisms of the SP FEL and the small signal gain obtained by different

*mkrtchian@ysu.am

approaches. Taking also into account that there is currently substantial interest in the implementation of a SP FEL, it is of interest to reexamine the theory of SP FEL.

In the present work, the operability of the SP FEL for the arbitrary grating is considered. The method described allows any complex geometry one may wish to consider so long as the geometry repeats itself periodically and has one axis of translational invariance. The consideration is based on the self-consistent set of Maxwell-fluid equations. An arbitrary grating is described in terms of impedance matrix. The electron beam is not assumed to be infinitely thin as it was considered in Refs. [12,16] and it does not occupy the entire space above the grating as was assumed in Ref. [13]. We consider an electron beam of finite thickness to be at a finite height above the grating and in contrast to Ref. [7] there is no external incident wave, and the electron beam is not magnetized. For comparison, we also analyze the case of a magnetized electron beam. We derive a general expression for the dispersion relation and examine various limits of small signal gain depending on the particle beam and grating parameters. Then, obtained results are applied to rectangular grating making comparisons with the results of the works [13,18].

The paper is organized as follows. In Sec. II we solve the self-consistent set of equations which describe FEL dynamics in general and as the most effective case the hydrodynamic instability of a cold electron beam is considered. Rectangular grating is considered in Sec. III. A concluding section summarizes the paper.

II. HIGH-GAIN REGIME OF A SP FEL

In this section, we develop a theory of a SP FEL for an arbitrary grating considering the linear stage of instability and calculate the small signal gain of a SP FEL in the regime of hydrodynamic instability. We take a Cartesian coordinate system: the electrons initially move to the z direction in the vacuum over the grating along the trajectories $d \le x \le d + \delta$ as it is shown in Fig. 1 and coupled with the TM mode of electromagnetic wave. The grating is

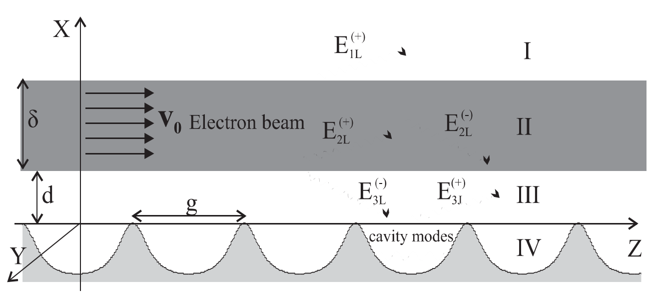

FIG. 1. The configuration of the Smith-Purcell FEL. The electron beam moves at a distance d parallel to the grating surface in the z direction. The grooves, oriented in the y direction, repeat periodically with the grating period g. The electron beam thickness is δ. The grating surface is at $x = 0$.

ruled parallel to the y axis and its ruled area is assumed to be large enough to ignore any boundary effect. We denote the grating period as g.

Our goal is to obtain the dispersion relation for the TM mode of electromagnetic waves. The roots of the dispersion equation give us the functional dependence $\omega(k)$. The presence of the electron beam leads to complex shift of the characteristic frequencies (or wave numbers). The roots with imaginary part indicate the collective instability, i.e., exponential gain of the corresponding wave.

Following the ansatz developed in the paper [20] for surface Cherenkov FEL, we proceed to calculation of the gain in the regime of hydrodynamic instability. Thus, we will arise from the self-consistent set of the Maxwell and fluid equations. For the TM mode ($E_y = H_x = H_z = 0$)

$$E_x(x, z, t) = E_{\omega x}(x, z)e^{-i\omega t} + \text{c.c.},$$
$$E_z(x, z, t) = E_{\omega z}(x, z)e^{-i\omega t} + \text{c.c.},$$
$$H_y(x, z, t) = H_{\omega y}(x, z)e^{-i\omega t} + \text{c.c.},$$

the Maxwell equations can be written as

$$\partial_x E_{\omega z} - \partial_z E_{\omega x} = -i\frac{\omega}{c}H_{\omega y}, \tag{1a}$$

$$-\partial_z H_{\omega y} = -i\frac{\omega}{c}E_{\omega x} + \frac{4\pi}{c}j_{\omega x}, \tag{1b}$$

$$\partial_x H_{\omega y} = -i\frac{\omega}{c}E_{\omega z} + \frac{4\pi}{c}j_{\omega z}, \tag{1c}$$

where

$$\mathbf{j}(x, z, t) = \mathbf{j}_\omega(x, z)e^{-i\omega t} + \text{c.c.} \tag{2}$$

is the current density. The fluid equations read (with $\mathbf{v}$ denoting the local fluid velocity)

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)(\mathbf{v}\gamma) = \frac{e}{m}\mathbf{E} + \frac{e}{mc}[\mathbf{v}\times\mathbf{H}], \tag{3a}$$

$$\frac{\partial n}{\partial t} + \text{div}(n\mathbf{v}) = 0, \tag{3b}$$

$$\mathbf{j}(x, z, t) = en(x, z, t)\mathbf{v}(x, z, t), \tag{3c}$$

where $\gamma = 1/\sqrt{1 - \text{v}^2/c^2}$ is the Lorenz factor, m is the electron mass, e is the electron charge, and $n(x, z, t)$ is the electron beam density. Before the interaction for $n(x, z, t)$ we assume

$$n(x, z, t)|_{t=0} \equiv n_0(x) = \begin{cases} 0, & x < d \\ n_0 = \text{const}, & d \le x \le d + \delta. \\ 0, & x > d + \delta \end{cases} \tag{4}$$

The set of Eqs. (1)–(3) holds as in the grooves of the grating as well as out of it ($x \ge 0$).

The linearization of the fluid equations (3)

$$\mathbf{v} = \mathbf{v}_0 + (\mathbf{v}_\omega e^{-i\omega t} + \text{c.c.}), \qquad \mathbf{v}_0 = \{0, 0, \text{v}_0\}, \quad \text{v}_0 = \text{const}, \tag{5}$$

$$n = n_0(x) + (n_\omega e^{-i\omega t} + \text{c.c.}) \tag{6}$$

gives

$$\text{v}_0 \frac{\partial \text{v}_{\omega x}}{\partial z} - i\omega \text{v}_{\omega x} = \frac{e}{m\gamma_0}\left(E_{\omega x} - \frac{\text{v}_0}{c} H_{\omega y}\right), \tag{7}$$

$$\text{v}_0 \frac{\partial \text{v}_{\omega z}}{\partial z} - i\omega \text{v}_{\omega z} = \frac{e}{m\gamma_0^3} E_{\omega z}, \tag{8}$$

$$\text{v}_0 \frac{\partial n_\omega}{\partial z} - i\omega n_\omega + \frac{\partial}{\partial x}(n_0(x)\text{v}_{\omega x}) + n_0(x)\frac{\partial}{\partial z}\text{v}_{\omega z} = 0, \tag{9}$$

$$j_{\omega x} = en_0(x)\text{v}_{\omega x}; \qquad j_{\omega z} = en_0(x)\text{v}_{\omega z} + en_\omega \text{v}_0. \tag{10}$$

Taking into account the periodicity of the grating, the solutions of Eqs. (1) above the grating are expanded in infinite series

$$E_{\omega x,z}(x, z) = \sum_{\text{L}=-\infty}^{\infty} E_{x,z\text{L}}(x)e^{ik_\text{L} z}, \tag{11a}$$

$$H_{\omega y}(x, z) = \sum_{\text{L}=-\infty}^{\infty} H_{y\text{L}}(x)e^{ik_\text{L} z}, \tag{11b}$$

where

$$k_\text{L} = k + \frac{2\pi}{g}\text{L}.$$

With the help of Eqs. (11), from Eqs. (7) and (8) we obtain

$$\text{v}_{\omega x,z} = \sum_{\text{L}=-\infty}^{\infty} \text{v}_{x,z\text{L}} e^{ik_\text{L} z}, \tag{12}$$

where

$$\text{v}_{z\text{L}} = \frac{ie}{m\gamma_0^3\Omega_\text{L}} E_{z\text{L}}(x), \tag{13}$$

$$\text{v}_{x\text{L}} = \frac{ie}{m\gamma_0\Omega_\text{L}}\left(E_{x\text{L}}(x) - \frac{\text{v}_0}{c} H_{y\text{L}}(x)\right). \tag{14}$$

Here

$$\Omega_\text{L} = \omega - \text{v}_0 k_\text{L} \tag{15}$$

is the resonance width.

Taking into account Eqs. (13) and (14), from Eqs. (9) and (10) for the amplitudes of the current density components we obtain

$$j_{\omega x,z} = \sum_{\text{L}=-\infty}^{\infty} j_{x,z\text{L}} e^{ik_\text{L} z}, \quad j_{x\text{L}}(x) = \frac{1}{4\pi\gamma_0} \frac{c^2\omega_p^2(x)}{c^2k_\text{L}^2 - \omega^2 + \frac{\omega_p^2(x)}{\gamma_0}} \frac{\chi_\text{L}}{\Omega_\text{L}} \frac{\partial E_{z\text{L}}(x)}{\partial x}, \tag{16}$$

$$j_{z\text{L}}(x) = \frac{i\omega_p^2(x)\omega}{4\pi\gamma_0^3\Omega_\text{L}^2} E_{z\text{L}}(x) + \frac{\text{v}_0}{i\Omega_\text{L}} \frac{\partial j_{x\text{L}}(x)}{\partial x}, \tag{17}$$

where

$$\omega_p(x) = \sqrt{\frac{4\pi e^2 n_0(x)}{m}}$$

is the plasma frequency of the electron beam and

$$\chi_\text{L} = k_\text{L} - \beta\frac{\omega}{c}; \qquad \beta = \frac{\text{v}_0}{c}. \tag{18}$$

Returning to the Maxwell equations (1) with the help of Eqs. (16) and (17), for the tangential component of the field amplitude we obtain the following equation:

$$\frac{\partial^2 E_{z\text{L}}(x)}{\partial x^2} + \left(\frac{\omega^2}{c^2} - k_\text{L}^2\right)\left(1 - \frac{\omega_p^2(x)}{\gamma_0^3\Omega_\text{L}^2}\right)E_{z\text{L}}(x) + \frac{c^2\chi_\text{L}^2}{\gamma_0\Omega_\text{L}^2}\frac{\partial}{\partial x}\left(\frac{\omega_p^2(x)}{\omega^2 - c^2k_\text{L}^2 - \frac{\omega_p^2(x)}{\gamma_0}}\frac{\partial E_{z\text{L}}(x)}{\partial x}\right) = 0. \tag{19}$$

In region (I) for the tangential component of the field from this equation we have

$$E_{z\text{L}}^{(\text{I})}(x) = E_{1\text{L}}^{(+)} e^{-q_\text{L} x}, \tag{20}$$

where $E_{1\text{L}}^{(+)}$ is a constant and

$$q_\text{L}^2 = k_\text{L}^2 - \frac{\omega^2}{c^2} = \left(k + \frac{2\pi}{g}\text{L}\right)^2 - \frac{\omega^2}{c^2}. \tag{21}$$

When q_L is imaginary, the solution (20) represents propagating waves. In this case we assume

$$q_\text{L} = -i\sqrt{\frac{\omega^2}{c^2} - k_\text{L}^2}$$

and Eq. (20) corresponds to an outgoing wave to $x = \infty$. When q_L is real, the solution (20) corresponds to an evanescent wave. To satisfy the boundary condition, that the wave vanish in the limit $x \to \infty$, we will choose the root with positive real part.

In region (II) $d \le x \le d + \delta$, where the electron beam exists, we have

$$\frac{\partial^2 E_{z\text{L}}(x)}{\partial x^2} - \varkappa_\text{L}^2 E_{z\text{L}}(x) = 0 \tag{22}$$

and the solution can be written in the form

$$E_{z\text{L}}^{(\text{II})}(x) = E_{2\text{L}}^{(+)} e^{-\varkappa_\text{L} x} + E_{2\text{L}}^{(-)} e^{\varkappa_\text{L} x}, \tag{23}$$

where

$$\varkappa_{\mathrm{L}}^2 = q_{\mathrm{L}}^2 + \frac{\omega_p^2}{\gamma_0 c^2}. \tag{24}$$

In region (III) the solution of Eq. (19) can be written in the form

$$E_{z\mathrm{L}}^{(\mathrm{III})}(x) = E_{3\mathrm{L}}^{(+)} e^{-q_{\mathrm{L}} x} + E_{3\mathrm{L}}^{(-)} e^{q_{\mathrm{L}} x}. \tag{25}$$

Hence the solutions of the Maxwell equations (1) in the gap between the electron beam and grating can be written in the form

$$E_{z\omega}^{(\mathrm{III})}(x, z) = \sum_{\mathrm{L}=-\infty}^{\infty} (E_{3\mathrm{L}}^{(+)} e^{-q_{\mathrm{L}} x} + E_{3\mathrm{L}}^{(-)} e^{q_{\mathrm{L}} x}) e^{ik_{\mathrm{L}} z}, \tag{26a}$$

$$H_{y\omega}^{(\mathrm{III})}(x, z) = \sum_{\mathrm{L}=-\infty}^{\infty} \frac{i\omega}{cq_{\mathrm{L}}} (E_{3\mathrm{L}}^{(+)} e^{-q_{\mathrm{L}} x} - E_{3\mathrm{L}}^{(-)} e^{q_{\mathrm{L}} x}) e^{ik_{\mathrm{L}} z}, \tag{26b}$$

$$E_{x\omega}^{(\mathrm{III})}(x, z) = \sum_{\mathrm{L}=-\infty}^{\infty} \frac{ik_{\mathrm{L}}}{q_{\mathrm{L}}} (E_{3\mathrm{L}}^{(+)} e^{-q_{\mathrm{L}} x} - E_{3\mathrm{L}}^{(-)} e^{q_{\mathrm{L}} x}) e^{ik_{\mathrm{L}} z}. \tag{26c}$$

Next we consider the boundary conditions. For this purpose we integrate Eq. (19) across the interface between the electron beam and vacuum

$$\lim_{\varepsilon\to 0} \int_{x_{1,2}-\varepsilon}^{x_{1,2}+\varepsilon} \frac{\partial}{\partial x}\left[\frac{\partial E_{z\mathrm{L}}(x)}{\partial x} + \frac{c^2 \chi_{\mathrm{L}}^2}{\gamma \Omega_{\mathrm{L}}^2}\left(\frac{\omega_p^2(x)}{\omega^2 - c^2 k_{\mathrm{L}}^2 - \frac{\omega_p^2(x)}{\gamma_0}} \frac{\partial E_{z\mathrm{L}}(x)}{\partial x}\right)\right] dx = 0, \tag{27}$$

where $x_1 = d + \delta$, $x_2 = d$, and we obtain the boundary condition for the normal component of the wave field

$$\frac{\partial E_{z\mathrm{L}}(x)}{\partial x}\Bigg|_{x=x_2\downarrow} = \frac{q_{\mathrm{L}}^2}{\varkappa_{\mathrm{L}}^2}\left(1 - \frac{\omega_p^2}{\gamma_0^3 \Omega_{\mathrm{L}}^2}\right) \frac{\partial E_{z\mathrm{L}}(x)}{\partial x}\Bigg|_{x=x_2\uparrow}, \tag{28}$$

$$\frac{\partial E_{z\mathrm{L}}(x)}{\partial x}\Bigg|_{x=x_1\uparrow} = \frac{q_{\mathrm{L}}^2}{\varkappa_{\mathrm{L}}^2}\left(1 - \frac{\omega_p^2}{\gamma_0^3 \Omega_{\mathrm{L}}^2}\right) \frac{\partial E_{z\mathrm{L}}(x)}{\partial x}\Bigg|_{x=x_1\downarrow}. \tag{29}$$

Taking also into account the matching conditions for the tangential fields

$$E_{z\mathrm{L}}(x)|_{x=x_2\downarrow} = E_{z\mathrm{L}}(x)|_{x=x_2\uparrow}, \tag{30}$$

$$E_{z\mathrm{L}}(x)|_{x=x_1\downarrow} = E_{z\mathrm{L}}(x)|_{x=x_1\uparrow}, \tag{31}$$

we obtain the coupling equation for $E_{3\mathrm{L}}^{(+)}$, $E_{3\mathrm{L}}^{(-)}$, $E_{2\mathrm{L}}^{(+)}$, $E_{2\mathrm{L}}^{(-)}$, and $E_{1\mathrm{L}}^{(+)}$. From these equations one can express the amplitude of the reflected wave, as

$$E_{3\mathrm{L}}^{(-)} = \Xi_{\mathrm{L}} E_{3\mathrm{L}}^{(+)}, \tag{32}$$

where

$$\Xi_{\mathrm{L}} = \frac{1 - \Theta_{\mathrm{L}}^2}{1 + \Theta_{\mathrm{L}}^2 + 2\Theta_{\mathrm{L}} \coth\varkappa_{\mathrm{L}}\delta} e^{-2q_{\mathrm{L}} d} \tag{33}$$

is the coupling constant and

$$\Theta_{\mathrm{L}} = \frac{q_{\mathrm{L}}}{\varkappa_{\mathrm{L}}}\left(1 - \frac{\omega_p^2}{\gamma_0^3 \Omega_{\mathrm{L}}^2}\right). \tag{34}$$

To guide electron beams near the grating surface, usually the strong static magnetic field is applied along the direction of the electron beam motion. At that, the electron beam dynamics is assumed to be one dimensional. The electron beam can be assumed to be magnetized and electron motion—one dimensional, if

$$\omega_c = \frac{|e|H_0}{mc\gamma_0} \gg |\Omega_0|,$$

and the Larmor gyration of the electron with radius R_c does not violate the electron wave coupling $R_c \ll \mathrm{v}_0\gamma_0/\omega$. The latter reads

$$H_0 \gg \frac{\omega m \mathrm{v}_\perp}{\beta|e|}.$$

Here H_0 is the static magnetic field strength, $\mathrm{v}_\perp$ is the electron transversal velocity, and ω_c is the cyclotron frequency. For magnetized electron beam, one can omit the density variation effect and the normal component of the current density in Eqs. (1)–(3). In this case Eq. (19) reads

$$\frac{\partial^2 E_{z\mathrm{L}}(x)}{\partial x^2} + \left(\frac{\omega^2}{c^2} - k_{\mathrm{L}}^2\right)\left(1 - \frac{\omega_p^2(x)}{\gamma_0^3 \Omega_{\mathrm{L}}^2}\right) E_{z\mathrm{L}}(x) = 0. \tag{35}$$

In region (II), where the electron beam exists, the solution can be written in the form

$$E_{z\mathrm{L}}^{(\mathrm{II})}(x) = E_{2\mathrm{L}}^{(+)} e^{-\alpha_{\mathrm{L}} x} + E_{2\mathrm{L}}^{(-)} e^{\alpha_{\mathrm{L}} x}, \tag{36}$$

where

$$\alpha_{\mathrm{L}} = q_{\mathrm{L}} \sqrt{1 - \frac{\omega_p^2}{\gamma_0^3 \Omega_{\mathrm{L}}^2}}. \tag{37}$$

With the same analysis performed for an unmagnetized electron beam, one can obtain coupling constant (32) for the magnetized electron beam:

$$\Xi_{(\mathrm{mag})\mathrm{L}} = \frac{q_{\mathrm{L}}^2 - \alpha_{\mathrm{L}}^2}{q_{\mathrm{L}}^2 + \alpha_{\mathrm{L}}^2 + 2\alpha_{\mathrm{L}} q_{\mathrm{L}} \coth\alpha_{\mathrm{L}}\delta} e^{-2q_{\mathrm{L}} d}. \tag{38}$$

Note that the latter is exactly the same as that obtained in Ref. [7].

For the boundary conditions on the grating surface, at $x = 0$, one needs the solution of the Maxwell equations in the grooves ($x < 0$). The latter is expressed as series of the cavity modes in the grooves. At this stage we do not concretize this solution and represent the boundary conditions imposed by the grating, in terms of an impedance matrix. As the most general relation between the amplitudes of electric and magnetic fields we take

$$E_{zL}^{(III)}(0) = \sum_{J=-\infty}^{\infty} \frac{cq_J Z_{LJ}}{i\omega} H_{zJ}^{(III)}(0), \qquad (39)$$

where Z_{LJ} is the impedance matrix. In terms of $E_{3J}^{(-)}$ and $E_{3J}^{(+)}$, the relation (39) with the help of Eqs. (26) becomes

$$E_{3L}^{(+)} + E_{3L}^{(-)} = \sum_{J=-\infty}^{\infty} Z_{LJ}(E_{3J}^{(+)} - E_{3J}^{(-)}). \qquad (40)$$

Here the elements of the impedance matrix are determined by the coupling of each incident harmonic ($E_{3J}^{(-)}$) with the entire manifold of reflected ones ($E_{3J}^{(+)}$). Taking also into account the coupling of reflected and incident waves due to the presence of the electron beam [see, Eq. (32)], we obtain

$$\sum_{J=-\infty}^{\infty} ((Z_{LJ} - \delta_{LJ}) - \Xi_J(Z_{LJ} + \delta_{LJ}))E_{3J}^{(+)} = 0. \qquad (41)$$

For a solution of Eq. (41) to exist, the determinant of the coefficients must be zero:

$$\det[(Z_{LJ} - \delta_{LJ}) - \Xi_J(Z_{LJ} + \delta_{LJ})] = 0. \qquad (42)$$

This is the most general dispersion relation, and its roots give us the functional dependence $\omega(k)$.

Now, we should take into account that the electrons resonantly interact with an evanescent mode traveling along the surface of a grating. Indeed, for the resonant interaction, the phase velocity of the wave should be synchronous with the mean velocity of the electron beam. The wave-electron synchronous implies that

$$\Omega_N = \omega - v_0 k_N \simeq 0, \qquad (43)$$

and from Eq. (21) we see that for this mode

$$q_N^2 = \frac{\omega^2}{\gamma_0^2 v_0^2} > 0. \qquad (44)$$

Without loss of generality, one can assume N = 0. Hence, near the particle-wave synchronism Ξ_J can be omitted, except the one for a mode J = 0, then

$$\Xi_J = \Xi_0 \delta_{0J},$$

and the dispersion relation (42) can be rewritten as

$$(1 - \Xi_0)\mathcal{D}(\omega, k) - 2\Xi_0 \mathcal{M}(\omega, k) = 0. \qquad (45)$$

Here we have introduced the notations

$$\mathcal{D}(\omega, k) \equiv \det(Z_{LJ} - \delta_{LJ}), \qquad (46)$$

and

$$\mathcal{M}(\omega, k) \equiv \det(Z_{LJ} - \delta_{LJ})_{00} \qquad (47)$$

is the corresponding minor. The dispersion relation (45) among other effects includes the space charge effects, the gain dependence on the beam thickness, and the beam height above the grating. Note that Eq. (45) can be written in terms of the reflection matrix:

$$1 + \Xi_0 R_{00} = 0, \qquad (48)$$

where

$$R_{00} = -\frac{\mathcal{D}(\omega, k) + 2\mathcal{M}(\omega, k)}{\mathcal{D}(\omega, k)} \qquad (49)$$

is the element of reflection matrix:

$$H_{3L}^{(+)} = \sum_{J=-\infty}^{\infty} R_{LJ} H_{3J}^{(-)}.$$

The dispersion relation in the form (48) with the coupling constant (38) corresponding to magnetized electron beam is exactly the same as that obtained in Refs. [7,16].

In the absence of the electron beam ($\Xi_0 = 0$), the dispersion relation is

$$\mathcal{D}(\omega, k) = 0. \qquad (50)$$

The solution of Eq. (50) gives us the functional dependence $\omega(k)$ of eigenmodes for a grating. It is important to note that the eigenmodes are evanescent waves. As we see from Eq. (49), the roots of Eq. (50) correspond to the singularities of the reflection matrix.

In general case Eq. (45) can be solved only numerically for different parameters of the beam. However, if the beam density is small enough

$$\omega_p^2 \ll \gamma_0^3 G^2, \qquad \omega_p^2 \ll \frac{\omega^2}{\gamma_0 \beta^2} \qquad (51)$$

(G is the small signal gain), then Ξ_0 can be expanded on powers of the small parameter $\sim\omega_p^2$:

$$\Xi_0 = \frac{\omega_p^2}{2\gamma_0^3 \Omega_0^2}(1 - e^{-2q_0\delta})e^{-2q_0 d}. \qquad (52)$$

For a magnetized electron beam under the condition (51), coupling constant (38) becomes

$$\Xi_{(mag)0} = \frac{\omega_p^2}{4\gamma_0^3 \Omega_0^2}(1 - e^{-2q_0\delta})e^{-2q_0 d}. \qquad (53)$$

As seen, the coupling constant for the magnetized electron beam under the condition (51) is 2 times smaller than that for the unmagnetized beam.

Dispersion relation (45) with Eqs. (52) and (53) becomes

$$\frac{\Omega_0^2 \mathcal{D}(\omega, k)}{\mathcal{D}(\omega, k) + 2\mathcal{M}(\omega, k)} = \frac{\mu \omega_p^2}{2\gamma_0^3}(1 - e^{-2q_0\delta})e^{-2q_0 d}. \qquad (54)$$

Here, for joint consideration of magnetized and unmagnetized electron beams, we have introduced a factor μ, i.e., $\mu = 1$ for the unmagnetized beam and $\mu = 1/2$ for the magnetized one.

In the case of evanescent wave amplification, we expand $\mathcal{D}(\omega, k)$ about the solution for the no-beam case: $\mathcal{D}(\omega(k_0), k_0) = 0$ and write

$$\frac{\mu\omega_p^2}{\gamma_0^3\Omega_0^2}(1 - e^{-2q_0\delta})e^{-2q_0 d} = \frac{\mathcal{D}_k(\omega(k_0), k_0)(k - k_0)}{\mathcal{M}(\omega(k_0), k_0)}, \tag{55}$$

where $\mathcal{D}_k \equiv \partial\mathcal{D}/\partial k$.

The dispersion relation (55) near the particle-wave synchronism

$$\omega(k_0) = c\beta k_0; \qquad \Omega_0 = c\beta(k_0 - k)$$

gives the cubic equation for small signal gain in the cold beam limit:

$$(k - k_0)^3 = \frac{\mu\omega_p^2}{\gamma_0^3 c^2\beta^2}(1 - e^{-(2k_0\delta/\gamma_0)})e^{-(2k_0 d/\gamma_0)} \times \frac{\mathcal{M}(c\beta k_0, k_0)}{\mathcal{D}_k(c\beta k_0, k_0)}.$$

Of the three roots, the root with the negative imaginary part gives rise to exponential gain of the corresponding wave, and we find that the amplitude growth rate is

$$\begin{aligned} G &= \mathrm{Im}(k_0 - k) \\ &= \frac{\sqrt{3}}{2}\left(\frac{\mu\omega_p^2}{\gamma_0^3 c^2\beta^2}(1 - e^{-(2k_0\delta/\gamma_0)})e^{-(2k_0 d/\gamma_0)} \times \left|\frac{\mathcal{M}(c\beta k_0, k_0)}{\mathcal{D}_k(c\beta k_0, k_0)}\right|\right)^{1/3}. \end{aligned} \tag{56}$$

As is seen from Eq. (56), the small signal gain decays exponentially with the beam height above the grating as $\exp(-2k_0 d/(3\gamma_0))$. The gain dependence on the beam thickness is given by the factor $[1 - \exp(-2k_0\delta/\gamma_0)]^{1/3}$. For the thick electron beam $k_0\delta/\gamma_0 \gg 1$, and when passing very close to a grating $k_0 d/\gamma_0 \ll 1$, from Eq. (56) we get

$$G = \frac{\sqrt{3}}{2}\left(\frac{\mu\omega_p^2}{\gamma_0^3 c^2\beta^2}\left|\frac{\mathcal{M}(c\beta k_0, k_0)}{\mathcal{D}_k(c\beta k_0, k_0)}\right|\right)^{1/3}. \tag{57}$$

This formula is analogous to a small signal gain obtained in [13] for a rectangular grating (with $\mu = 1$).

For the thin electron beam $k_0\delta/\gamma_0 \ll 1$, which for magnetized electron beam ($\mu = 1/2$) corresponds to the setup of Ref. [16], from Eq. (56) we get

$$G = \frac{\sqrt{3}}{2}\left(\frac{\mu}{\gamma_0^4\beta^3}\frac{8\pi k_0}{\delta_y}\frac{I_b}{I_A}e^{-(2k_0 d/\gamma_0)}\left|\frac{\mathcal{M}(c\beta k_0, k_0)}{\mathcal{D}_k(c\beta k_0, k_0)}\right|\right)^{1/3}. \tag{58}$$

Here the plasma frequency associated with the electron beam is expressed by the beam current as follows:

$$\omega_p^2 = \frac{4\pi c^2}{\beta\delta\delta_y}\frac{I_b}{I_A}, \tag{59}$$

where $I_A = mc^3/e = 17$ kA is the Alfvén current and δ_y is the beam width in the y direction.

If one assumes the existence of a propagating mode, then for those ω and k some of q_{L} ($\mathrm{L} = -1, -2, \ldots$) are imaginary. In this case $\mathcal{D}(\omega, k) \neq 0$ and Eq. (54) becomes

$$G^2 = R_{00}(c\beta k, k)\frac{\mu\omega_p^2}{2\gamma_0^3 c^2\beta^2}(1 - e^{-(2k_0\delta/\gamma_0)})e^{-(2k_0 d/\gamma_0)}.$$

Assuming that $\mathrm{Im}R_{00}(c\beta k, k) = 0$, and $\mathrm{Re}R_{00}(c\beta k, k) > 0$ for the small signal gain, one obtains

$$G = \left(R_{00}(c\beta k, k)\frac{\mu\omega_p^2}{2\gamma_0^3 c^2\beta^2}(1 - e^{-(2k_0\delta/\gamma_0)})e^{-(2k_0 d/\gamma_0)}\right)^{1/2}. \tag{60}$$

Far away from the grating, the propagating modes correspond to SP radiation. In this regime, the electron beam amplifies the evanescent mode which scatters into the propagating radiation mode. The wavelength of the radiation, observed at the angle ϑ from the direction of the electron beam, is

$$\lambda = \frac{g}{|\mathrm{L}|}\left(\frac{1}{\beta} - \cos\vartheta\right). \tag{61}$$

As follows from Eqs. (51) and (60), the formula for the gain (60) is valid when $|R_{00}(c\beta k, k)| \gg 1$. The latter is difficult to realize and presumes severe restriction on the grating parameters.

For the thin electron beam $k_0\delta/\gamma_0 \ll 1$, the small signal gain (60) becomes

$$G = \frac{1}{\gamma_0^2\beta^2}\left(\frac{4\pi\mu}{\delta_y}\frac{\omega}{c}\frac{I_b}{I_A}R_{00}(c\beta k, k)e^{-(2k_0 d/\gamma_0)}\right)^{1/2}. \tag{62}$$

Note that the latter with $\mu = 1/2$ is exactly the same as that obtained in Ref. [12]. This can be understood as follows. In [12] the electron beam dynamics is assumed to be one dimensional which is applicable for magnetized electron beam.

For the high-gain regime it is also necessary to take into account the conditions

$$G \gg \max\left\{\frac{1}{c\beta}\left|\frac{\partial\Omega_0}{\partial\eta_i}\delta\eta_i + \frac{1}{2}\frac{\partial^2\Omega_0}{\partial\eta_i^2}(\delta\eta_i)^2\right|, \frac{1}{L_{\mathrm{int}}}\right\}, \tag{63}$$

where L_{int} is the interaction length in the z direction. Here by η_i we denote the set of quantities characterizing the electron beam and by $\delta\eta_i$ their spreads. The condition (63) can be written as

$$G \gg \max\left\{k\frac{\delta\gamma}{\beta^2\gamma_0^3}, \frac{k}{2}\delta\theta^2, \frac{1}{L_{\mathrm{int}}}\right\}. \tag{64}$$

The first and second terms in the curly brackets of Eq. (64)

are the resonance widths due to energetic and angular spreads, and the last term expresses the resonance width due to limited interaction length.

III. RECTANGULAR GRATING

In this section, as an important example of the application of the obtained results, we present the small signal gain of a SP FEL in the case of a rectangular grating. The grating is assumed to have ideal conductivity. We denote width and depth of a groove as w and a, respectively. The solution of the Maxwell equations in the grooves ($-a \leq x \leq 0$) is expressed as series of the cavity modes in the grooves. With the help of these solutions it is easy to calculate the impedance matrix Z_{LJ}. The latter is presented in the Appendix of this work. On inserting the impedance matrix Z_{LJ} Eq. (A11) into Eq. (50), we obtain the free dispersion relation for a rectangular grating:

$$\det\left[\frac{2w}{g}\sum_{m=0}^{\infty}\frac{p_m \tan p_m a}{(1+\delta_{m0})}\frac{\Psi^*_{m\mathrm{L}}\Psi_{m\mathrm{J}}}{q_{\mathrm{L}}} - \delta_{\mathrm{LJ}}\right] = 0. \tag{65}$$

Note that the coefficients of Eq. (65) show the existence of the similarity relation in the functional dependence $\omega(k)$ with respect to the grating parameters. In other words, the dependence of the normalized angular frequency $\omega g/(2\pi c)$ on the normalized wave number $kg/(2\pi)$ for one grating coincides with that of another grating if the ratios w/g and a/g are the same for the two gratings.

To find out the functional dependence $\omega(k)$ for rectangular grating we solve Eq. (65) using a 5×5 approximation to infinite determinant (we have kept elements with J, L $= -2, -1, 0, 1, 2$). The result of the numerical solution of Eq. (65) is shown in Fig. 2 for a rectangular grating with $w/g = 0.5$ and $a/g = 0.5$. This corresponds to the setup of [18]. The abscissa is the normalized wave number and the ordinate is the normalized angular frequency. As is seen from this figure, the beam line $\omega = c\beta k$ for $\beta = 0.548$ (at the kinetic energy 100 keV) intersects the dispersion curve at the wave number 0.5684 and frequency 0.3115 of the resonant evanescent wave. The resonant frequency and wave number are close to those of the paper [18]. As we see the group velocity ($d\omega/dk$) of the resonant evanescent wave is negative and for this energy of an electron beam the SP FEL operates like a backward wave oscillator.

Next we solve Eq. (45) to find out small signal gain. In Fig. 3 the small signal gain versus the beam current (I) per meter in the y direction is plotted for $d/g = 0.1$ and $\delta/g = 0.25$. We have also plotted gain calculated by the approximate solution (56) (dashed line). As we see, the numerical solution is in agreement with the analytical one, except the region for high current values because of the space charge effects contribution in the amplification process.

For the comparison with the results of PIC simulations [18] where the gain is estimated to be $Gg/(2\pi) \simeq 0.0045 \times I^{1/3}$ cm^{-1}, we solve Eq. (45) with a coupling constant corresponding to the magnetized electron beam [see Eq. (38)]. The latter comes from the fact that in [18] a superimposed constant magnetic field of 2 T (in the z direction) was assumed. That is, in fact they have magnetized electron beam. In Fig. 4 the small signal gain versus the beam current is plotted for the magnetized electron beam. The solid line corresponds to the numerical solution of Eq. (45), while the dashed line to PIC simulations [18]. As we see, the agreement is quite good.

We have also made calculations for the parameters close to those of the experiment at Dartmouth [10]. We assume that the electron beam fills a region of width $\delta = 24$ μm, equal to the diameter of the beam used at experiment. At that, beam height above the grating is assumed to be $d = 0$

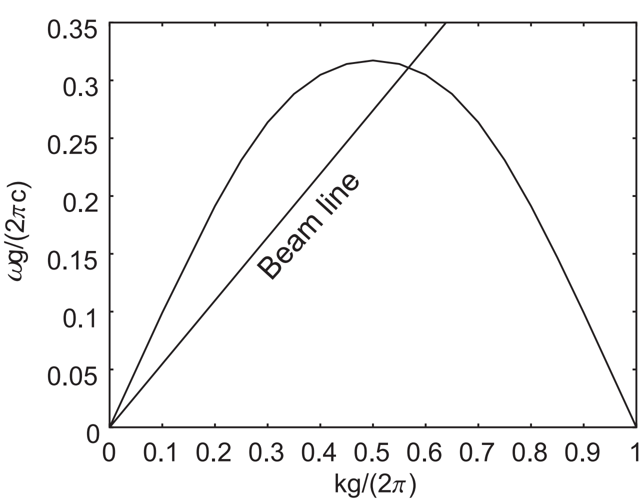

FIG. 2. Dispersion relation for the rectangular grating with $w/g = 0.5$, $a/g = 0.5$ and intersection with the beam line for $\beta = 0.548$.

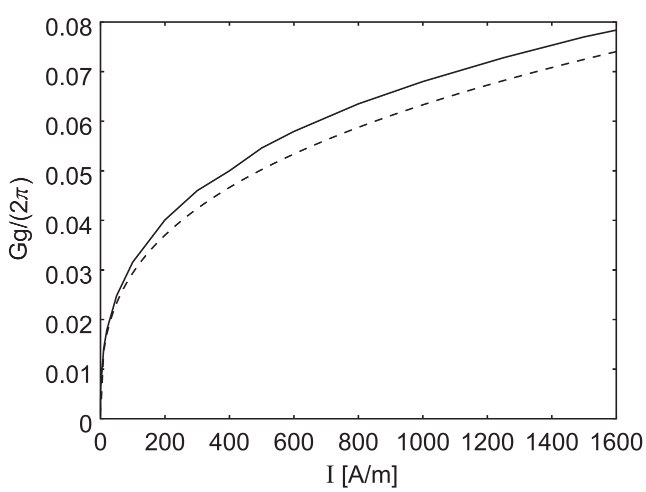

FIG. 3. Normalized small signal gain versus the beam current per meter in the y direction. The solid line corresponds to numerical solution of the dispersion relation Eq. (45), while the dashed line corresponds to approximate solution Eq. (56).

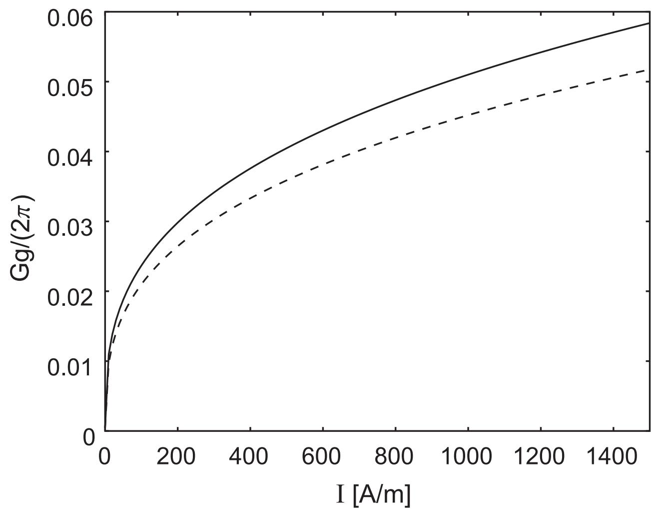

FIG. 4. Normalized small signal gain for magnetized electron beam. The solid line corresponds to numerical solution of the dispersion relation Eq. (45) for magnetized electron beam, while the dashed line corresponds to PIC simulations [18].

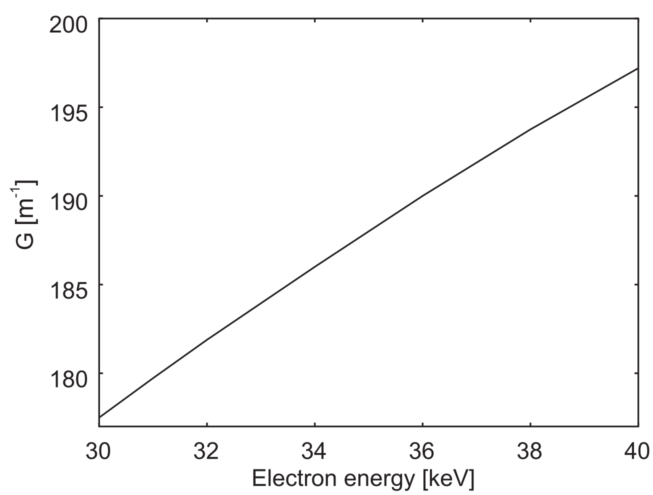

FIG. 5. Small signal gain for the experimental parameters of Ref. [10].

and beam current per meter is $I \simeq 53$ A/m (this corresponds to beam current $I_b = 1$ mA). In Fig. 5 the small signal gain versus the beam energy is plotted for $g = 173\ \mu$m, $w = 62\ \mu$m, and $a = 100\ \mu$m. Within factor 1.2 this result is close to that of [13], where it is assumed that the entire space above the grating is filled by an electron beam. It is easy to see that difference comes from the "filling factor" $\sim[1 - e^{-2k_0\delta/\gamma_0}]^{1/3}$ for a beam of finite height.

IV. CONCLUSION

We have presented a theoretical treatment of the high-gain regime of a SP FEL. Considering an electron beam of finite thickness being at a finite height above the grating, we have established the dispersion relation for the TM mode of electromagnetic waves. An arbitrary grating is described in terms of impedance matrix. We have also taken into account electron motion in the normal to the grating surface direction. The dispersion relation has been studied considering amplification of the evanescent as well as radiation modes. An analytical expression for the small signal gain has been derived. The gain dependence on the beam thickness and the beam height above the grating has also been analyzed. We have examined various limits of small signal gain depending on the electron beam and grating parameters. For comparison, we have also analyzed the case of a magnetized electron beam. Then, obtained results have been applied to rectangular grating, making comparisons with the works [13,18]. The numerical results in general are in agreement with analytical ones. The analysis of obtained results shows that the obtained small signal gain is in good agreement with the results of numerical investigations [18].

ACKNOWLEDGMENTS

The author would like to acknowledge helpful discussions with Professor H. K. Avetissian. This work was supported by International Science and Technology Center (ISTC) Project No. A-1307.

APPENDIX

In this Appendix we present expression of the impedance matrix for the rectangular grating. The latter is necessary for the solution of dispersion relation (45). We denote the grating period, width, and depth of the groove as g, w, and a, respectively. The solution of the Maxwell equations in the grooves ($-a \le x \le 0$) is expressed as series of the cavity modes in the grooves

$$H_{\omega y}^{(w)} = e^{iskg} \sum_{m=0}^{\infty} H_m \cos\frac{m\pi(z - sg)}{w} \cos p_m(x + a), \tag{A1}$$

$$E_{\omega z}^{(w)} = e^{iskg} \sum_{m=0}^{\infty} E_{zm} \cos\frac{m\pi(z - sg)}{w} \sin p_m(x + a), \tag{A2}$$

$$E_{\omega x}^{(w)} = e^{iskg} \sum_{m=0}^{\infty} E_{xm} \sin\frac{m\pi(z - sg)}{w} \cos p_m(x + a), \tag{A3}$$

where H_m, E_{zm}, and E_{xm} are constants, and $s = (0, \pm1, \pm2, \ldots)$ indicates the number of the groove. These expressions satisfy the boundary conditions that $E_{\omega z}^{(w)}$ vanishes at the bottom of the groove ($x = -a$), and $E_{\omega x}^{(w)}$ vanishes at the sides of the groove ($z = sg$; $w + sg$). From the Maxwell equations (1), we find that

$$p_m^2 = \frac{\omega^2}{c^2} - \left(\frac{m\pi}{w}\right)^2 \tag{A4}$$

and

$$H_m = i\frac{\omega}{c p_m} E_{zm}; \qquad E_{xm} = -\frac{m\pi}{w p_m} E_{zm}. \tag{A5}$$

Across the interface between the grating and the vacuum, the tangential component of the electric field is continuous. Since the tangential field vanishes on the surface of the conductor, with the help of Eqs. (26a) and (A2) we will have the following boundary condition:

$$\sum_{\mathrm{J}=-\infty}^{\infty} (E_{3\mathrm{J}}^{(+)} + E_{3\mathrm{J}}^{(-)}) e^{ik_{\mathrm{J}} z} = \begin{cases} \sum_{m=0}^{\infty} E_{zm} \cos\frac{m\pi z}{w} \sin p_m a, & 0 < z < w \\ 0, & w < z < g. \end{cases} \tag{A6}$$

Multiplying Eq. (A6) by $e^{-ik_{\mathrm{L}} z}$ and integrating over z in the range $0 < z < g$, we get

$$E_{3\mathrm{J}}^{(+)} + E_{3\mathrm{J}}^{(-)} = \frac{w}{g} \sum_{m=0}^{\infty} E_{zm} \Psi_{m\mathrm{J}} \sin p_m a, \tag{A7}$$

where

$$\Psi_{m\mathrm{J}} = \frac{(-1)^m e^{-ik_{\mathrm{J}} w} - 1}{w} \frac{ik_{\mathrm{J}}}{k_{\mathrm{J}}^2 - (\frac{m\pi}{w})^2}. \tag{A8}$$

The tangential component of the magnetic field must be continuous across the interface, so with the help of Eqs. (26b) and (A1) we have

$$\sum_{m'=0}^{\infty} \frac{1}{p_{m'}} E_{zm'} \cos p_{m'} a \cos\frac{m'\pi z}{w} = \sum_{\mathrm{L}=-\infty}^{\infty} \frac{1}{q_{\mathrm{L}}} (E_{3\mathrm{L}}^{(+)} - E_{3\mathrm{L}}^{(-)}) e^{ik_{\mathrm{L}} z}. \tag{A9}$$

Multiplying Eq. (A9) by $\cos\frac{m\pi z}{w}$ and integrating over z in the range $0 < z < w$, we get

$$E_{zm} = \frac{2p_m}{(1+\delta_{m0})\cos p_m a} \sum_{\mathrm{L}=-\infty}^{\infty} \frac{\Psi_{m\mathrm{L}}^*}{q_{\mathrm{L}}} (E_{3\mathrm{L}}^{(+)} - E_{3\mathrm{L}}^{(-)}). \tag{A10}$$

Combining Eqs. (A7) and (A10) with definition (40), we obtain the impedance matrix for a rectangular grating:

$$Z_{\mathrm{JL}} = \frac{2w}{g} \sum_{m=0}^{\infty} \frac{p_m \tan p_m a}{(1+\delta_{m0})} \frac{\Psi_{m\mathrm{L}}^* \Psi_{m\mathrm{J}}}{q_{\mathrm{L}}}. \tag{A11}$$

[1] S. J. Smith and E. M. Purcell, Phys. Rev. **92**, 1069 (1953).
[2] Toraldo di Francia, Nuovo Cimento **16**, 61 (1960).
[3] P. M. van den Berg, J. Opt. Soc. Am. **63**, 689 (1973); **63**, 1588 (1973); P. M. van den Berg and T. H. Tan, J. Opt. Soc. Am. **64**, 325 (1974).
[4] B. M. Bolotovski and G. V. Voskresenskii, Sov. Phys. Usp. **11**, 143 (1968).
[5] A. S. Kesar, Phys. Rev. ST Accel. Beams **8**, 072801 (2005); D. V. Karlovets and A. P. Potylitsyn, Phys. Rev. ST Accel. Beams **9**, 080701 (2006).
[6] A. Yariv and C. C. Shih, Opt. Commun. **24**, 233 (1978); A. Gover and Z. Livni, Opt. Commun. **26**, 375 (1978); J. M. Wachtel, J. Appl. Phys. **50**, 49 (1979); D. E. Wortman, R. P. Leavitt, H. Dropkin, and C. A. Morrison, Phys. Rev. A **24**, 1150 (1981).
[7] L. Schachter and A. Ron, Phys. Rev. A **40**, 876 (1989).
[8] K. J. Woods, J. E. Walsh, R. E. Stoner, H. G. Kirk, and R. C. Fernow, Phys. Rev. Lett. **74**, 3808 (1995); K. Ishi, Y. Shibata, T. Takahashi, S. Hasebe, M. Ikezawa, K. Takami, T. Matsuyama, K. Kobayashi, and Y. Fujita, Phys. Rev. E **51**, R5212 (1995); Y. Shibata *et al.*, Phys. Rev. E **57**, 1061 (1998); G. Kube *et al.*, Phys. Rev. E **65**, 056501 (2002); S. E. Korbly, A. S. Kesar, J. R. Sirigiri, and R. J. Temkin, Phys. Rev. Lett. **94**, 054803 (2005); Amit S. Kesar, Roark A. Marsh, and Richard J. Temkin, Phys. Rev. ST Accel. Beams **9**, 022801 (2006).
[9] M. C. Lampel, Nucl. Instrum. Methods Phys. Res., Sect. A **385**, 19 (1997); D. C. Nguyen, Nucl. Instrum. Methods Phys. Res., Sect. A **393**, 514 (1997); G. Doucas, M. F. Kimmitt, J. H. Brownell, S. R. Trotz, and J. E. Walsh, Nucl. Instrum. Methods Phys. Res., Sect. A **474**, 10 (2001); A. Doria, G. P. Gallerano, E. Giovenale, G. Messina, G. Doucas, M. F. Kimmitt, H. L. Andrews, and J. H. Brownell, Nucl. Instrum. Methods Phys. Res., Sect. A **483**, 263 (2002); G. Doucas, M. F. Kimmitt, A. Doria, G. P. Gallerano, E. Giovenale, G. Messina, H. L. Andrews, and J. H. Brownell, Phys. Rev. ST Accel. Beams **5**, 072802 (2002).
[10] J. Urata, M. Goldstein, M. F. Kimmitt, A. Naumov, C. Platt, and J. E. Walsh, Phys. Rev. Lett. **80**, 516 (1998).
[11] A. Bakhtyari, J. E. Walsh, and J. H. Brownell, Phys. Rev. E **65**, 066503 (2002).
[12] K.-J. Kim and S.-B. Song, Nucl. Instrum. Methods Phys. Res., Sect. A **475**, 158 (2001).
[13] H. L. Andrews and C. A. Brau, Phys. Rev. ST Accel. Beams **7**, 070701 (2004).
[14] H. L. Andrews, C. H. Boulware, C. A. Brau, and J. D. Jarvis, Phys. Rev. ST Accel. Beams **8**, 050703 (2005).
[15] H. L. Andrews, C. H. Boulware, C. A. Brau, J. T. Donohue, J. Gardelle, and J. D. Jarvis, New J. Phys. **8**, 289 (2006).
[16] V. Kumar and K.-J. Kim, Phys. Rev. E **73**, 026501 (2006).
[17] J. A. Swegle, Phys. Fluids **30**, 1201 (1987).
[18] J. T. Donohue and J. Gardelle, Phys. Rev. ST Accel. Beams **8**, 060702 (2005).
[19] D. Li, Z. Yang, K. Imasaki, and Gun-Sik Park, Phys. Rev. ST Accel. Beams **9**, 040701 (2006).
[20] H. K. Avetissian, K. Z. Hatsagortsian, and G. F. Mkrtchian, IEEE J. Quantum Electron. **33**, 897 (1997).